Chapter 26

Current and Resistance

1. Electric Current

Electric current through a given surface is the net flow of charge though that surface.

The free electrons in a copper wire are in random motion. Electrons pass though a hypothetical surface across the wire in both directions. The net electron transport through the surface is zero and thus no current in the wire. When we connect the ends of the wire to a battery, we create an electric field across the wire that results in a net electron transport in one direction and thus electric current through the wire.

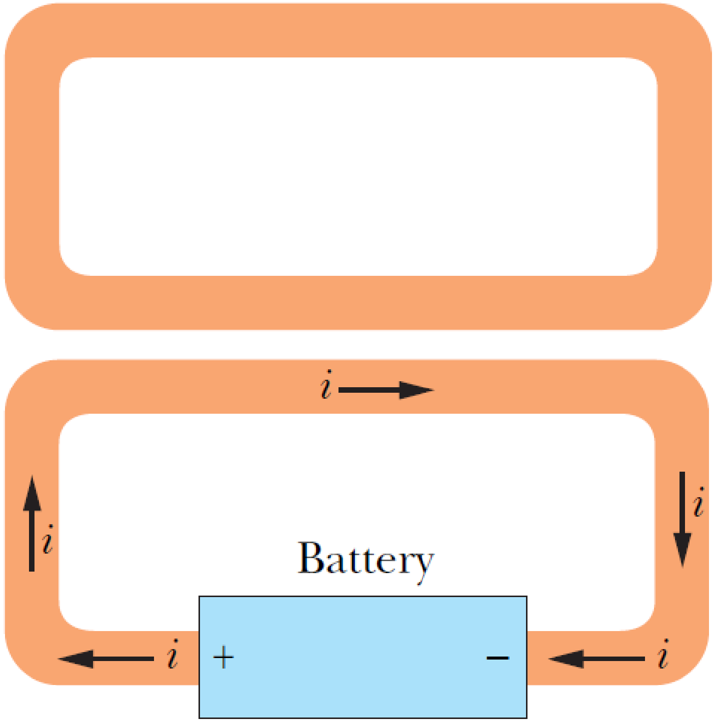
As water flows through a pipe, positive charge flow (protons) in the same direction as that of the water. However, there is no net transport of charge because there is an exactly equal flow of negative charge (electrons) in the same direction.

1. Electric Current

In this chapter we discuss steady currents of conduction electrons moving through metallic conductors such as a copper wire.

The figure shows an isolated conducting loop over which the potential is constant. Although conduction electrons are available, no net electric force acts on them and thus no current.

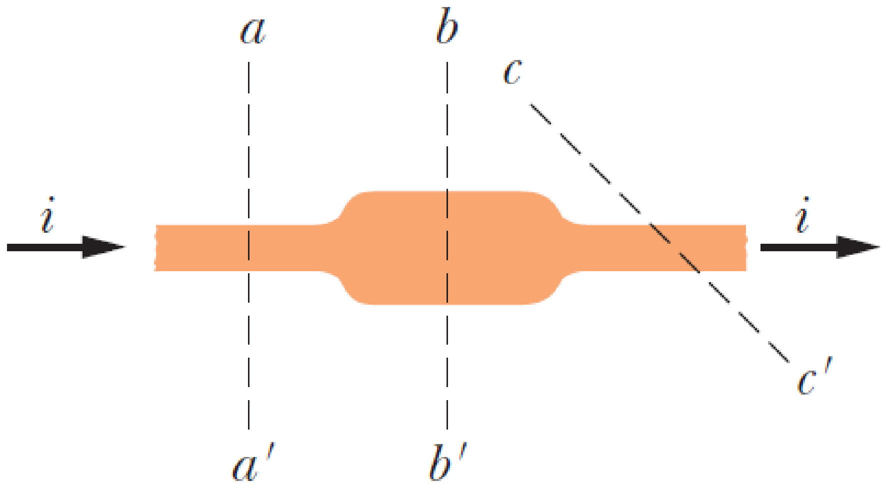
Now we insert a battery in the loop. The conducting loop is no longer at a single potential difference and electric fields acts on the conducting electrons causing them to move and thus establishing a current. The electrons flow reaches a constant value (steady state) in very short time.



