Chapter 30

Induction

1. Introduction

We have seen that a current produces a magnetic field. The reverse is true: A magnetic field can produce an electric field that can drive a current.

This link between a magnetic field and the electric field it produces (*induces*) is called **Faraday's law of induction**.

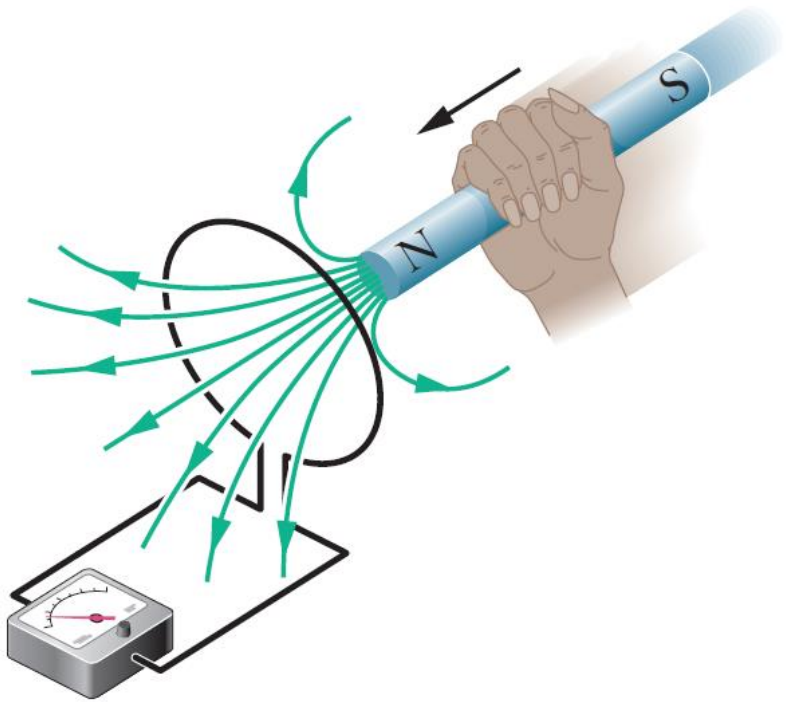
2. Two experiments

https://www.youtube.com/watch?v=3hj98E5K_c4

We start by discussing two simple experiments before discussing Faraday’s law of induction.

First Experiment

The figure shows a conducting loop connected to an ammeter. A current suddenly appears in the circuit if we move a bar magnet toward the loop. The current stops when the magnet stops moving. If we move the magnet away, again a sudden current appears, but in the opposite direction.

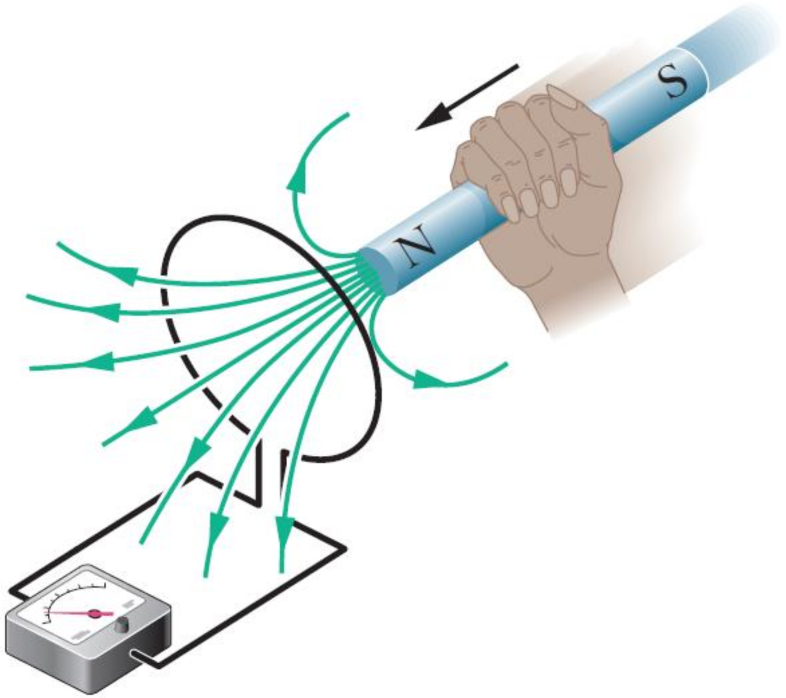


2. Two experiments

First Experiment

By experimenting this set up we can conclude the following:

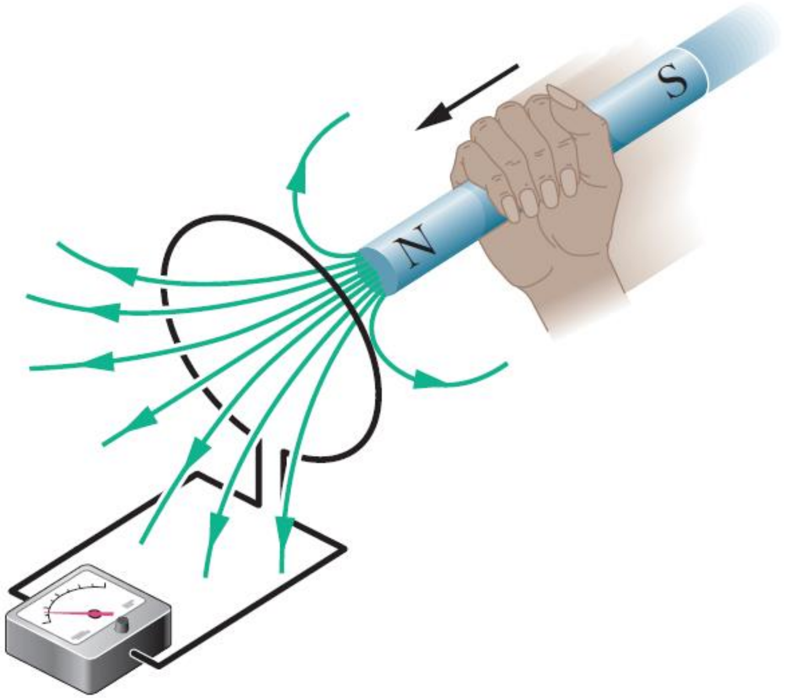
1. A current appears only if the loop and the magnet are in relative motion.
2. Faster motion produces greater current.
3. If moving the magnet's north pole away produces a clockwise current, then moving the same north pole toward the loop causes a counterclockwise current. Moving the south pole toward or away also causes currents in the reversed direction.



