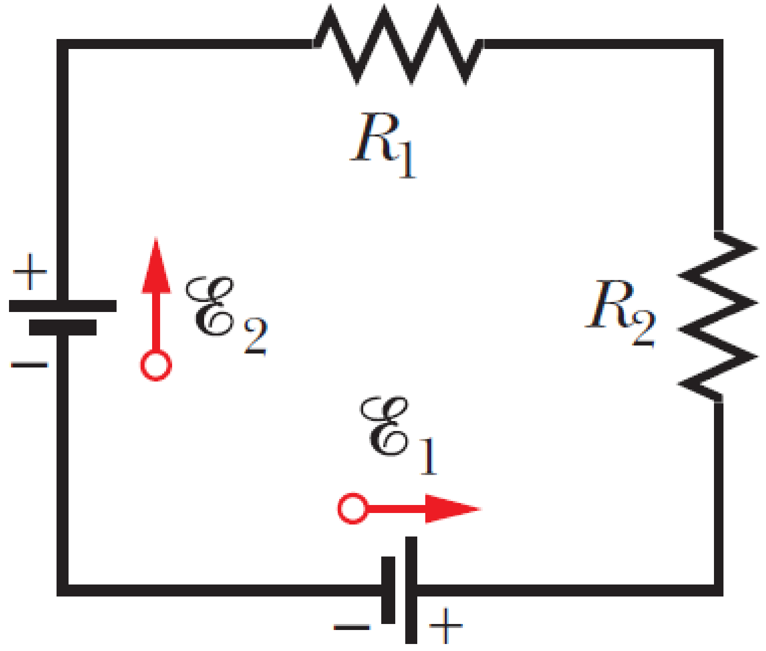
Recitation 11

•1 SSM WWW In Fig. 27-25, the ideal batteries have emfs $\mathscr{E}_1 = 12$ V and $\mathscr{E}_2 = 6.0$ V. What are (a) the current, the dissipation rate in (b) resistor 1 (4.0 Ω) and (c) resistor 2 (8.0 Ω), and the energy transfer rate in (d) battery 1 and (e) battery 2? Is energy being supplied or absorbed by (f) battery 1 and (g) battery 2?

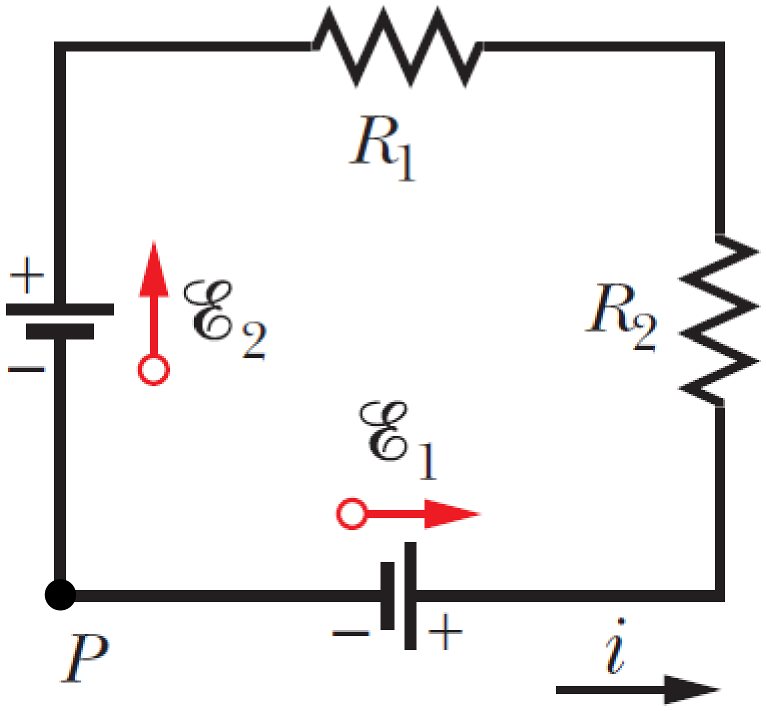


a) Applying the loop rule counterclockwise starting at point P gives

$$\mathcal{E}_1 - iR_2 - iR_1 - \mathcal{E}_2 = 0.$$

It gives

$$i = \frac{\mathcal{E}_1 - \mathcal{E}_2}{R_1 + R_2} = \frac{12\text{ V} - 6.0\text{ V}}{4.0\ \Omega + 8.0\ \Omega} = 0.50\text{ A}.$$



