Chapter 27

Circuits

1. “Pumping” Chagres

We need to establish a potential difference between the ends of a device to make charge carriers follow through the device.

To generate a steady flow of charges, you need a “charge pump”; a device that maintains a potential difference between a pair of terminals. We call such a device an **emf device**, and the device is said to provide an emf $\mathcal{E}$.

Very common emf devices include the battery, electric generator and solar cells.

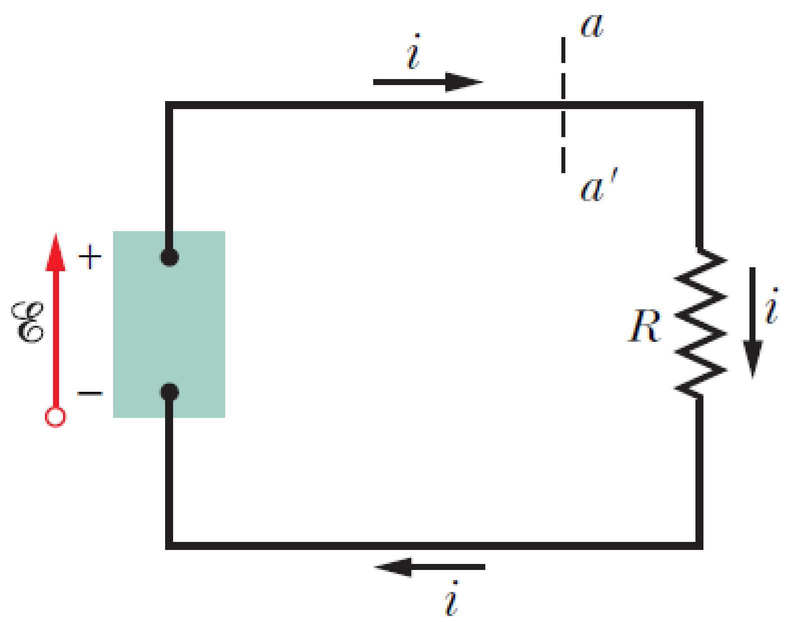
The term emf stands for electromotive force.

Previously, we discussed the motion of charge carriers in terms of the electric field set up in a circuit. In this chapter, we discuss the motion of charge carriers in terms of the required work.

2. Work, Energy, and Emf

The figure shows a simple circuit consisting of an emf device (battery) and a resistor R. The emf device keeps one of its terminals (the positive terminal $+$) at a higher electric potential than the other terminal (the negative terminal $-$). We represent the emf of the device with an arrow, with a small circle in the tail, that points from the negative terminal toward the positive terminal.

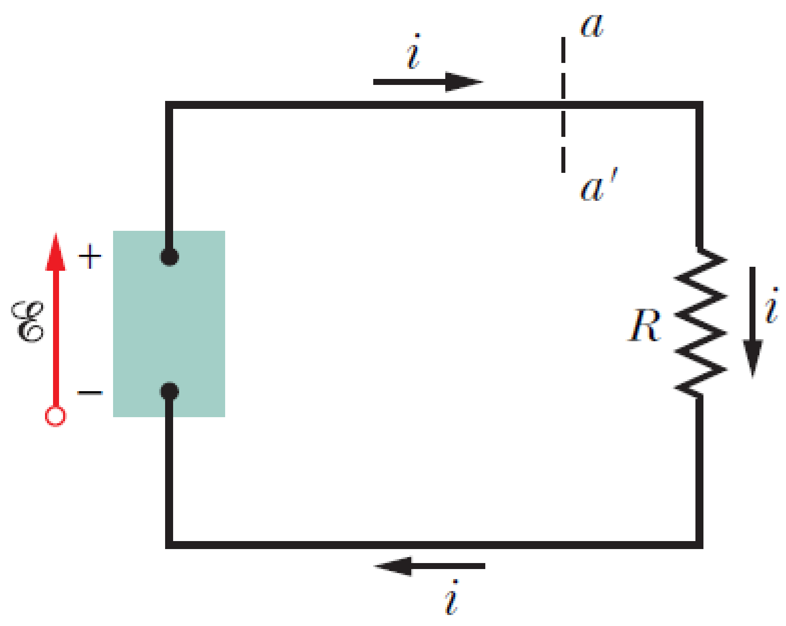
When an emf device is connected to a circuit, its internal chemistry causes a net flow of positive charge carriers from the negative terminal to the positive terminal in the direction of the emf arrow. This flow is part of the current that is set up around the circuit in that same direction.



2. Work, Energy, and Emf

Within the emf device, positive charge carriers move from a region of low electric potential to a region of higher electric potential. This motion is opposite to the electric field between the terminals!

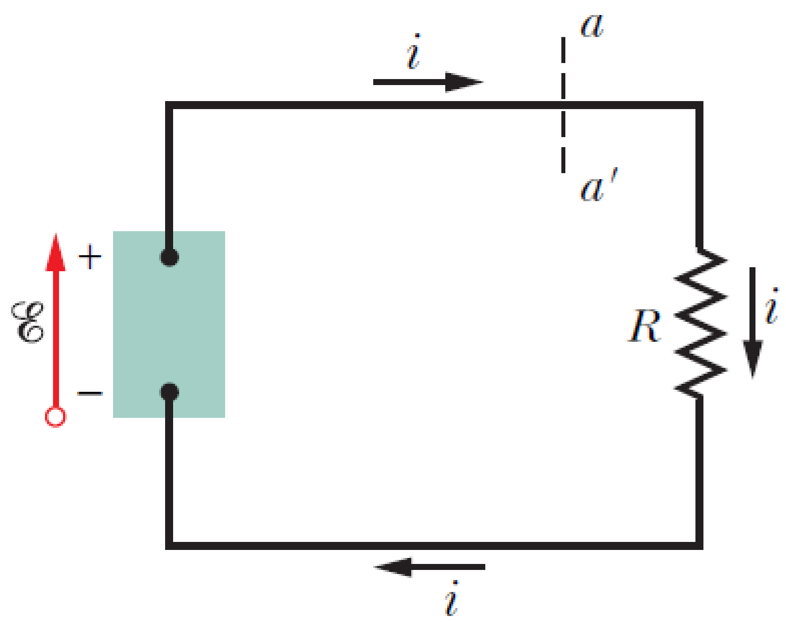
There must be some source of energy within that device that enables it to do work on the charges by forcing them to move the way they do. The source of energy may be chemical, mechanical, thermal, etc.



2. Work, Energy, and Emf

Here we analyze the circuit from the point of view of work and energy transfers. In a time interval dt, a charge dq passes through any cross section of this circuit. The same amount of charge must enter the emf device at its low potential end and leave at its high potential end. The device must do work dW on the charge dq to force it to move in this way. In terms of this work, the emf of the device is defined as

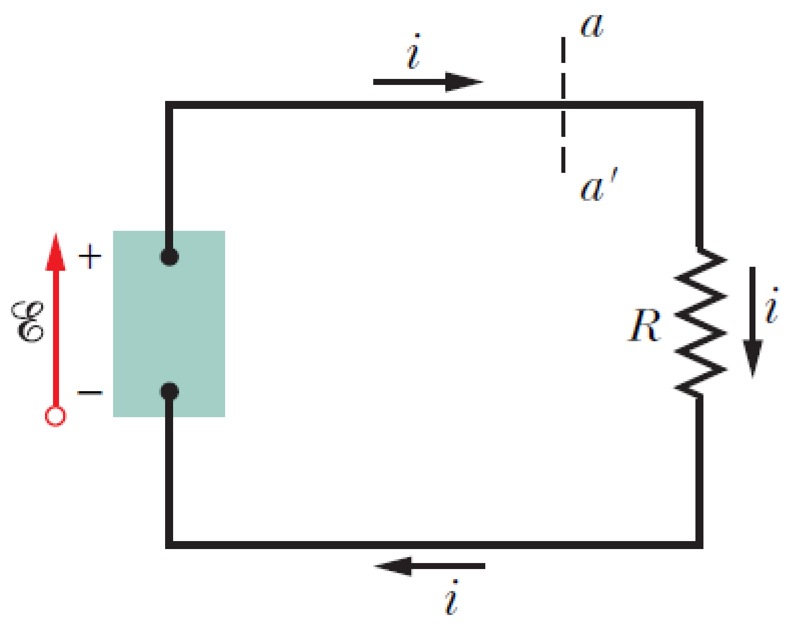
$$\mathcal{E} = \frac{dW}{dq}.$$



2. Work, Energy, and Emf

The emf of an emf device is the work per unit charge that the device does to move charge from its low potential terminal to its high potential terminal.

The SI unit of emf is joule per coulomb or volt.



2. Work, Energy, and Emf

An **ideal emf device** is one in which charge moves from terminal to terminal without any **internal resistance**. This makes the potential difference between the terminals of an ideal device equal to the emf of the device.

For example, an ideal battery with an emf of $12\ \mathrm{V}$ always has a potential difference of $12\ \mathrm{V}$ between its terminals.

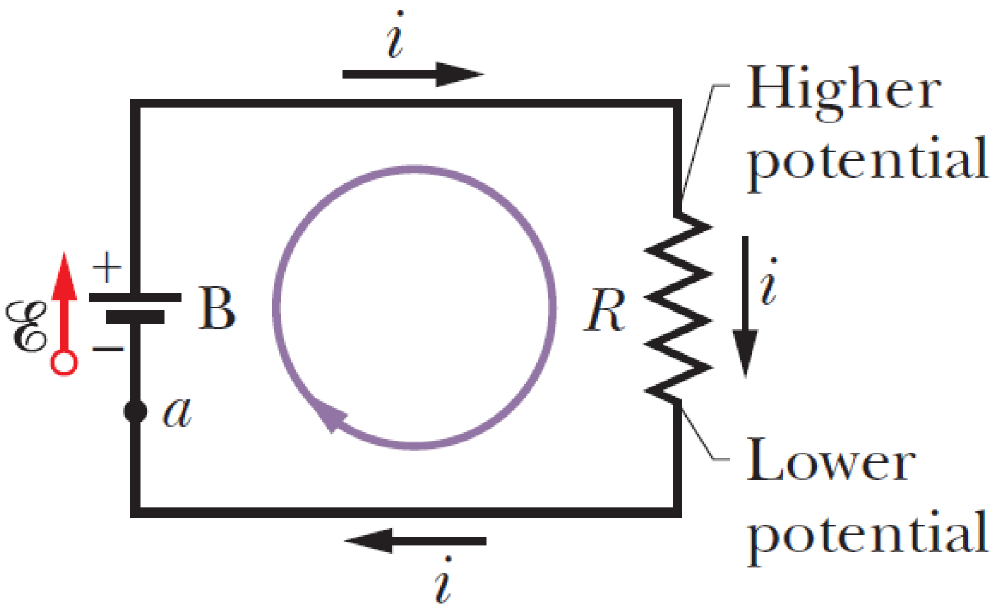
A **real emf device** has **internal resistance** to the internal movement of charges.

When an emf device is connected to a circuit, the device transfers energy to the charge carriers passing through it. The charge carriers then transfer this energy to the devices in the circuit.

3. Calculating current in a Single-Loop Circuit

We will study two ways to find the current in the simple single-loop circuit shown in the figure. The first method is based on energy conservation considerations. The second is based on the concept of potentials.

The circuit shown consists of an *ideal* battery with emf $\mathcal{E}$, a resistor of resistance R, and two connecting wires that have negligible resistances.



3. Calculating current in a Single-Loop Circuit

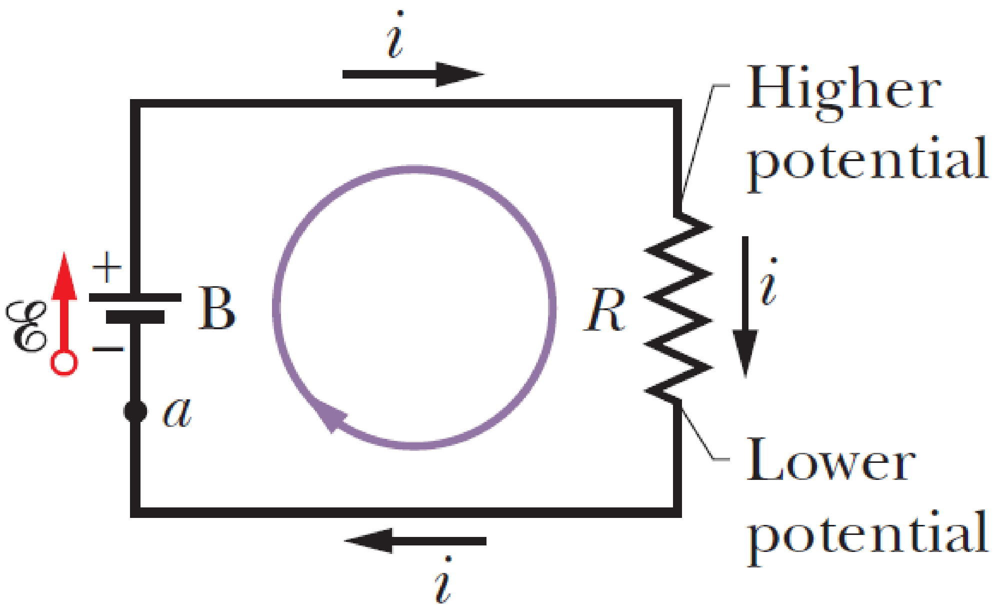
<u>Energy Method</u>

In a time interval dt an amount of energy given by $Pdt = i^2Rdt$ will be dissipated in the resistor. At the same time, a charge $dq = idt$ would have moved through the battery B, and the work that the battery would have done on this charge is

$$dW = \mathcal{E}dq = \mathcal{E}idt.$$

The work done by the ideal battery must equal the transferred energy dissipated in the resistor:

$$\mathcal{E}idt = i^2Rdt.$$



3. Calculating current in a Single-Loop Circuit

<u>Energy Method</u>

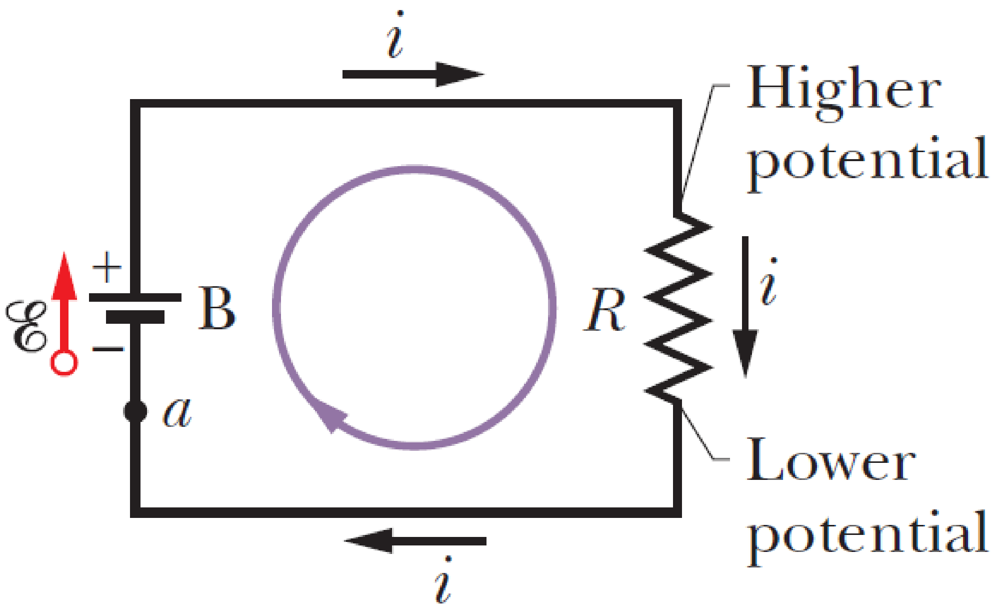
$$\mathcal{E} i dt = i^2 R dt,$$

which gives

$$\mathcal{E} = iR.$$

This equation tells us that the energy per unit charge transferred to the moving charges is equal to the energy per unit charge transferred from them. Solving for i gives