2. Two experiments

First Experiment

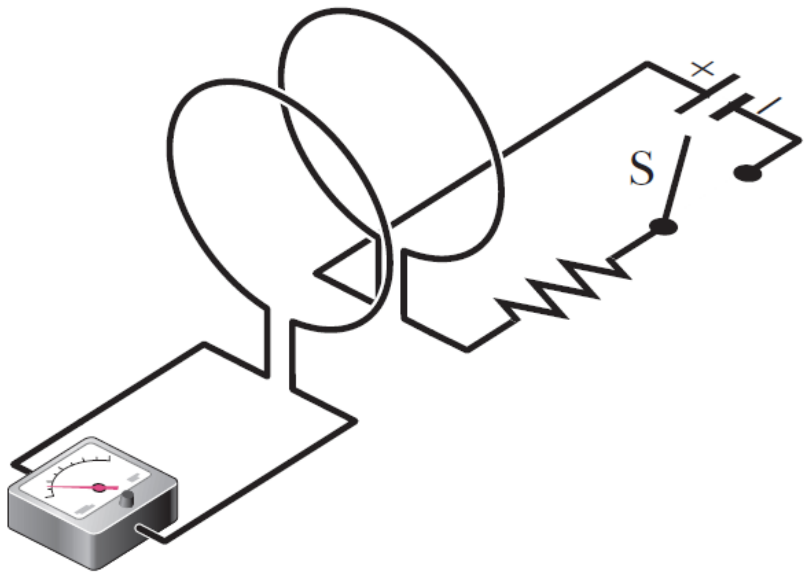
The current produced in the loop is called an **induced current**. The work done per unit charge that produces that current is called an **induced emf**. The process of producing the current and emf is called **induction**.



2. Two experiments

Second Experiment

The figure shows two conducting loops close to each other. If we close switch S, a sudden brief induced current appears in the left-hand loop. Another brief induced current appears in the left-hand loop if we open the switch, but in the opposite direction. An induced current appears only when the current in the right loop is changing.



The induced emf and current in the two experiments appears when some physical quantity is changing. Let us find out what that quantity is.

3. Faraday’s Law of Induction

Faraday concluded that an emf and a current can be induced in a loop by changing the *amount of magnetic field* passing through the loop. He also concluded that the amount of magnetic field can be visualized in terms of the magnetic field lines passing through the loop.

Faraday’s law of induction can be stated in terms of the two experiments as follows:

An emf is induced in the loop at the left in the two figures above when the number of magnetic field lines that pass through the loop is changing.

3. Faraday’s Law of Induction

Quantitative Treatment

We need a way to calculate the amount of magnetic field that passes through a loop. In analogy with the electric flux $\Phi_E = \int \vec{E} \cdot d\vec{A}$, we define magnetic flux. The **magnetic flux** through a loop of area A is

$$\Phi_B = \int \vec{B} \cdot d\vec{A}.$$

As before, $d\vec{A}$ is a vector of magnitude dA that is perpendicular to a differential area dA.

3. Faraday's Law of Induction

Quantitative Treatment

If the magnetic field is perpendicular to the plane of the loop, then $\vec{B} \cdot d\vec{A} = BdA\cos 0 = BdA$. If $\vec{B}$ is also uniform then we can take B out of the integral sign, and $\int dA$ is just the loop's area. Thus,

$$\Phi_B = BA. \qquad (\vec{B} \perp A \text{ of the loop}, \vec{B} \text{ uniform})$$

The SI unit for magnetic flux is the tesla-square meter, which has the special name **weber** (Wb):

$$1 \text{ weber} = 1 \text{ Wb} = 1 \text{ T} \cdot \text{m}^2.$$

3. Faraday's Law of Induction

Quantitative Treatment

With the notion of magnetic flux, we can state Faraday's law as:

The magnitude of the emf $\mathcal{E}$ induced in a conducting loop is equal to the rate at which the magnetic flux Φ_B through that loop changes with time.

You will see in the next section that the induced emf $\mathcal{E}$ tends to oppose the flux change. Faraday's law is therefore written as

$$\mathcal{E} = -\frac{d\Phi_B}{dt}.$$

For an ideal N turns coil, the total emf induced in the loop is

$$\mathcal{E} = -N\frac{d\Phi_B}{dt}.$$

3. Faraday’s Law of Induction

Quantitative Treatment

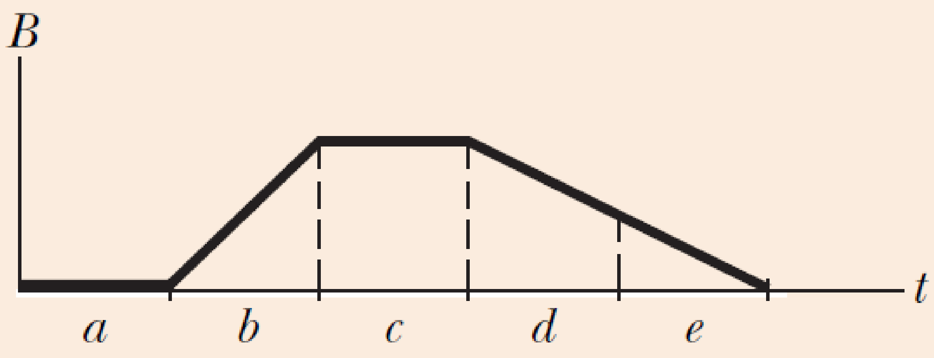
Here are the general means by which we can change the magnetic flux through a coil:

1. Changing the magnitude B of the magnetic field within the coil.
2. Changing either the total area of the coil or the portion of that area that lies within magnetic field.
3. Changing the angle between the direction of the magnetic field $\vec{B}$ and the plane of the coil.

3. Faraday's Law of Induction

CHECKPOINT 1

The graph gives the magnitude $B(t)$ of a uniform magnetic field that exists throughout a conducting loop, with the direction of the field perpendicular to the plane of the loop. Rank the five regions of the graph according to the magnitude of the emf induced in the loop, greatest first.



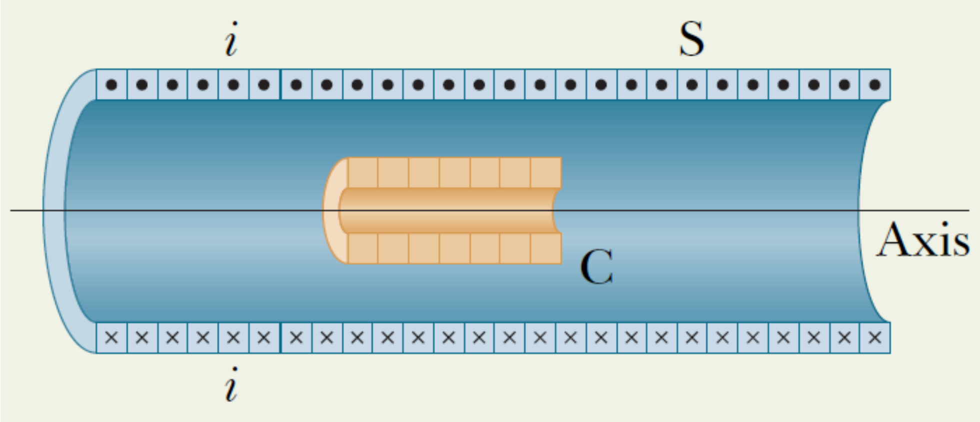
b, d & e tie, a & c tie.

$$\mathcal{E} = -\frac{d\Phi_B}{dt}$$

$$\Phi_B = BA$$

3. Faraday's Law of Induction

Example 1: The long solenoid S shown (in cross section) in the figure has 220 turns/cm and carries a current $i = 1.5\text{ A}$; its diameter D is 3.2 cm. At its center we place a 130-turn closely packed coil C of diameter $d = 2.1\text{ cm}$. The current in the solenoid is reduced to zero at a steady rate in 25 ms. What is the magnitude of the emf that is induced in coil C while the current in the solenoid is changing?



