Recitation 12

•1 SSM ILW A proton traveling at 23.0° with respect to the direction of a magnetic field of strength 2.60 mT experiences a magnetic force of 6.50×10^{-17} N. Calculate (a) the proton's speed and (b) its kinetic energy in electron-volts.

a)

$$F = |q|vB\sin\theta$$

$$v = \frac{F}{|q|B\sin\theta} = \frac{6.50 \times 10^{-17}\ \text{N}}{(1.60 \times 10^{-19}\ \text{C})(2.60 \times 10^{-3}\ \text{T})\sin 23.0^\circ}$$

$$= 4.00 \times 10^{5}\ \text{m/s}.$$

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b)

$$K = \frac{1}{2} m_p v^2 = \frac{1}{2}(1.67 \times 10^{-27}\ \text{kg})(4.00 \times 10^{5}\ \text{m/s})^2$$
$$= 1.34 \times 10^{-16}\ \text{J}.$$

In electron-volts,

$$K = (1.33 \times 10^{-16}\ \text{J})\left(\frac{1\ \text{eV}}{1.60 \times 10^{-19}\ \text{J}}\right) = 835\ \text{eV}.$$

••5 GO An electron moves through a uniform magnetic field given by $\vec{B} = B_x\hat{\mathrm{i}} + (3.0B_x)\hat{\mathrm{j}}$. At a particular instant, the electron has velocity $\vec{v} = (2.0\hat{\mathrm{i}} + 4.0\hat{\mathrm{j}})$ m/s and the magnetic force acting on it is $(6.4 \times 10^{-19}\ \mathrm{N})\hat{\mathrm{k}}$. Find B_x.

a)

$$\vec{v} \times \vec{B} = \begin{vmatrix} \hat{\mathrm{i}} & \hat{\mathrm{j}} & \hat{\mathrm{k}} \\ 2.0 & 4.0 & 0 \\ B_x & 3.0B_x & 0 \end{vmatrix} = \left[(0-0)\hat{\mathrm{i}} - (0-0)\hat{\mathrm{j}} + (6.0B_x - 4.0B_x)\hat{\mathrm{k}}\right]$$

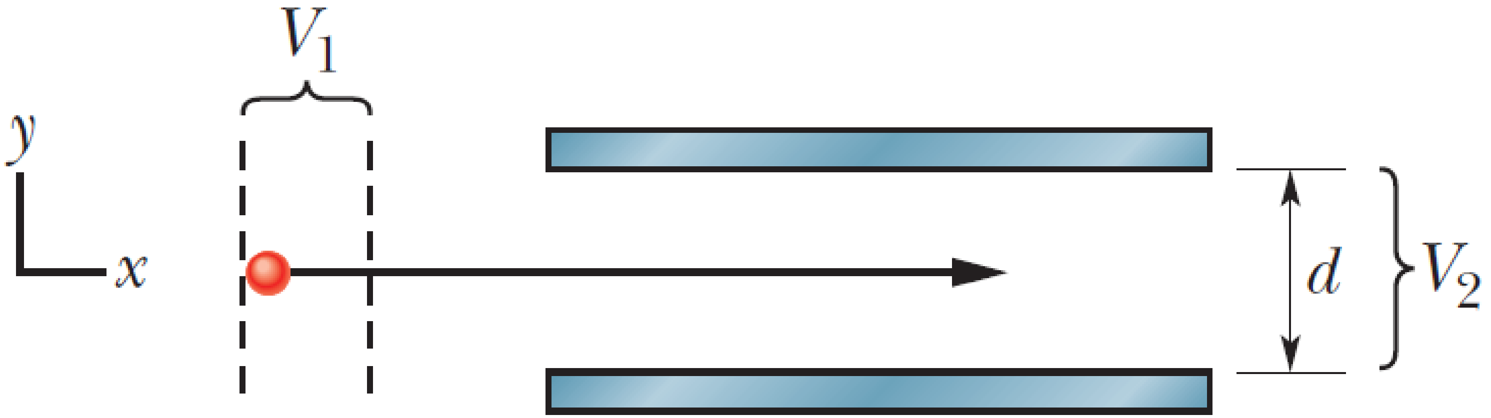
$$= 2.0B_x\,\hat{\mathrm{k}}.$$

$$\vec{F} = q\vec{v} \times \vec{B} = 2.0qB_x\ \hat{\mathrm{k}} = 6.4 \times 10^{-19}\,\hat{\mathrm{k}}.$$

Thus,

$$B_x = \frac{6.4 \times 10^{-19}}{2.0(-1.6 \times 10^{-19})} = -2.0\ \mathrm{T}.$$

•9 ILW In Fig. 28-32, an electron accelerated from rest through potential difference $V_1 = 1.00$ kV enters the gap between two parallel plates having separation $d = 20.0$ mm and potential difference



$V_2 = 100$ V. The lower plate is at the lower potential. Neglect fringing and assume that the electron's velocity vector is perpendicular to the electric field vector between the plates. In unit-vector notation, what uniform magnetic field allows the electron to travel in a straight line in the gap?