$$i = \frac{\mathcal{E}}{R}.$$



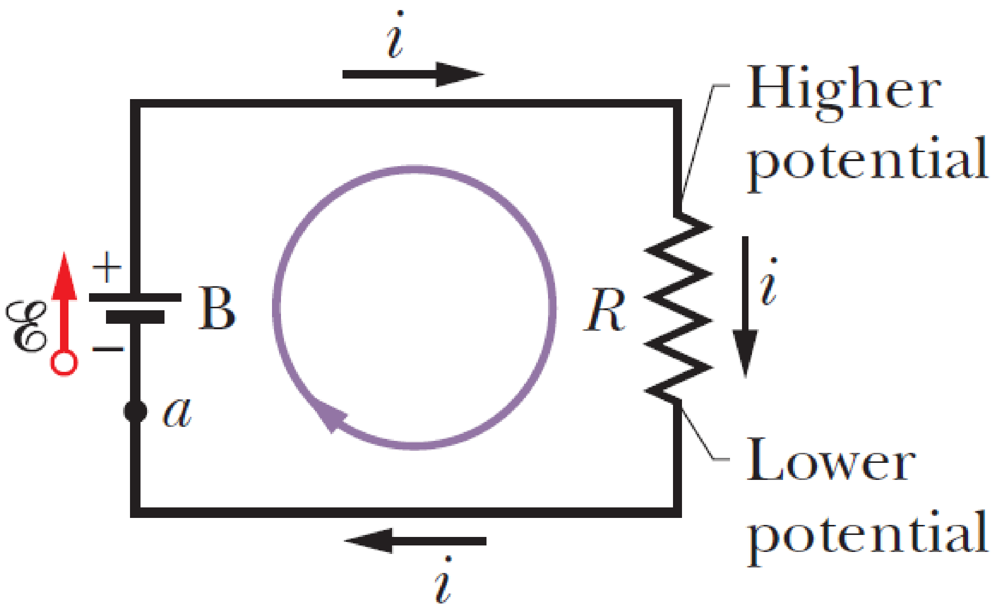
3. Calculating current in a Single-Loop Circuit

<u>Potential Method</u>

Suppose you start at any point in the circuit and proceed around it in either direction, adding the potential differences you encounter. When you return to the starting point, you must have returned to the starting potential.

LOOP RULE: The algebraic sum of the changes in potential encountered in a complete traversal of any loop of a circuit must be zero.

This is referred to as **Kirchhoff's loop rule**.

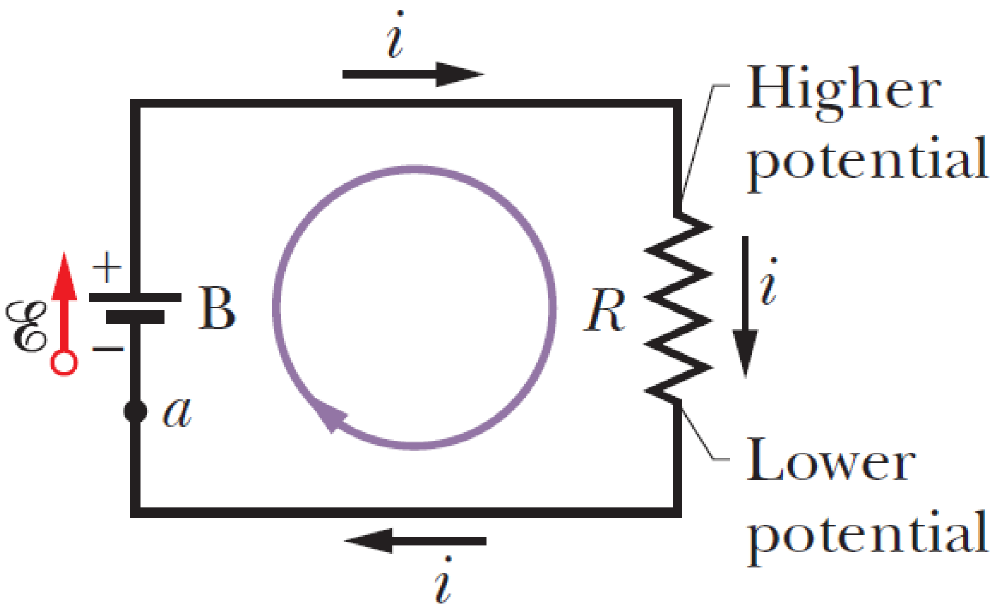


3. Calculating current in a Single-Loop Circuit

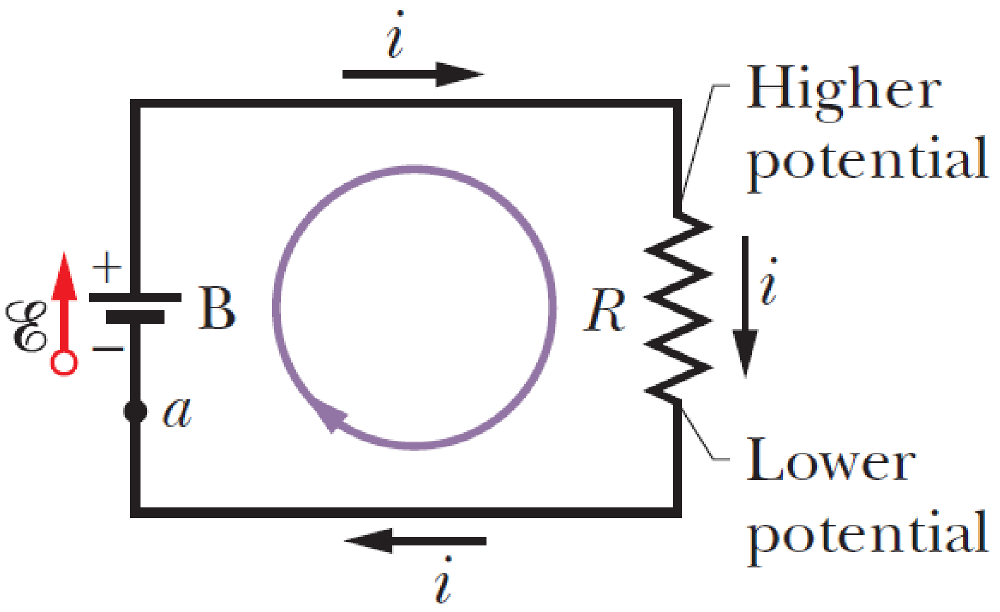
<u>Potential Method</u>

In the figure, we start at point a, whose potential is V_a and go clockwise around the circuit, back to point a. The potential change is $+\mathcal{E}$ as we pass through the battery to the high-potential terminal.

There is no potential change as we process through the wires. The potential changes by $V = iR$ as we proceed through the resistor. The potential decreases because we proceed from the high potential side of the resistor to the low potential side.



3. Calculating current in a Single-Loop Circuit



Potential Method

When we return to point a, the potential is again V_a.

Adding all potential changes we can write

$$V_a + \mathcal{E} - iR = V_a,$$

or

$$\mathcal{E} - iR = 0.$$

Solving for i gives $i = \mathcal{E}/R$.

3. Calculating current in a Single-Loop Circuit

<u>Potential Method</u>

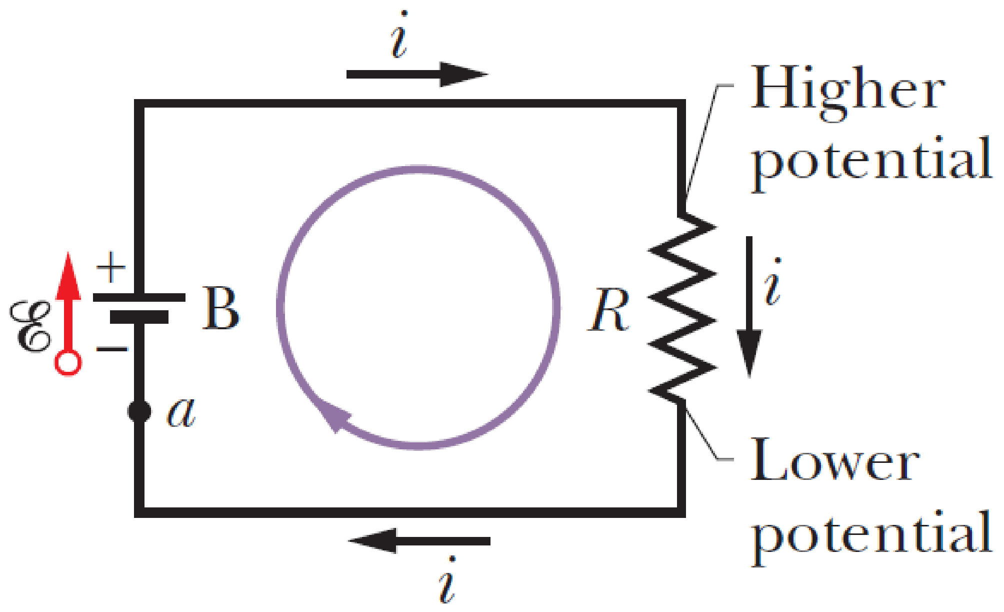
If we proceed around the circuit counterclockwise, get instead

$$V_a + iR - \mathcal{E} = V_a,$$

or

$$iR - \mathcal{E} = 0,$$

which gives the same result.

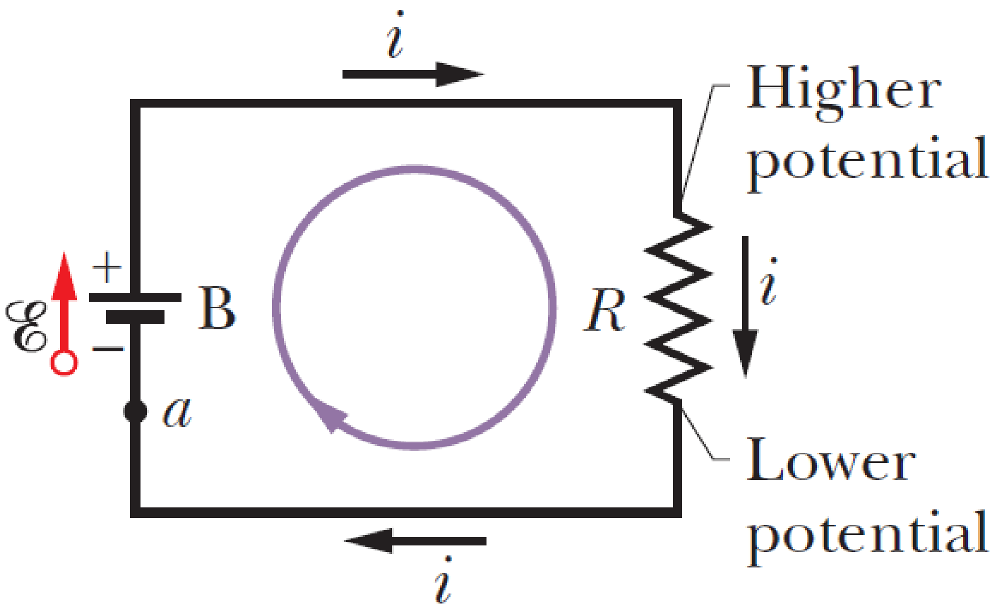


3. Calculating current in a Single-Loop Circuit

Potential Method

RESISTANCE RULE: For a move through a resistance in the direction of the current, the change in potential is $-iR$; in the opposite direction it is iR.

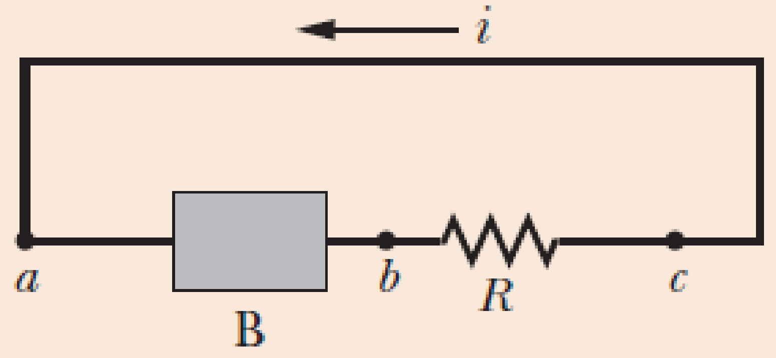
EMF RULE: For a move through an ideal emf device in the direction of the emf arrow, the change in potential is $+\mathcal{E}$; in the opposite direction it is $-\mathcal{E}$.



3. Calculating current in a Single-Loop Circuit

CHECKPOINT 1

The figure shows the current i in a single-loop circuit with a battery B and a resistance R (and wires of negligible resistance). (a) Should the emf arrow at B be drawn pointing leftward or rightward? At points a, b, and c, rank (b) the magnitude of the current, (c) the electric potential, and (d) the electric potential energy of the charge carriers, greatest first.



a) Rightward.

b) All tie.

c) b, then c and a tie.

d) b, then c and a tie.

4. Other Single-Loop Circuits

We now extend the simple circuit of the previous section in two ways.

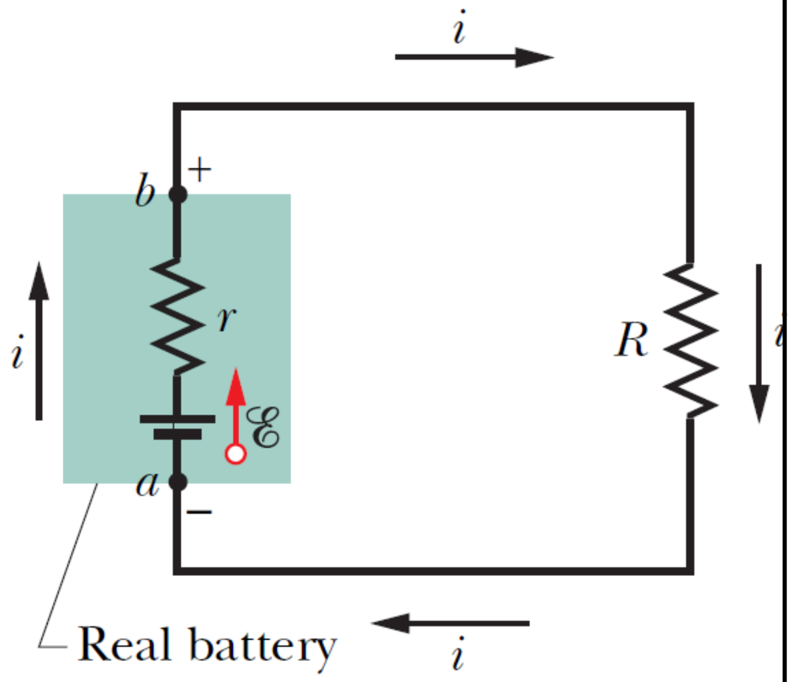
<u>Internal Resistance</u>

The figure shows a real battery, with **internal resistance** r, wired to an external resistor R. The battery's internal resistance is due to the electrical resistance of the conducting materials of the battery.

