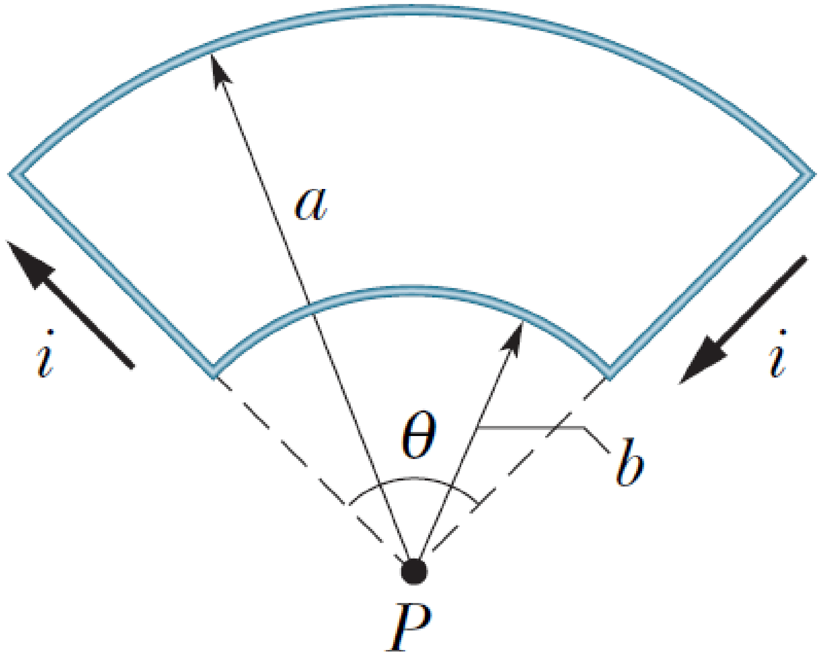
Recitation 13

•7 In Fig. 29-38, two circular arcs have radii $a = 13.5$ cm and $b = 10.7$ cm, subtend angle $\theta = 74.0°$, carry current $i = 0.411$ A, and share the same center of curvature P. What are the (a) magnitude and (b) direction (into or out of the page) of the net magnetic field at P?



a) The field due to the straight segments at point P is zero. The right hand rule tells us that the magnetic field due the small and large arcs at point P are out of the page and into the page, respectively.

The net field B is then

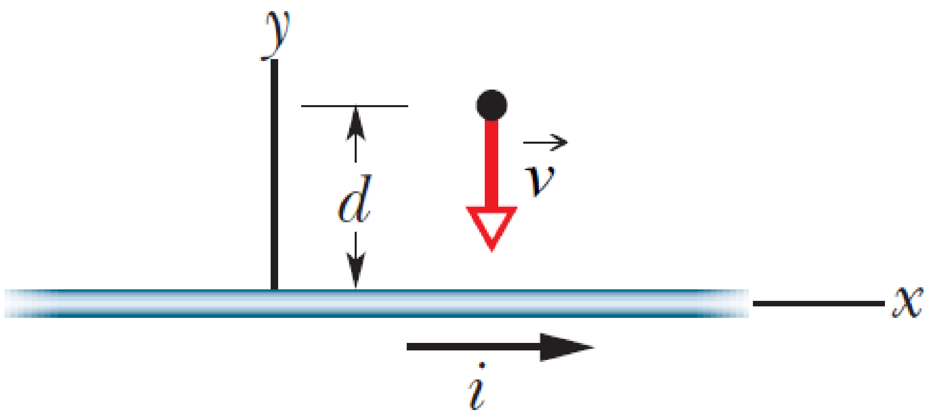
$$B = B_b - B_a = \frac{\mu_0 i\theta}{4\pi b} - \frac{\mu_0 i\theta}{4\pi a} = \frac{\mu_0 i\theta}{4\pi}\left(\frac{1}{b} - \frac{1}{a}\right)$$

$$= \frac{(4\pi \times 10^{-7}\ \mathrm{T \cdot m/A})(0.411\ \mathrm{A})\left(74.0° \times \frac{\pi}{180°}\right)}{4\pi}\left(\frac{1}{0.107\ \mathrm{m}} - \frac{1}{0.135\ \mathrm{m}}\right)$$

$$= 1.03 \times 10^{-7}\ \mathrm{T}.$$

b) Out of the page since the outward field (B_b) is larger than the field inward (B_a).

••23 ILW Figure 29-50 shows a snapshot of a proton moving at velocity $\vec{v} = (-200 \text{ m/s})\hat{\text{j}}$ toward a long straight wire with current $i = 350$ mA. At the instant shown, the proton's distance from the wire is $d = 2.89$ cm. In unit-vector notation, what is the magnetic force on the proton due to the current?