1. Electric Current

The figure shows a part of a conductor in which current has been established. If charge dq passes through a hypothetical plane (such as aa') in time dt, the current i through the plane is defined as

$$i = \frac{dq}{dt}.$$

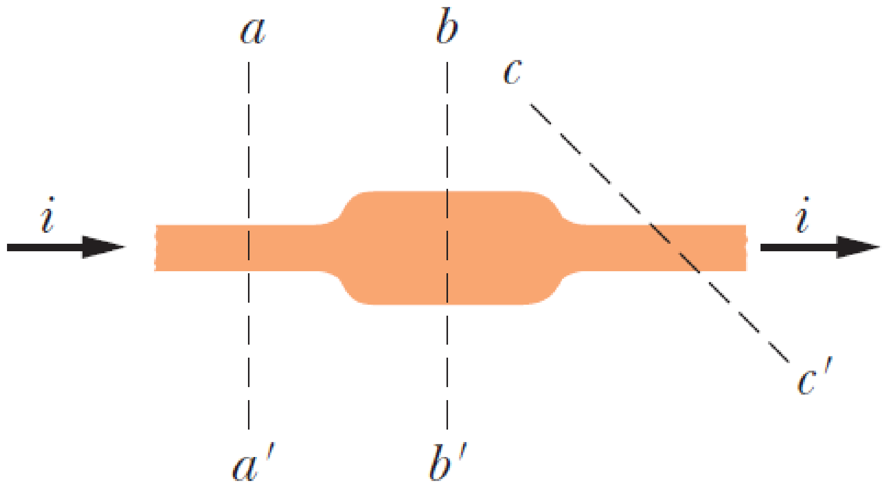


The charge q that passes through the plane in a time interval, between 0 and t, is

$$q = \int dq = \int_0^t i dt.$$

1. Electric Current

Under steady state conditions, the current is the same for planes aa', bb' and cc' or any other current that passes completely though the conductor. This follows from charge conservation.



The SI unit for current is coulomb per second, or the ampere (A):

$$1\ \text{ampere} = 1\ \text{A} = 1\ \text{C/s}.$$

1. Electric Current

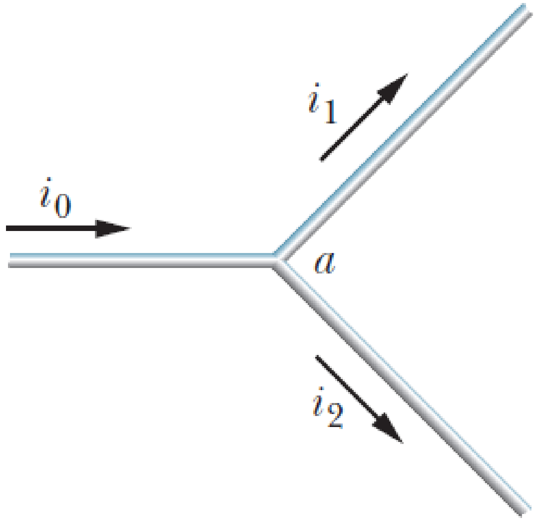
We often represent a current with an arrow to indicate that charge is moving. The arrow does not mean that current is a vector.

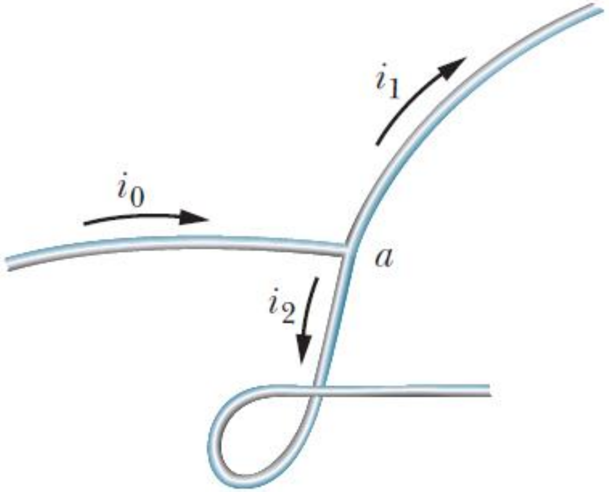
Consider the situation shown in the upper figure. Because charge is conserved

$$i_0 = i_1 + i_2.$$

If we bend the wires, as shown in the second figure, the currents remain the same and $i_0 = i_1 + i_2$ remains valid.