Applying the loop rule clockwise, starting at point a, gives

$$\mathcal{E} - ir - iR = 0.$$

Solving



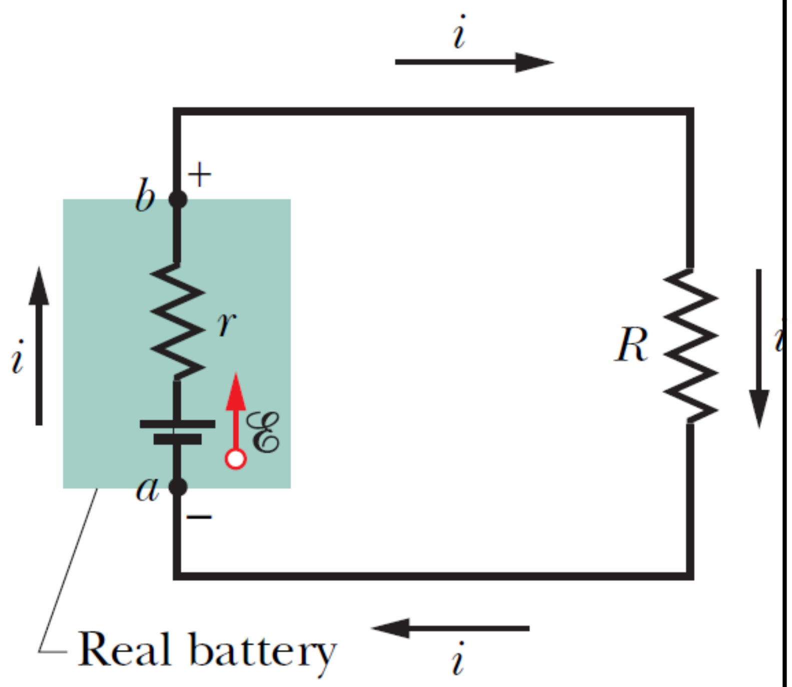
4. Other Single-Loop Circuits

<u>Internal Resistance</u>

$$\mathcal{E} - ir - iR = 0.$$

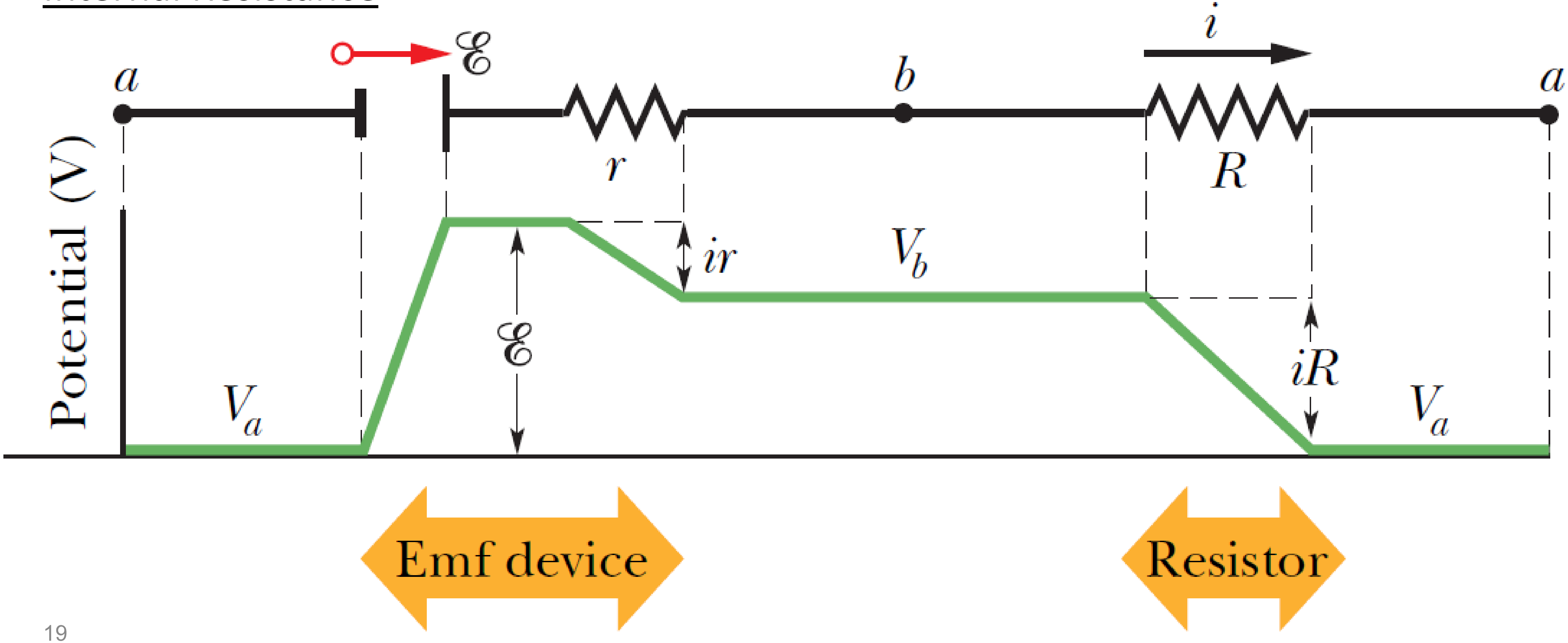
Solving for i gives

$$i = \frac{\mathcal{E}}{R + r}.$$



4. Other Single-Loop Circuits

Internal Resistance

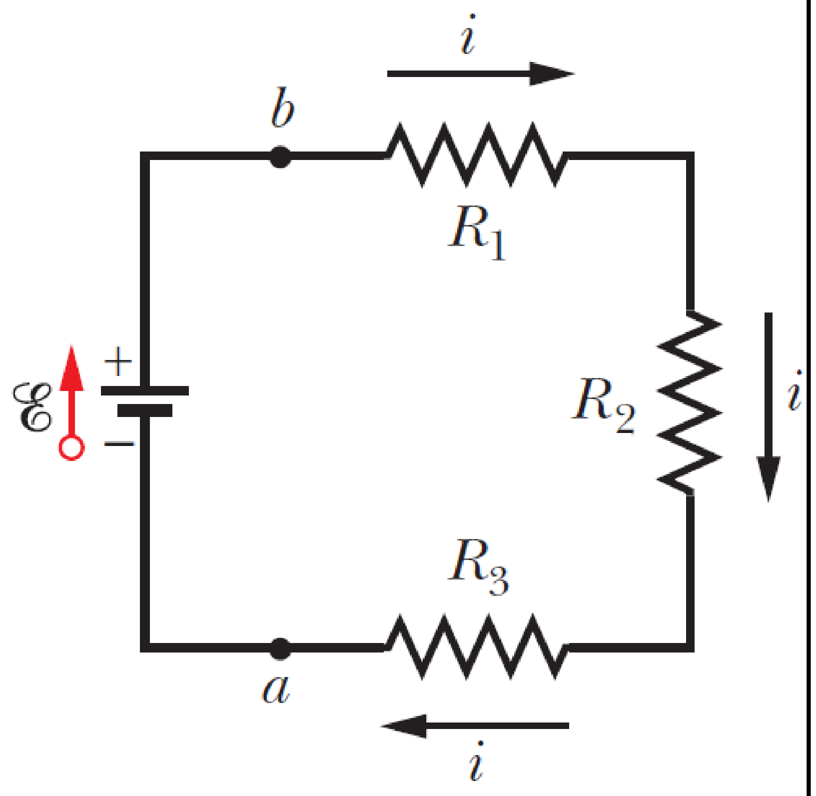


4. Other Single-Loop Circuits

<u>Resistances in Series</u>

The figure shows three resistors connected **in series** to an ideal battery with emf $\mathcal{E}$. The resistors are connected one another between points a and b, and a potential difference is maintained across the two points by the battery. The potential difference across the resistors produces current i through all of them.

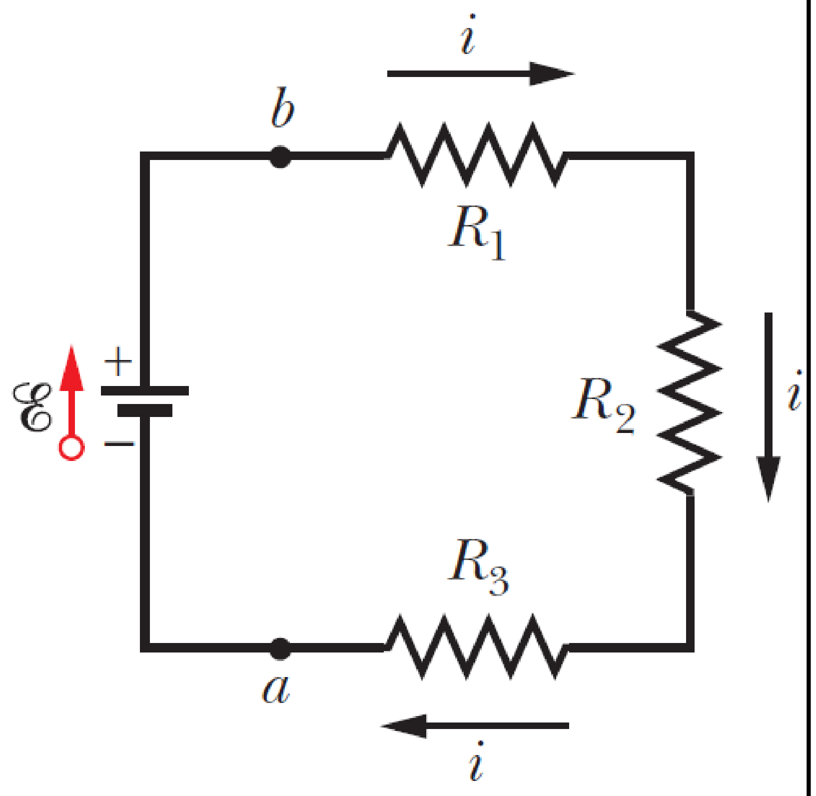
In general, when a potential difference V is applied across resistances connected in series, the resistances have identical currents i. The sum of the potential differences across the resistances is equal to the applied potential difference V.



4. Other Single-Loop Circuits

<u>Resistances in Series</u>

Resistances connected in series can be replaced with an equivalent resistance R_{eq} that has the same current i and the same *total* potential difference V as the actual resistances.



Let us derive an expression for R_{eq}. We first apply the loop rule to the circuit shown, starting at point a and going clockwise. We get

$$\mathcal{E} - iR_1 - iR_2 - iR_3 = 0,$$

or

$$i = \frac{\mathcal{E}}{R_1 + R_2 + R_3}.$$

4. Other Single-Loop Circuits

<u>Resistances in Series</u>

We next apply the loop rule to circuit with R_{eq}, starting at point a and going clockwise. We obtain

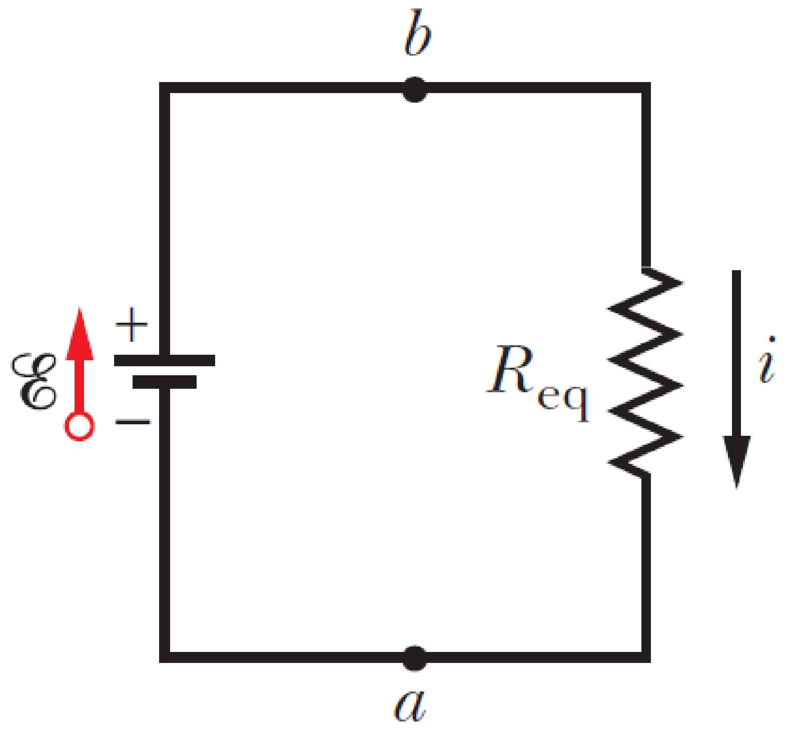
$$\mathcal{E} - iR_{\mathrm{eq}} = 0,$$

or

$$i = \frac{\mathcal{E}}{R_{\mathrm{eq}}}.$$

Comparing the two expressions of i shows that

$$R_{\mathrm{eq}} = R_1 + R_2 + R_3.$$



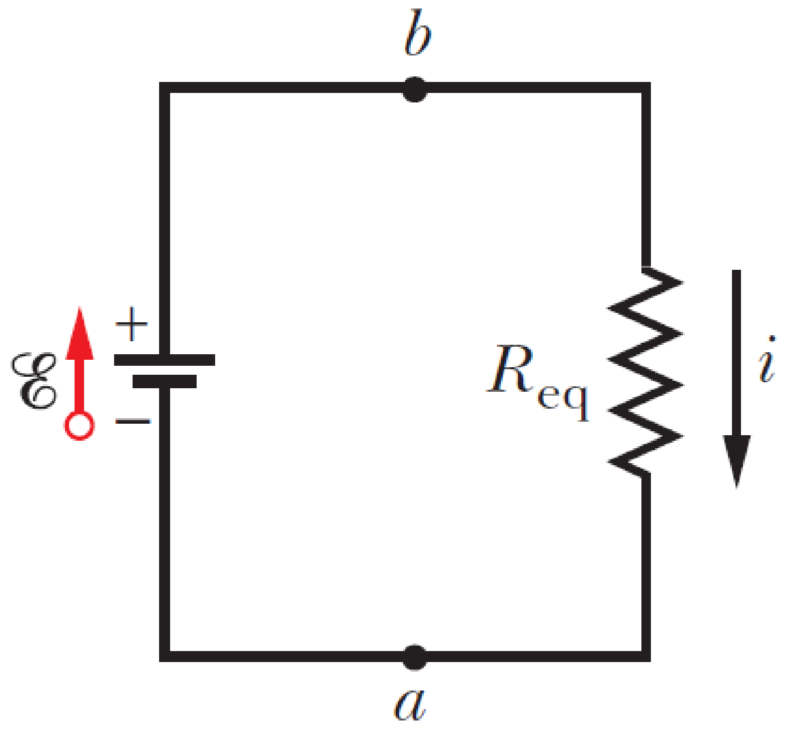
4. Other Single-Loop Circuits

<u>Resistances in Series</u>

In general, for n resistors,

$$R_{\text{eq}} = \sum_{j=1}^{n} R_j \,.$$

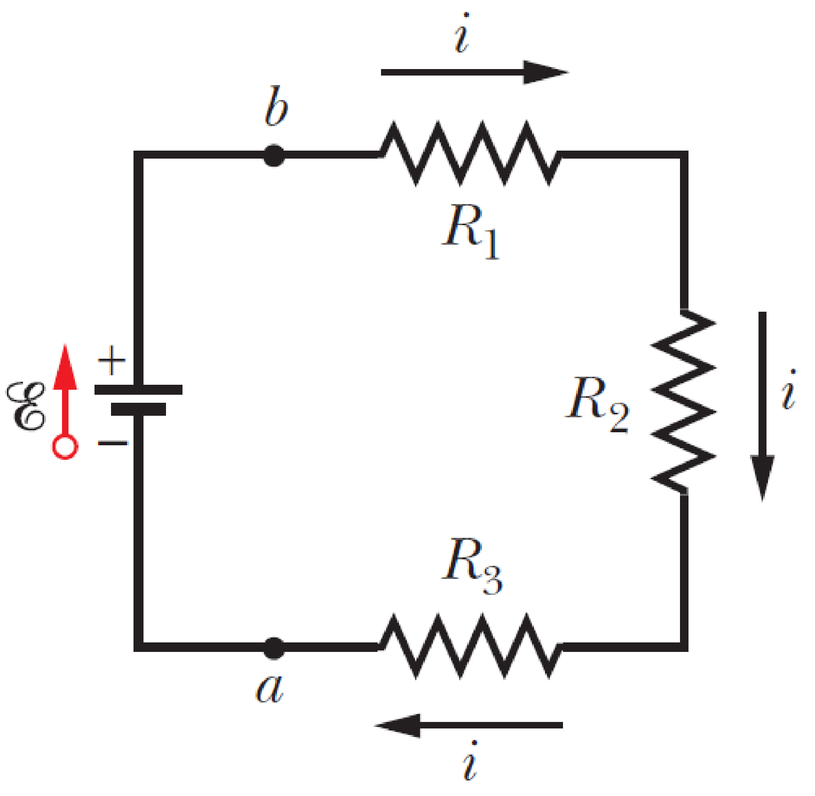
Note that R_{eq} is greater than any of the individual resistances.