b)

$$P_{R_1} = i^2R_1 = (0.50\text{ A})^2(4.0\ \Omega) = 1.0\text{ W}.$$

c)

$$P_{R_2} = i^2R_2 = (0.50\text{ A})^2(8.0\ \Omega) = 2.0\text{ W}.$$

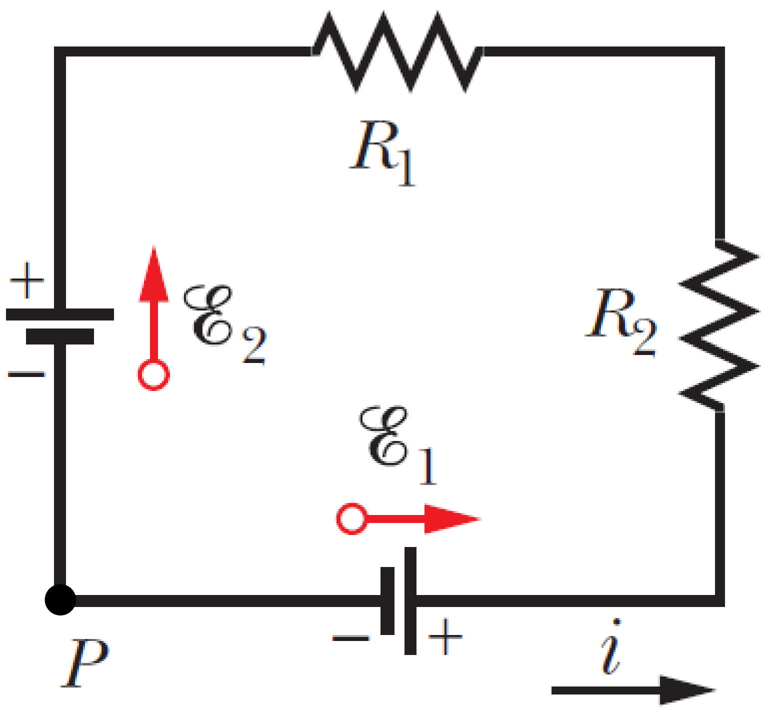
d)

$$P_{\mathcal{E}_1} = i\mathcal{E}_1 = (0.50 \text{ A})(12 \text{ V}) = 6.0 \text{ W}.$$

e)

$$P_{\mathcal{E}_2} = i\mathcal{E}_2 = (0.50 \text{ A})(6.0 \text{ V}) = 3.0 \text{ W}.$$

f,g) Energy is supplied by battery 1 and absorbed by battery 2.



••15 ILW The current in a single-loop circuit with one resistance R is 5.0 A. When an additional resistance of 2.0 Ω is inserted in series with R, the current drops to 4.0 A. What is R?