The magnetic field is determined by the requirement that $F_m = F_e$. Therefore,

$$evB = eE$$

$$evB = e\frac{V_2}{d}$$

$$B = \frac{V_2}{vd}$$

$$v = \sqrt{\frac{2K}{m_e}} = \sqrt{\frac{2eV_1}{m_e}} = \sqrt{\frac{2(1.60 \times 10^{-19}\text{C})(1000\text{ V})}{9.11 \times 10^{-31}\text{kg}}} = 1.87 \times 10^7 \text{m/s}.$$

$$B = \frac{100\text{ V}}{(1.87 \times 10^7 \text{m/s})(20.0 \times 10^{-3}\text{ m})} = 2.67 \times 10^{-4}\text{ T}.$$

The magnetic field must be into the page such that $\vec{F}_m$ and $\vec{F}_e$ are opposite in direction. Thus,

$$\vec{B} = -(2.67 \times 10^{-4}\text{ T})\,\hat{\text{k}}.$$

••33 SSM WWW A positron with kinetic energy 2.00 keV is projected into a uniform magnetic field $\vec{B}$ of magnitude 0.100 T, with its velocity vector making an angle of 89.0° with $\vec{B}$. Find (a) the period, (b) the pitch p, and (c) the radius r of its helical path.

a)

$$T = \frac{2\pi m}{|q|B} = \frac{2\pi m_e}{eB} = \frac{2\pi(9.11\times10^{-31}\text{kg})}{(1.60\times10^{-19}\text{ C})(0.100\text{ T})} = 3.58\times10^{-10}\text{ s.}$$

b)

$$v = \sqrt{\frac{2K}{m_e}} = \sqrt{\frac{2(2.00\times10^{3}\text{ eV})(1.60\times10^{-19}\text{ J/eV})}{9.11\times10^{-31}\text{kg}}} = 2.65\times10^{7}\text{ m/s.}$$

$$v_{||} = v\cos\theta = (2.65\times10^{7}\text{ m/s})\cos 89.0° = 4.63\times10^{5}\text{ m/s.}$$

$$p = v_{||}T = (4.63\times10^{5}\text{m/s})(3.58\times10^{-10}\text{ s}) = 1.65\times10^{-4}\text{ m.}$$

••33 **SSM** **WWW** A positron with kinetic energy 2.00 keV is projected into a uniform magnetic field $\vec{B}$ of magnitude 0.100 T, with its velocity vector making an angle of 89.0° with $\vec{B}$. Find (a) the period, (b) the pitch p, and (c) the radius r of its helical path.

c)

$$v_\perp = v\sin\theta = (2.65\times10^7\ \text{m/s})\sin 89.0° = 2.65\times10^7\ \text{m/s}.$$

$$r = \frac{mv_\perp}{|q|B} = \frac{m_e v_\perp}{eB} = \frac{(9.11\times10^{-31}\text{kg})(2.65\times10^7\ \text{m/s})}{(1.60\times10^{-19}\ \text{C})(0.100\ \text{T})}$$

$$= 1.51\times10^{-3}\ \text{m}.$$

•39 SSM A horizontal power line carries a current of 5000 A from south to north. Earth's magnetic field (60.0 μT) is directed toward the north and inclined downward at 70.0° to the horizontal. Find the (a) magnitude and (b) direction of the magnetic force on 100 m of the line due to Earth's field.

a)

$$F_B = iLB\sin\phi = (5000\text{ A})(100\text{ m})(60.0\times10^{-6})\sin 70.0° = 28.2\text{ N}.$$

b)

Westward

••65 SSM ILW A wire of length 25.0 cm carrying a current of 4.51 mA is to be formed into a circular coil and placed in a uniform magnetic field $\vec{B}$ of magnitude 5.71 mT. If the torque on the coil from the field is maximized, what are (a) the angle between $\vec{B}$ and the coil's magnetic dipole moment and (b) the number of turns in the coil? (c) What is the magnitude of that maximum torque?

a) τ is maximum when $\theta = 90°$.

b)

$$L = 2\pi r N \Longrightarrow r = L/(2\pi N)$$

$$A = \pi r^2 = L^2/(4\pi N^2)$$

$$\tau = (NiA)B\sin\theta = \frac{L^2}{4\pi N} iB\sin\theta\,.$$

τ is maximum when $N = 1$.

••65 SSM ILW A wire of length 25.0 cm carrying a current of 4.51 mA is to be formed into a circular coil and placed in a uniform magnetic field $\vec{B}$ of magnitude 5.71 mT. If the torque on the coil from the field is maximized, what are (a) the angle between $\vec{B}$ and the coil's magnetic dipole moment and (b) the number of turns in the coil? (c) What is the magnitude of that maximum torque?

c)

$$\tau = (NiA)B\sin\theta = \frac{L^2}{4\pi N} iB\sin\theta$$
$$= \frac{(0.250\text{ m})^2}{4\pi(1)}(4.51\times10^{-3}\text{ A})(5.71\times10^{-3}\text{ A})\sin 90°$$
$$= 1.28\times10^{-7}\text{ N}\cdot\text{m}.$$