The initial flux through solenoid C is

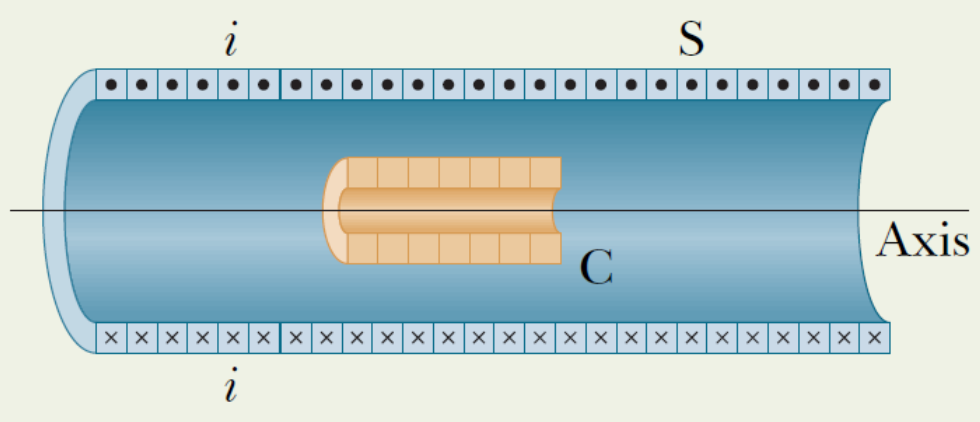
$$\Phi_{Bi} = BA_\text{C} = \mu_0 i n_\text{S} A_\text{C} = \pi\mu_0 i n_\text{S} r_C^2$$

3. Faraday's Law of Induction

Now we can write

$$\frac{d\Phi_B}{dt} = \frac{\Delta\Phi_B}{\Delta t} = \frac{\Phi_{Bf} - \Phi_{Bi}}{\Delta t}$$

$$= \frac{0 - \pi\mu_0 i n_S r_C^2}{\Delta t} = -\frac{\pi\mu_0 i n_S r_C^2}{\Delta t}.$$



Substituting gives

$$\frac{d\Phi_B}{dt} = -\frac{\pi\left(4\pi \times 10^{-7}\ \mathrm{T}\cdot\frac{\mathrm{m}}{\mathrm{A}}\right)(1.5\ \mathrm{A})}{25\ \mathrm{ms}}$$

$$\times \left(22000\frac{\mathrm{turn}}{\mathrm{m}}\right)(0.0105\ \mathrm{m})^2$$

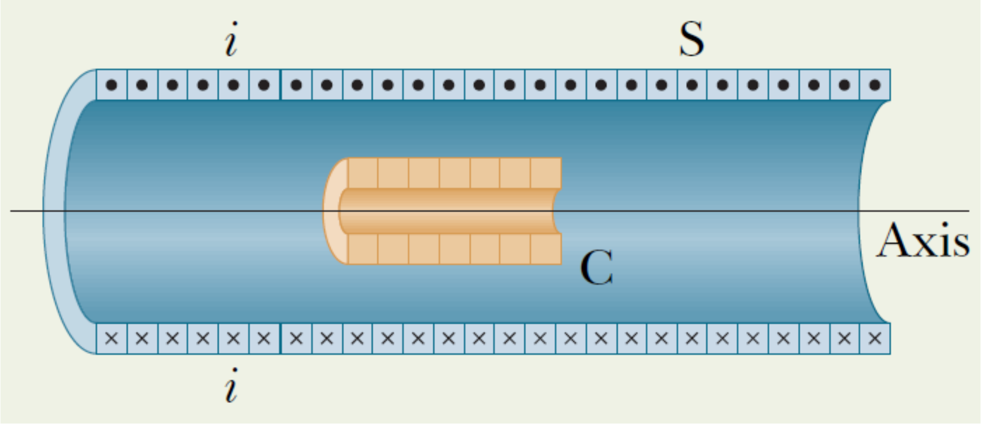
$$= -5.76 \times 10^{-4}\ \mathrm{V}.$$

3. Faraday's Law of Induction

The magnitude of the induced emf is then

$$\mathcal{E} = N\left|\frac{d\Phi_B}{dt}\right| = (130)(5.76 \times 10^{-4}\ \text{V})$$

$$= 75\ \text{mV}.$$



3. Faraday's Law of Induction

•2 A certain elastic conducting material is stretched into a circular loop of 12.0 cm radius. It is placed with its plane perpendicular to a uniform 0.800 T magnetic field. When released, the radius of the loop starts to shrink at an instantaneous rate of 75.0 cm/s. What emf is induced in the loop at that instant?

3. Faraday’s Law of Induction

$$\Phi_B = BA = \pi B r^2.$$

$$\frac{d\Phi_B}{dt} = 2\pi B r \frac{dr}{dt} = 2\pi(0.800\ \mathrm{T})(0.120\ \mathrm{m})\left(-0.750\ \frac{\mathrm{m}}{\mathrm{s}}\right) = -0.452\frac{\mathrm{Wb}}{\mathrm{s}}.$$

$$\mathcal{E} = -\frac{d\Phi_B}{dt} = 0.452\ \mathrm{V}.$$

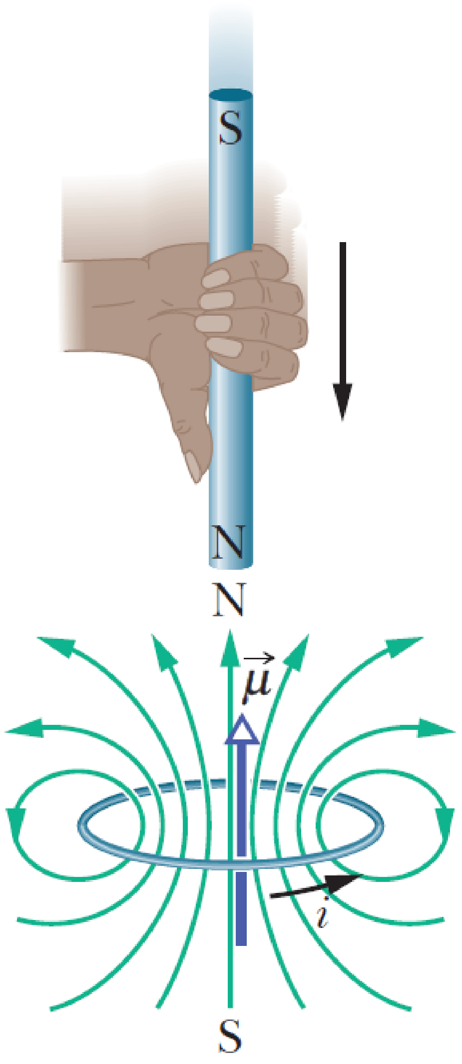
4. Lenz’s Law

https://www.youtube.com/watch?v=3-1hiBBWgSA

The direction of an induced current can be determined using **Lenz’s law**. It can be stated as follows:

An induced current has a direction such that the magnetic field due to *the current* opposes the change in the magnetic flux that induces the current.