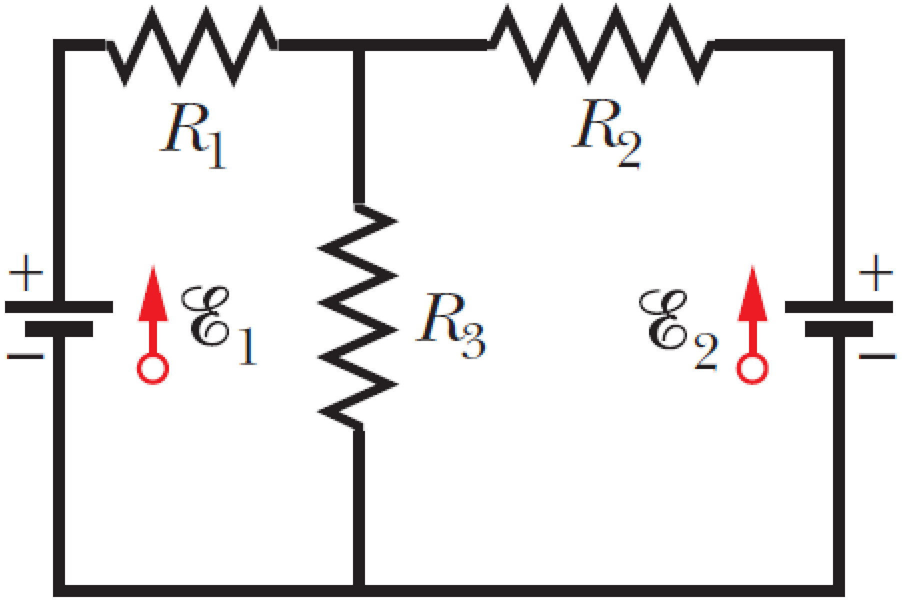
$$i = \frac{\mathcal{E}}{R}$$

$$i_2 = \frac{\mathcal{E}}{R + 2.0}$$

$$\frac{i}{i_2} = \frac{\mathcal{E}/R}{\frac{\mathcal{E}}{R + 2.0}} = \frac{R + 2.0}{R} = 1 + \frac{2.0}{R}$$

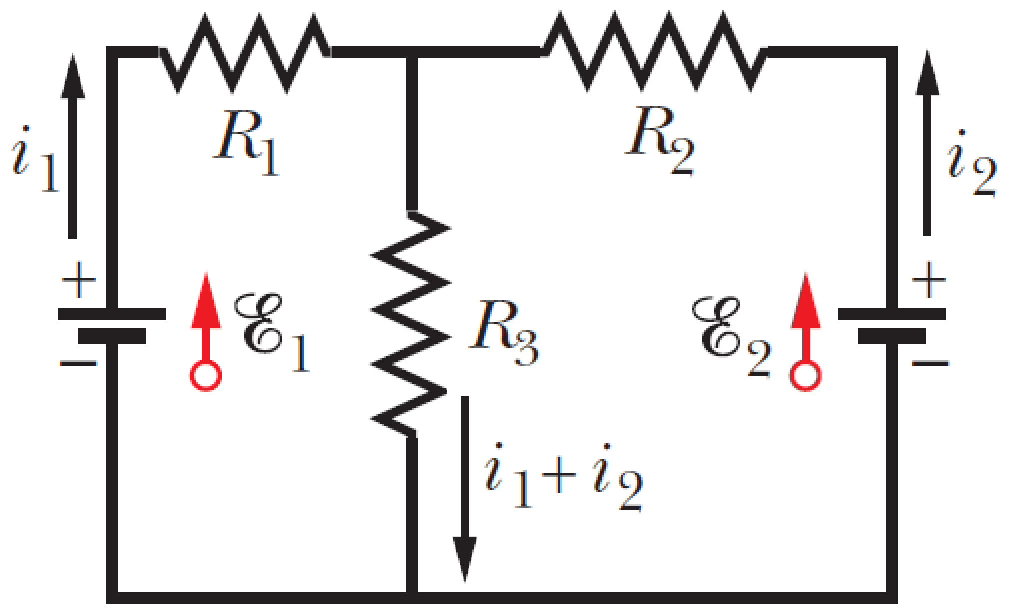
$$R = \frac{2.0}{i/i_2 - 1} = \frac{2.0}{5.0/4.0 - 1} = 8.0\ \Omega.$$

••30 GO In Fig. 27-41, the ideal batteries have emfs $\mathscr{E}_1 = 10.0$ V and $\mathscr{E}_2 = 0.500\mathscr{E}_1$, and the resistances are each 4.00 Ω. What is the current in (a) resistance 2 and (b) resistance 3?