Currents arrows show only a direction of flow along a wire, not a direction in space.





1. Electric Current

The Direction of Currents

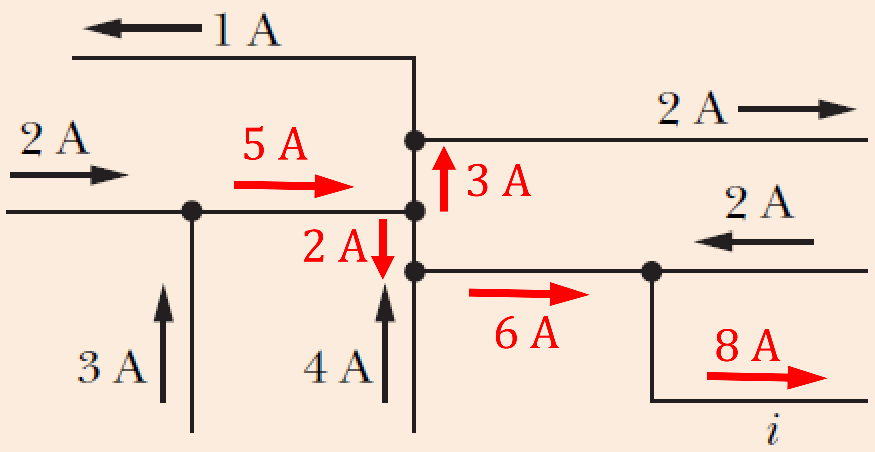
The current arrows are in the direction in which *positively charged particles would be forced to move* due to an electric field. In conductors, it is electrons that can move. However, we will use the following convention:

A current arrow is drawn in the direction in which positive charge carriers would move, even if the actual charge carriers are negative and move in the opposite direction.

1. Electric Current

CHECKPOINT 1

The figure here shows a portion of a circuit. What are the magnitude and direction of the current *i* in the lower right-hand wire?



1. Electric Current

Example 1: Water flows through a garden hose at a volume flow rate dV/dt of 450 cm^3/s. What is the current of negative charge?

$$i = \begin{pmatrix}\text{charge}\\\text{per}\\\text{electron}\end{pmatrix}\begin{pmatrix}\text{electrons}\\\text{per}\\\text{molecule}\end{pmatrix}\begin{pmatrix}\text{molecules}\\\text{per}\\\text{second}\end{pmatrix} = e\,(10)\,\frac{dN}{dt}.$$

$$\frac{dN}{dt} = \begin{pmatrix}\text{molecules}\\\text{per}\\\text{mole}\end{pmatrix}\begin{pmatrix}\text{moles}\\\text{per unit}\\\text{mass}\end{pmatrix}\begin{pmatrix}\text{mass}\\\text{per unit}\\\text{volume}\end{pmatrix}\begin{pmatrix}\text{volume}\\\text{per}\\\text{second}\end{pmatrix}$$

$$= N_A\left(\frac{1}{M}\right)(\rho_w)\left(\frac{dV}{dt}\right) = \frac{N_A\rho_w}{M}\frac{dV}{dt}.$$

1. Electric Current

Example 1: Water flows through a garden hose at a volume flow rate dV/dt of 450 $\mathrm{cm^3/s}$. What is the current of negative charge?

$$i = \frac{10eN_A\rho_w}{M}\frac{dV}{dt}$$

$$= \frac{10(1.60 \times 10^{-19}\ \mathrm{C})(6.02 \times 10^{23}\ \mathrm{mol^{-1}})(1.00\ \mathrm{g/cm^3})}{18.0\ \mathrm{g/mol}}\frac{450\ \mathrm{cm^3}}{\mathrm{s}}$$

$$= 2.41 \times 10^7\ \mathrm{A}.$$

2. Current Density

Sometimes we are interested in the flow of charge through a cross section of the conductor at a particular point. To describe this flow, we use the current density $\vec{J}$. It has the same direction as that of the velocity of the moving charges if they are positive and the opposite direction if they negative.

The magnitude J gives the current per unit area through an element of the cross section.

