Chapter 28

Magnetic Fields

1. What Produces a Magnetic Field?

There are to ways in which magnetic fields can be produced:

1) Using moving charged particles, such as a current in a wire, to make an **electromagnet**.
2) By means of elementary particles, such as electrons. These particles have an intrinsic magnetic field around them. The magnetic fields of the electrons in certain materials add together to produce a net magnetic field surround the material. Such addition makes the martial a **permanent magnetic**. In other martials, the magnetic fields of the electrons cancel out, giving no net magnetic field surrounding the material.

2. The Definition of $\vec{B}$

We define the magnetic field $\vec{B}$ in terms of the magnetic force $\vec{F}_B$ exerted on a moving electrically charged particle.

The magnetic force $\vec{F}_B$ on a charged particle of charge q and velocity vector $\vec{v}$ is given by

$$\vec{F}_B = q\vec{v} \times \vec{B}.$$

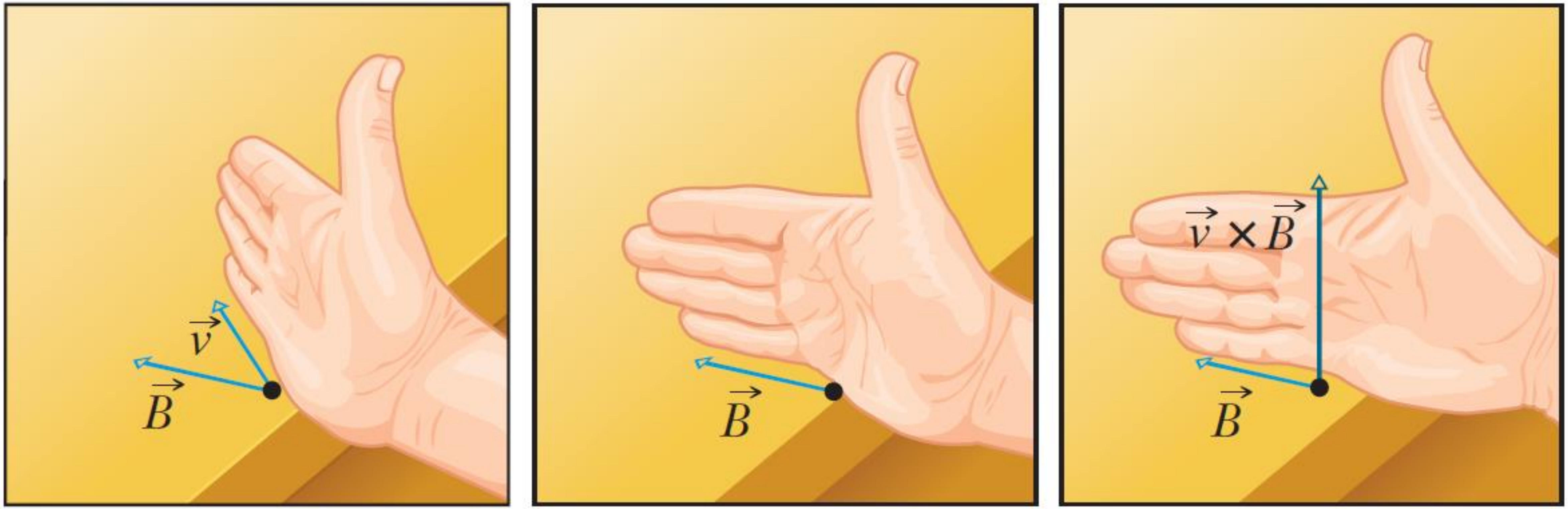
The magnitude of $\vec{F}_B$ is

$$F_B = |q|vB \sin\phi\,,$$

where ϕ is the angle between the directions of the velocity vector $\vec{v}$ and the magnetic field vector $\vec{B}$.

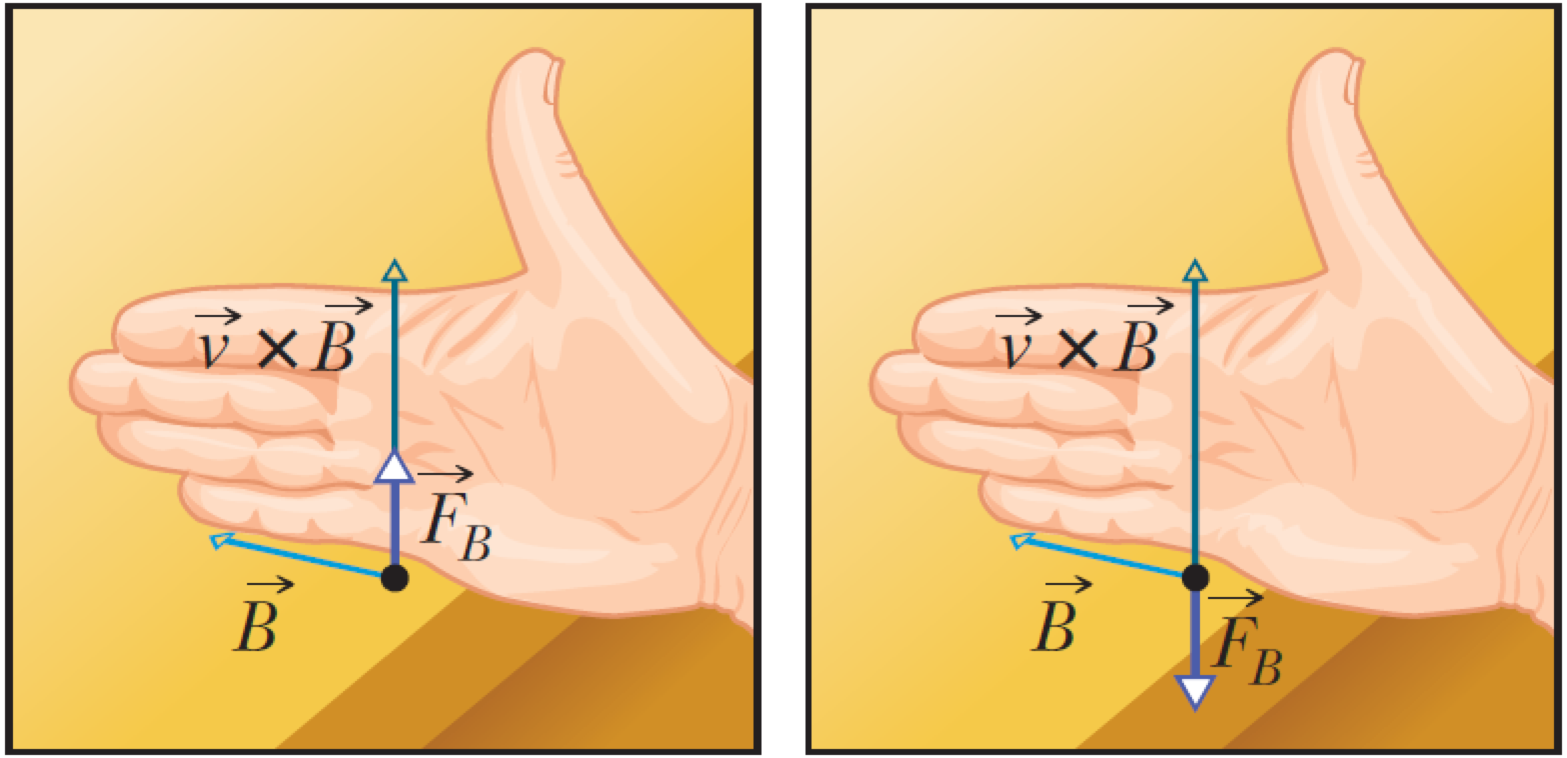
2. The Definition of $\vec{B}$

Finding the Magnetic Force on a Particle



2. The Definition of $\vec{B}$

<u>Finding the Magnetic Force on a Particle</u>



$q > 0$ $\quad$ $q < 0$

2. The Definition of $\vec{B}$

The force $\vec{F}_B$ acting on a charged particle moving with velocity $\vec{v}$ through a magnetic field $\vec{B}$ is *always* perpendicular to $\vec{v}$ and $\vec{B}$.

The SI unit for $\vec{B}$ is $\mathrm{N/(C \cdot m/s)}$, which is called **tesla** (T):

$$1\ \mathrm{T} = 1\,\frac{\mathrm{N}}{\mathrm{C \cdot m/s}}.$$

Using that $1\ \mathrm{A} = 1\ \mathrm{C}/s$, we write

$$1\ \mathrm{T} = 1\,\frac{\mathrm{N}}{\mathrm{C/s \cdot m}} = 1\,\frac{\mathrm{N}}{\mathrm{A \cdot m}}.$$

2. The Definition of $\vec{B}$

Another commonly used magnetic field unit is the gauss (G), where

$$1 \text{ tesla} = 10^4 \text{ gauss.}$$

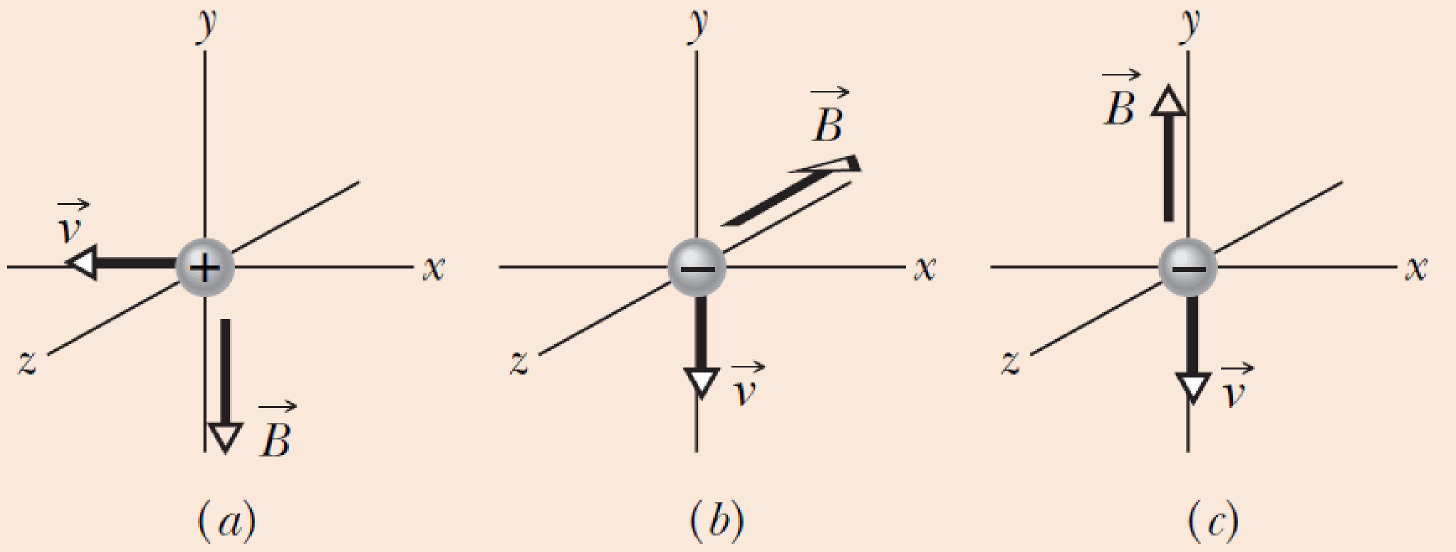
Some Approximate Magnetic Fields

At surface of neutron star	10^8 T
Near big electromagnet	1.5 T
Near small bar magnet	10^{-2} T
At Earth's surface	10^{-4} T
In interstellar space	10^{-10} T
Smallest value in magnetically shielded room	10^{-14} T

2. The Definition of $\vec{B}$

CHECKPOINT 1

The figure shows three situations in which a charged particle with velocity $\vec{v}$ travels through a uniform magnetic field $\vec{B}$. In each situation, what is the direction of the magnetic force $\vec{F}_B$ on the particle?



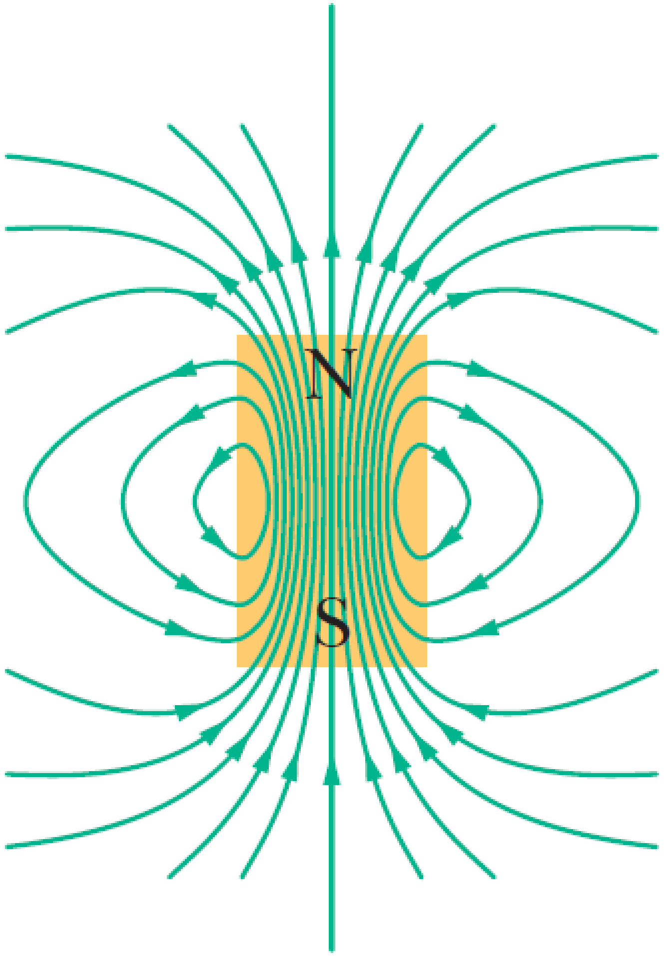
a) $\hat{k}$
b) $-\hat{i}$
c) The force is zero.