4. Other Single-Loop Circuits

CHECKPOINT 2

In Fig. 27-5*a*, if $R_1 > R_2 > R_3$, rank the three resistances according to (a) the current through them and (b) the potential difference across them, greatest first.



a) All tie.

b) R_1 then R_2 then R_3.

5. Potential Difference Between Two Points

We often want to find the potential difference between two points. For example, let us find the potential difference $V_b - V_a$ between points a and b.

Starting at point a and moving through the battery to point b, while keeping track of the potential changes we encounter, we obtain

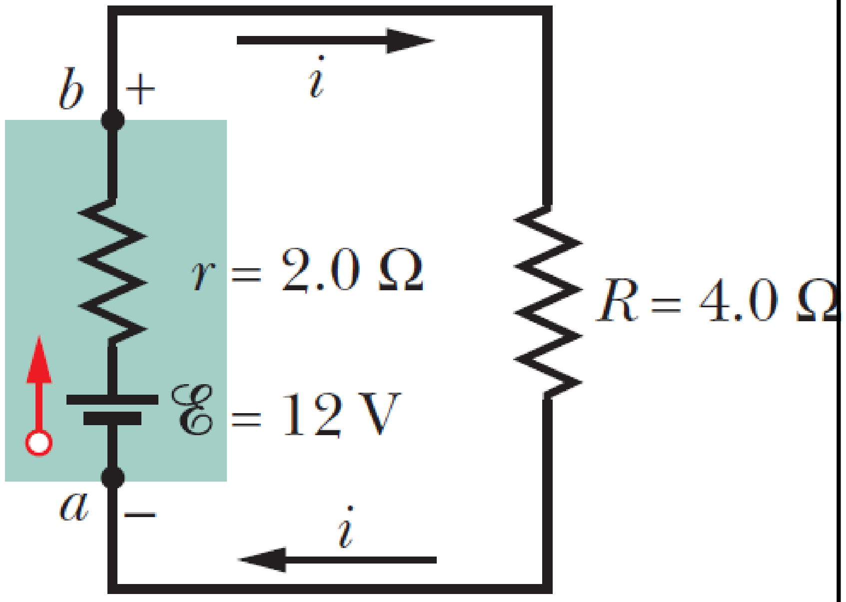
$$V_a + \mathcal{E} - ir = V_b.$$

or

$$V_b - V_a = \mathcal{E} - ir.$$

We already found that

$$i = \frac{\mathcal{E}}{R + r}.$$



5. Potential Difference Between Two Points

The potential difference becomes

$$V_b - V_a = \mathcal{E} - \frac{\mathcal{E}}{R+r}r = \frac{\mathcal{E}}{R+r}R.$$

Substituting the data in the figure gives

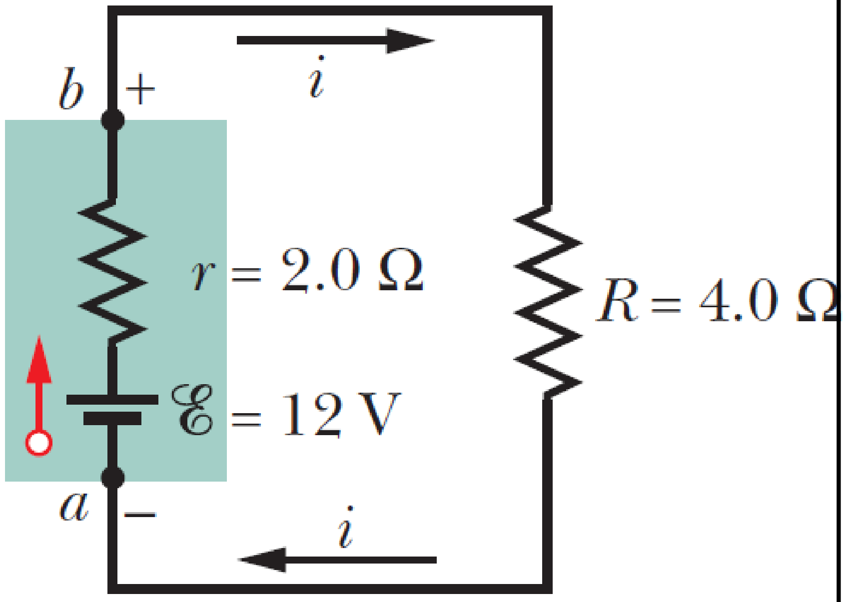
$$V_b - V_a = \frac{\mathcal{E}}{R+r}R = \frac{12\ \mathrm{V}}{4.0\ \Omega + 2.0\ \Omega}(4.0\ \Omega) = 8.0\ \mathrm{V}.$$

Let us now move from a to b counterclockwise. We get

$$V_a + iR = V_b.$$

or

$$V_b - V_a = iR.$$



5. Potential Difference Between Two Points

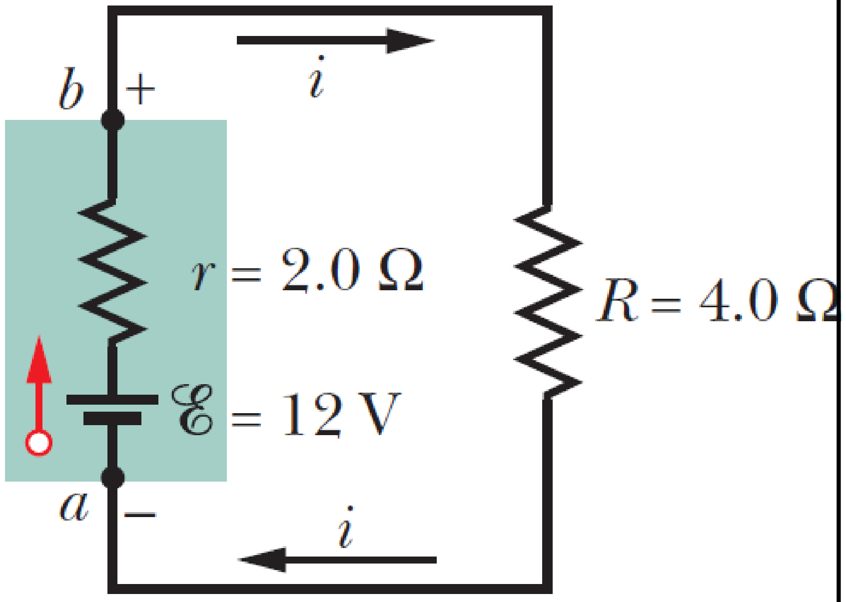
$$V_b - V_a = iR.$$

Substituting for i using

$$i = \frac{\mathcal{E}}{R + r},$$

gives us the same expression for $V_b - V_a$ as before.

In general, to find the potential between any two points in a circuit, start at one point and traverse the circuit to the other point, following any path, and add algebraically the changes in potential you encounter.



5. Potential Difference Between Two Points

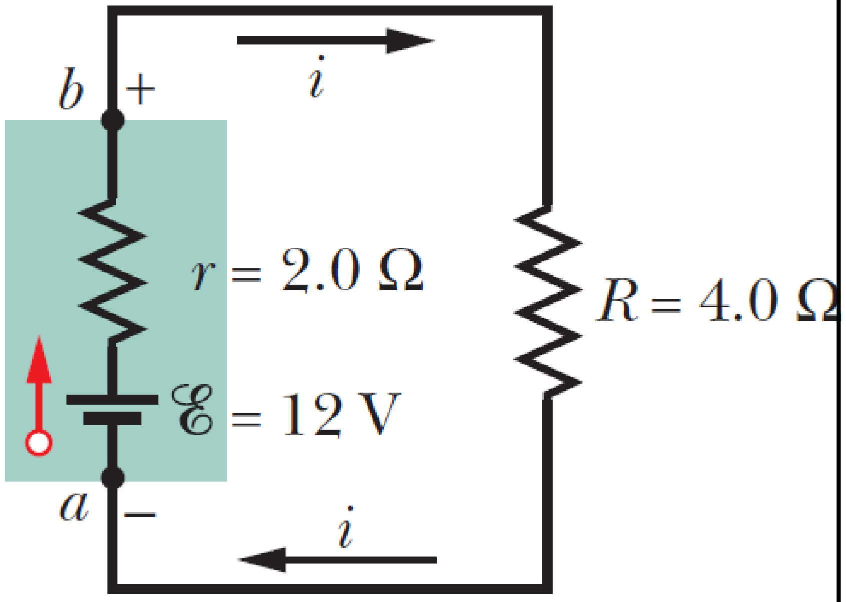
<u>Potential Difference Across a Real Battery</u>

Points a and b are located at the terminals of the battery. The potential difference $V_b - V_a$ is the terminal-to-terminal potential difference V across the battery. We thus write

$$V = \mathcal{E} - ir.$$

If r were zero, V would be the emf $\mathcal{E} = 12$ V. However, because $r = 2.0\ \Omega$, $V = 8.0$ V.

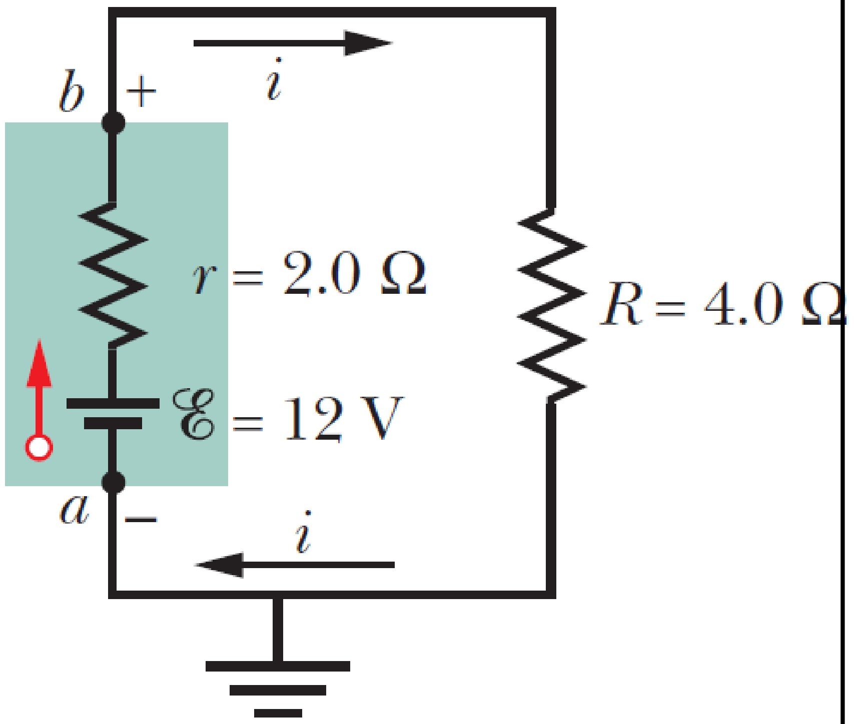
Note that V depends on the value of the current i.



5. Potential Difference Between Two Points

<u>Grounding a Circuit</u>

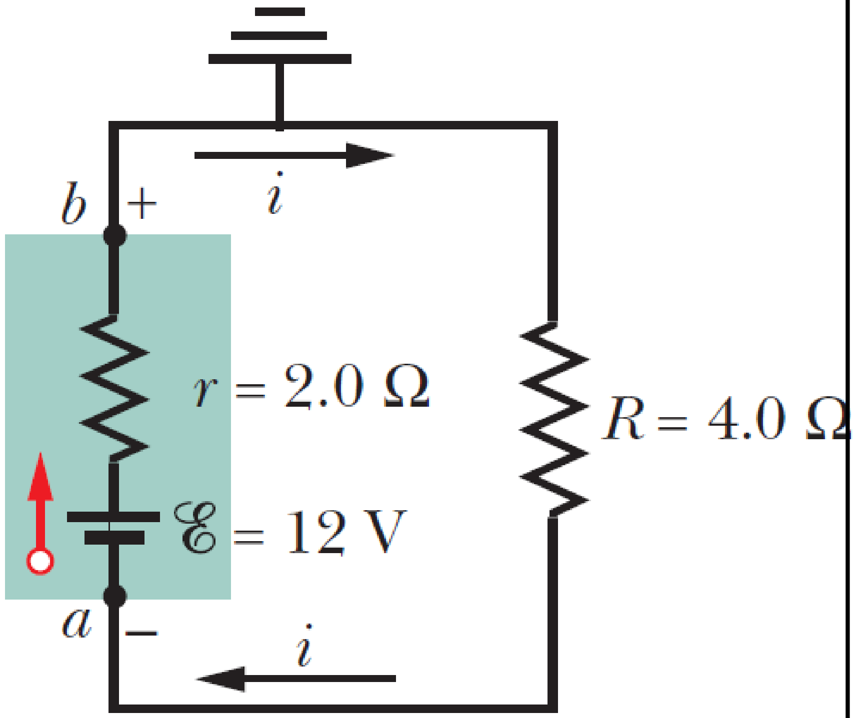
The figure shows the same previous circuit but with point a connected to ground. The grounding here means only that the potential is defined to be zero at the grounding point in the circuit. Thus, the potential at a is defined to be $V_a = 0$, which implies that $V_b = 8.0$ V.



5. Potential Difference Between Two Points

<u>Grounding a Circuit</u>

The figure is the same as before but with point b connected to ground. Thus, $V_b = 0$, which tells that $V_a = -8.0$ V.



5. Potential Difference Between Two Points

<u>Power, Potential, and Emf</u>

An emf device does work on the charge carriers to establish a current i, transferring energy to the charge carriers. A real battery emf device also transfers energy to the internal thermal energy via resistive dissipation.

The net rate P of energy transfer from the emf device to the charge carriers is

$$P = iV,$$

where V is the potential across the terminals of the emf device. Using $V = \mathcal{E} - ir$, we obtain

$$P = i(\mathcal{E} - ir) = i\mathcal{E} - i^2 r.$$

5. Potential Difference Between Two Points

<u>Power, Potential, and Emf</u>

The term i^2r is the rate P_r of energy transfer to thermal energy within the emf device:

$$P_r = i^2r.$$

The term $i\mathcal{E}$ must be the rate P_{emf} at which the emf device transfers energy both to charge carriers and to internal thermal energy:

$$P_{\text{emf}} = i\mathcal{E}.$$

If a battery is being recharged, with a "wrong way " current through it, the energy transfer is then from the charge carriers to the battery. The rate of change of the chemical energy is $P_{\text{emf}} = i\mathcal{E}$, the rate of dissipation is given by i^2r, and the rate at which carriers supply energy is given by $P = iV$.