The amount of current through an area element dA is $\vec{J} \cdot d\vec{A}$. The total current through the surface is then

$$i = \int \vec{J} \cdot d\vec{A}.$$

2. Current Density

If $\vec{J}$ is uniform across a surface and parallel to $d\vec{A}$ then

$$i = \int J dA = J \int dA = JA,$$

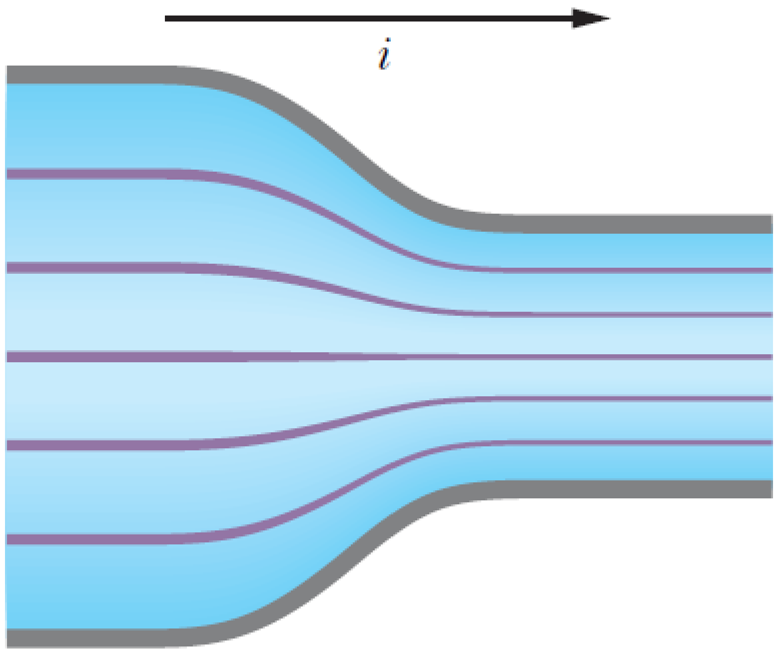
or

$$J = \frac{i}{A}.$$

The SI unit of current density is $(\mathrm{A/m^2})$.

2. Current Density

We can represent current density with a set of lines, called **streamlines**, similar to field lines. The amount of current cannot change during the transition, however, the current density can change. It is greater in narrower regions.



2. Current Density

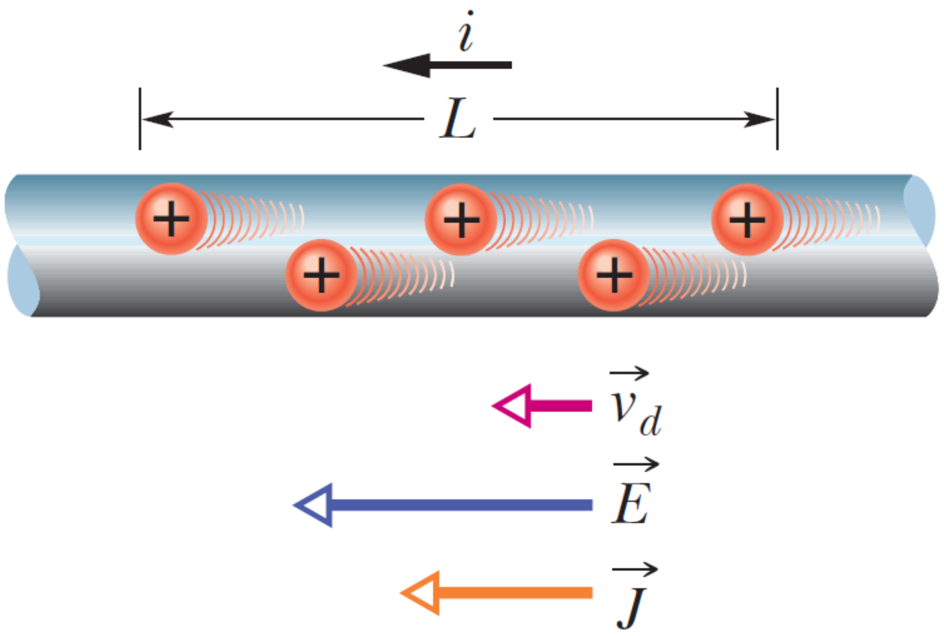
<u>Drift Speed</u>

When an electric field is established in a conductor, the charge carriers (assumed positive) acquire a drift speed v_d in the direction of the velocity is related to the current density by

$$\vec{J} = (ne)\vec{v}_d,$$

where ne is the charge carrier density (in $\mathrm{C/m^3}$).