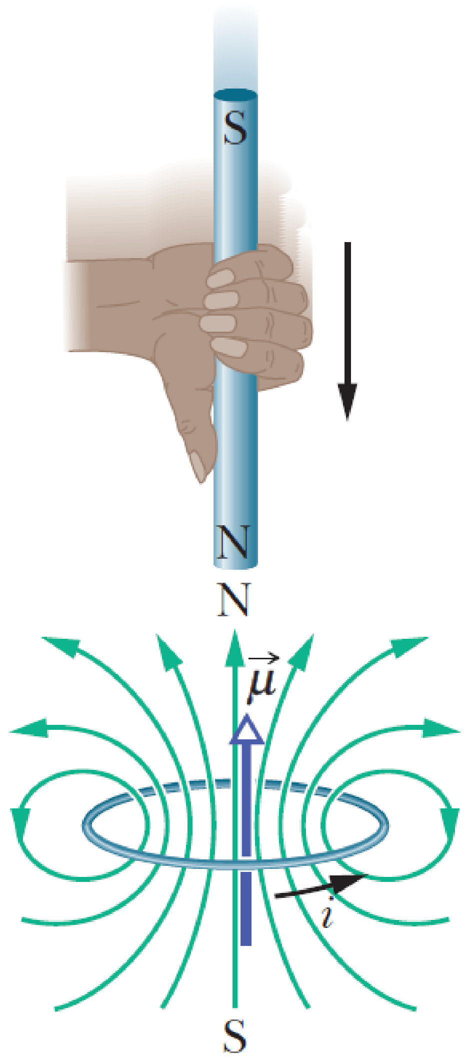
To get a feel of Lenz’s law, we apply it in two different but equivalent ways to the situation shown in the figure.



4. Lenz’s Law

1. Opposition to Pole Movement:

The approach of the magnet’s north pole increases the magnetic flux through the loop and thereby induces a current in the loop. The loop then acts as a magnetic dipole, directed from south to north. The loop’s north pole must face toward the approaching north pole of the magnet so as to repel it, to oppose the magnetic flux increase caused by the approaching magnet. The curled-straight right-hand rule tells us that the induced current in the loop must be counterclockwise.

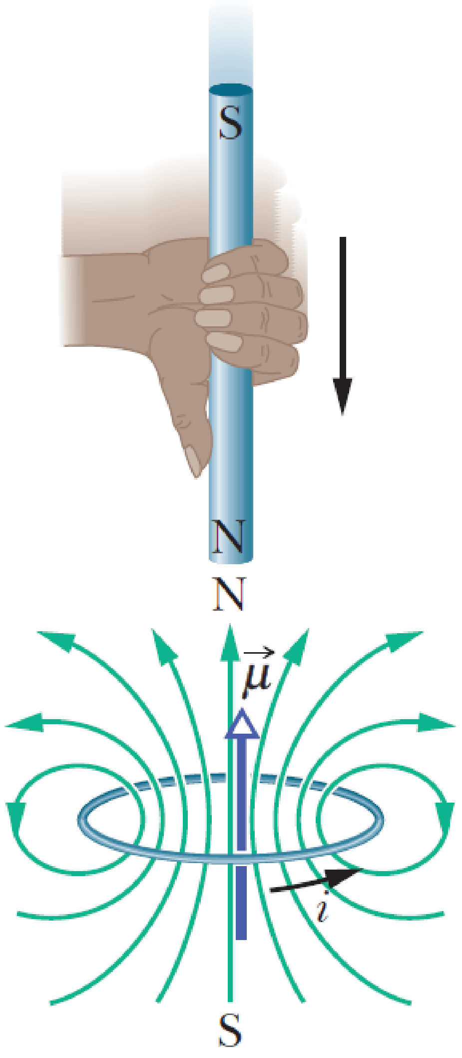


4. Lenz's Law

2. Opposition to Flux Change:

As the north pole of the magnet nears the loop with its field $\vec{B}$ directed down, the flux through the loop increases. To oppose this increase in flux, the induced current i must set up its own magnetic field $\vec{B}_{\text{ind}}$, directed upward inside the loop. The curled-straight right-hand rule tells us the i must be counterclockwise.

Note that the flux of $\vec{B}_{\text{ind}}$ always opposes the change in the flux of $\vec{B}$, but that does not always mean that $\vec{B}_{\text{ind}}$ is opposite to $\vec{B}$.



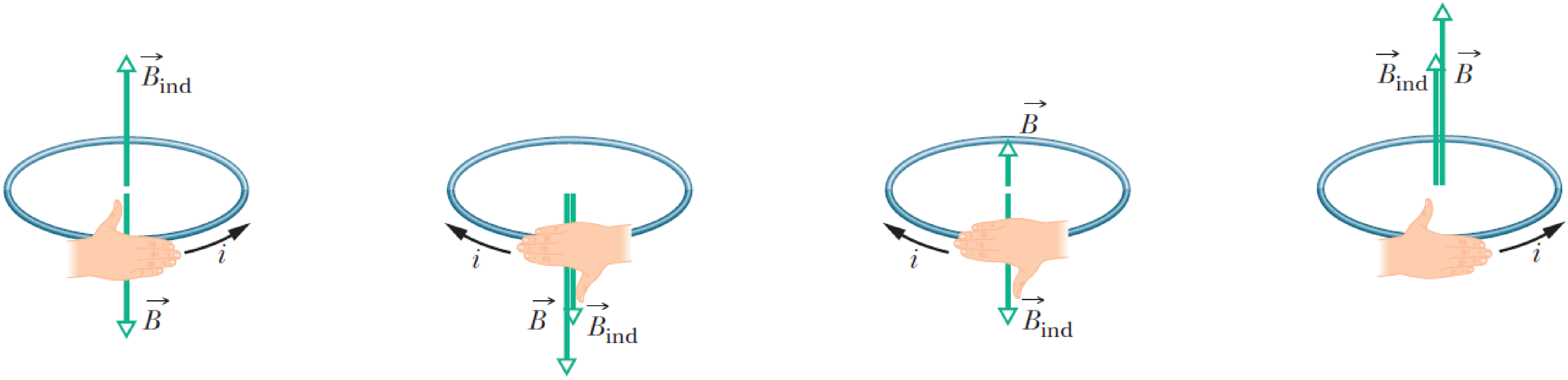
4. Lenz's Law

Increasing the external field $\vec{B}$ induces a current with a field $\vec{B}_{ind}$ that *opposes the change.*

Decreasing the external field $\vec{B}$ induces a current with a field $\vec{B}_{ind}$ that *opposes the change.*

Increasing the external field $\vec{B}$ induces a current with a field $\vec{B}_{ind}$ that *opposes the change.*