2. The Definition of $\vec{B}$

<u>Magnetic Field Lines</u>

We can visualize magnetic fields with field lines. They obey two rules:

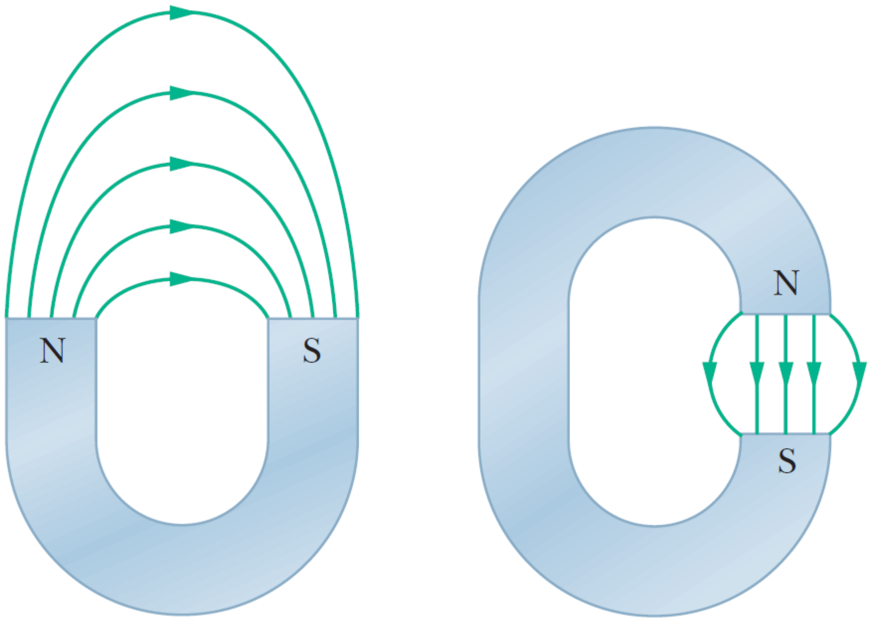
1) The direction of the tangent to a magnetic field line at any point gives the direction of $\vec{B}$ at that point.
2) The spacing of the lines represents the magnitude of $\vec{B}$. The magnetic field is stronger where the lines are closer together, and vice versa.

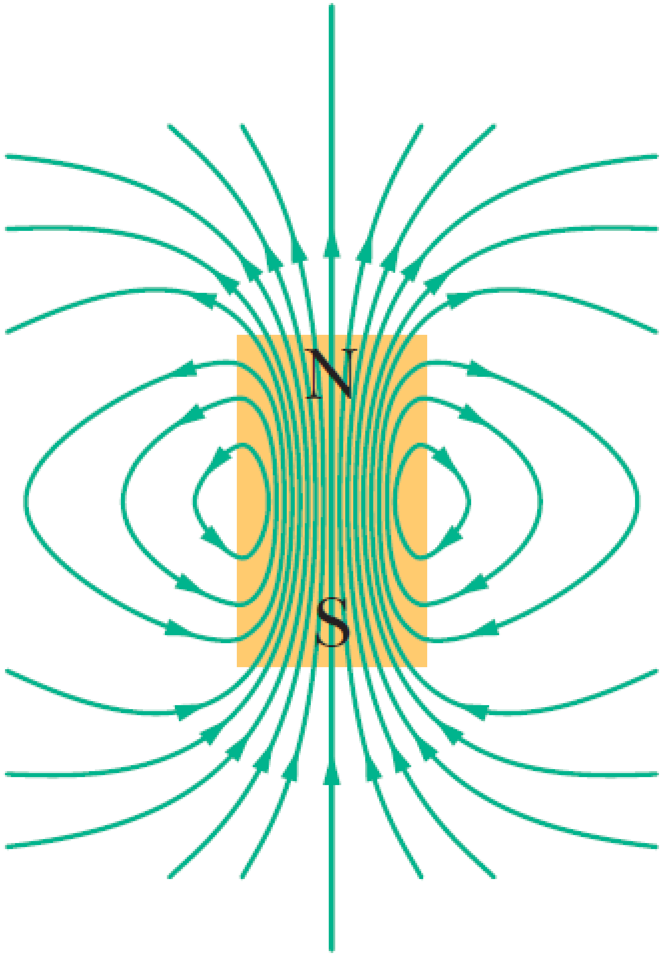


2. The Definition of $\vec{B}$

Magnetic Field Lines

The lines pass through the magnet. They form closed loops. The lines leave from one of the ends (**north pole**) and enter the other end (**south pole**).





2. The Definition of $\vec{B}$

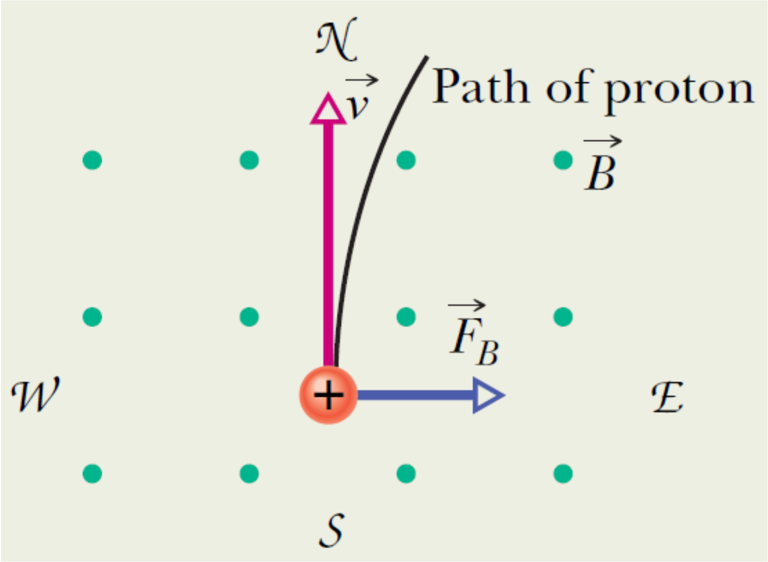
Magnetic Field Lines

A magnet is called a **magnetic dipole** because it has two poles.

If we place two magnets of any shape near each other we find that opposite magnetic poles attract each other, and like magnetic poles repel each other.

2. The Definition of $\vec{B}$

Example 1: A uniform magnetic field, with magnitude $1.2\ \mathrm{mT}$, is directed vertically upward throughout the volume of a laboratory chamber. A proton with kinetic energy $5.3\ \mathrm{MeV}$ enters the chamber, moving horizontally from south to north. What magnetic deflecting force acts on the proton as it enters the chamber? The proton mass is $1.67 \times 10^{-27}\ \mathrm{kg}$. (Neglect Earth's magnetic field.)



The speed of the proton is

$$v = \sqrt{\frac{2K}{m_p}} = \sqrt{\frac{2(5.3\ \mathrm{MeV})(1.60 \times 10^{-19}\ \mathrm{J/eV})}{1.67 \times 10^{-27}\ \mathrm{kg}}}$$

$$= 3.2 \times 10^{7}\ \mathrm{m/s}.$$

2. The Definition of $\vec{B}$

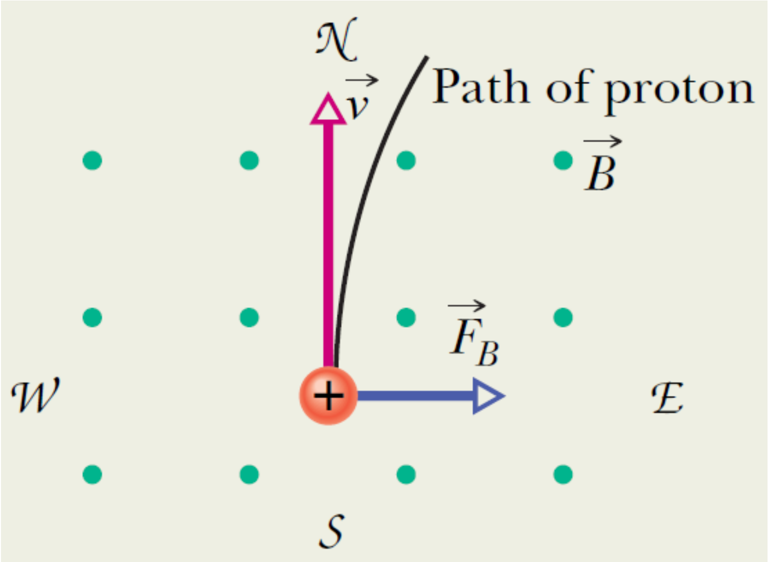
The magnetic force is then

$$F_B = |q|vB\sin\phi$$

$$= (1.60 \times 10^{-19}\ \mathrm{C})\left(3.2 \times 10^{7}\ \frac{\mathrm{m}}{\mathrm{s}}\right)$$

$$\times (1.2\ \mathrm{mT})\sin 90°$$

$$= 6.1 \times 10^{-15}\ \mathrm{N}.$$



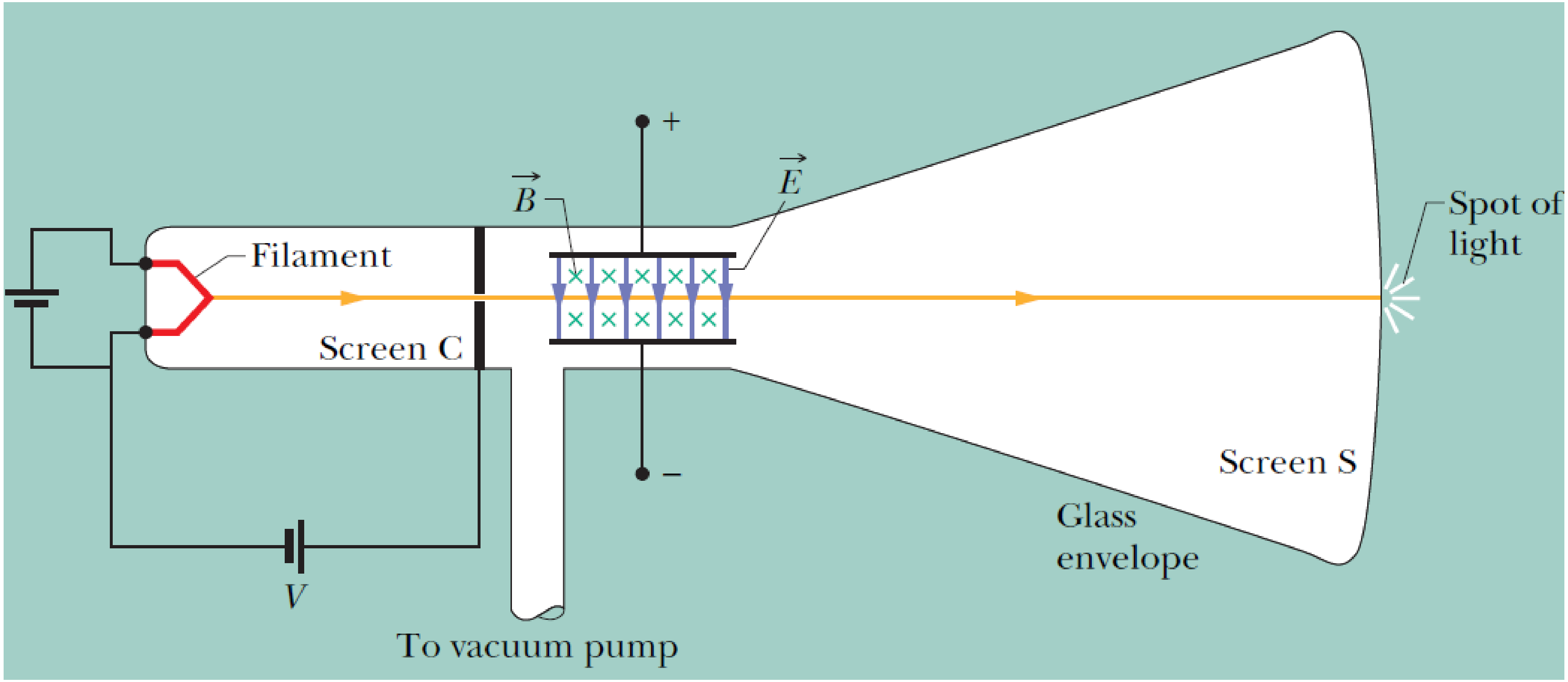
3. Crossed Fields: Discovery of Electrons

Both an electric field $\vec{E}$ and a magnetic field $\vec{B}$ can produce force on a charged particle. The fields are said to be **crossed fields** when they are perpendicular to each other.

We will examine what happens to charged particles, namely electrons, as they move through crossed fields via studying the experiment that led to the discovery of the electron.

The figure in the next slide shows a modern, simplified version of Thomson's experiment apparatus; a cathode ray tube.

3. Crossed Fields: Discovery of Electrons



3. Crossed Fields: Discovery of Electrons

Thomson procedure was equivalent to the following steps:

1. Set $E = 0$ and $B = 0$ and note the position (y) of the spot on screen S due to the undeflected beam.
2. Turn on $\vec{E}$ and measure the resulting beam deflection.
3. Maintaining $\vec{E}$, now turn on $\vec{B}$ and adjust its value until the beam returns to the undeflected position.