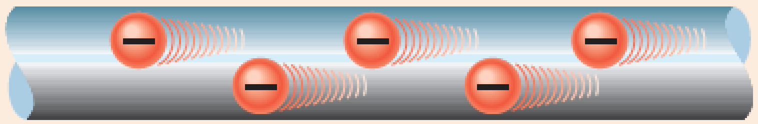
For positive charge carriers, ne is positive and $\vec{J}$ and $\vec{v}_d$ have the same direction. For negative charge carriers, ne is negative and $\vec{J}$ and $\vec{v}_d$ are opposite in direction.



2. Current Density

CHECKPOINT 2

The figure shows conduction electrons moving leftward in a wire. Are the following leftward or rightward: (a) the current i, (b) the current density $\vec{J}$, (c) the electric field $\vec{E}$ in the wire?



(a) Rightward.

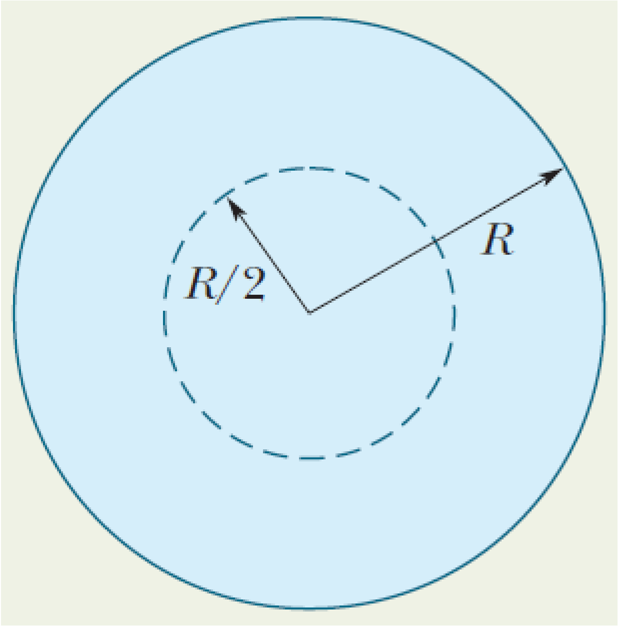
(b) Rightward.

(c) Rightward.

2. Current Density

Example 2:

(a) The current density in a cylindrical wire of radius R $= 2.0$ mm is uniform across a cross section of the wire and is $J = 2.0 \times 10^5\ \mathrm{A/m^2}$. What is the current through the outer portion of the wire between radial distances $R/2$ and R?

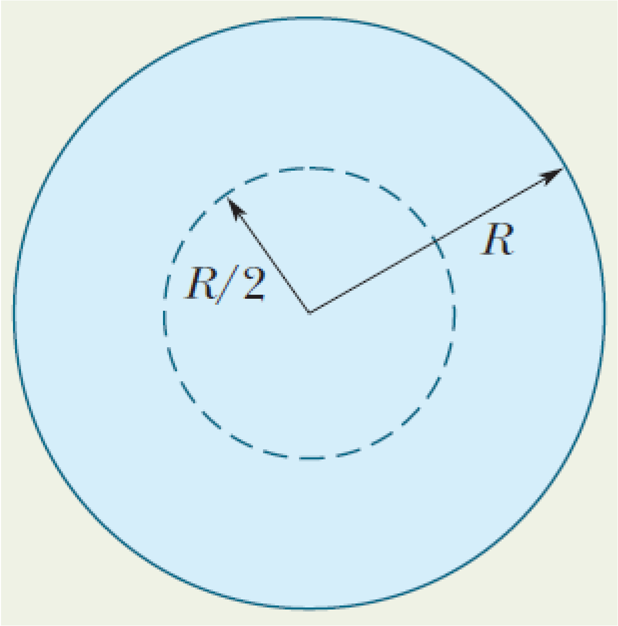


$$i = JA' = J\left[\pi R^2 - \pi\left(\frac{R}{2}\right)^2\right] = J\left(\frac{3}{4}\pi R^2\right)$$

$$= (2.0 \times 10^5\ \mathrm{A/m^2})\left[\frac{3}{4}\pi(2.0\ \mathrm{mm})^2\right] = 1.9\ \mathrm{A}.$$

2. Current Density

(b) Suppose, instead, that the current density through a cross section varies with radial distance r as $J = ar^2$, in which $a = 3.0 \times 10^{11}\ \mathrm{A/m^4}$ and r is in meters. What now is the current through the same outer portion of the wire?