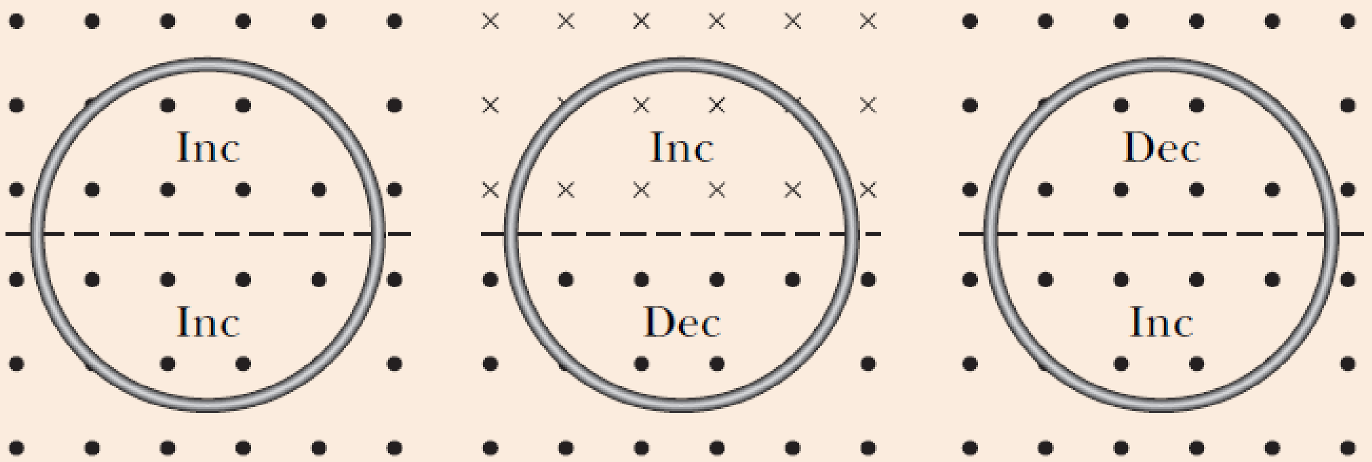
Decreasing the external field $\vec{B}$ induces a current with a field $\vec{B}_{ind}$ that *opposes the change.*



4. Lenz's Law

CHECKPOINT 2

The figure shows three situations in which identical circular conducting loops are in uniform magnetic fields that are either increasing (Inc) or decreasing (Dec) in magnitude at identical rates. In each, the dashed line coincides with a diameter. Rank the situations according to the magnitude of the current induced in the loops, greatest first.

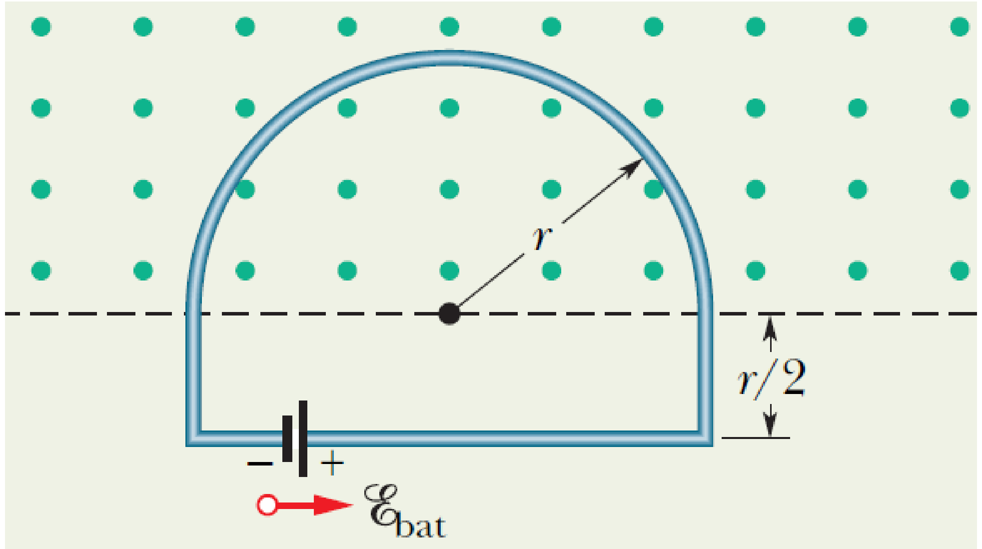


(*a*) (*b*) (*c*)

a & b, c.

4. Lenz's Law

Example 2: The figure shows a conducting loop consisting of a half-circle of radius $r = 0.20\ \mathrm{m}$ and three straight sections. The half-circle lies in a uniform magnetic field $\vec{B}$ that is directed out of the page; the field magnitude is given by $B = 4.0t^2 + 2.0t + 3.0$, with B in teslas and t in seconds. An ideal battery with emf $\mathcal{E}_{\mathrm{bat}} = 2.0\ \mathrm{V}$ is connected to the loop. The resistance of the loop is $2.0\ \Omega$.



(a) What are the magnitude and direction of the emf $\mathcal{E}_{\mathrm{ind}}$ induced around the loop by field $\vec{B}$ at $t = 10\ \mathrm{s}$?

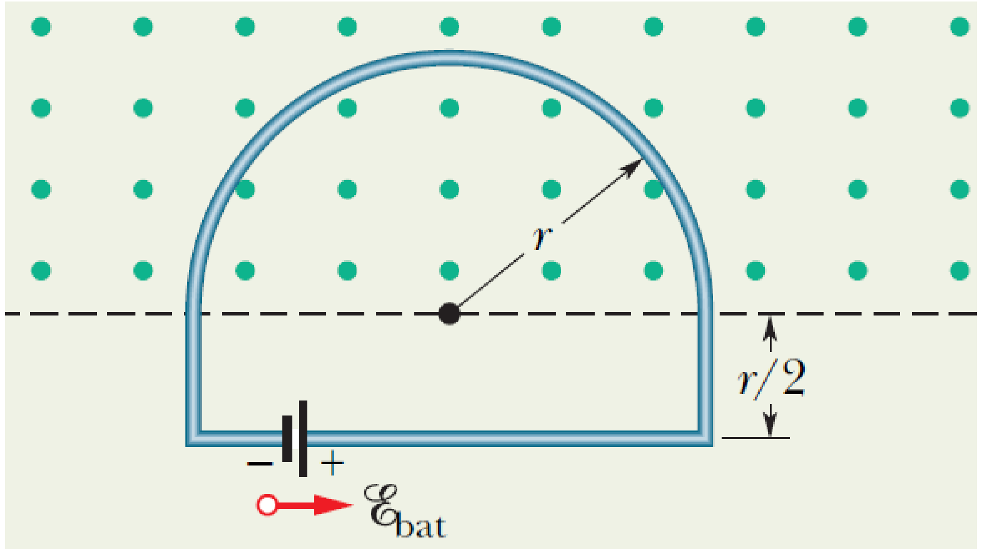
4. Lenz's Law

$$\mathcal{E}_{\text{ind}} = \frac{d\Phi_B}{dt} = \frac{d(BA)}{dt} = A\frac{dB}{dt}$$
$$= \left(\frac{\pi r^2}{2}\right)(8.0t + 2.0).$$

At $t = 10$ s,

$$\mathcal{E}_{\text{ind}} = \frac{\pi(0.20\text{ m})^2}{2}[8.0(10\text{ s}) + 2.0] = 5.2\text{ V}.$$

By Lenz's law, the direction of the emf is clockwise.