5. Potential Difference Between Two Points

CHECKPOINT 3

A battery has an emf of 12 V and an internal resistance of 2 Ω. Is the terminal-to-terminal potential difference greater than, less than, or equal to 12 V if the current in the battery is (a) from the negative to the positive terminal, (b) from the positive to the negative terminal, and (c) zero?

$$V = \mathcal{E} - ir$$
$$= \frac{\mathcal{E}}{R + r} R$$

a) Less.
b) Greater.
c) Equal.

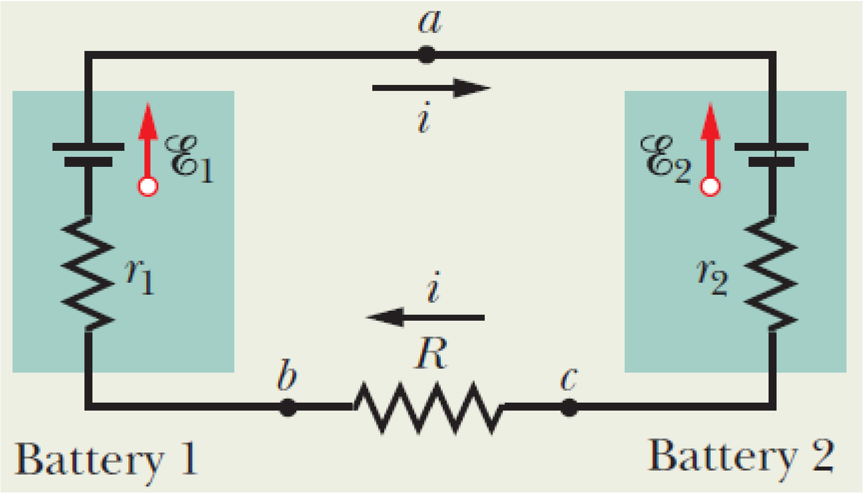
5. Potential Difference Between Two Points

Example 1: The emfs and resistances in the circuit of the figure have the following values:

$$\mathcal{E}_1 = 4.4 \text{ V}, \qquad \mathcal{E}_2 = 2.1 \text{ V}.$$

$$r_1 = 2.3\ \Omega, \qquad r_2 = 1.8\ \Omega, \qquad R = 5.5\ \Omega.$$

(a) What is the current i in the circuit?



Applying the loop rule by going counterclockwise and starting at point a gives

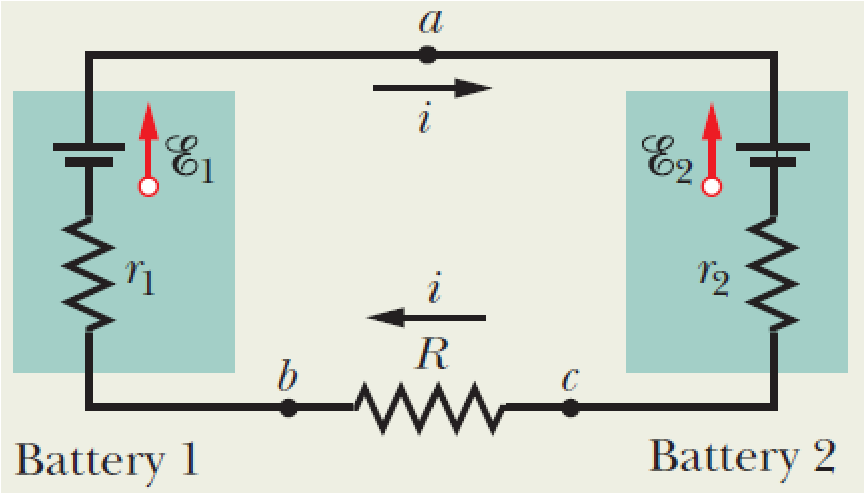
$$-\mathcal{E}_1 + ir_1 + iR + ir_2 + \mathcal{E}_2 = 0.$$

Solving for i and substituting give

$$i = \frac{\mathcal{E}_1 - \mathcal{E}_2}{r_1 + r_2 + R} = \frac{4.4 \text{ V} - 2.1 \text{ V}}{2.3\ \Omega + 1.8\ \Omega + 5.5\ \Omega} = 240 \text{ mA}.$$

5. Potential Difference Between Two Points

b) What is the potential difference between the terminals of battery 1 in the figure?



We now start at point b and travel clockwise to point a, keeping track of potential changes. We find

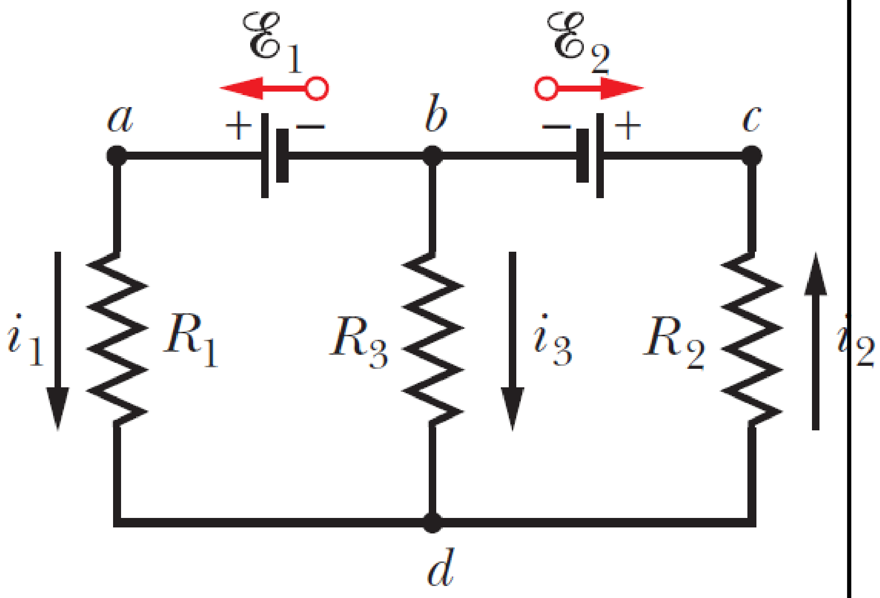
$$V_b - ir_1 + \mathcal{E}_1 = V_a,$$

which gives

$$V_a - V_b = \mathcal{E}_1 - ir_1 = 4.4\ \mathrm{V} - (0.2396\ \mathrm{A})(2.3\ \Omega)$$
$$= 3.8\ \mathrm{V}.$$

6. Multiloop Circuit

The figure shows a circuit with more than one loop. There are two **junctions**, at b and d, and there are three **branches** connecting these junctions. The branches are the left branch (bad), the right branch (bcd), and the central branch (bd). We want to find the current in the three branches.



We arbitrarily label the currents in each branch. A current has the same value everywhere in a branch. The directions of the currents i_1, i_2 and i_3 are assumed arbitrarily.

The total current coming into a junction must be equal to the total current leaving the junction, by charge conservation.

6. Multiloop Circuit

For junction d,

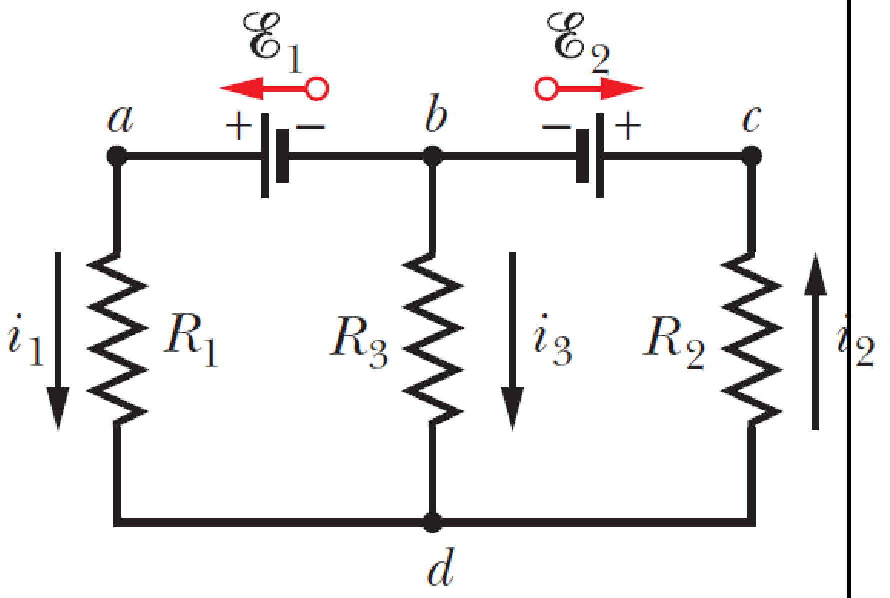
$$i_1 + i_3 = i_2.$$

Applying this condition to junction b leads to the same equation. This equation suggests a general principle:

JUNCTION RULE: The sum of the currents entering any junction must be equal to the sum of the currents leaving that junction.

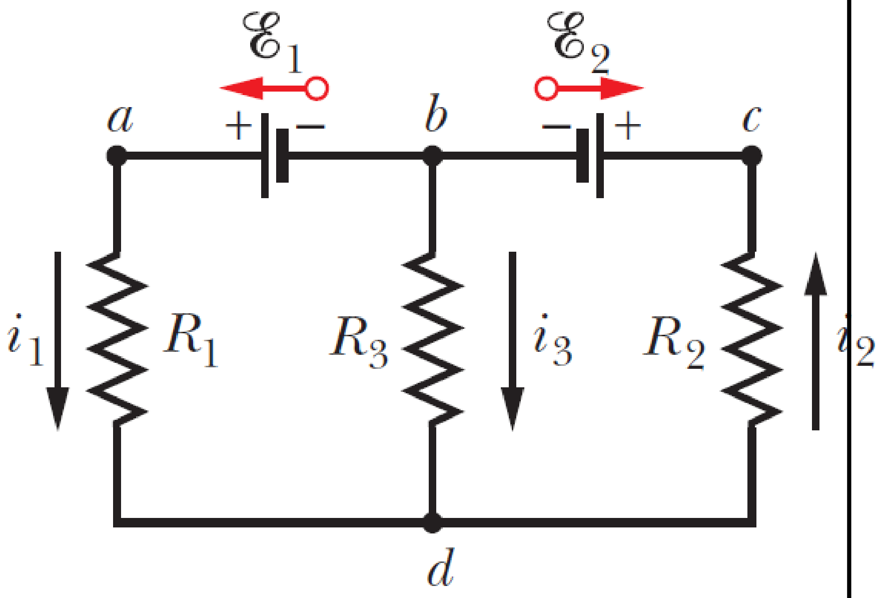
This rule is known as **Kirchhoff's junction rule**, or **Kirchhoff's currents law**.

The equation above involves three unknowns. We need two more equations involving the same unknown.



6. Multiloop Circuit

We obtain those two equations by applying the loop rule twice. In this circuit, we have thee loops from which we may choose any two. Let us choose the left-hand and the right-hand loops.



Traversing the left-hand loop counterclockwise, starting at point b, gives

$$\mathcal{E}_1 - i_1R_1 + i_3R_3 = 0.$$

Traversing the right-hand loop counterclockwise, starting at point b, yields

$$-i_3R_3 - i_2R_2 - \mathcal{E}_2 = 0.$$

Now, we have thee equations in three unknown.

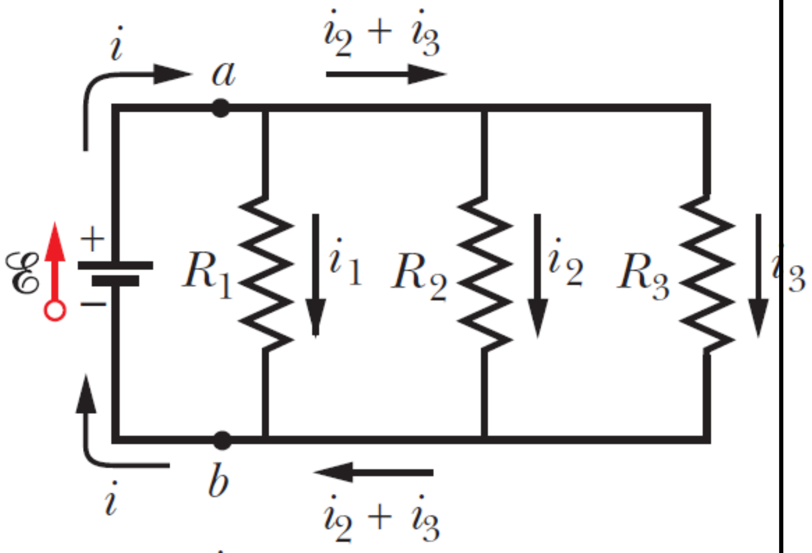
6. Multiloop Circuit

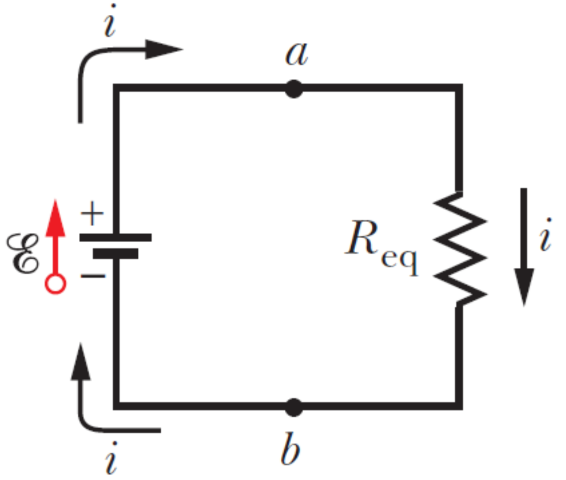
<u>Resistances in Parallel</u>

The figure shows three resistors connected in parallel to an ideal emf. A potential difference V is applies across each of them, producing a current through each of them.

In general, when a potential difference V is applied across resistances connected in parallel, the resistances all have that same potential difference V.