The current density is not constant across a cross section of the wire. We need to use the integral $i = \int \vec{J} \cdot d\vec{A}$.

Both $\vec{J}$ and $d\vec{A}$ are perpendicular to a cross section of the wire. Thus,

$$\vec{J} \cdot d\vec{A} = J dA \cos 0 = J dA.$$

2. Current Density

The differential area reads

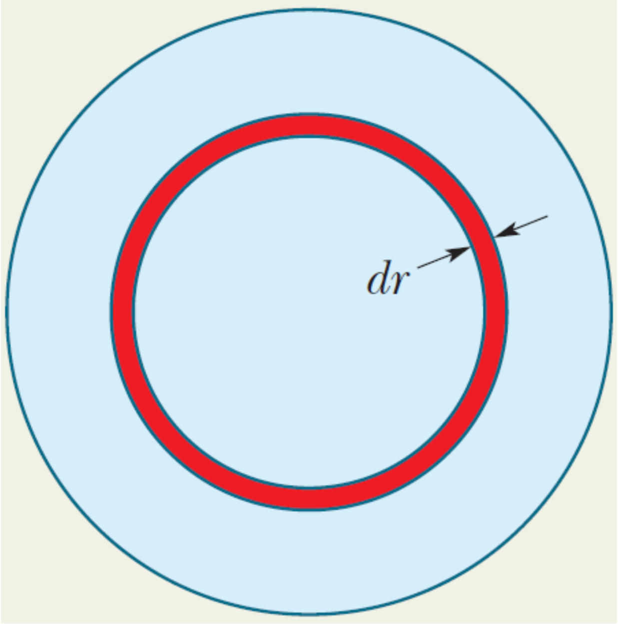
$$dA = (2\pi r)dr = 2\pi r dr.$$

The current now becomes

$$i = \int \vec{J} \cdot d\vec{A} = \int_{R/2}^{R} (ar^2)(2\pi r dr) = 2\pi a \int_{R/2}^{R} r^3 dr$$

$$= 2\pi a \left[\frac{r^4}{4}\right]_{R/2}^{R} = \frac{\pi a}{2}\left[R^4 - \left(\frac{R}{2}\right)^4\right] = \frac{15}{32}\pi a R^4$$

$$= \frac{15}{32}\pi\left(3.0 \times 10^{11}\ \frac{\mathrm{A}}{\mathrm{m}^4}\right)(0.0020\ \mathrm{m})^4 = 7.1\ \mathrm{A}.$$



3. Resistance and Resistivity

The current in a conductor, which we apply a potential difference between its ends, is determined by the electrical **resistance** of the conductor.

We can determine the resistance between any two points of a conductor by applying a potential difference V between those two points and measure the resulting current i. The resistance R is then

$$R = \frac{V}{i}.$$

The SI unit for resistance is the volt per ampere, which has the special name ohm (Ω):

$$1 \text{ ohm} = 1\ \Omega = 1 \text{ volt per ampere} = 1 \text{ V/A}.$$

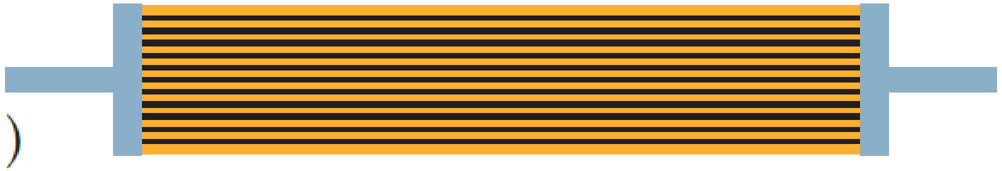
3. Resistance and Resistivity

A conductor whose function is to provide a specified resistance is called a resistor. We represent a resistor and resistance with the symbol  or  .