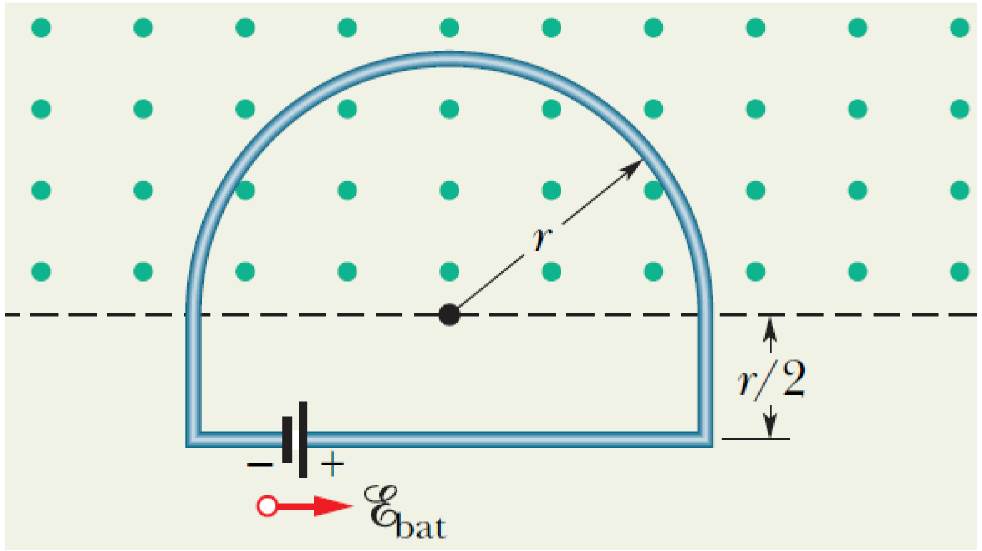


4. Lenz's Law

b) What is the current in the loop at $t = 10$ s?

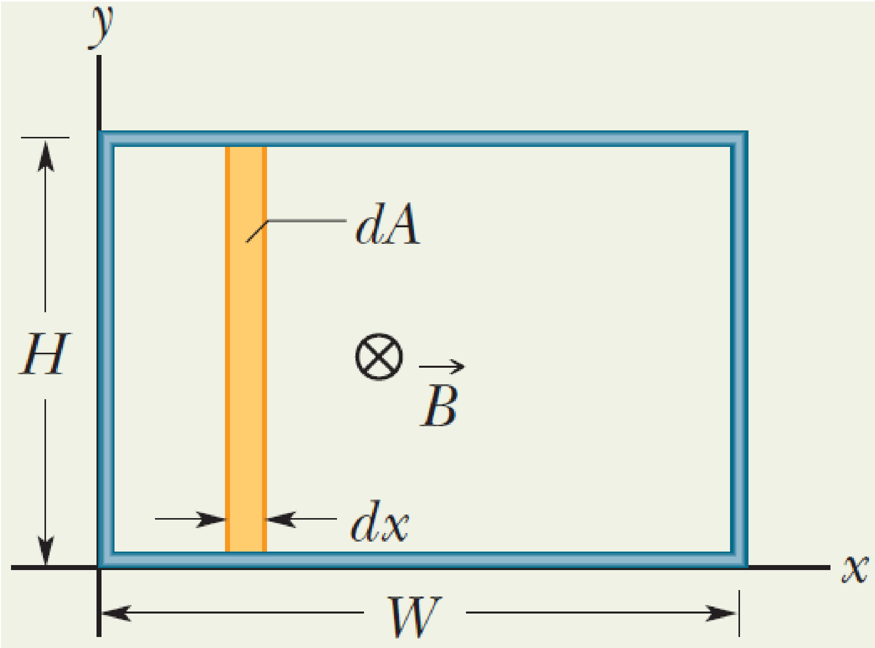
$$i = \frac{\mathcal{E}_{net}}{R} = \frac{\mathcal{E}_{ind} - \mathcal{E}_{bat}}{R} = \frac{5.15\ \text{V} - 2.0\ \text{V}}{2.0\ \Omega}$$

$$= 1.6\ A.$$



4. Lenz's Law

Example 3: The figure shows a rectangular loop of wire immersed in a nonuniform and varying magnetic field $\vec{B}$ that is perpendicular to and directed into the page. The field's magnitude is given by $B = 4t^2x^2$, with B in teslas, t in seconds, and x in meters. (Note that the function depends on *both* time and position.) The loop has width $W = 3.0\ \text{m}$ and height $H = 2.0\ \text{m}$. What are the magnitude and direction of the induced emf $\mathcal{E}$ around the loop at $t = 0.10\ \text{s}$?