Applying the loop rule gives

$$\mathcal{E}_1 - i_1 R_1 - (i_1 + i_2) R_3 = 0,$$
$$\mathcal{E}_2 - i_2 R_2 - (i_1 + i_2) R_3 = 0.$$



Simplifying the two gives

$$i_1(R_1 + R_3) + i_2 R_3 = \mathcal{E}_1,$$
$$i_2(R_2 + R_3) + i_1 R_3 = \mathcal{E}_2,$$

or

$$8.00 i_1 + 4.00 i_2 = 10.0,$$
$$8.00 i_2 + 4.00 i_1 = 5.00,$$

Solving for the currents gives $i_1 = 1.25$ A, $i_2 = 0$.

a) $i_2 = 0$.

b) $i_3 = i_1 + i_2 = 1.25$ A.

••31 SSM GO In Fig. 27-42, the ideal batteries have emfs $\mathscr{E}_1 = 5.0$ V and $\mathscr{E}_2 = 12$ V, the resistances are each 2.0 Ω, and the potential is defined to be zero at the grounded point of the circuit. What are potentials (a) V_1 and (b) V_2 at the indicated points?

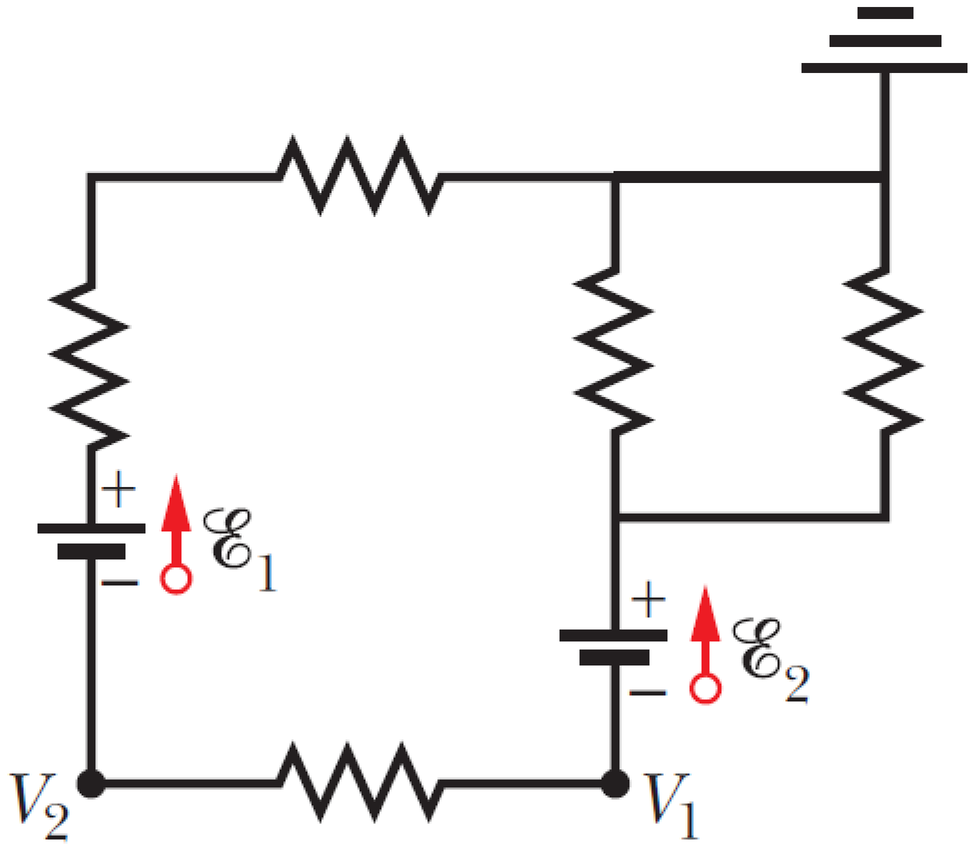


Figure 27-42 Problem 31.