For a given V, $i = V/R$. The greater the resistance the smaller the current.

The resistance of a conductor depends on the way in which the potential difference is applied.

(a) 

(b) 

Unless otherwise stated, we will assume that any potential difference is applied as in figure (b).

3. Resistance and Resistivity

The **resistivity** ρ of an isotropic (having the same properties in all directions) resistive material at a point is related to the electric field $\vec{E}$ and current density $\vec{J}$ at that point by

$$\rho = \frac{E}{J}.$$

The SI unit of resistivity is ohm-meter ($\Omega \cdot \mathrm{m}$).

Resistivities of Some Materials at Room Temperature (20°C)

Material	Resistivity, ρ ($\Omega \cdot \mathrm{m}$)	Temperature Coefficient of Resistivity, α (K^{-1})
	Typical Metals	
Silver	1.62×10^{-8}	4.1×10^{-3}
Copper	1.69×10^{-8}	4.3×10^{-3}
Gold	2.35×10^{-8}	4.0×10^{-3}
Aluminum	2.75×10^{-8}	4.4×10^{-3}
Manganin[a]	4.82×10^{-8}	0.002×10^{-3}
Tungsten	5.25×10^{-8}	4.5×10^{-3}
Iron	9.68×10^{-8}	6.5×10^{-3}
Platinum	10.6×10^{-8}	3.9×10^{-3}
	Typical Semiconductors	
Silicon, pure	2.5×10^{3}	-70×10^{-3}
Silicon, *n*-type[b]	8.7×10^{-4}	
Silicon, *p*-type[c]	2.8×10^{-3}	
	Typical Insulators	
Glass	$10^{10} - 10^{14}$	
Fused quartz	$\sim 10^{16}$	

3. Resistance and Resistivity

We can rewrite the last equation in vector form:

$$\vec{J} = \frac{\vec{E}}{\rho}.$$

The **conductivity** σ of a material is defined as

$$\sigma = \frac{1}{\rho}.$$

The SI unit of conductivity is $(\Omega \cdot \mathrm{m})^{-1}$ (mohs per meter ℧/m). We can rewrite the first equation as

$$\vec{J} = \sigma\vec{E}.$$

3. Resistance and Resistivity

Calculating Resistance from Resistivity

Resistance is a property of an object. Resistivity is a property of a material.

Let us write the resistance of a resistor made of material of resistivity ρ, cross-sectional area A and length L. Assuming the electric field and current density are constant over the resistor, we write

$$\rho = \frac{E}{J} = \frac{V/L}{i/A} = \frac{A}{L}\frac{V}{i} = R\frac{A}{L}.$$

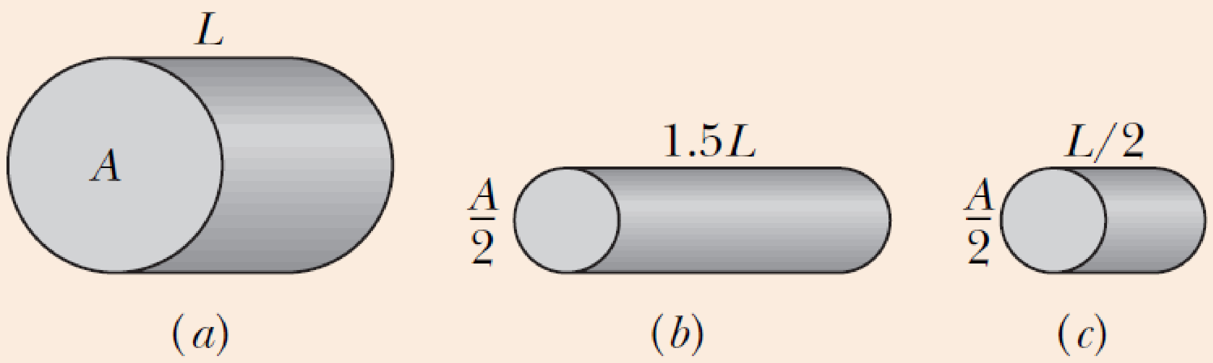
or

$$R = \rho\frac{L}{A}.$$

3. Resistance and Resistivity

CHECKPOINT 3

The figure here shows three cylindrical copper conductors along with their face areas and lengths. Rank them according to the current through them, greatest first, when the same potential difference V is placed across their lengths.