Recall that the deflection of a charged particle, of mass m and charge q, moving through an electric field $\vec{E}$ with speed v between two plates of length L is given by

$$y = \frac{|q|EL^2}{2mv^2},$$

3. Crossed Fields: Discovery of Electrons

When the two fields are adjusted so that the two defecting forces cancel, we have

$$|q|E = |q|vB \sin 90° = |q|vB,$$

or

$$v = \frac{E}{B}.$$

Thus, the crossed fields allow us to measure the speed of the charged particles passing through them. Substituting for v in the expression for the defection and rearranging give

$$\frac{m}{|q|} = \frac{B^2 L^2}{2yE}.$$

Thomson concluded that the $m/|q|$ ratio for these particles (electrons) was less than that of hydrogen by more than 1000.

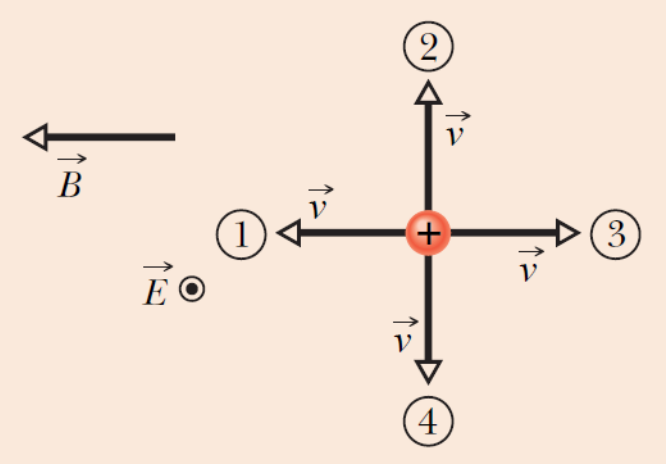
3. Crossed Fields: Discovery of Electrons

CHECKPOINT 2

The figure shows four directions for the velocity vector $\vec{v}$ of a positively charged particle moving through a uniform electric field $\vec{E}$ (directed out of the page and represented with an encircled dot) and a uniform magnetic field $\vec{B}$. (a) Rank directions 1, 2, and 3 according to the magnitude of the net force on the particle, greatest first. (b) Of all four directions, which might result in a net force of zero?

a) 2, 1 & 3 tie.

b) 4

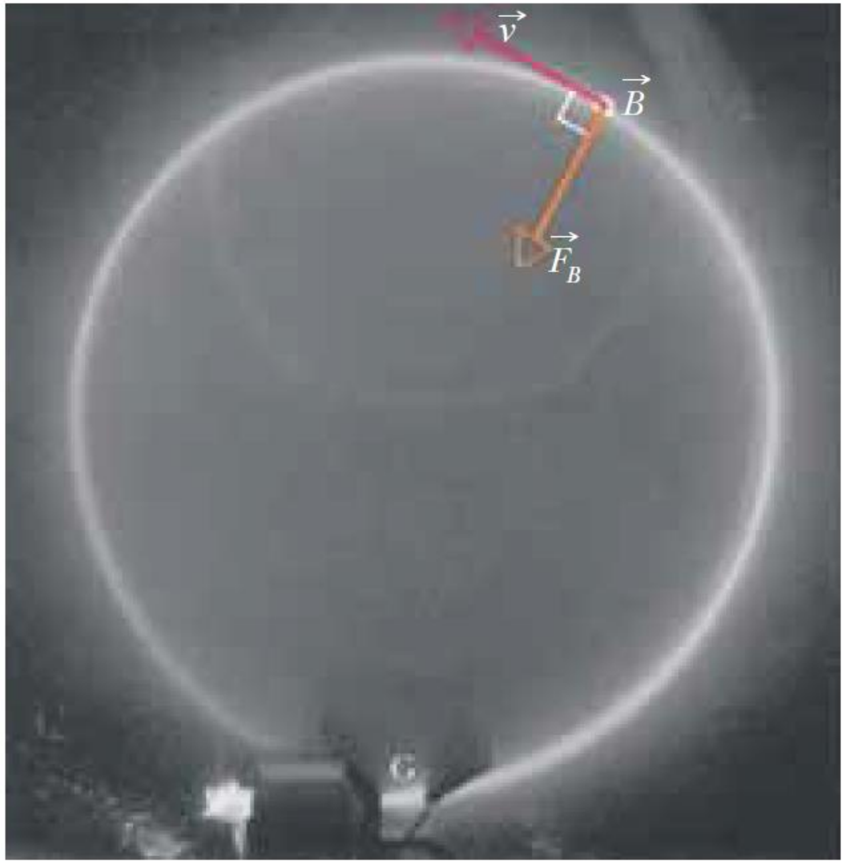


4. A Circulating Charged Particle

<u>Circulating Charged Particles</u>

If a particle moves in a circle at a constant speed, the net force acting on the particle is constant in magnitude and points radially inward, perpendicular to the particle's velocity.

The figure shows an example of such a motion, where a beam of electrons enter the plane of the page with speed v and move in a uniform magnetic field $\vec{B}$, directed out of the plane. Consequently, a radially directed magnetic force $\vec{F}_B = q\vec{v} \times \vec{B}$ defects the electrons, causing them to follow a circular path.



4. A Circulating Charged Particle

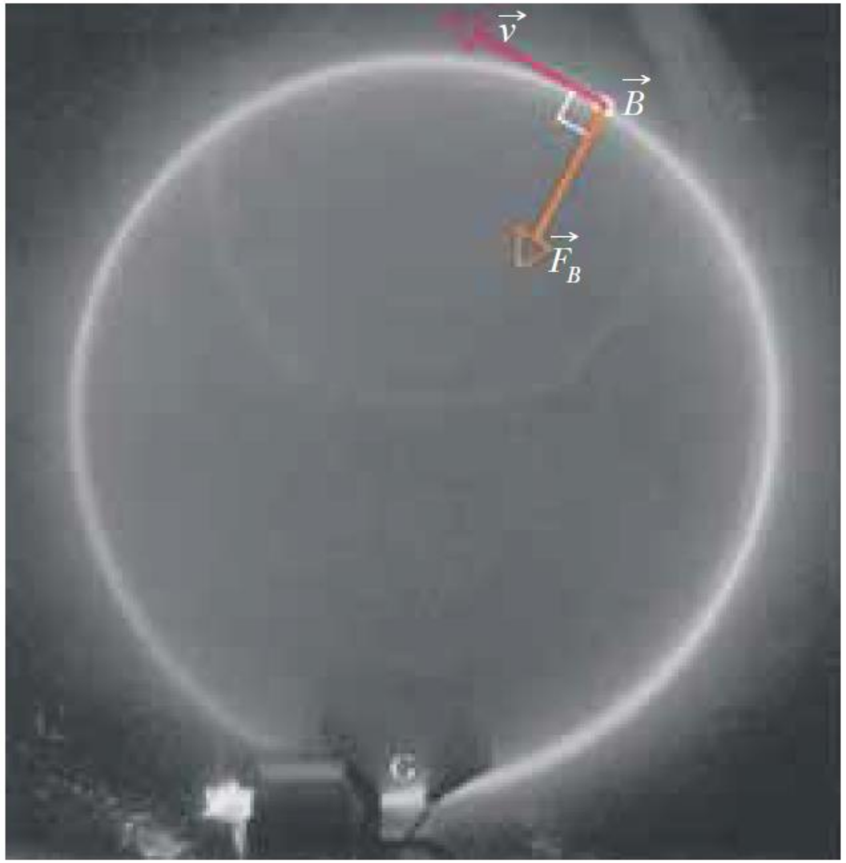
<u>Circulating Charged Particles</u>

Let us determine the parameters that characterize the circular motion of any particle of charge $|q|$ and mass m, perpendicular to a uniform magnetic field $\vec{B}$ at speed v. Newton's 2nd law ($\vec{F} = m\vec{a}$) applied to uniform circular motion reads

$$F = m\frac{v^2}{r}.$$

With $F = F_B = |q|mv$, we obtains

$$|q|vB = \frac{mv^2}{r}.$$



4. A Circulating Charged Particle

The radius of the circular path is therefore

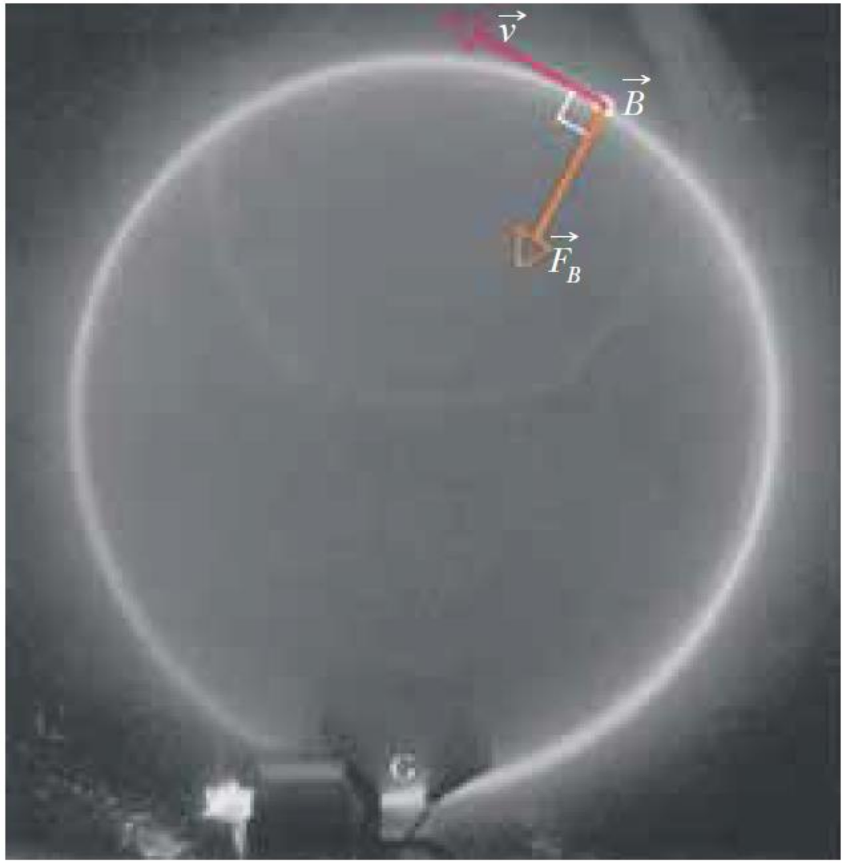
$$r = \frac{mv}{|q|B}.$$

We can write the period T as

$$T = \frac{2\pi r}{v} = \frac{2\pi}{v}\frac{mv}{|q|B} = \frac{2\pi m}{|q|B}.$$

The frequency of the motion is then

$$f = \frac{1}{T} = \frac{|q|B}{2\pi m}.$$

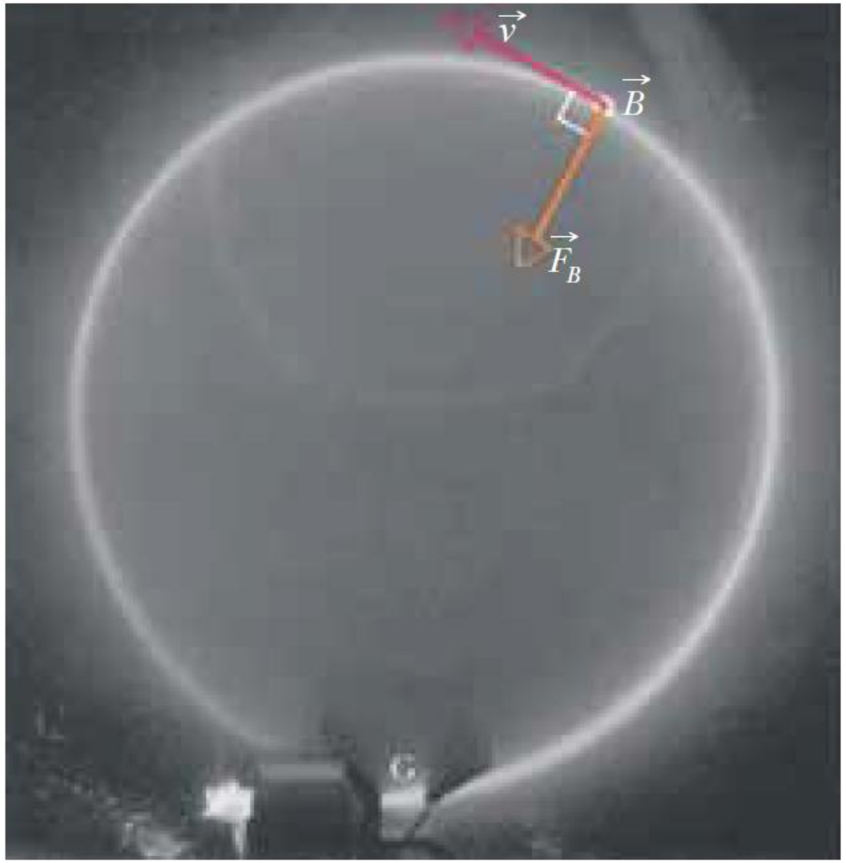


4. A Circulating Charged Particle

The angular frequency of the motion is

$$\omega = 2\pi f = \frac{|q|B}{m}.$$

Note that T, f and ω do not depend on v.