Applying the loop rule to this equivalent single loop circuit gives, starting at point V_1 and going counter clockwise gives

$$\mathcal{E}_2 - iR - iR - iR_{\parallel} - \mathcal{E}_1 - iR = 0,$$

or

$$\mathcal{E}_2 - \mathcal{E}_1 - (3R + R_{\parallel})i = 0.$$

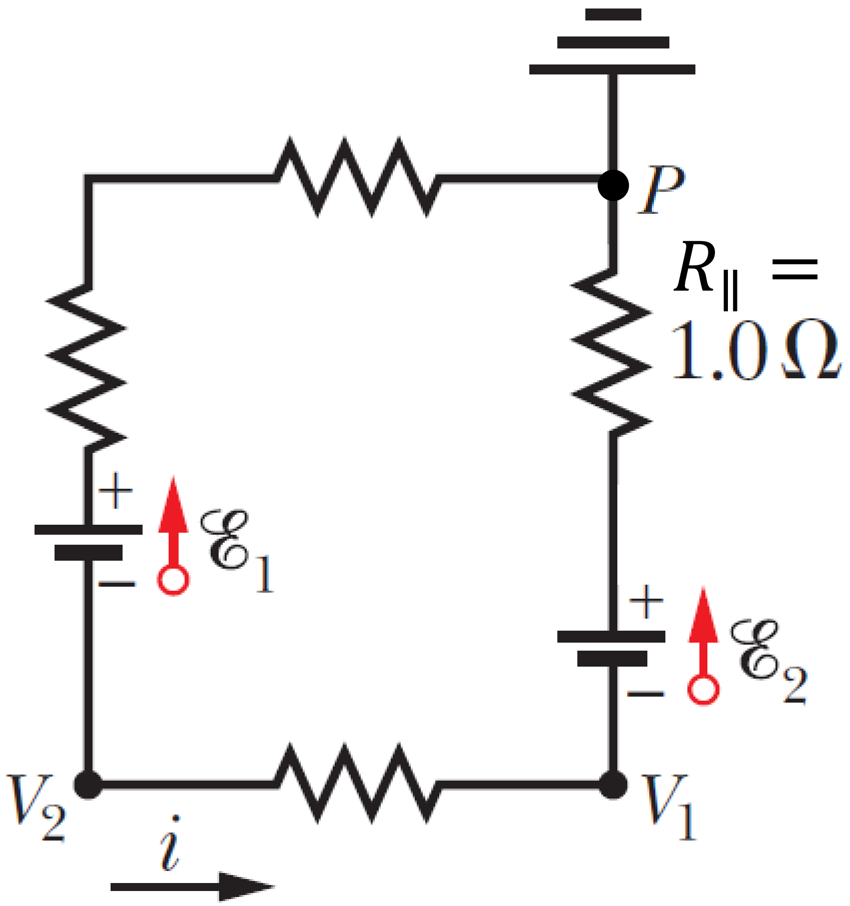
Thus

$$i = \frac{\mathcal{E}_2 - \mathcal{E}_1}{3R + R_{\parallel}} = \frac{12V - 5.0V}{3(2.0\ \Omega) + 1.0\ \Omega} = 1.0\ \text{A}.$$

a)