Resistances connected in parallel can be replaced with an equivalent resistance R_{eq} that has the same potential difference V and the same *total* current i as the actual resistances





6. Multiloop Circuit

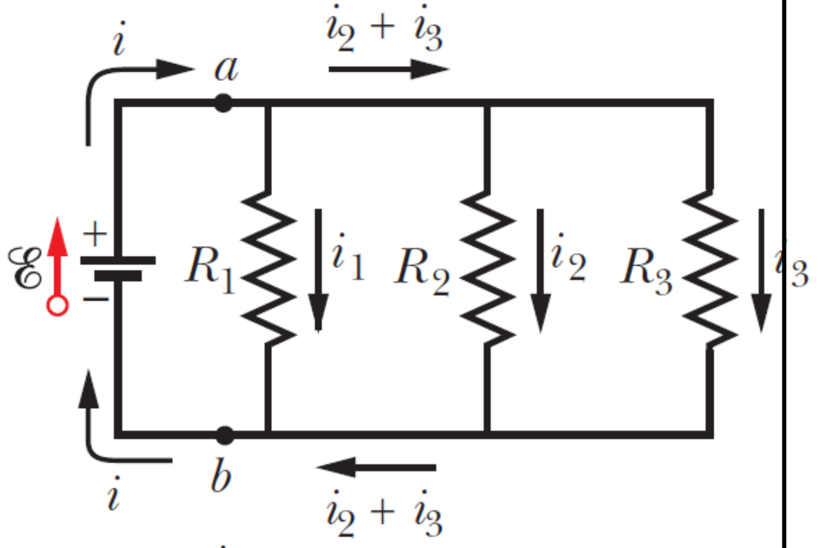
<u>Resistances in Parallel</u>

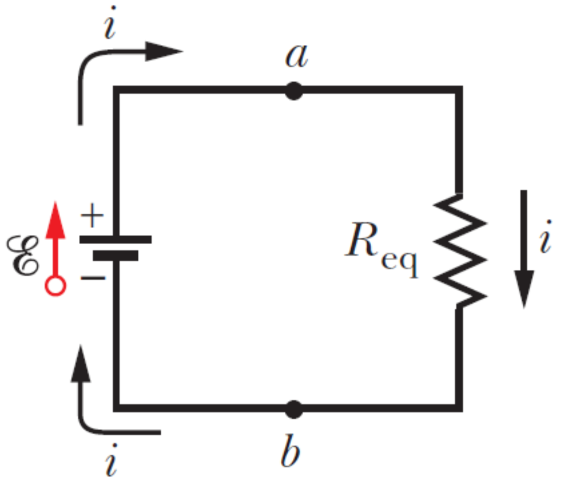
Let us write an expression for R_{eq}. The currents in the actual resistors, are

$$i_1 = \frac{V}{R_1}, \qquad i_2 = \frac{V}{R_2}, \qquad i_3 = \frac{V}{R_3}.$$

If we apply the junction rule at point a in the real circuit we find

$$i = i_1 + i_2 + i_3 = V\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right).$$





6. Multiloop Circuit

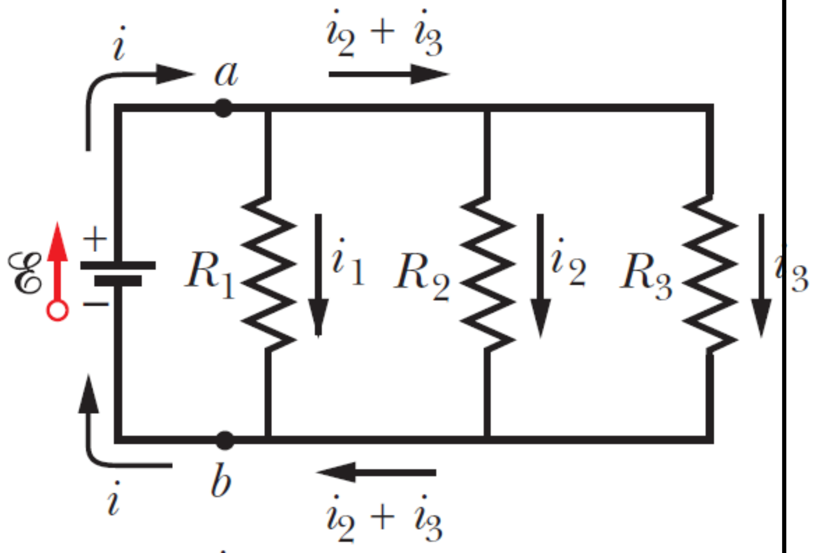
Resistances in Parallel

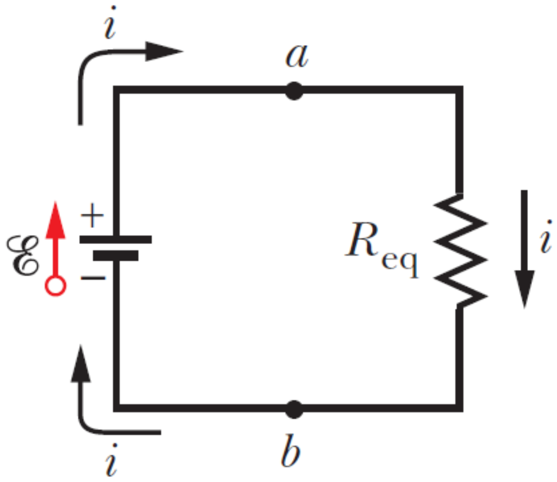
In terms of the equivalent resistances R_{eq}, the current i reads

$$i = \frac{V}{R_{\text{eq}}}.$$

Equating the two expressions for i gives

$$\frac{1}{R_{\text{eq}}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}.$$





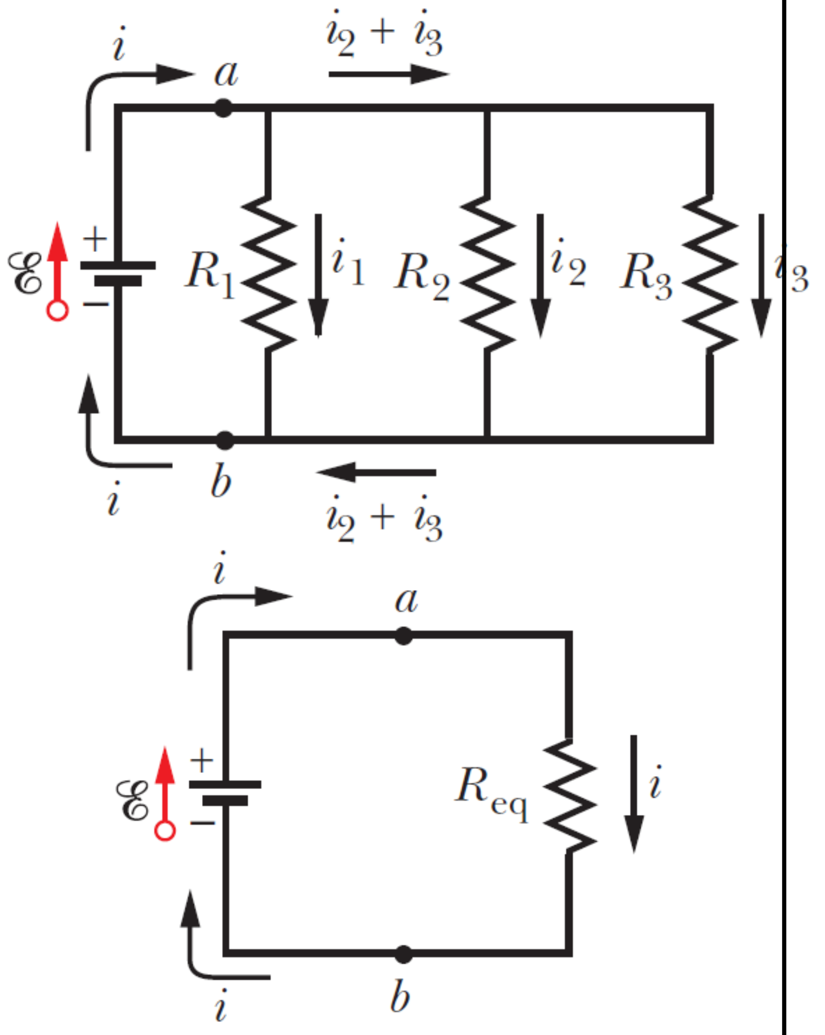
6. Multiloop Circuit

<u>Resistances in Parallel</u>

In general, for n resistors, the equivalent resistance is given by

$$\frac{1}{R_{\text{eq}}} = \sum_{j=1}^{n} \frac{1}{R_j}.$$

Note that the equivalent resistance is smaller than any of the combing resistances.



6. Multiloop Circuit

CHECKPOINT 4

A battery, with potential V across it, is connected to a combination of two identical resistors and then has current i through it. What are the potential difference across and the current through either resistor if the resistors are (a) in series and (b) in parallel?

a) $V/2, i$.

b) $V, i/2$.

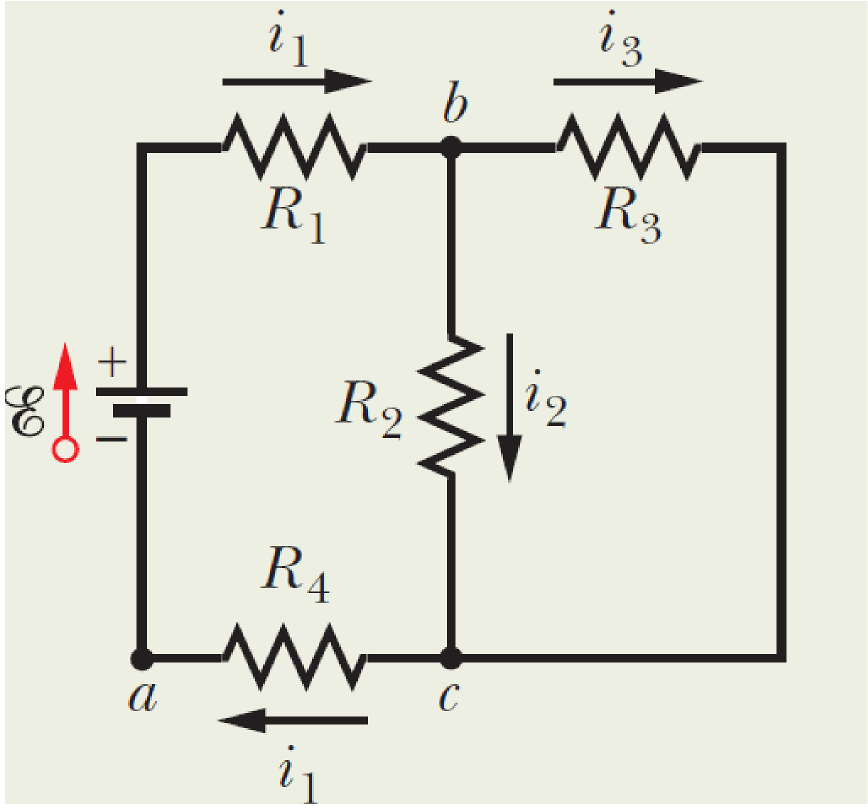
6. Multiloop Circuit

Example 2: The figure shows a multiloop circuit containing one ideal battery and four resistances with the following values:

$$\mathcal{E} = 12\text{ V}, \qquad R_1 = R_2 = 20\ \Omega.$$

$$R_3 = 30\ \Omega, \qquad R_4 = 8.0\ \Omega.$$

(a) What is the current through the battery?



R_2 and R_3 are connected in parallel. Their equivalent resistance is

$$R_{23} = \frac{R_2 R_3}{R_2 + R_3} = \frac{(20\ \Omega)(30\ \Omega)}{20\ \Omega + 30\Omega} = 12\ \Omega.$$

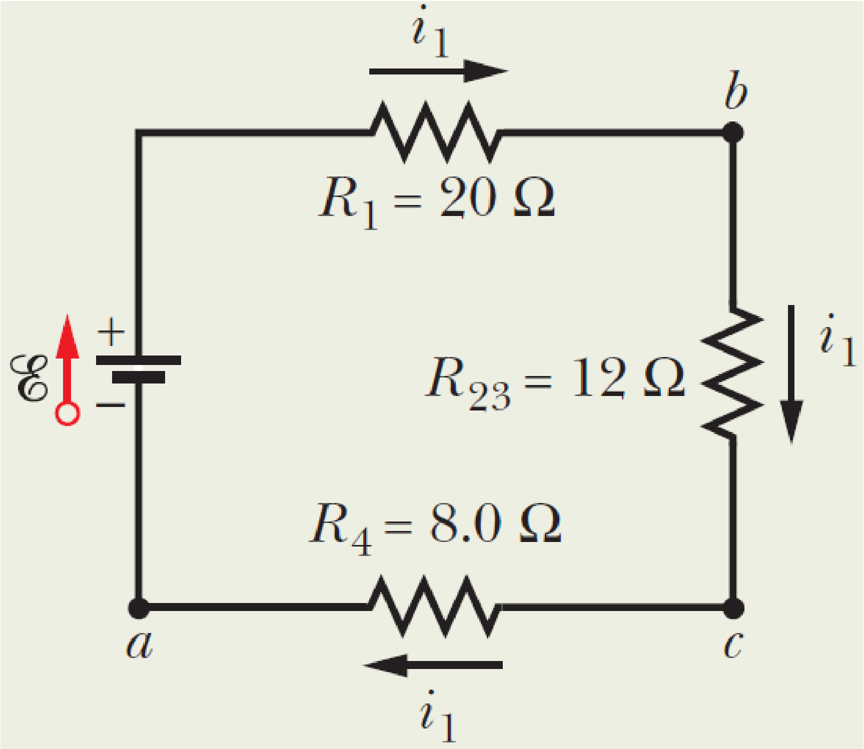
6. Multiloop Circuit

Applying the loop rule clockwise from point a yields

$$\mathcal{E} - i_1 R_1 - i_1 R_{23} - i_1 R_4 = 0.$$

Therefore,

$$i = \frac{\mathcal{E}}{R_1 + R_{23} + R_4} = \frac{12\text{ V}}{20\Omega + 12\Omega + 8.0\Omega} = 0.30\text{ A}.$$



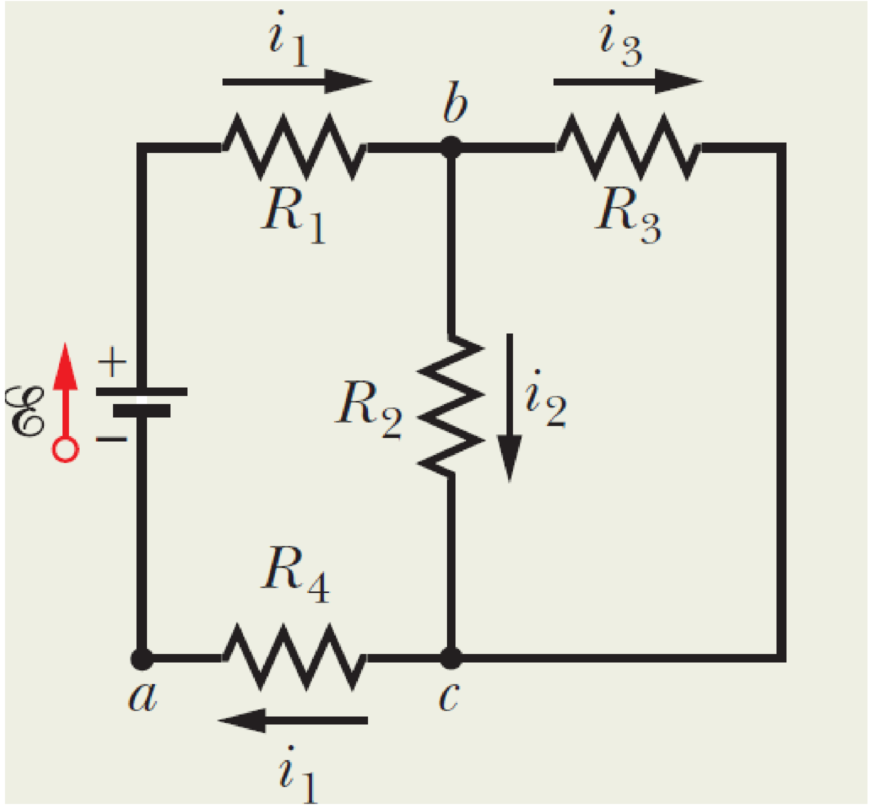
6. Multiloop Circuit

b) What is the current i_2 through R_2?