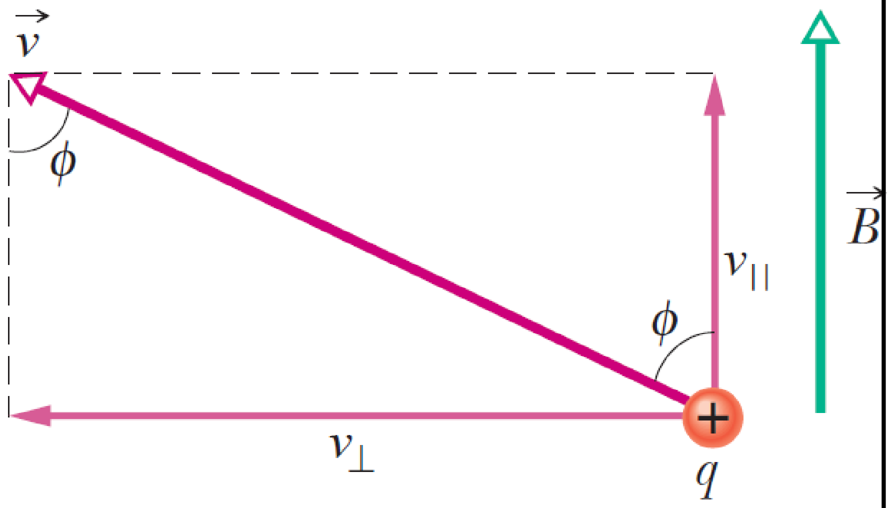
4. A Circulating Charged Particle

<u>Helical Paths</u>

If the velocity of a charged particle has a component parallel to the magnetic field, the particle will move in a helical path about the direction of the field vector.

The velocity vector $\vec{v}$ of such a particle can be resolved into two components, one parallel to $\vec{B}$ and one perpendicular to it:

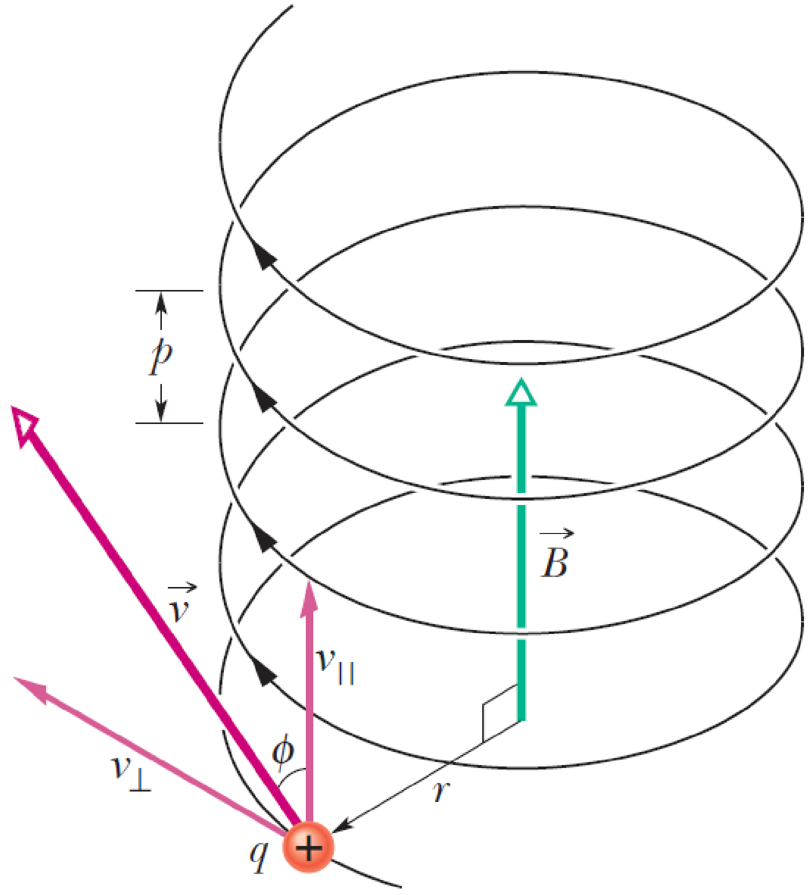
$$v_{\parallel} = v\cos\phi \quad \text{and} \quad v_{\perp} = v\sin\phi\,.$$



4. A Circulating Charged Particle

Helical Paths

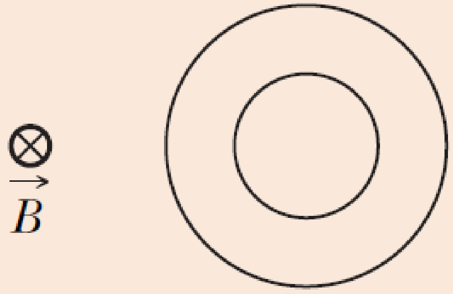
The parallel component $v_{\parallel}$ determines the pitch p of the helix (the distance between two adjacent turns). The perpendicular component $v_{\perp}$ determines the radius of the helix; $v_{\perp}$ replaces v in all the equations in the previous subsection.



4. A Circulating Charged Particle

CHECKPOINT 3

The figure here shows the circular paths of two particles that travel at the same speed in a uniform magnetic field $\vec{B}$, which is directed into the page. One particle is a proton; the other is an electron (which is less massive). (a) Which particle follows the smaller circle, and (b) does that particle travel clockwise or counterclockwise?



a) The electron.

b) Clockwise.

$$r = \frac{mv}{|q|B}$$

4. A Circulating Charged Particle

Example 3: An electron with a kinetic energy of $22.5\ \mathrm{eV}$ moves into a region of uniform magnetic field $\vec{B}$ of magnitude $4.55 \times 10^{-4}\ \mathrm{T}$. The angle between the directions of $\vec{B}$ and the electron's velocity $\vec{v}$ is $65.5°$. What is the pitch of the helical path taken by the electron?

The pitch p is the distance the electron travels parallel to the magnetic field $\vec{B}$ during one period T of circulation. Thus,

$$p = v_{\parallel} T = (v \cos \phi)\left(\frac{2\pi m}{|q|B}\right).$$

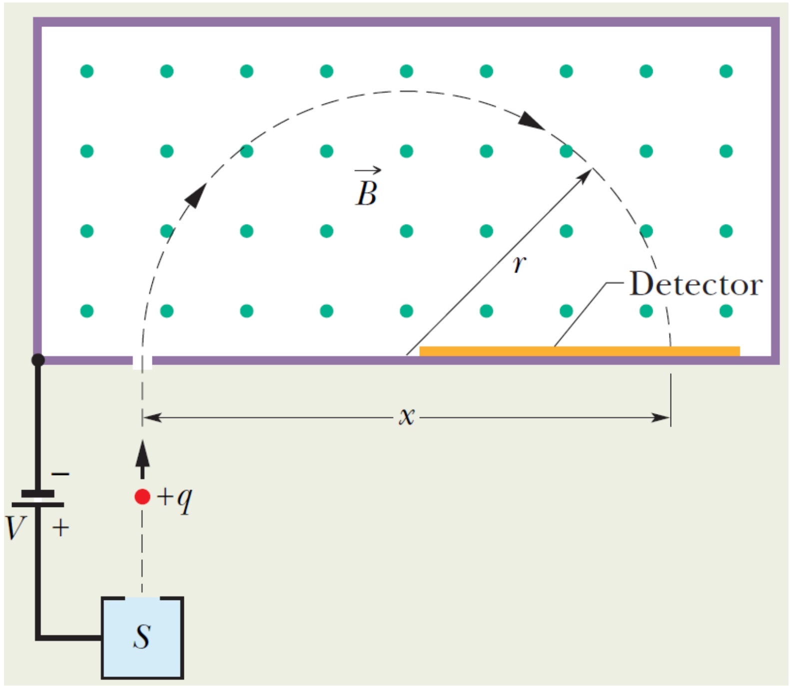
$$v = \sqrt{2K/m} = 2.81 \times 10^{6}\ \mathrm{m/s}.$$

Thus,

$$p = \left(2.81 \times 10^{6}\,\frac{\mathrm{m}}{\mathrm{s}}\right)(\cos 65.5°)\left[\frac{2\pi(9.11 \times 10^{-31}\ \mathrm{kg})}{(1.60 \times 10^{-19}\ \mathrm{C})(4.55 \times 10^{-4}\ \mathrm{T})}\right] = 9.16\ \mathrm{cm}.$$

4. A Circulating Charged Particle

Example 4: The figure shows the essentials of a *mass spectrometer,* which can be used to measure the mass of an ion; an ion of mass m (to be measured) and charge q is produced in source S. The initially stationary ion is accelerated by the electric field due to a potential difference V. The ion leaves S and enters a separator chamber in which a uniform magnetic field is perpendicular to the path of the ion. A wide detector lines the bottom wall of the chamber, and the $\vec{B}$ causes the ion to move in a semicircle and thus strike the detector. Suppose that $B = 80.000\ \mathrm{mT}$, $V = 1000.0\ \mathrm{V}$, and ions of charge $q = +1.6022 \times 10^{-19}\ \mathrm{C}$ strike the detector at a point that lies at $x = 1.6254\ \mathrm{m}$. What is the mass m of the individual ions, in atomic mass units ($1\ \mathrm{u} = 1.6605 \times 10^{-27}\ \mathrm{kg}$)?



4. A Circulating Charged Particle

The speed of an ion as it reaches the chamber is given by

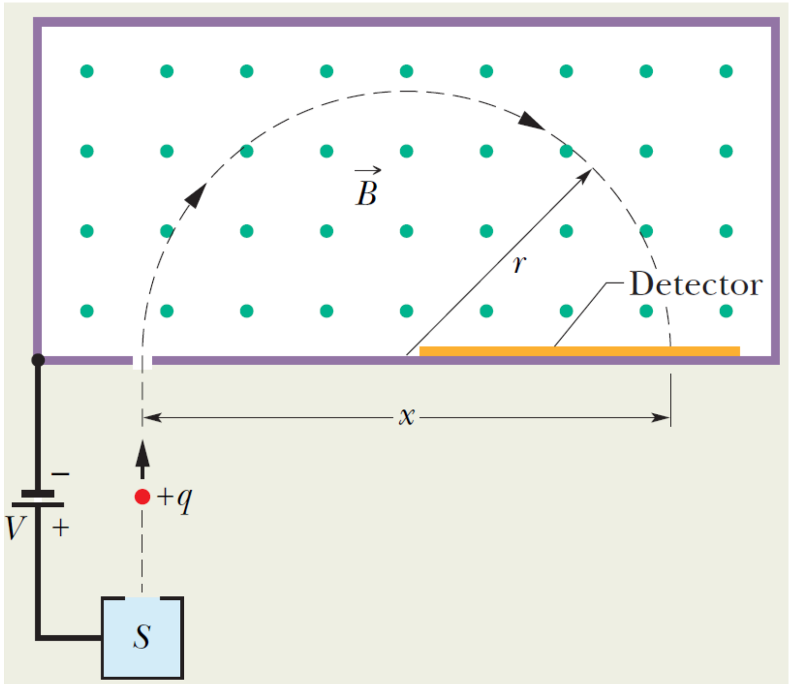
$$\frac{1}{2}mv^2 = qV,$$

which gives

$$v = \sqrt{\frac{2qV}{m}}.$$

The ion's mass m and the radius of the circular path $r = x/2$ are related by

$$r = \frac{mv}{qB} = \frac{m}{qB}\sqrt{\frac{2qV}{m}} = \frac{1}{B}\sqrt{\frac{2mV}{q}}.$$



4. A Circulating Charged Particle

Thus,

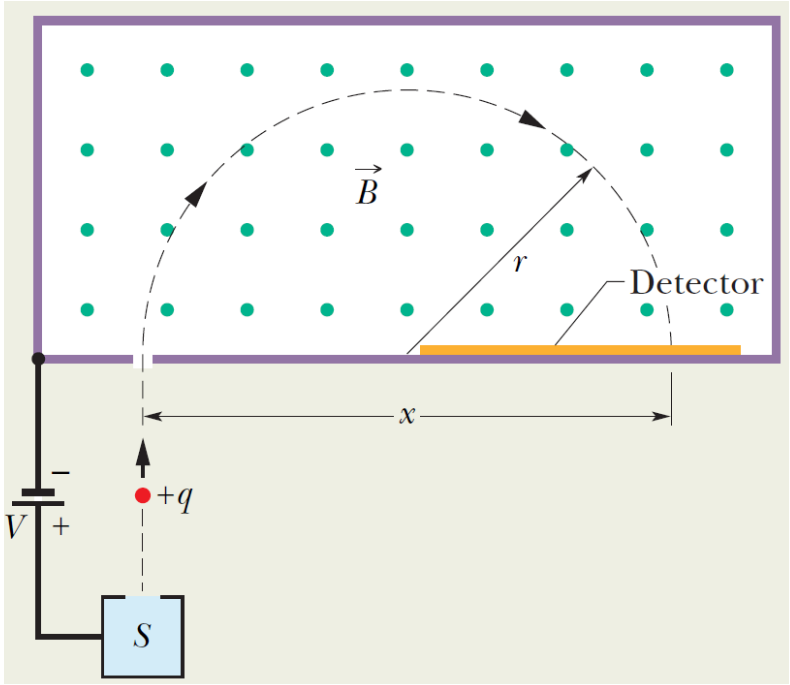
$$x = \frac{2}{B}\sqrt{\frac{2mV}{q}}.$$