4. Lenz's Law

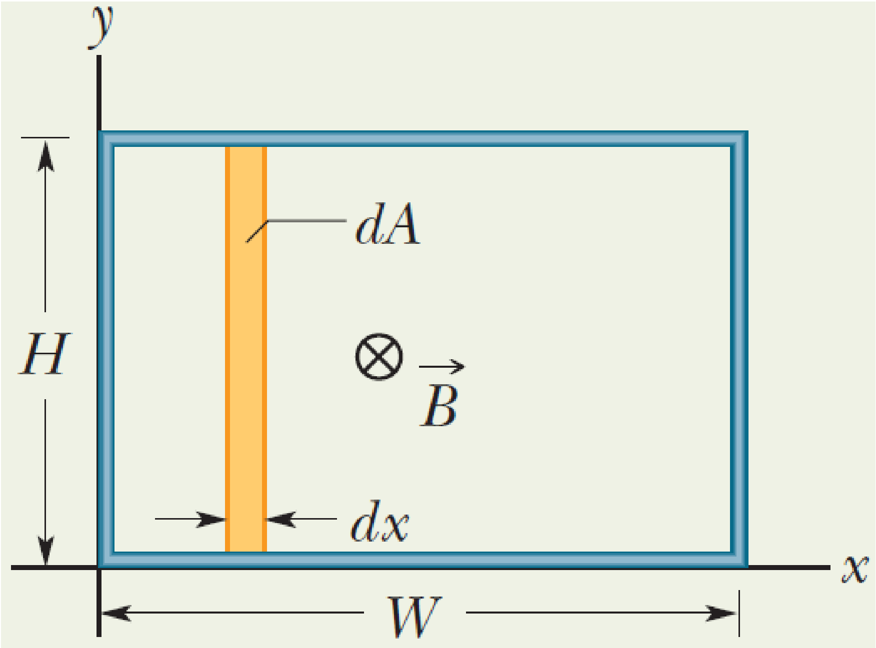
The flux through the loop is

$$\Phi_B = \int \vec{B} \cdot d\vec{A} = \int B dA = \int_0^W (4t^2x^2)(H dx)$$

$$= 4Ht^2 \int_0^W x^2 dx = \frac{4}{3} H t^2 W^3.$$

Substituting for H and W gives,

$$\Phi_B = \frac{4}{3}(2.0 \text{ m})(3.0 \text{ m})^3 t^2 = 72t^2.$$



4. Lenz's Law

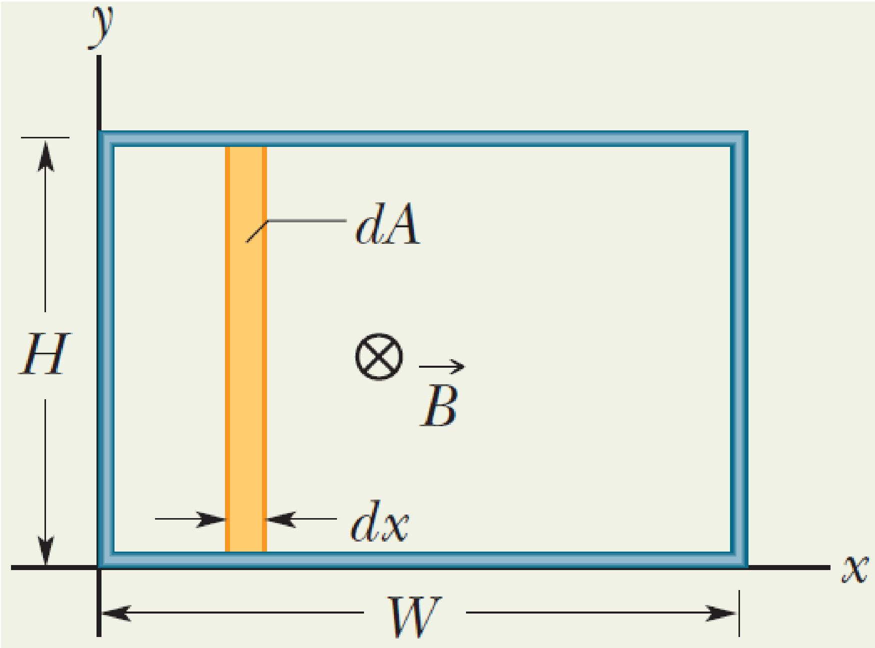
The magnitude of the induced emf is

$$\mathcal{E} = \frac{d\Phi_B}{dt} = 144\,t.$$

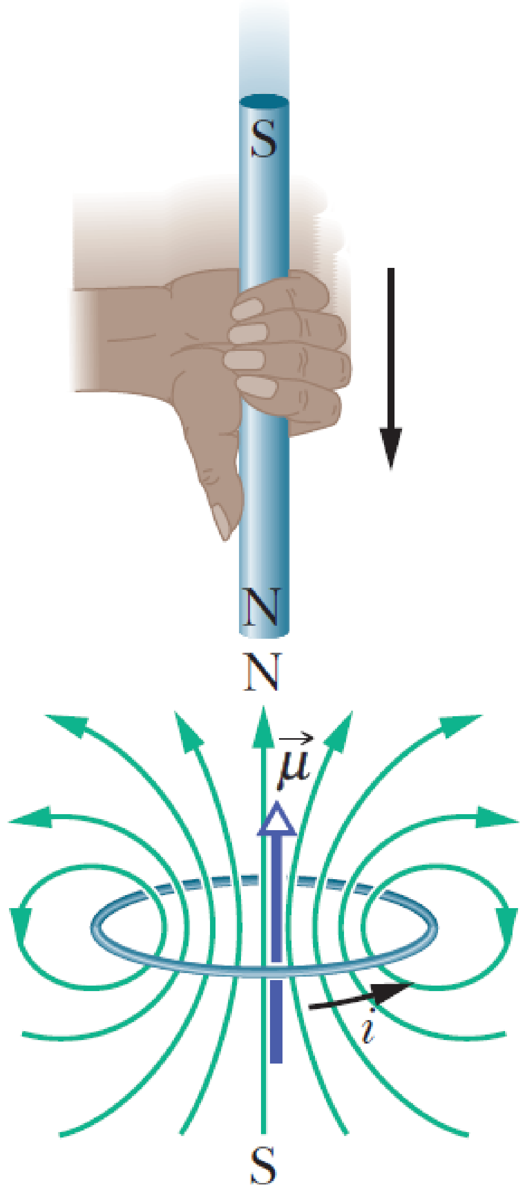
At $t = 0.10$ s,

$$\mathcal{E} = (144)(0.10\text{ s}) = 14\text{ V}.$$

The direction of the induced emf is counterclockwise, by Lenz's law.



5. Induction and Energy Transfers



According to Lenz's law, a magnetic force resists the moving magnet away or toward the loop, requiring your applied force that moves the magnet to do positive work.

Thermal energy is then produced in the material of the loop due to the material's electrical resistance to the induced current. The energy transferred to the closed loop + magnet system while you move the magnet ends up in this thermal energy.

The faster the magnet is moved, the more rapidly the applied force does work and the greater the rate at which energy is transferred to thermal energy in the loop; the power transfer is greater.

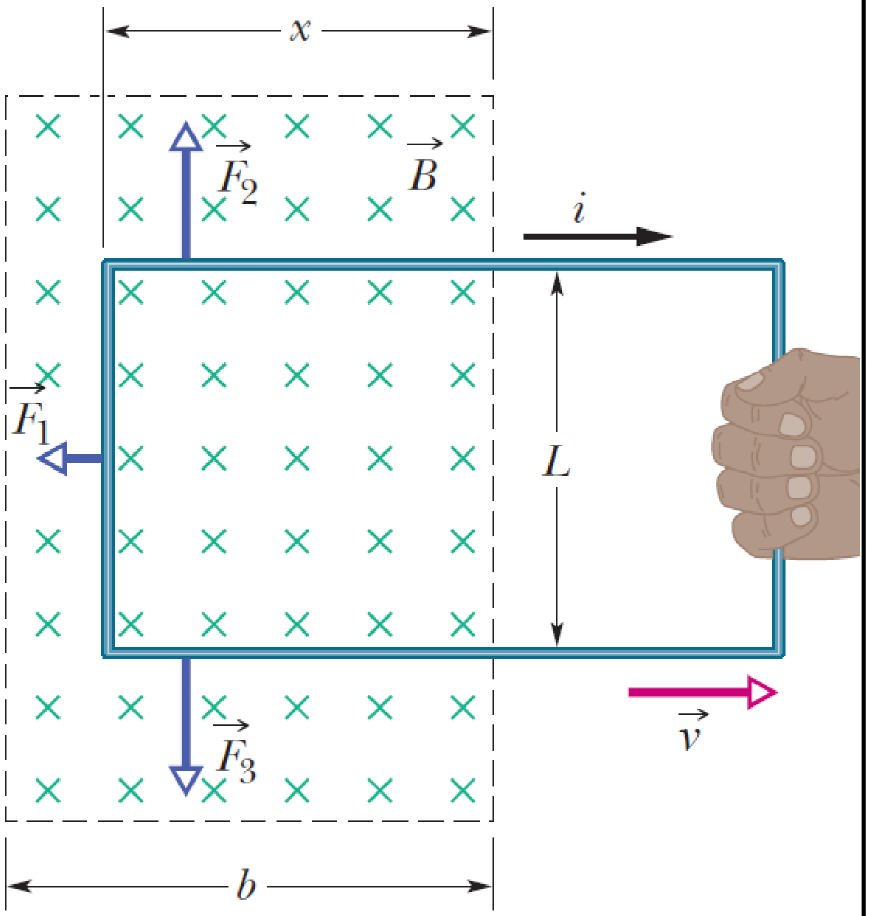
5. Induction and Energy Transfers

The figure shows a situation involving an induced current. You are to pull this loop to the right at a constant velocity $\vec{v}$. Let us calculate the rate at which you do mechanical work as you pull steadily on the loop.

The rate P at which you do work is given by

$$P = Fv,$$

where F is the magnitude of your force. The magnitude of this force is equal to the magnitude of the magnetic force on the loop due to the emergence of the induced current.