$$V_1 + \mathcal{E}_2 - iR_{\parallel} = V_P = 0$$

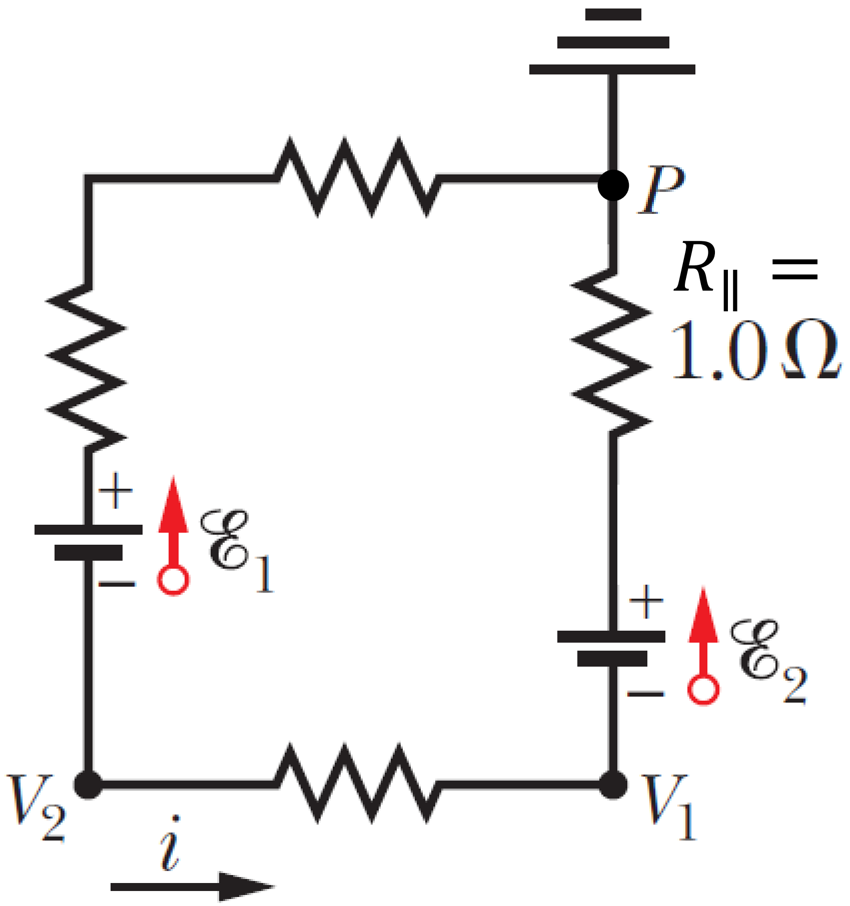
$$V_1 = iR_{\parallel} - \mathcal{E}_2 = (1.0\ \Omega)(1.0\ \text{A}) - 12\ \text{V} = -11\ \text{V}.$$



b)

$$V_2 - R\,i + \mathcal{E}_2 - R_\parallel i = V_P = 0$$

$$\begin{aligned} V_2 &= (R + R_\parallel)i - \mathcal{E}_2 \\ &= (2.0\ \Omega + 1.0\ \Omega)(1.0\ \text{A}) - 12\ \text{V} \\ &= -9.0\ \text{V}. \end{aligned}$$



•61 ILW A 15.0 kΩ resistor and a capacitor are connected in series, and then a 12.0 V potential difference is suddenly applied across them. The potential difference across the capacitor rises to 5.00 V in 1.30 μs. (a) Calculate the time constant of the circuit. (b) Find the capacitance of the capacitor.

a)

$$V_C = \frac{q_C}{C} = \mathcal{E}\left(1 - e^{-t/\tau}\right)$$

Solve for τ gives

$$\tau = \frac{-t}{\ln(1 - V_C/\mathcal{E})} = \frac{-1.30 \times 10^{-6}\ \mathrm{s}}{\ln(1 - 5.00\ \mathrm{V}/12.0\ \mathrm{V})} = 2.41 \times 10^{-6}\ \mathrm{s}.$$

b)

$$\tau = RC \Longrightarrow C = \frac{\tau}{R} = \frac{2.41 \times 10^{-6}\ \mathrm{s}}{15.0 \times 10^{3}\ \Omega} = 1.61 \times 10^{-10}\ \mathrm{F} = 161\ \mathrm{pF}.$$

••64 A capacitor with an initial potential difference of 100 V is discharged through a resistor when a switch between them is closed at $t = 0$. At $t = 10.0$ s, the potential difference across the capacitor is 1.00 V. (a) What is the time constant of the circuit? (b) What is the potential difference across the capacitor at $t = 17.0$ s?

a)

$$V_C = \frac{q_C}{C} = V_0 e^{-t/\tau}$$

Solving for τ gives

$$\tau = \frac{-t}{\ln V_C/V_0} = \frac{-10.0\ \text{s}}{\ln(1.00\ \text{V}/100\ \text{V})} = 2.17\ \text{s}.$$

b)

$$V_C(17.0\ \text{s}) = (100\ \text{V})e^{-17.0\ \text{s}/2.17\ \text{s}} = 0.0398\ \text{V}.$$