The potential difference V_2 across R_2 is the same as that across R_{23}. We have

$$V_2 = V_{23} = i_1 R_{23} = (0.30\ \text{A})(12\ \Omega) = 3.6\ \text{V}.$$

$$i_2 = \frac{V_2}{R_2} = \frac{3.6\ \text{V}}{20\ \Omega} = 0.18\ \text{A}.$$



6. Multiloop Circuit

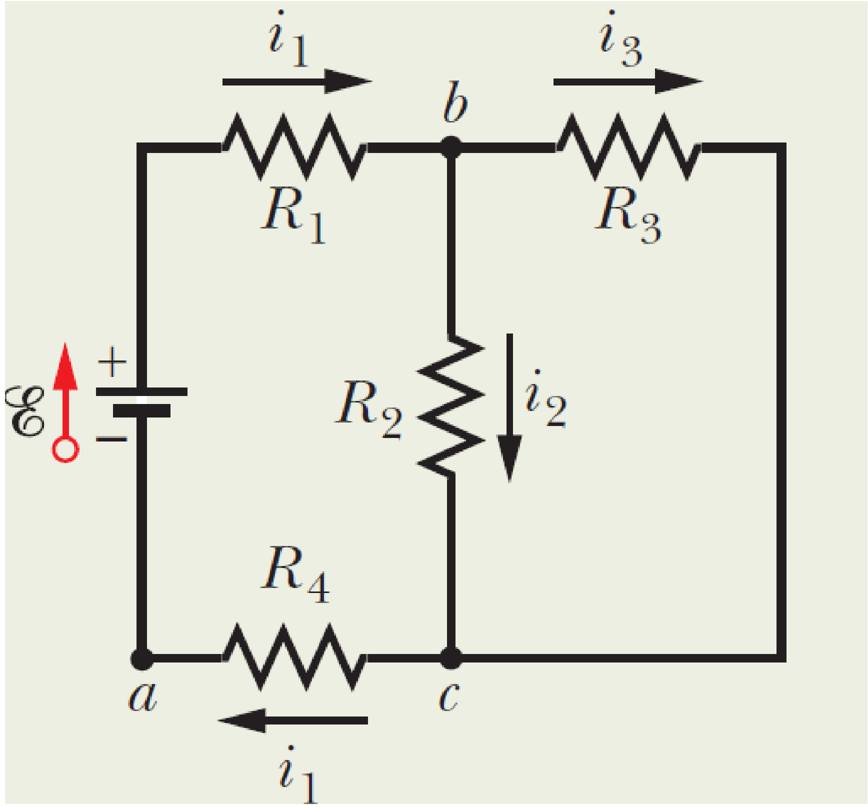
c) What is the current i_3 through R_3?

Applying the junction rule at point b gives

$$i_1 = i_2 + i_3.$$

Thus,

$$i_3 = i_1 - i_2 = 0.30\ \text{A} - 0.18\ \text{A} = 0.12\ \text{A}.$$

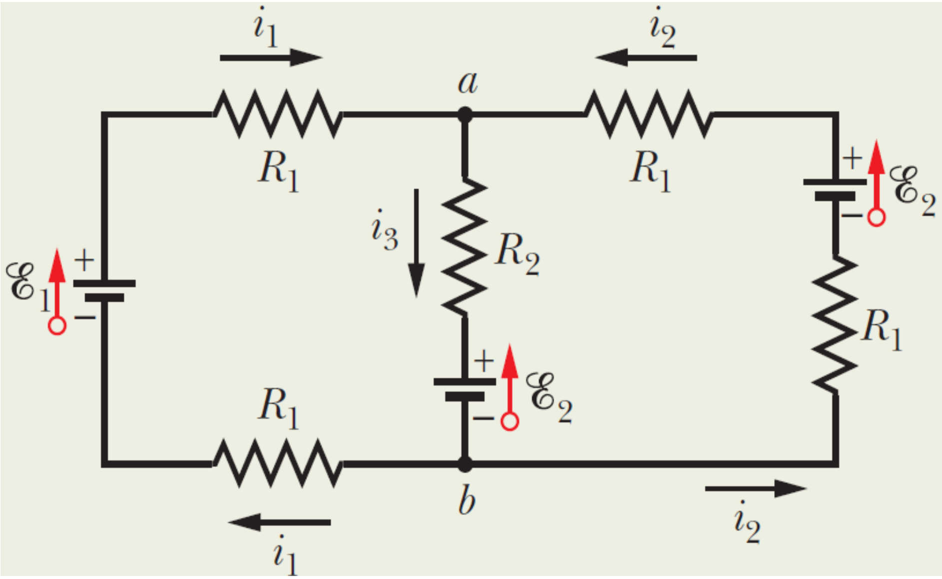


6. Multiloop Circuit

Example 3: The figure shows a circuit whose elements have the following values:

$$\mathcal{E}_1 = 3.0 \text{ V}, \qquad \mathcal{E}_2 = 6.0 \text{ V},$$

$$R_1 = 2.0\ \Omega, \qquad R_2 = 4.0\ \Omega.$$



The three batteries are ideal batteries. Find the magnitude and direction of the current in each of the three branches.

We assume that the three currents' directions are as in the figure. Applying the junction rule at point a gives

$$i_1 + i_2 = i_3.$$

6. Multiloop Circuit

Applying the loop rule to the left-hand loop clockwise, starting at point b, gives

$$-i_1R_1 + \mathcal{E}_1 - i_1R_1 - i_3R_2 - \mathcal{E}_2 = 0.$$

Using $i_1 + i_2 = i_3$ and substituting, we find

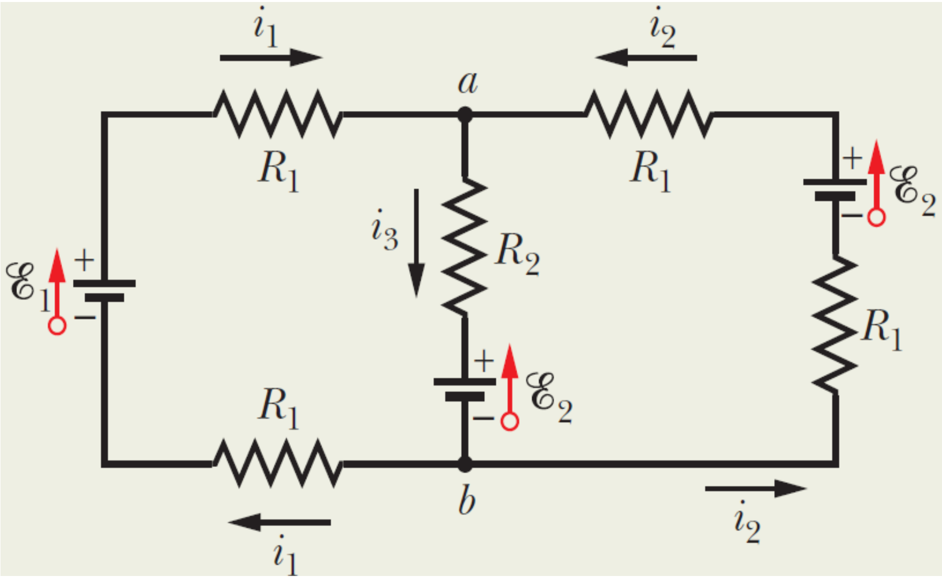
$$i_1(8.0\ \Omega) + i_2(4.0\ \Omega) = -3.0\ \text{V}.$$

Applying the loop rule to the right-hand loop counterclockwise, starting at point b, gives

$$-i_2R_1 + \mathcal{E}_2 - i_2R_1 - i_3R_2 - \mathcal{E}_2 = 0.$$

Using $i_1 + i_2 = i_3$ and substituting, we obtain

$$i_1(4.0\ \Omega) + i_2(8.0\ \Omega) = 0.$$



6. Multiloop Circuit

We have now two equations in two variable:

$$i_1(8.0\ \Omega) + i_2(4.0\ \Omega) = -3.0\ \text{V}.$$

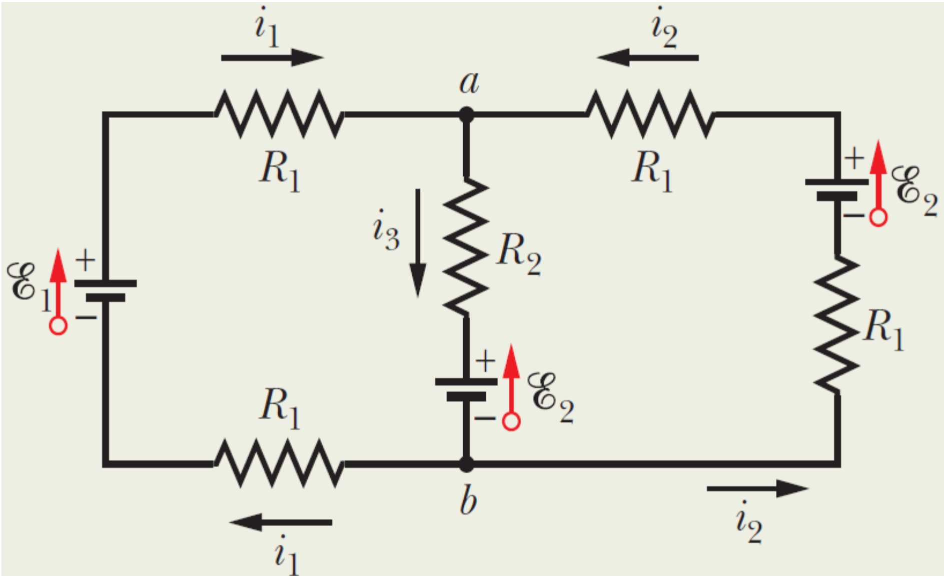
$$i_1(4.0\ \Omega) + i_2(8.0\ \Omega) = 0.$$

Solving for the current gives

$$i_1 = -0.50\ \text{A}, \qquad i_2 = 0.25\ \text{A}.$$

We then find that

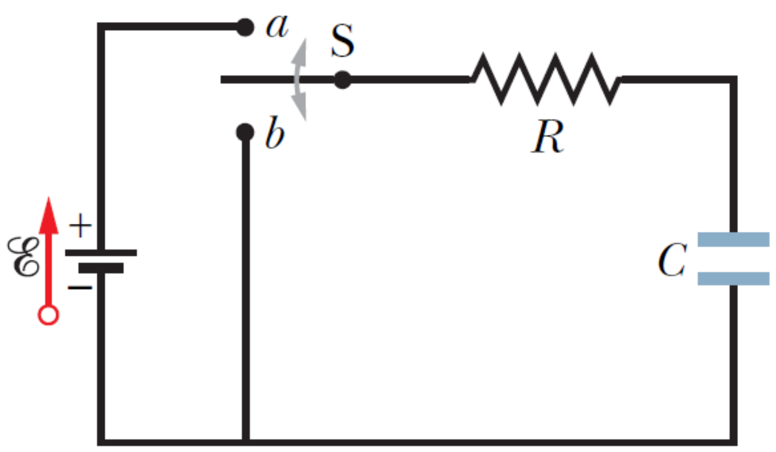
$$i_3 = i_1 + i_2 = -0.50\ \text{A} + 0.25\ \text{A} = -0.25\ \text{A}.$$



7. *RC* Circuits

We here start to discuss circuits of time-varying currents.

<u>Charging Capacitors</u>



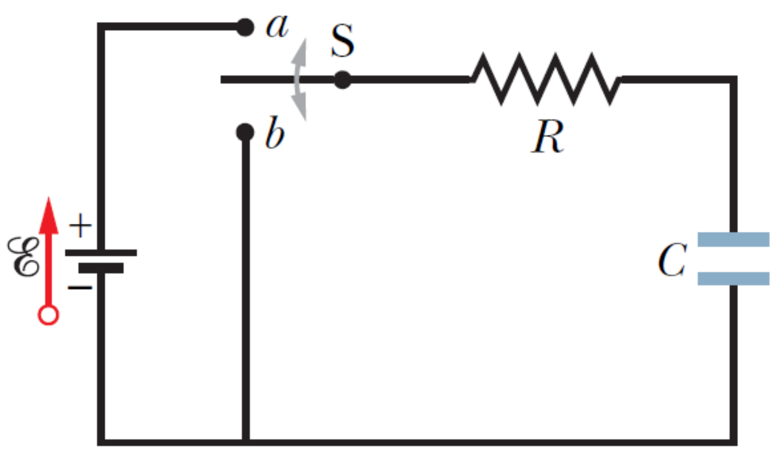
The figure shows an RC series circuit, consisting of a capacitor C, an ideal battery of emf $\mathcal{E}$ and a resistance R. The capacitor starts getting charged when we close the switch S on point a.

The charge begins to flow between the battery plates and the battery terminals. This flow of charge (current) increases the charge q on the capacitor and the potential difference $V_C (= q/C)$. The current becomes zero when the potential difference across the capacitor becomes equal to the emf $\mathcal{E}$. The charge on the capacitor is then $q = C\mathcal{E}$.

7. *RC* Circuits

<u>Charging Capacitors</u>

Here we want to know how the charge $q(t)$ in the capacitor plates, the potential difference $V_C(t)$ across the capacitor, and the current $i(t)$ in the circuit vary with time.



Applying the loop rule to the circuit, clockwise, from negative terminal of the battery gives us

$$\mathcal{E} - iR - \frac{q}{C} = 0.$$

Note that the last term, representing the potential difference across the capacitor, is negative because the potential drops as we traverse the capacitor clockwise.

7. *RC* Circuits

<u>Charging Capacitors</u>

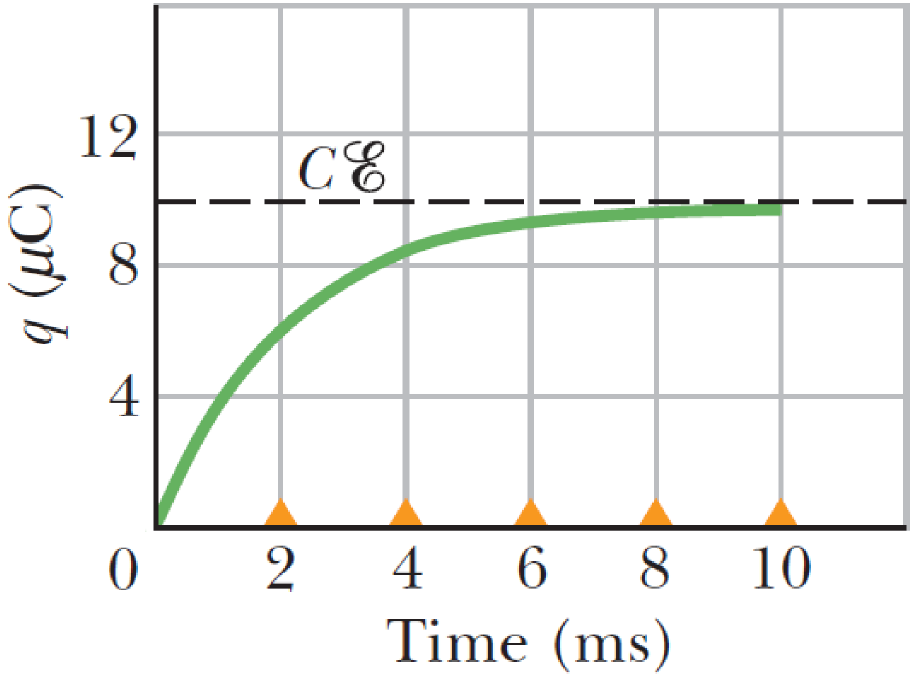
Using $i = dq/dt$, we obtain after some rearrangement

$$R\frac{dq}{dt} + \frac{q}{C} = \mathcal{E}.$$

A solution to this equation is

$$q(t) = C\mathcal{E}\left(1 - e^{-t/RC}\right).$$

This solution satisfies the initial condition $q(0) = 0$. As t approaches ∞, q approaches $C\mathcal{E}$.