Solving for m and substituting give

$$m = \frac{qB^2x^2}{8V}$$

$$= \frac{(1.6022 \times 10^{-19}\ \mathrm{C})(80.000\ \mathrm{mT})^2(1.6254\ \mathrm{m})^2}{8(1000.0\ \mathrm{V})}$$

$$= 3.3863 \times 10^{-25}\ \mathrm{kg} = 203.93\ \mathrm{u}.$$

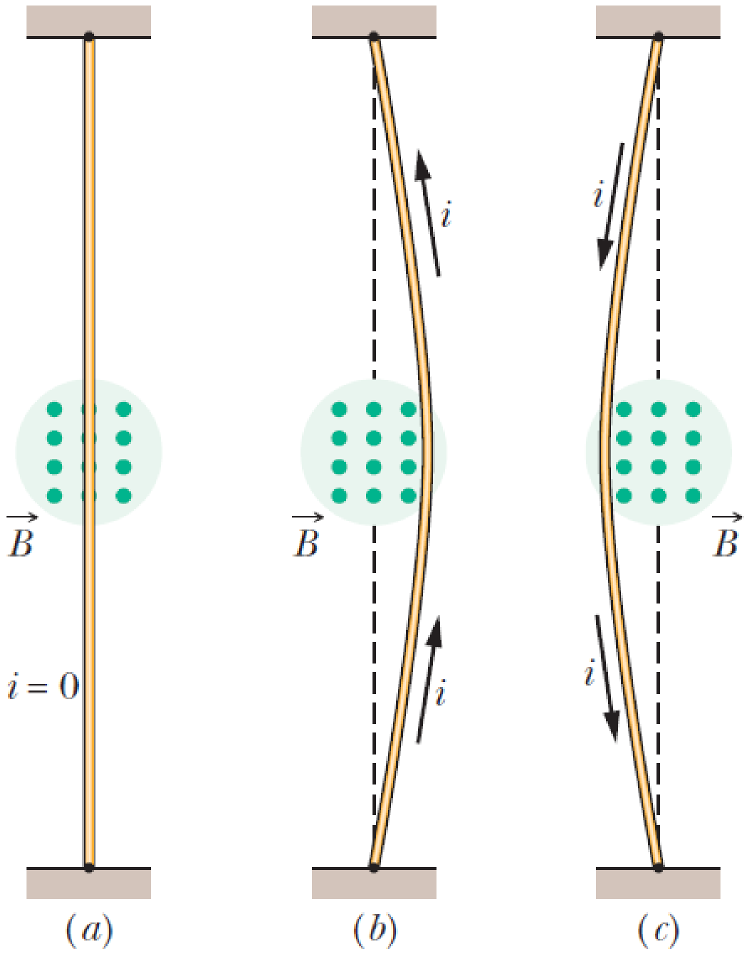


5. Magnetic Force on a Current-Carrying Wire

Figure (a) shows a vertical wire, carrying no current. A local magnetic field near the middle of the wire, directed out of the page, is shown.

In figure (b), a current is sent upward through the wire; the wire deflects to the right.

In figure (c), we reverse the current direction and the wire deflects to the left.



5. Magnetic Force on a Current-Carrying Wire

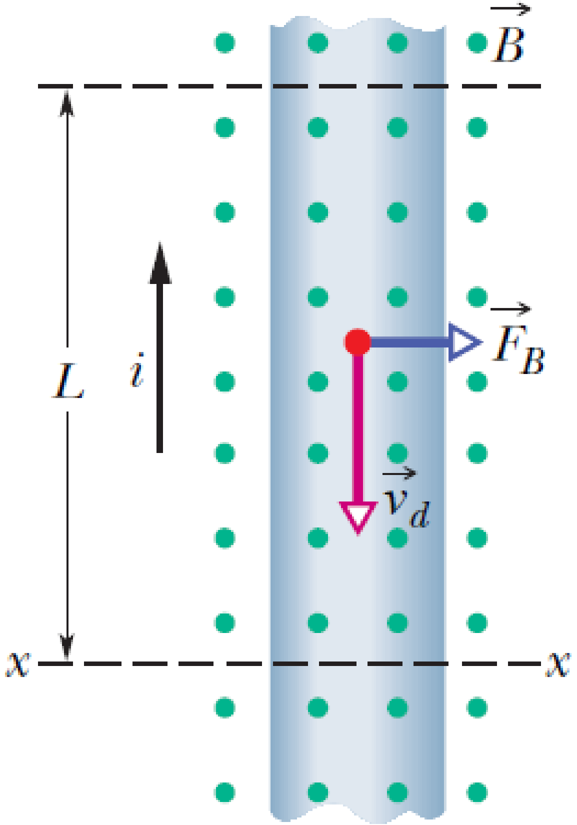
The figure shows what happens inside the wire.

To find the force on the wire, we consider a length L of the wire. All the conduction electrons in this section of the wire drift past plane xx in the figure in a time $t = L/v_d$. Thus, in that time a charge given by

$$q = it = i\frac{L}{v_d}$$

will pass through that plane. The magnetic force F_B is then

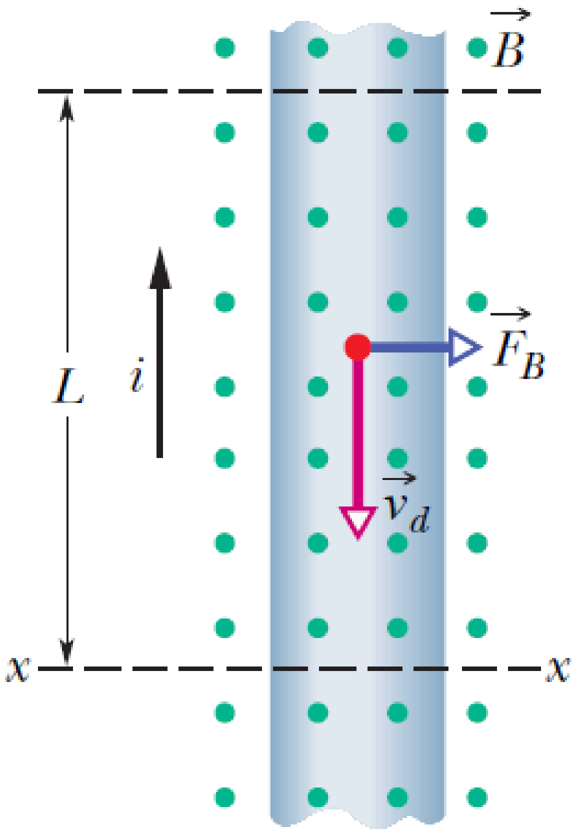
$$F_B = qvB = i\frac{L}{v_d}v_dB = iLB.$$



5. Magnetic Force on a Current-Carrying Wire

$$F_B = iLB.$$

This equation gives the magnetic force that acts on a length L of a straight wire carrying a current i and immersed in a uniform magnetic field $\vec{B}$ that is *perpendicular* to the wire.



5. Magnetic Force on a Current-Carrying Wire

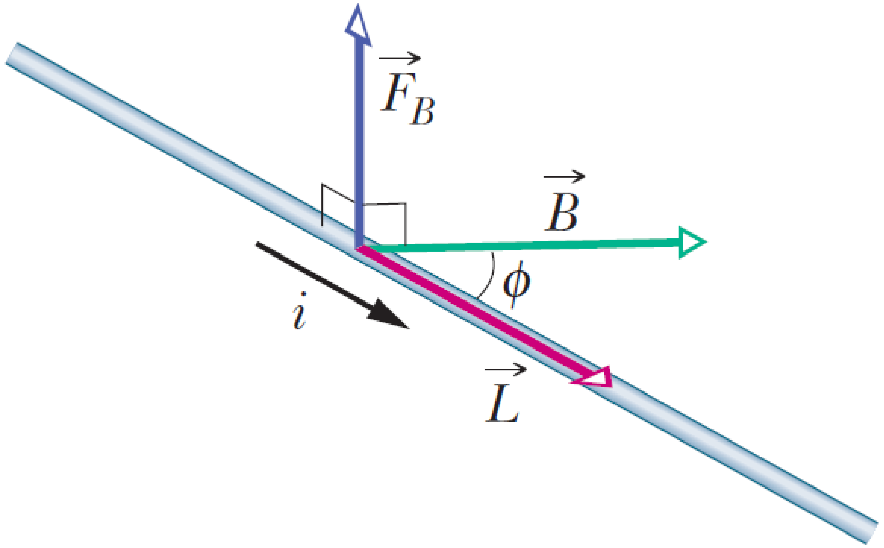
In general, the magnetic field is not necessarily perpendicular to the wire, and the magnetic force is given by the generalization

$$\vec{F}_B = i\vec{L} \times \vec{B},$$

where $\vec{L}$ is a length vector that has magnitude L and is directed along the wire segment. The force magnitude is

$$F_B = iLB \sin\phi\,.$$

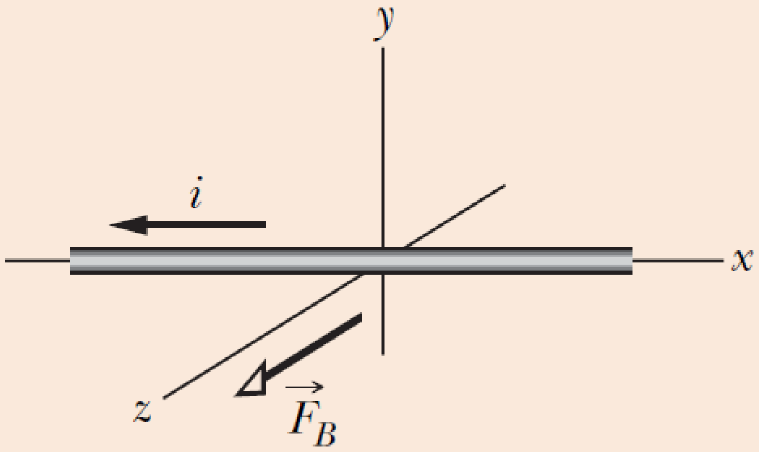
The direction of $\vec{F}_B$ is that of the cross product $\vec{L} \times \vec{B}$ since we take the current i to be positive.



5. Magnetic Force on a Current-Carrying Wire

CHECKPOINT 4

The figure shows a current i through a wire in a uniform magnetic field $\vec{B}$, as well as the magnetic force $\vec{F}_B$ acting on the wire. The field is oriented so that the force is maximum. In what direction is the field?



$-\hat{j}$ direction

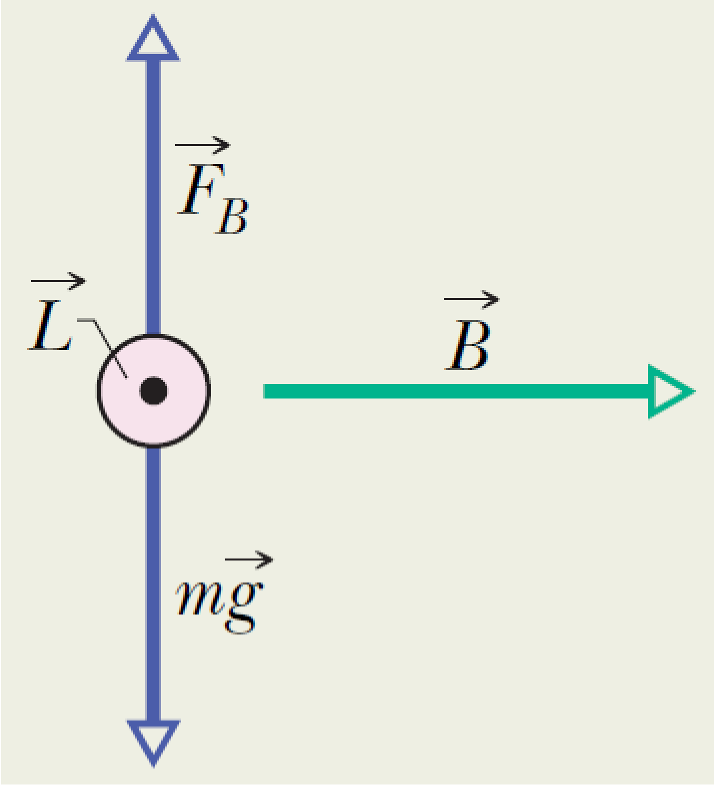
5. Magnetic Force on a Current-Carrying Wire

Example 5: A straight, horizontal length of copper wire has a current $i = 28\ \mathrm{A}$ through it. What are the magnitude and direction of the minimum magnetic field $\vec{B}$ needed to suspend the wire—that is, to balance the gravitational force on it? The linear density (mass per unit length) of the wire is $46.6\ \mathrm{g/m}$.

To have an upward magnetic force, the magnetic field direction must be as shown in the figure.

