Recitation 10

••3 A charged belt, 50 cm wide, travels at 30 m/s between a source of charge and a sphere. The belt carries charge into the sphere at a rate corresponding to 100 μA. Compute the surface charge density on the belt.

$$i = \frac{dq}{dt} = \frac{d(\sigma A)}{dt} = \frac{d(\sigma w x)}{dt} = \sigma w \frac{dx}{dt} = \sigma w v.$$

$$\sigma = \frac{i}{wv} = \frac{100 \times 10^{-6}\,\mathrm{A}}{(0.50\ \mathrm{m})(30\ \mathrm{m/s})} = 6.7 \times 10^{-6}\,\frac{\mathrm{C}}{\mathrm{m}^2}.$$

••11 What is the current in a wire of radius $R = 3.40$ mm if the magnitude of the current density is given by (a) $J_a = J_0 r/R$ and (b) $J_b = J_0(1 - r/R)$, in which r is the radial distance and $J_0 = 5.50 \times 10^4$ A/m^2? (c) Which function maximizes the current density near the wire's surface?

a)

$$i_a = \int_0^R \vec{J}_a \cdot d\vec{a} = \int_0^R J_a\, da = \int_0^R (J_0 r/R)\,(2\pi r dr)$$

$$= \frac{2\pi J_0}{R}\int_0^R r^2 dr = \frac{2\pi J_0}{R}\left(\frac{R^3}{3}\right) = \frac{2\pi J_0 R^2}{3}$$

$$= \frac{2\pi(5.50 \times 10^4 \mathrm{A/m^2})(3.40 \times 10^{-3}\ \mathrm{m})^2}{3} = 1.33\ \mathrm{A}.$$

b)

$$i_b = \int_0^R \vec{J}_b \cdot d\vec{a} = \int_0^R J_b \, da = \int_0^R J_0(1 - r/R)\,(2\pi r dr)$$

$$= 2\pi J_0 \int_0^R r dr - \frac{2\pi J_0}{R} \int_0^R r^2 dr = 2\pi J_0 \left(\frac{R^2}{2}\right) - i_a$$

$$= \pi J_0 R^2 - i_a = \frac{3}{2} i_a - i_a = \frac{i_a}{2} = 0.666 \text{ A.}$$

c)

$$J_a.$$

••27 SSM WWW Two conductors are made of the same material and have the same length. Conductor A is a solid wire of diameter 1.0 mm. Conductor B is a hollow tube of outside diameter 2.0 mm and inside diameter 1.0 mm. What is the resistance ratio R_A/R_B, measured between their ends?

$$\frac{R_A}{R_B} = \frac{\rho L/A_A}{\rho L/A_B} = \frac{A_B}{A_A} = \frac{\pi r_2^2 - \pi r_1^2}{\pi r_1^2} = \frac{r_2^2}{r_1^2} - 1 = \left(\frac{1.0\ \text{mm}}{0.50\ \text{mm}}\right)^2 - 1 = 3.0.$$

••31 An electrical cable consists of 125 strands of fine wire, each having 2.65 $\mu\Omega$ resistance. The same potential difference is applied between the ends of all the strands and results in a total current of 0.750 A. (a) What is the current in each strand? (b) What is the applied potential difference? (c) What is the resistance of the cable?

a)

$$i_{\text{str.}} = \frac{0.750\ \text{A}}{125} = 6.00\ \text{mA}.$$

b)

$$V = i_{\text{str.}} R_{\text{str.}} = (6.00 \times 10^{-3}\ \text{A})(2.65 \times 10^{-6}\ \Omega) = 15.9\ \text{nV}.$$

c)

$$R = \frac{V}{i} = \frac{15.9 \times 10^{-9}\ \text{V}}{0.750\ \Omega} = 21.2\ \text{n}\Omega.$$

••51 SSM WWW Wire C and wire D are made from different materials and have length $L_C = L_D = 1.0$ m. The resistivity and diameter of wire C are 2.0×10^{-6} $\Omega \cdot m$ and 1.00 mm, and those of wire D are 1.0×10^{-6} $\Omega \cdot m$ and 0.50 mm. The wires are joined as shown in Fig. 26-34, and a current of 2.0 A is set up in them. What is the electric potential difference between (a) points 1 and 2 and (b) points 2 and 3? What is the rate at which energy is dissipated between (c) points 1 and 2 and (d) points 2 and 3?

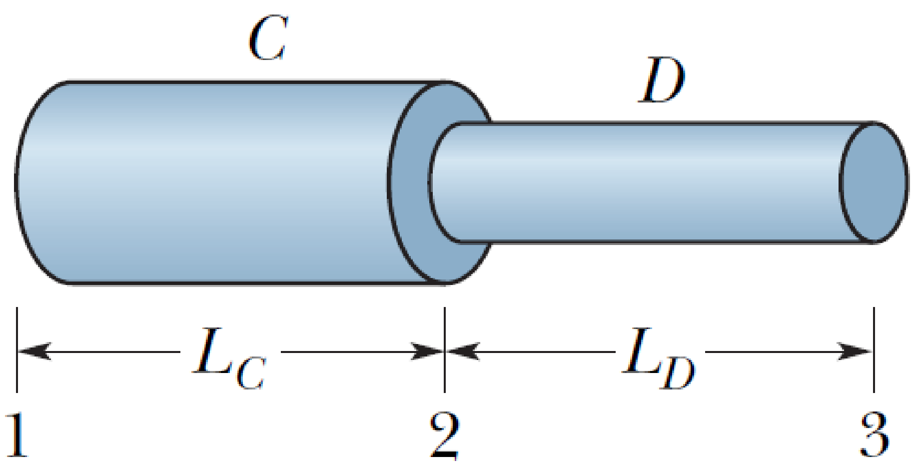

Fig. 26-34 Problem 51.