7. *RC* Circuits

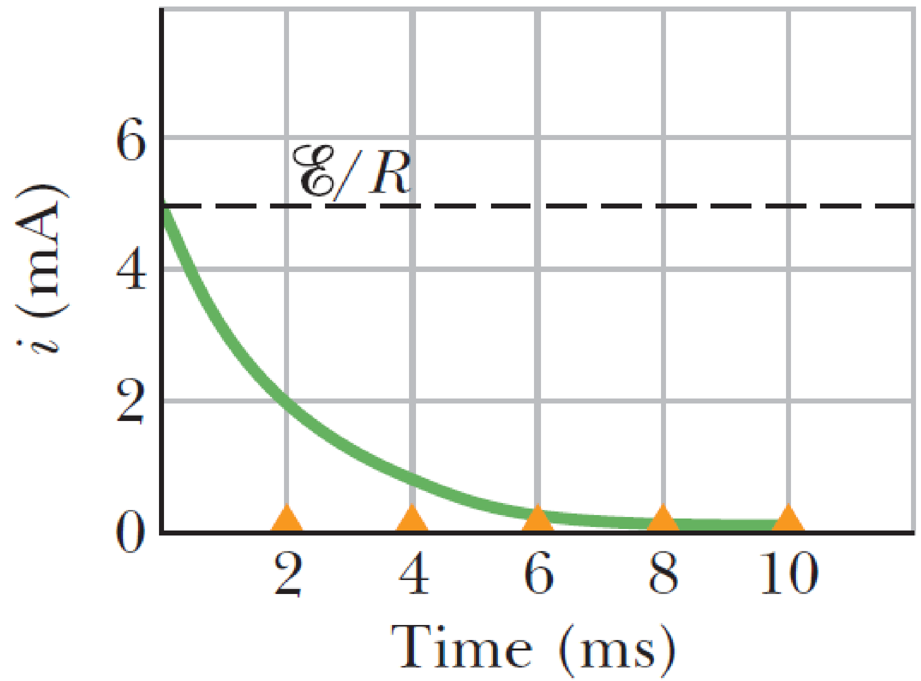
<u>Charging Capacitors</u>

The derivative of $q(t)$ is the current $i(t)$ charging the capacitor:

$$i(t) = \frac{dq}{dt} = \frac{\mathcal{E}}{R} e^{-t/RC}.$$

Note that $i(0) = \mathcal{E}/R$ and $i \to 0$ as $t \to \infty$.

A capacitor that is being charged initially acts like ordinary connecting wire relative to the charging current. A long time later, it acts like a broken wire.



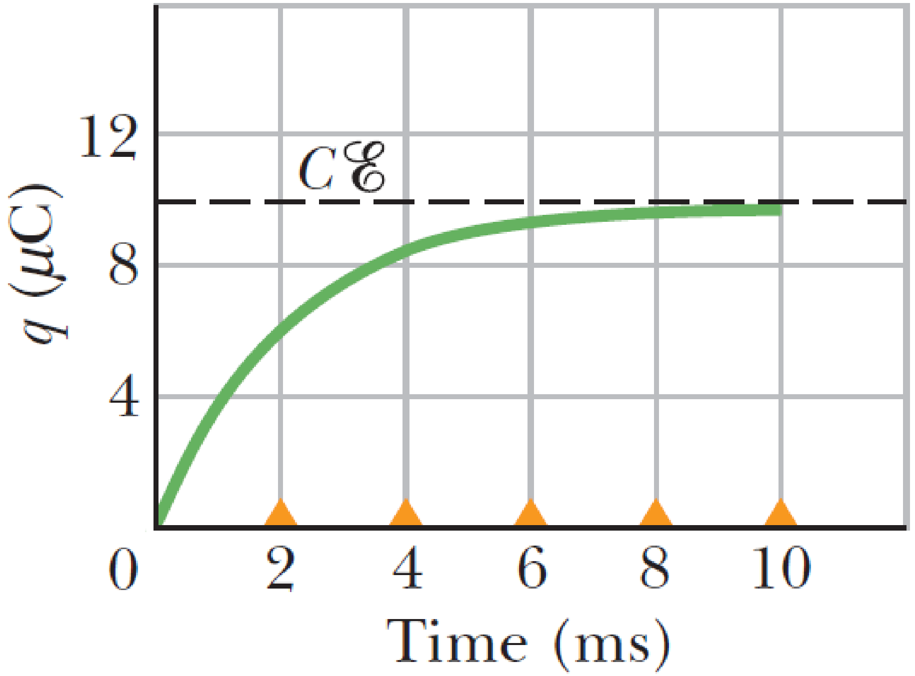
7. *RC* Circuits

Charging Capacitors

The potential difference $V_C(t)$ across the capacitor is

$$V_C(t) = \frac{q(t)}{C} = \mathcal{E}\left(1 - e^{-t/RC}\right).$$

Note that $V_C(0) = 0$ and $V_C(t) \to \mathcal{E}$ as $t \to \infty$.



7. *RC* Circuits

<u>The Time Constant</u>

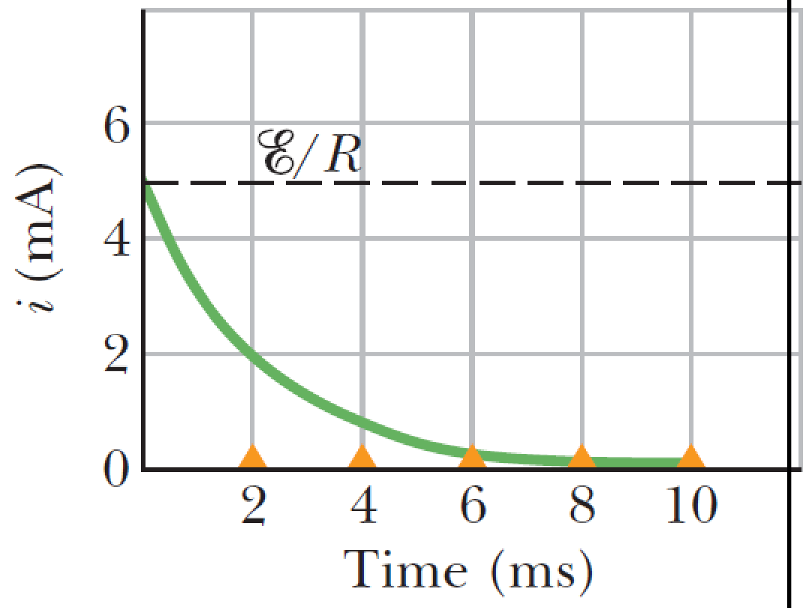
The product RC has the dimension of time ($1\,\Omega \times 1\,\mathrm{F} = 1\,\mathrm{s}$). It is called the **capacitive time constant** of the circuit and represented with the symbol τ:

$$\tau = RC.$$

At time $t = \tau$, the charge on the capacitor become

$$q = C\mathcal{E}(1 - e^{-1}) = 0.63\, C\mathcal{E}.$$

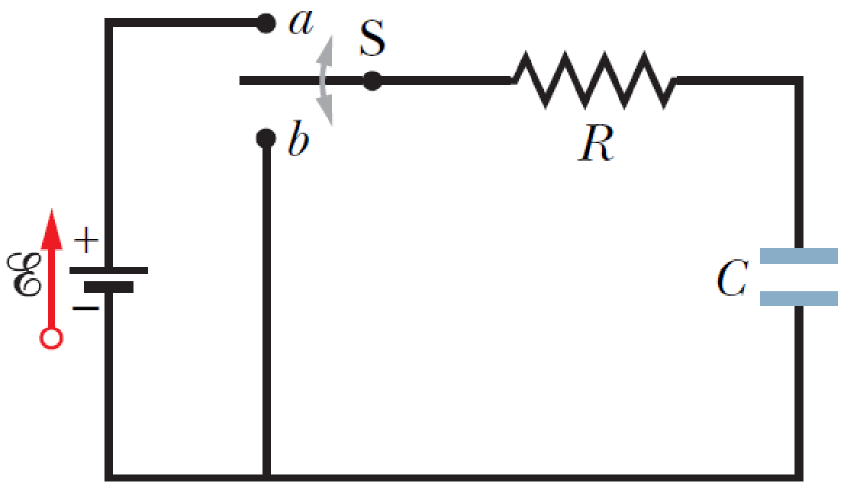
The charging times for RC circuits are often stated in terms of τ.



7. *RC* Circuits

Discharging a Capacitor

Assume now that the capacitor is fully charged to a potential V_0, equal to the emf $\mathcal{E}$. The switch is thrown from a to b, so that the capacitor can discharge through the resistor R.



The differential equation describing $q(t)$ is as before, but with $\mathcal{E} = 0$. Thus,

$$R\frac{dq}{dt} + \frac{q}{C} = 0.$$

7. *RC* Circuits

Discharging a Capacitor

A solution to this differential equation is

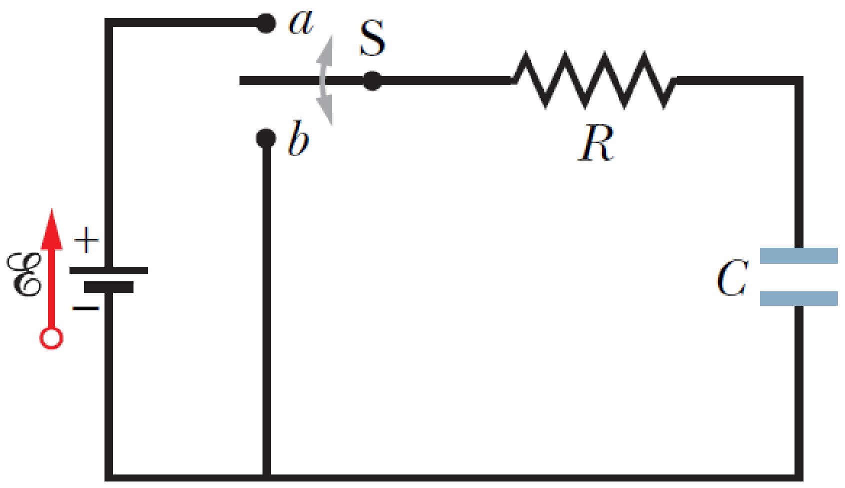
$$q(t) = q_0 e^{-t/RC},$$

where $q_0 = CV_0$. Note that $q(0) = q_0$. Additionally, $q(\tau) = q_0/e = 0.37q_0$.

Differentiating $q(t)$ gives us $i(t)$:

$$i = \frac{dq}{dt} = -\left(\frac{q_0}{RC}\right)e^{-\frac{t}{RC}}.$$

Note that $i(0) = i_0 = q_0/RC = V_0/R$. The minus sign means that the capacitor's charge is decreasing.



7. *RC* Circuits

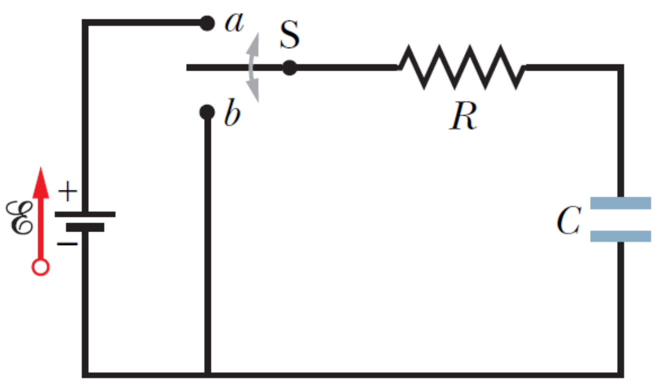
CHECKPOINT 5

The table gives four sets of values for the circuit elements in Fig. 27-15. Rank the sets according to (a) the initial current (as the switch is closed on *a*) and (b) the time required for the current to decrease to half its initial value, greatest first.

	1	2	3	4
$\mathscr{E}$ (V)	12	12	10	10
R (Ω)	2	3	10	5
C (μF)	3	2	0.5	2

a) 1,2,4,3.

b) 4, 1 & 2 tie, 3.

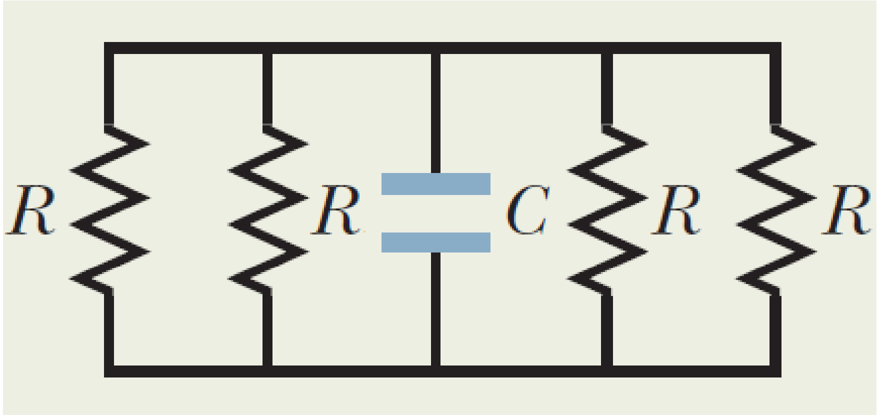


7. *RC* Circuits

Example 4: The figure shows a circuit whose elements have the following values:

$$C = 500\ \mathrm{pF}, \qquad R = 100\ \mathrm{G\Omega},$$

At time $t = 0$, the potential V_0 between the capacitor plates is $30\ \mathrm{kV}$. How much time does the capacitor take to discharge through the resistors so that its stored potential energy be $50\ \mathrm{mJ}$?

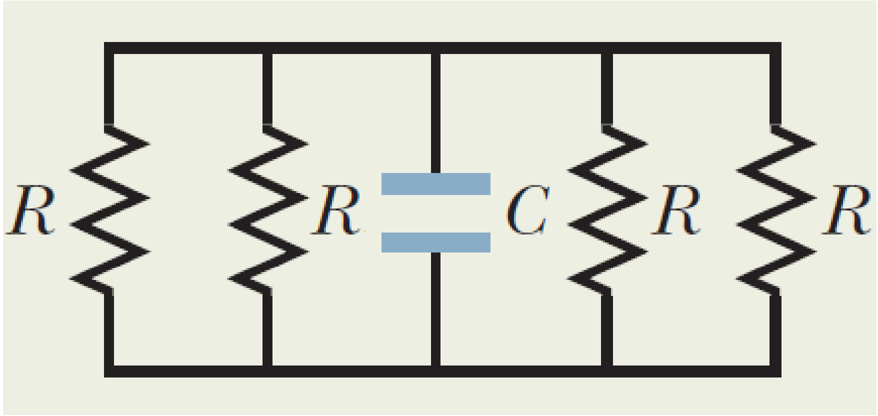


7. *RC* Circuits

The four resistances are connected in parallel. Their equivalent resistance is given by

$$\frac{1}{R_{\text{eq}}} = \frac{4}{R}.$$

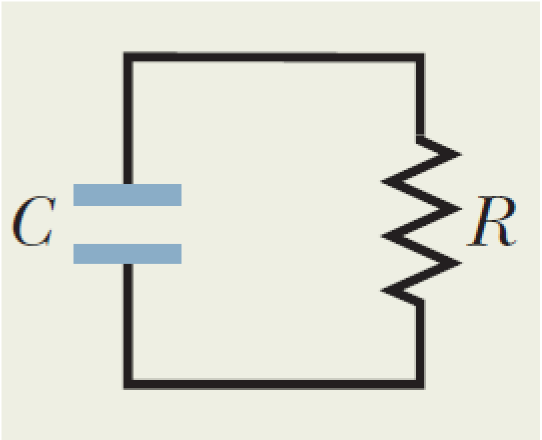
Thus, $R_{\text{eq}} = R/4 = 25\ \text{G}\Omega$.



The stored potential energy U and the charge q on the capacitor's plates are related by

$$U = \frac{q^2}{2C} = \frac{\left(q_0 e^{-t/RC}\right)^2}{2C} = \frac{\left(CV_0 e^{-t/RC}\right)^2}{2C}$$

$$= \frac{CV_0^2 e^{-2t/RC}}{2}.$$



7. *RC* Circuits

Solving for t and substituting we find

$$t = -\frac{RC}{2}\ln\frac{2U}{CV_0^2}$$

$$= -\frac{(25\ \mathrm{G\Omega})(500\ \mathrm{pF})}{2}\ln\frac{2(50\ \mathrm{mJ})}{(500\ \mathrm{pF})(30\ \mathrm{kV})^2}$$

$$= 9.4\ \mathrm{s}.$$