We want F_B to balance F_g. Thus,

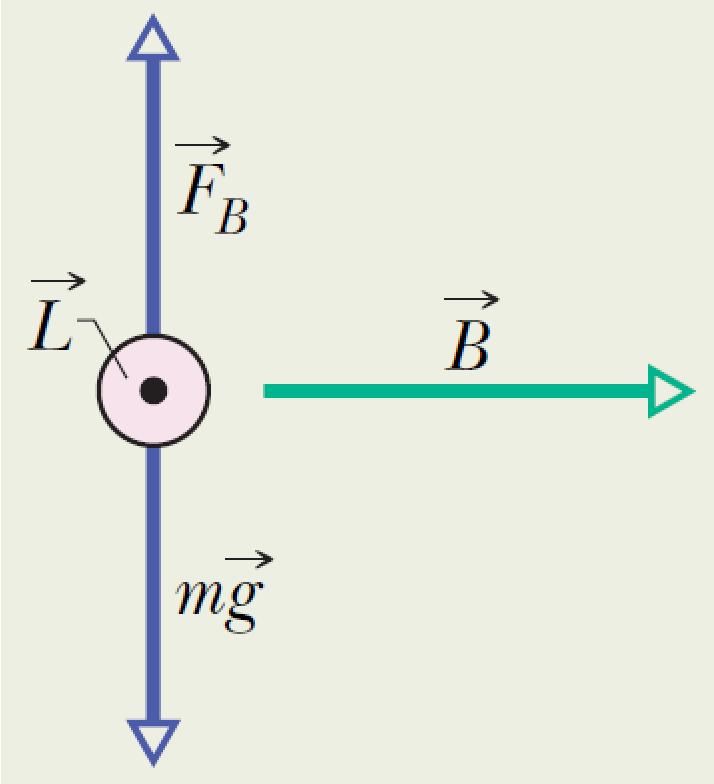
$$iLB = mg.$$



5. Magnetic Force on a Current-Carrying Wire

Solving for B and substituting give

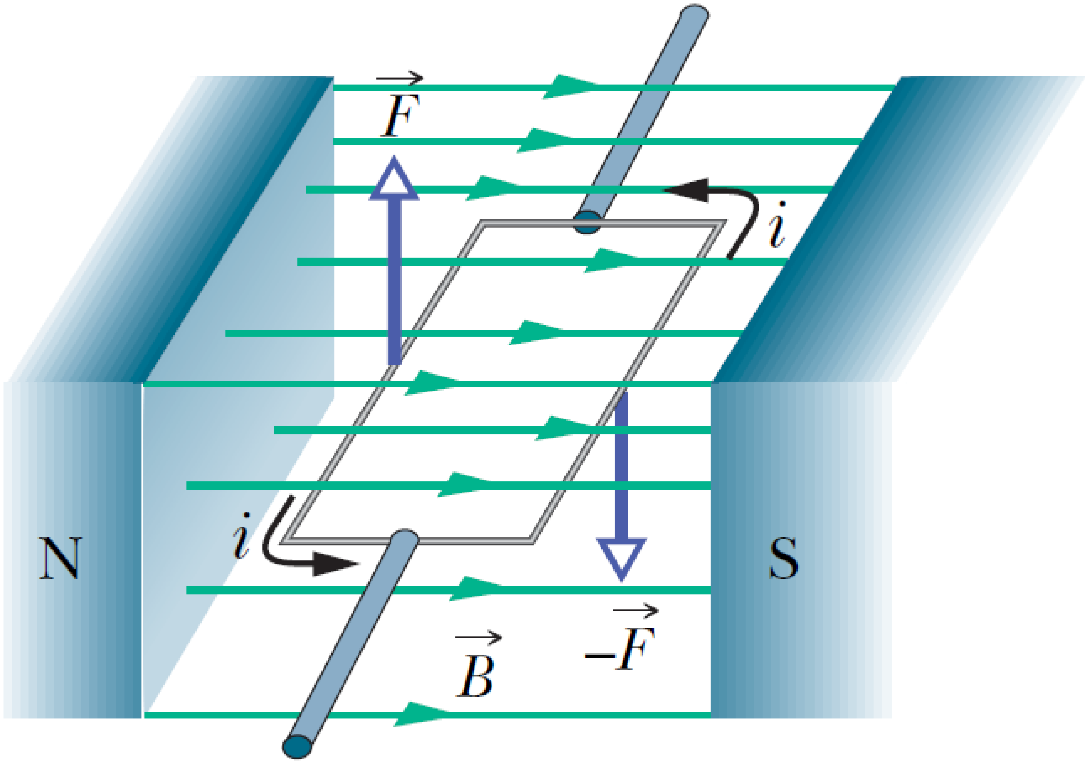
$$B = \frac{mg}{iL} = \frac{\mu g}{i} = \frac{(0.0466\ \mathrm{kg/m})(9.80\ \mathrm{m/s^2})}{28\ \mathrm{A}}$$
$$= 1.6 \times 10^{-2}\ \mathrm{T}.$$



6. Torque on a Current Loop

Current-carrying wires, immersed in magnetic fields, are at the heart of every electric motor.

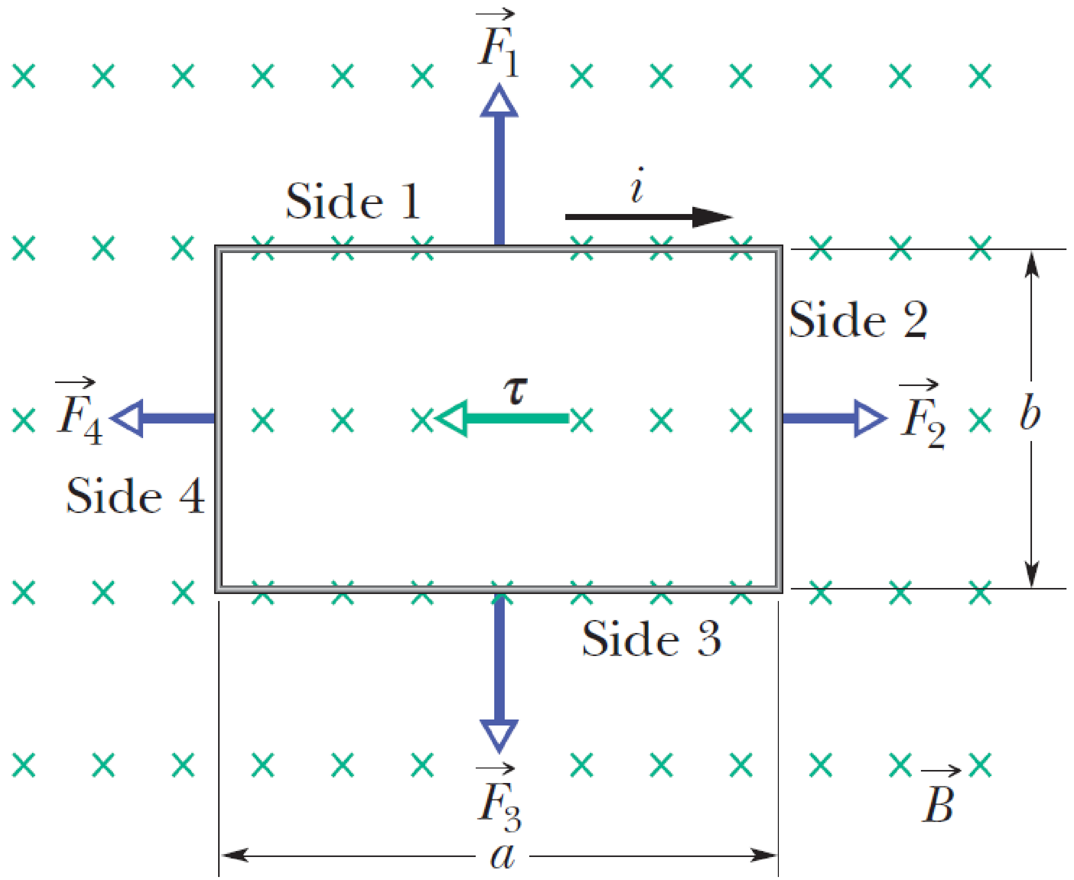
The figure shows a simple motor, which consists of a single current-carrying loop immersed in magnetic field $\vec{B}$. The two magnetic forces $\vec{F}$ and $-\vec{F}$ produce a torque on the loop, tending to rotate it about its central axis . Let us analyze that motion.



6. Torque on a Current Loop

The figure show a rectangular loop of sides a and b, carrying current i through a uniform magnetic field $\vec{B}$.

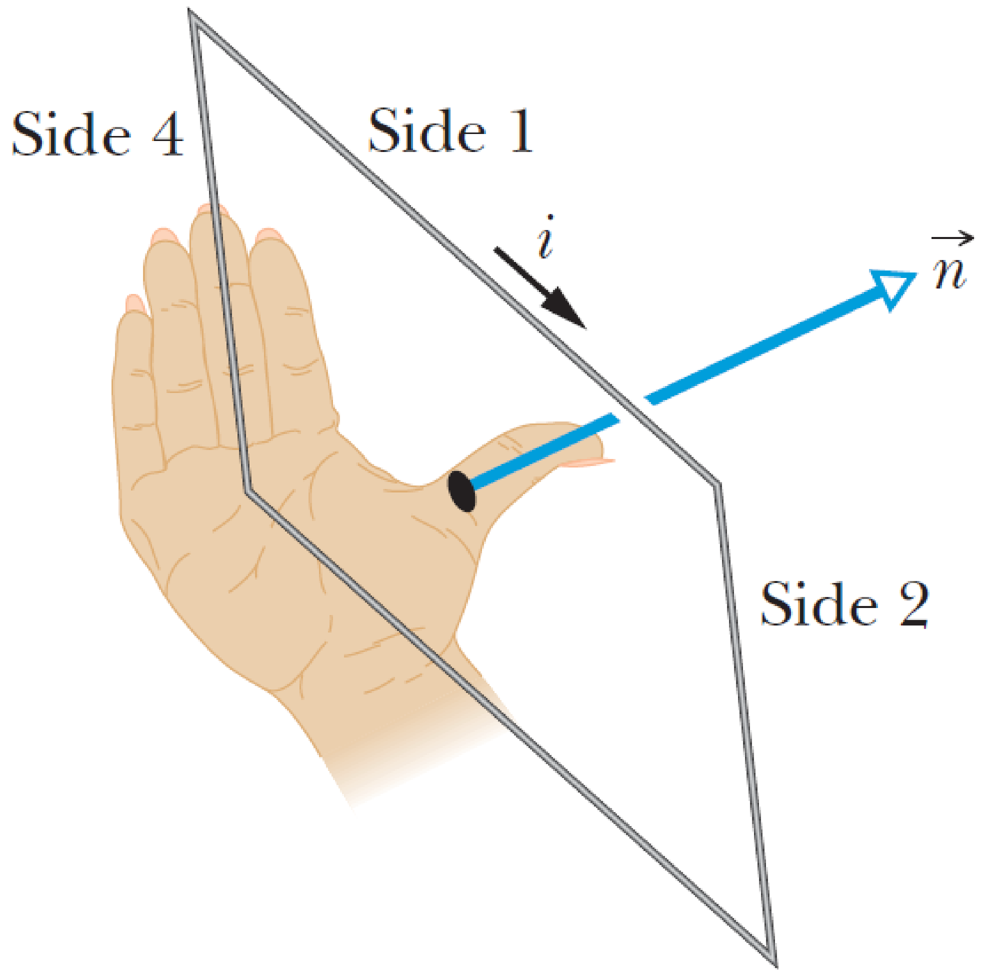
We place the loop in the field so that the long sides (1 and 3) are always perpendicular to $\vec{B}$, but the short sides (2 and 4) are not.



6. Torque on a Current Loop

We use a normal vector $\vec{n}$ that is perpendicular to the plane of the loop in order to define the orientation of the loop in the magnetic field.

Curl the fingers of your right hand in the direction of the current at any point on the loop. Your extended thumb then points in the direction of the normal vector $\vec{n}$.

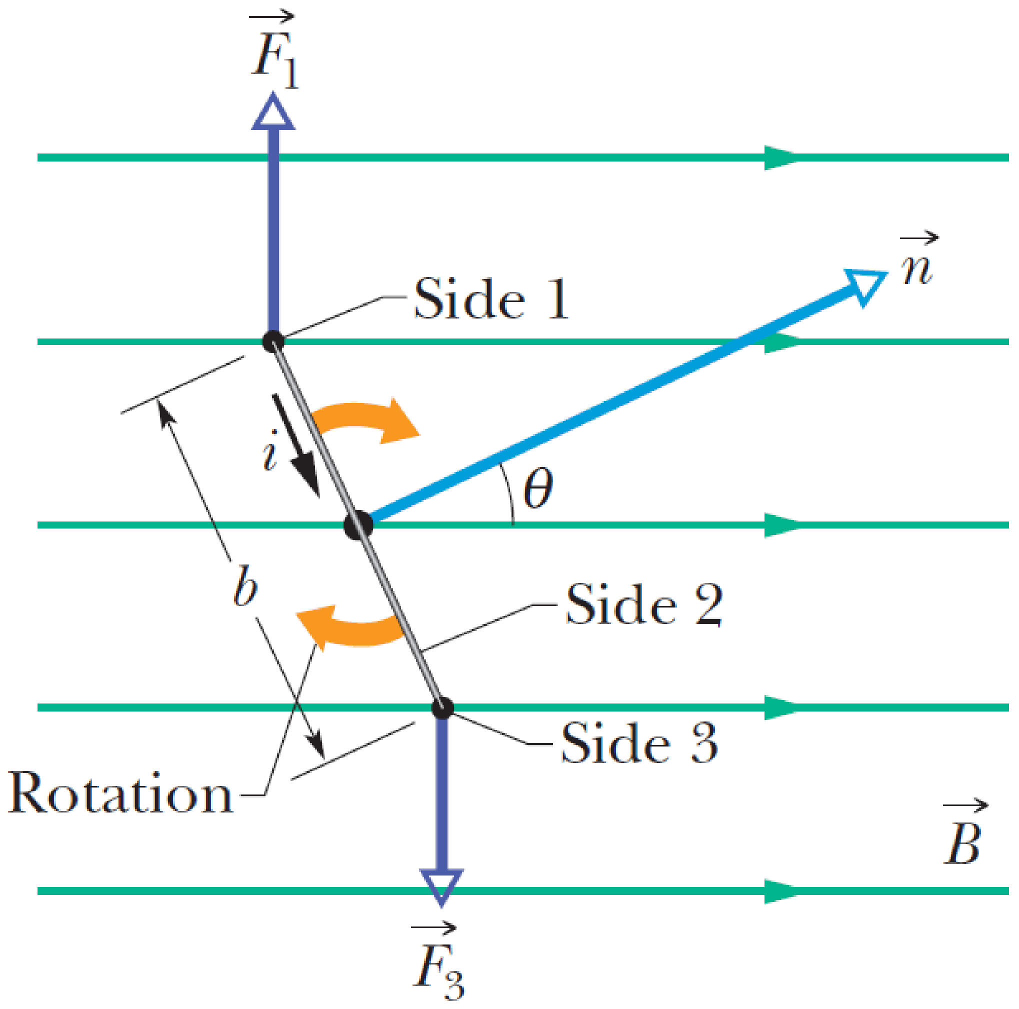


6. Torque on a Current Loop

In this figure, the normal vector $\vec{n}$ of the loop is shown at an arbitrary angle θ to the direction of $\vec{B}$.