5. Induction and Energy Transfers

We thus write

$$F = F_1 = iLB.$$

The induced current i is related to the induced emf by

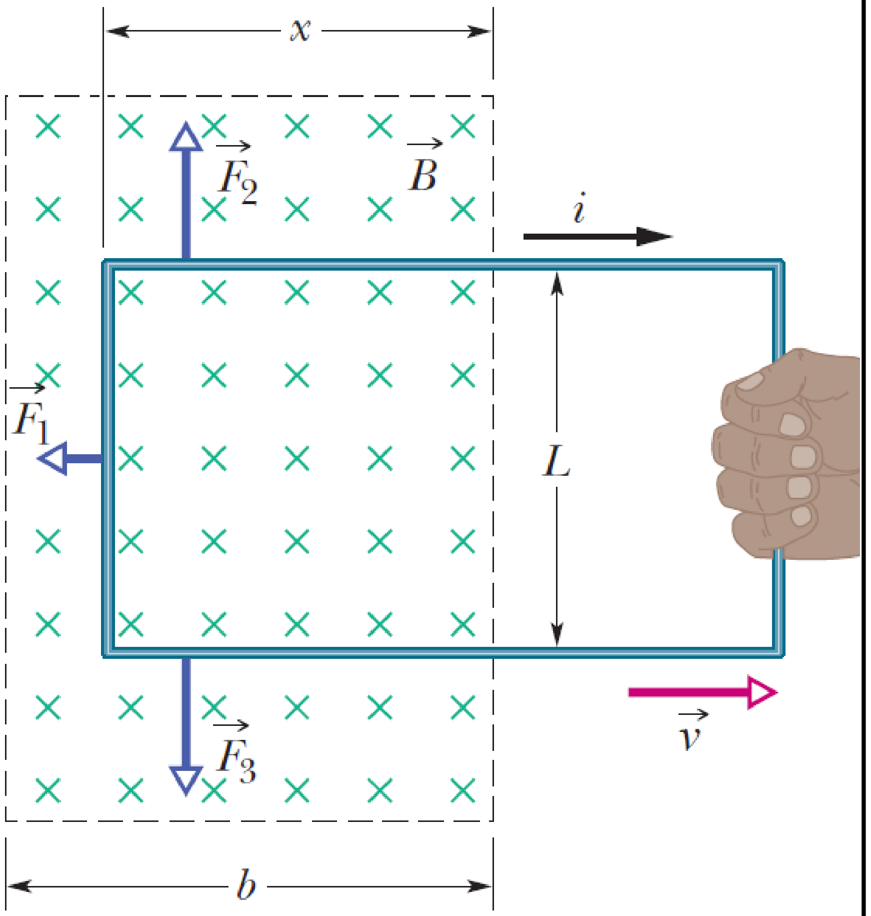
$$i = \frac{\mathcal{E}}{R}.$$

The induced emf is given by

$$\mathcal{E} = \frac{d\Phi_B}{dt} = \frac{d}{dt}BLx = BL\frac{dx}{dt} = BLv.$$

Thus,

$$i = \frac{BLv}{R}.$$



5. Induction and Energy Transfers

The force is then

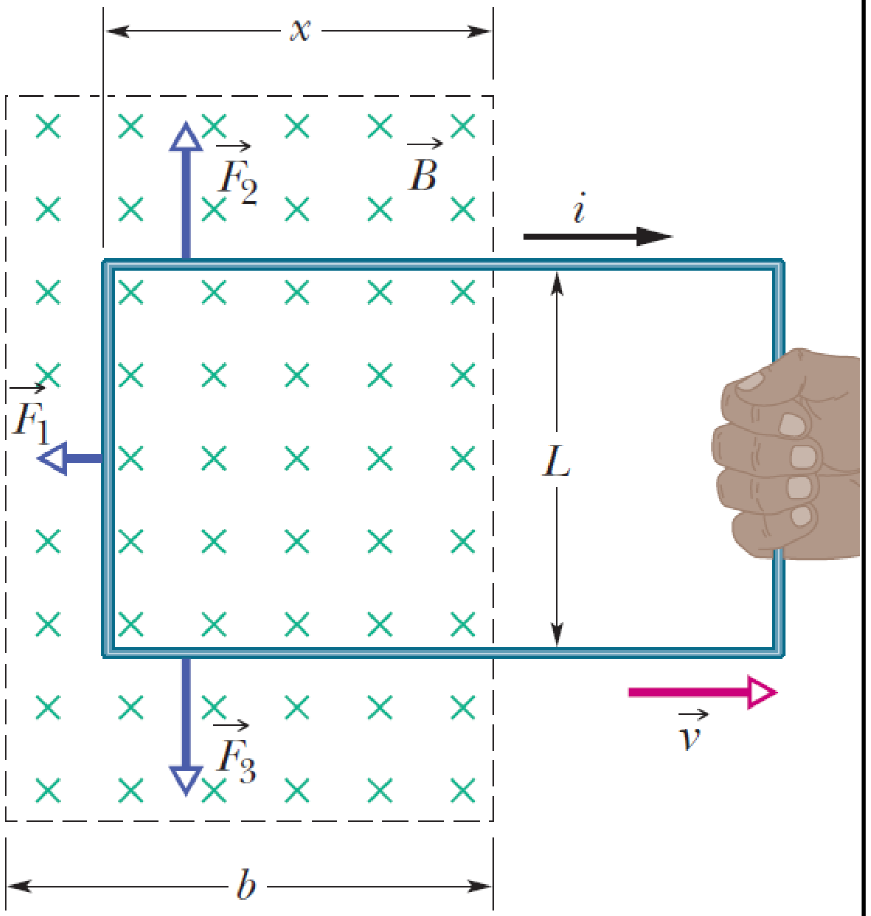
$$F = \frac{B^2L^2v}{R}.$$

The rate at which you do work is now

$$P = Fv = \frac{B^2L^2v^2}{R}.$$

This rate must be equal to the rate at which thermal energy appears in the loop:

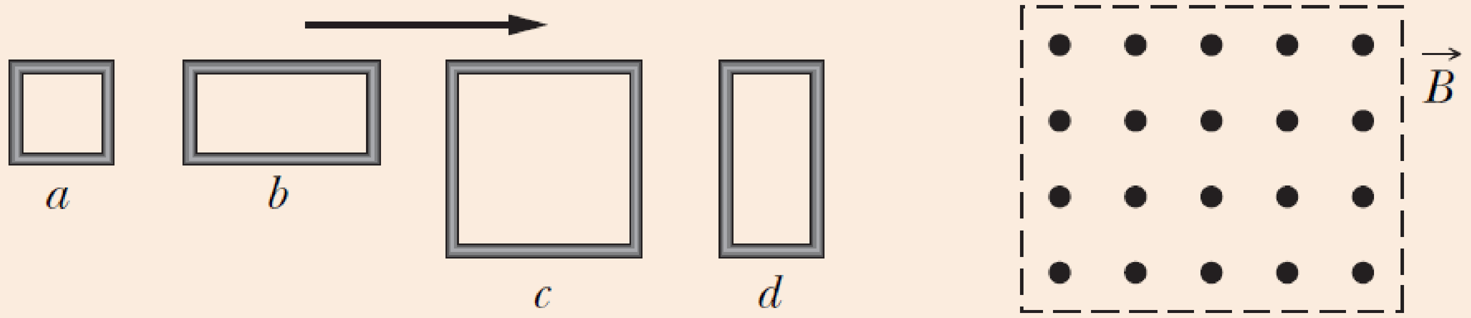
$$P = \frac{B^2L^2v^2}{R^2}R = i^2R.$$



5. Induction and Energy Transfers

CHECKPOINT 3

The figure shows four wire loops, with edge lengths of either L or $2L$. All four loops will move through a region of uniform magnetic field $\vec{B}$ (directed out of the page) at the same constant velocity. Rank the four loops according to the maximum magnitude of the emf induced as they move through the field, greatest first.



c & d, a & b.

$$\mathcal{E} = BLv$$