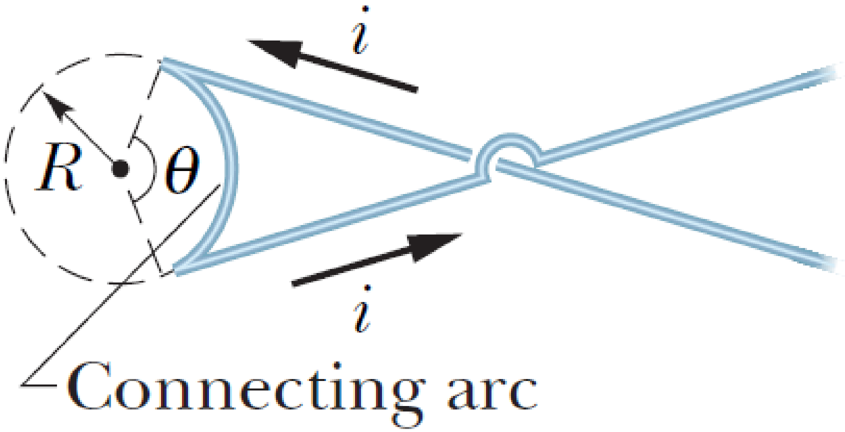
The magnetic field above the wire is out of the page. At the proton's location

$$\vec{B} = \frac{\mu_0 i}{2\pi d}\hat{\mathrm{k}} = \frac{(4\pi \times 10^{-7}\ \mathrm{T \cdot m/A})(0.350\ \mathrm{A})}{2\pi(0.0289\ \mathrm{m})}\hat{\mathrm{k}} = 2.42 \times 10^{-6}\ \mathrm{T}\ \hat{\mathrm{k}}.$$

$$\vec{F}_B = q\vec{v} \times \vec{B} = (1.60 \times 10^{-19}\ \mathrm{C})\left(200\,\frac{\mathrm{m}}{\mathrm{s}}\right)(2.42 \times 10^{-6}\ \mathrm{T})(-\hat{\mathrm{j}} \times \hat{\mathrm{k}})$$

$$= -7.75 \times 10^{-23}\mathrm{N}\ \hat{\mathrm{i}}.$$

••25 SSM A wire with current $i = 3.00$ A is shown in Fig. 29-52. Two semi-infinite straight sections, both tangent to the same circle, are connected by a circular arc that has a central angle θ and runs along the circumference of the circle. The arc and the two straight sections all lie in the same plane. If $B = 0$ at the circle's center, what is θ?



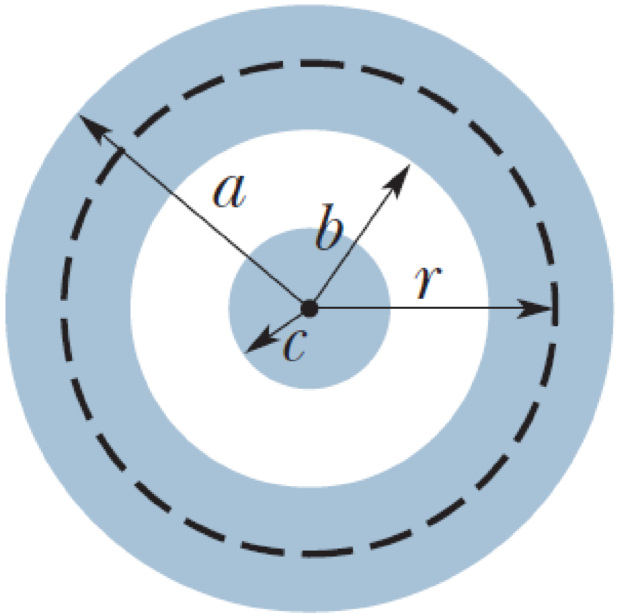
The outward field (the field of the two semi-infinite wires) is equal to the field of the arc but opposite to it. Thus,

$$\frac{\mu_0 i}{4\pi R} + \frac{\mu_0 i}{4\pi R} = \frac{\mu_0 i\theta}{4\pi R}.$$

This gives that

$$\theta = 2 \text{ rad} = 115°.$$

87 Figure 29-88 shows a cross section of a long conducting coaxial cable and gives its radii (a, b, c). Equal but opposite currents i are uniformly distributed in the two conductors. Derive expressions for $B(r)$ with radial distance r in the ranges (a) $r < c$, (b) $c < r < b$, (c) $b < r < a$, and (d) $r > a$.



Part (c) is optional.

Ampere's law gives

a) $r < c$

$$2\pi r B = \mu_0 i \frac{r^2}{c^2} \Longrightarrow B = \frac{\mu_0 i r}{2\pi c^2}.$$

b) $c < r < b$

$$2\pi r B = \mu_0 i \Longrightarrow B = \frac{\mu_0 i}{2\pi r}.$$

c) $b < r < a$

$$2\pi r B = \mu_0 \left(i - i \frac{r^2 - b^2}{a^2 - b^2} \right) \Longrightarrow B = \left(1 - \frac{r^2 - b^2}{a^2 - b^2} \right) \frac{\mu_0 i}{2\pi r}.$$

d) $r > a$

$$2\pi r B = 0 \Longrightarrow B = 0.$$