We want to find the net force and net torque acting on the loop in this orientation.

The net force on the loop is the vector sum of the forces acting on its four sides.

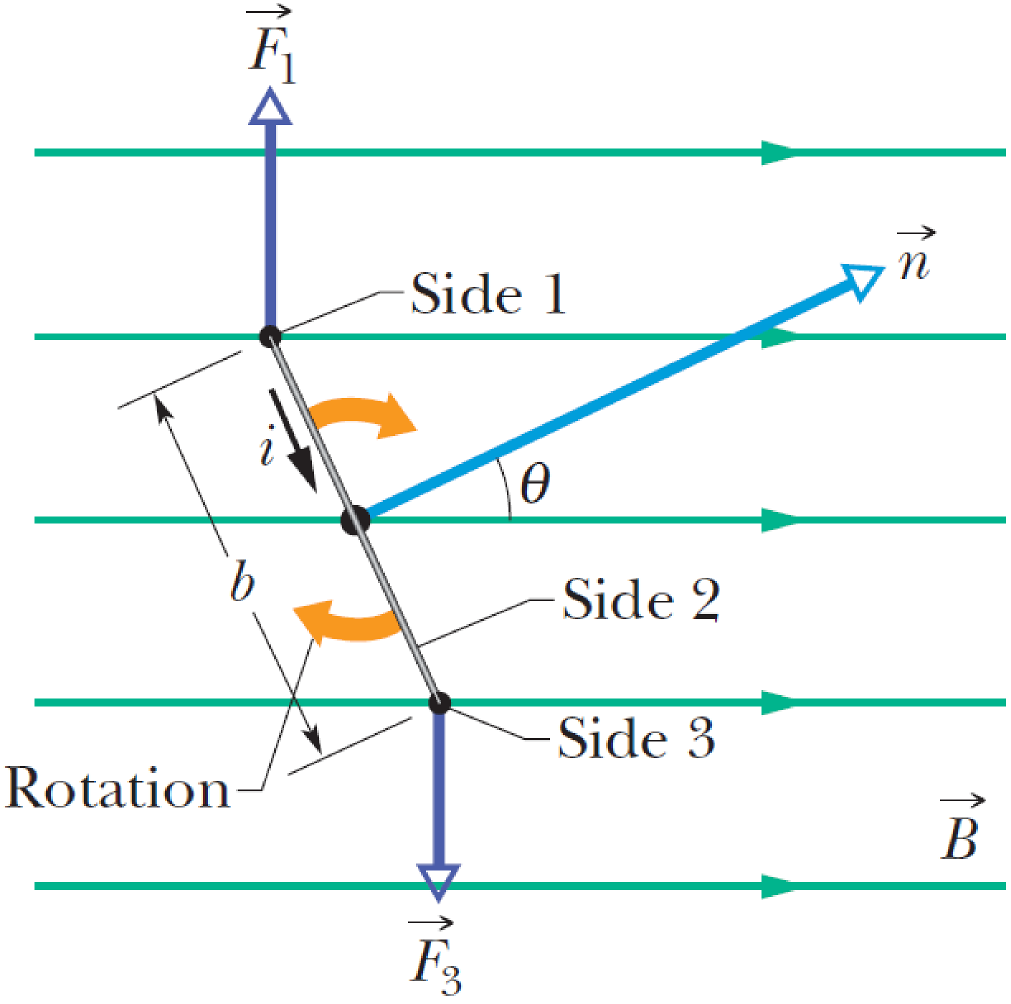


6. Torque on a Current Loop

For side 2, the vector $\vec{L}$ points in the direction of the current and has magnitude b. The angle between $\vec{L}$ and $\vec{B}$ for side 2 is $90° - \theta$. The magnitude of the force acting on this side is

$$F_2 = ibB\sin(90° - \theta) = ibB\cos\theta\,.$$

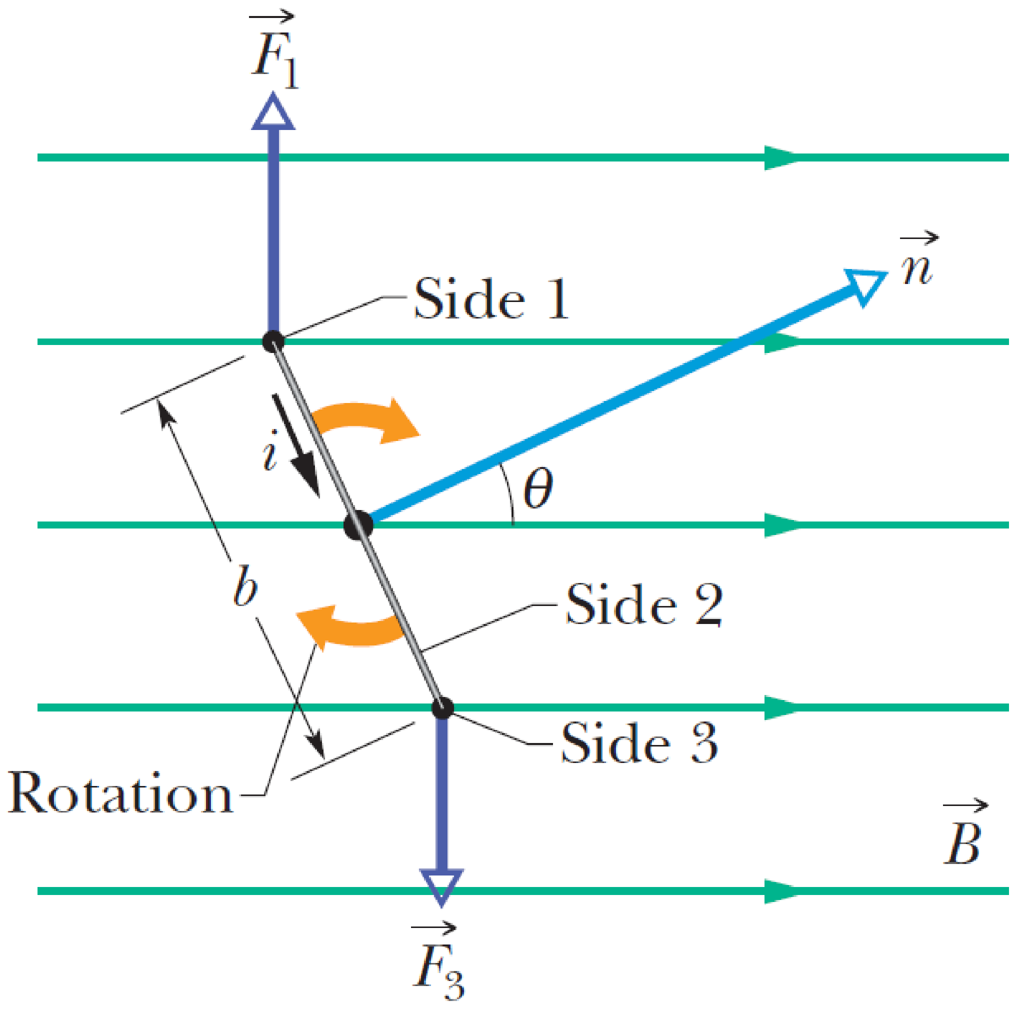
The force $\vec{F}_4$ acting on side 4 has the same magnitude as $\vec{F}_2$ but the opposite direction. Thus, $\vec{F}_2$ and $\vec{F}_4$ cancel out. Their net torque is zero too.



6. Torque on a Current Loop

For sides 1 and 3, $\vec{L}$ is perpendicular to $\vec{B}$. Therefore, the forces $\vec{F}_1$ and $\vec{F}_3$ have the common magnitude iaB.

The forces do not tend to move the loop up or down, since they have opposite directions. However, the two forces do not share the same line of action and thus produce a net torque. The torque tends to rotate the loop so as to align its normal vector $\vec{n}$ with the direction of the magnetic field $\vec{B}$.

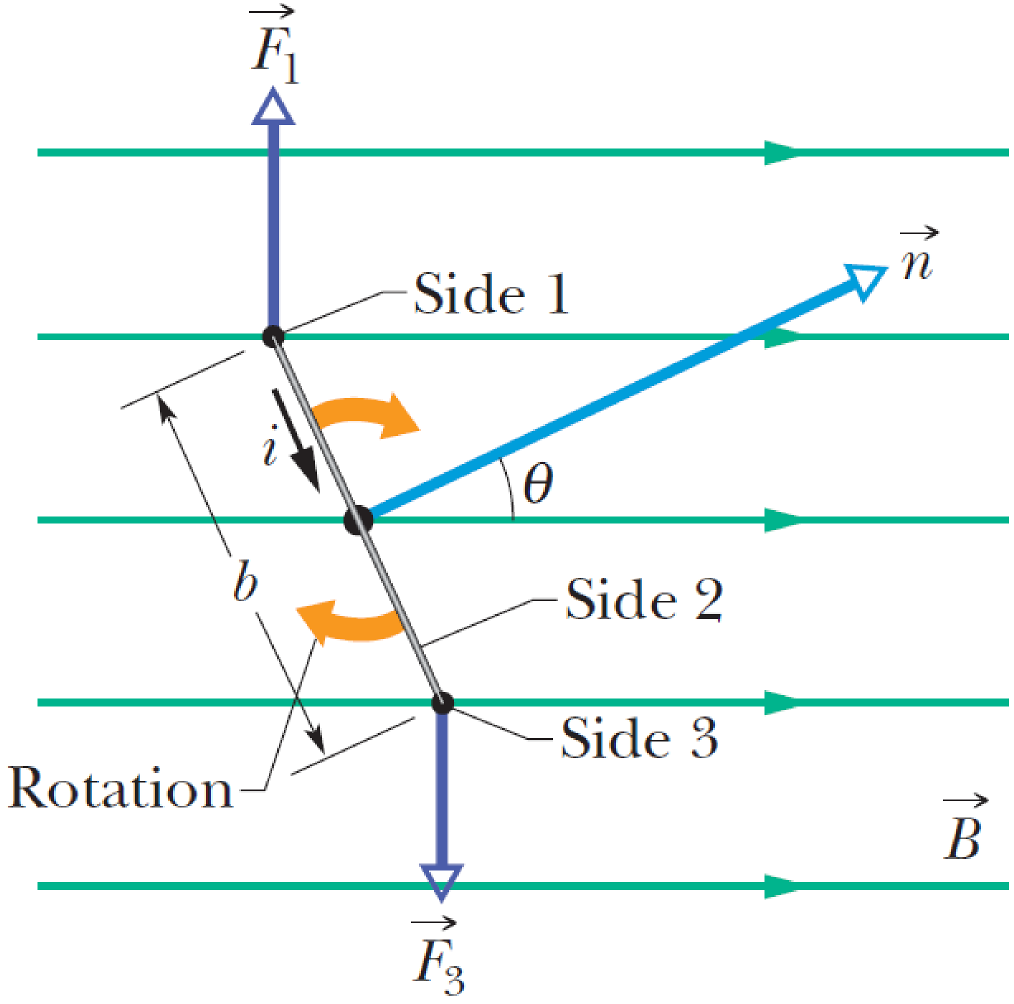


6. Torque on a Current Loop

The moment arm of that torque about the central axis is $(b/2)\sin\theta$.

The magnitude τ' of the torque due to forces $\vec{F}_1$ and $\vec{F}_3$ is then

$$\tau' = \left(iaB\frac{b}{2}\sin\theta\right) + \left(iaB\frac{b}{2}\sin\theta\right)$$
$$= iabB\sin\theta\,.$$



6. Torque on a Current Loop

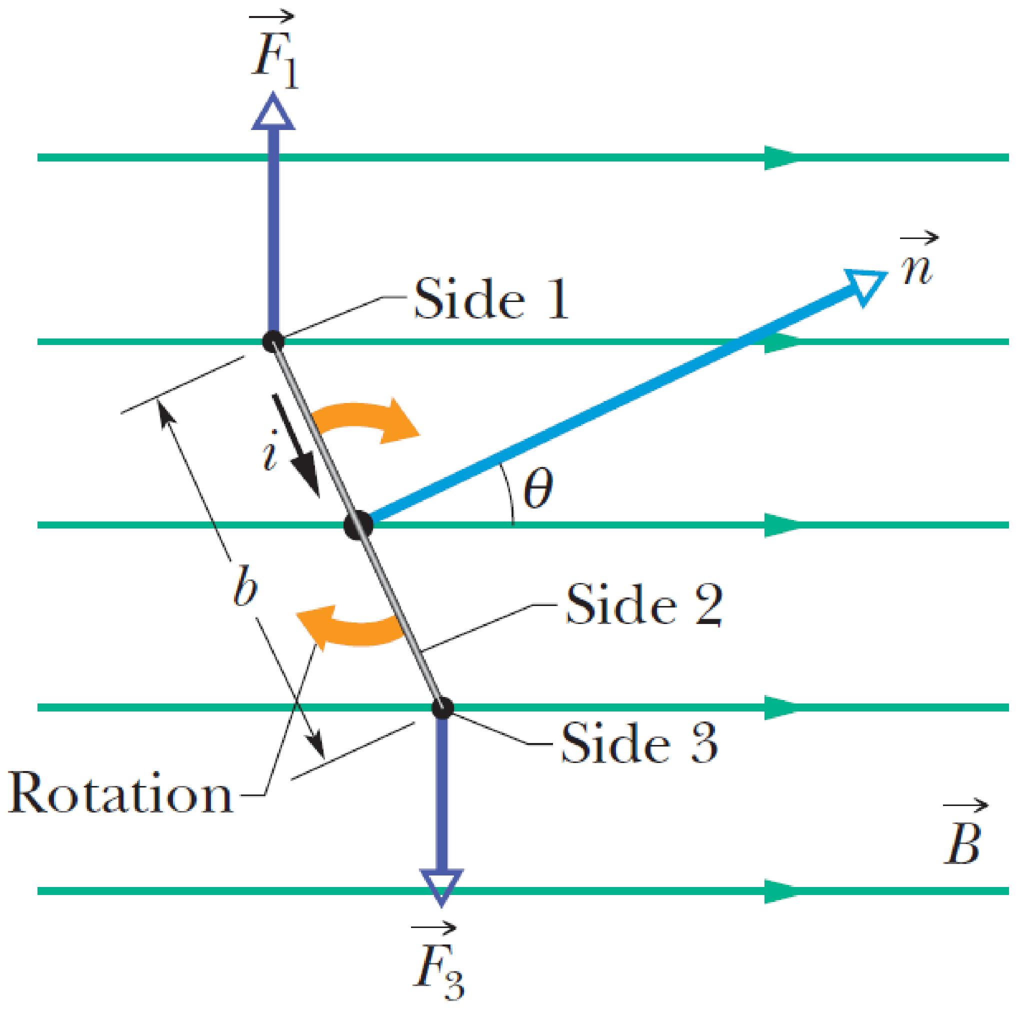
Suppose we replace the single loop with a coil of N loops, that are wound tightly enough that they can be a flat coil that they can be approximated as all having the same dimensions and lying in a plane.

The turns then form a flat coil, and a torque τ' acts on each turn.

The total torque on the coil has the magnitude

$$\tau = N\tau' = NiabB\sin\theta = (NiA)B\sin\theta\,,$$

where $A(= ab)$ is the area enclosed by the coil.



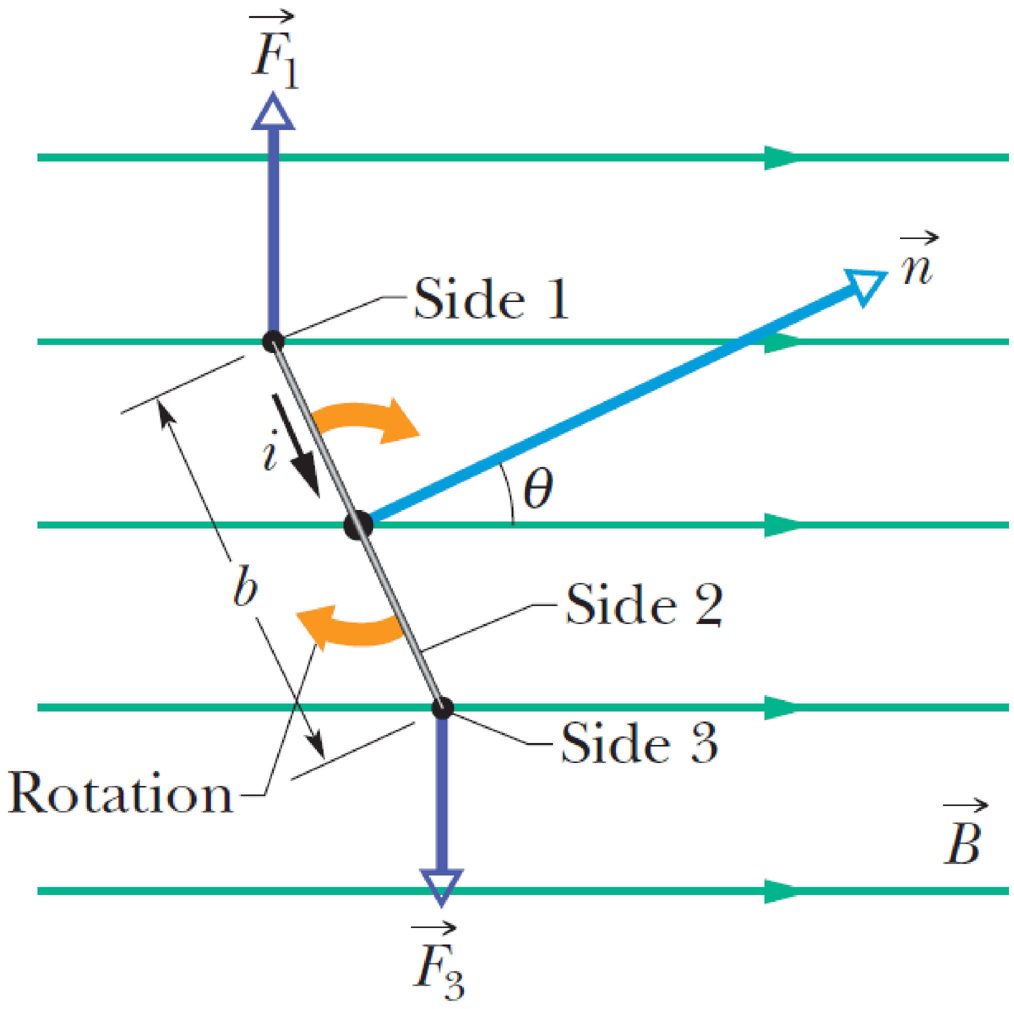
6. Torque on a Current Loop

The expression $\tau = (NiA)B\sin\theta$ is valid for a coil of any shape. For example, for a circular coil, of radius r,

$$\tau = (Ni\pi r^2)B\sin\theta\,.$$

A current-carrying flat coil placed in a magnetic field tends to rotate so that $\vec{n}$ has the same direction as that of $\vec{B}$.

In a motor, the current in the coil is reversed as $\vec{n}$ begins to line up with $\vec{B}$, so that the torque continues to rotate the coil.



7. The Magnetic Dipole Moment

We have just seen that a torque acts to rotate a current-carrying coil placed in a magnetic field. In this sense, the coil behaves like a bar magnet placed in the magnetic field. Thus, a current-carrying coil is said to be a **magnetic dipole**.

We assign a **magnetic dipole moment** $\vec{\mu}$ to account for the torque on the current-carrying loop due to the magnetic field. The direction of $\vec{\mu}$ is the same as that of $\vec{n}$. The magnitude of $\vec{\mu}$ is given by

$$\mu = NiA.$$

The SI unit of $\vec{\mu}$ is the ampere-square meter $(\mathrm{A} \cdot \mathrm{m}^2)$.

In terms of $\vec{\mu}$, we can rewrite the torque on the coil due to a magnetic field as

$$\tau = \mu B \sin\theta\,,$$

where θ is the angle between the vectors $\vec{\mu}$ and $\vec{B}$.

7. The Magnetic Dipole Moment

Generalizing to vector equation, we obtain

$$\vec{\tau} = \vec{\mu} \times \vec{B}.$$

This equation is analogous to that of the torque exerted by an electric field $\vec{E}$ on an electric dipole moment $\vec{p}$, namely,

$$\vec{\tau} = \vec{p} \times \vec{E}.$$

A magnetic dipole in an external magnetic field has an energy that depends on the dipole's orientation. For an electric dipole $\vec{p}$ in an electric field $\vec{E}$, we have shown that

$$U(\theta) = -\vec{p} \cdot \vec{E}.$$

7. The Magnetic Dipole Moment

Analogously, for a magnetic dipole $\vec{\mu}$ in an external magnetic field $\vec{B}$, the dipole's energy is

$$U(\theta) = -\vec{\mu} \cdot \vec{B}.$$

The energy of a magnetic dipole is minimum $(= -\mu B \cos 0° = -\mu B)$ when its dipole moment $\vec{\mu}$ is lined up with $\vec{B}$. The dipole's energy is maximum $(= -\mu B \cos 180° = +\mu B)$ when $\vec{\mu}$ is opposite to the field.

The unit of $\vec{\mu}$ can be written is joule per tesla $(\mathrm{J/T})$, instead of ampere-square meter $(\mathrm{A} \cdot \mathrm{m}^2)$.

7. The Magnetic Dipole Moment

If an applied torque rotated the magnetic dipole from an initial orientation θ_i to another orientation θ_f, then work W_a is done on the dipole by the applied torque. If the dipole is stationary before and after the change in its orientation, the work W_a is

$$W_a = U_f - U_i.$$

A magnetic dipole does not need to be a current-carrying loop. For example, a simple bar magnet, rotating charged sphere, Earth, electron, proton and neutron have magnetic dipole moments.

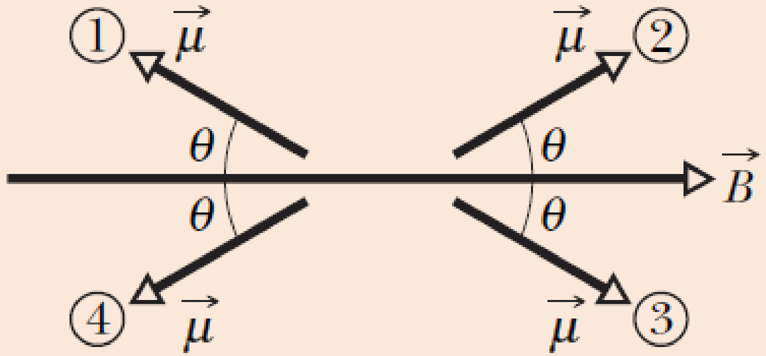
Some Magnetic Dipole Moments

Small bar magnet	5 J/T
Earth	8.0×10^{22} J/T
Proton	1.4×10^{-26} J/T
Electron	9.3×10^{-24} J/T

7. The Magnetic Dipole Moment

CHECKPOINT 5

The figure shows four orientations, at angle θ, of a magnetic dipole moment $\vec{\mu}$ in a magnetic field. Rank the orientations according to (a) the magnitude of the torque on the dipole and (b) the orientation energy of the dipole, greatest first.

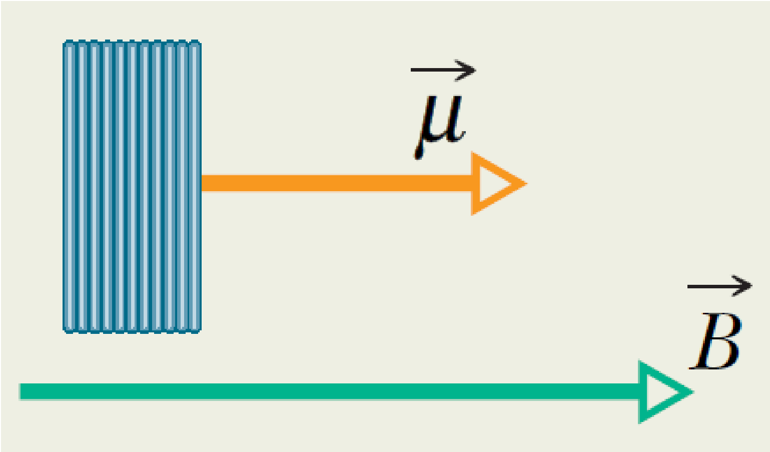


(a) All tie.

(b) 1 & 4, then 2 & 3.

7. The Magnetic Dipole Moment

Example 6: The figure shows a circular coil with 250 turns, an area $A = 2.52 \times 10^{-4}\ \mathrm{m}^2$, and a current of $100\ \mu\mathrm{A}$. The coil is at rest in a uniform magnetic field of magnitude $B = 0.85\ \mathrm{T}$, with its magnetic dipole moment $\vec{\mu}$ initially aligned with $\vec{B}$.

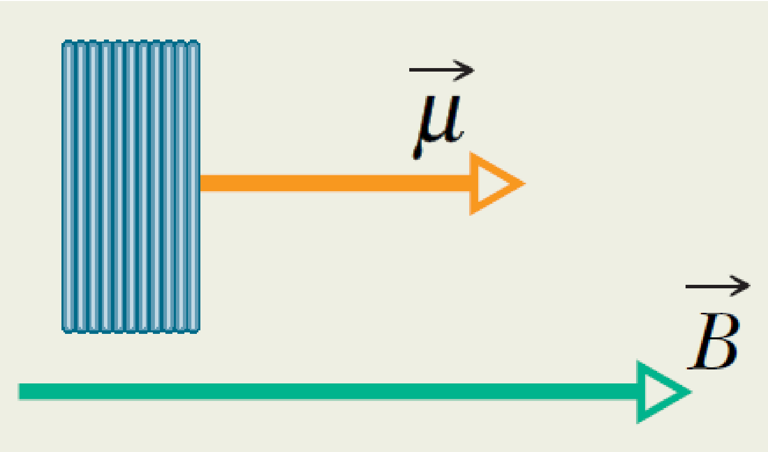


What is the direction of the current in the coil?

The current direction in the loop is counterclockwise, by the right hand loop.

7. The Magnetic Dipole Moment

(b) How much work would the torque applied by an external agent have to do on the coil to rotate it 90° from its initial orientation, so that $\vec{\mu}$ is perpendicular to $\vec{B}$ and the coil is again at rest?



$$W_a = U(90°) - U(0°) = -\mu B \cos 90° - (-\mu B \cos 0°)$$
$$= \mu B.$$

Using $\mu = NiA$, we have

$$W_a = \mu B = NiAB$$
$$= (250)(100\ \mu\text{A})(2.52 \times 10^{-4}\ \text{m}^2)(0.85\ \text{T})$$
$$= 5.4\ \mu\text{J}.$$