(a) and (c) tie, then (b).

$$i = \frac{V}{R} = \frac{A}{\rho L} V$$

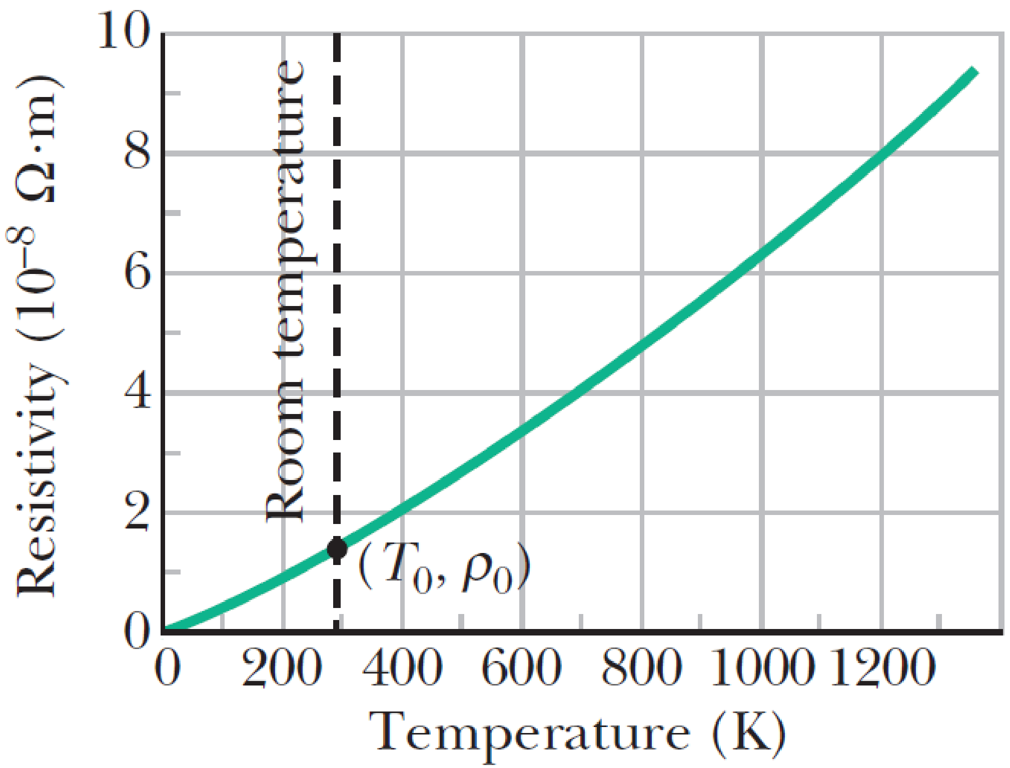
3. Resistance and Resistivity

Variation with Temperature

The resistivity of a material varies with temperature. The relation between temperature and resistivity for metals is approximately linear. Thus, we can write an approximate relation between resistivity and temperature:

$$\rho - \rho_0 = \rho_0 \alpha (T - T_0).$$

Here T_0 is a selected reference temperature and ρ_0 is the corresponding resistivity. The quantity α is the temperature coefficient of resistivity.



3. Resistance and Resistivity

Example 3: A rectangular block of iron has dimensions $1.2\ \mathrm{cm} \times 1.2\ \mathrm{cm} \times 15\ \mathrm{cm}$. A potential difference is to be applied to the block between parallel sides and in such a way that those sides are equipotential surfaces.

(a) What is the resistance of the block if the two parallel sides are the square ends (with dimensions $1.2\ \mathrm{cm} \times 1.2\ \mathrm{cm}$)?

$$A = 1.2\ \mathrm{cm} \times 1.2\ \mathrm{cm} = 1.44 \times 10^{-4}\ \mathrm{m}^2.$$

$$R = \rho \frac{L}{A} = (9.68 \times 10^{-8}\ \Omega \cdot \mathrm{m}) \frac{0.15\ \mathrm{m}}{1.44 \times 10^{-4}\ \mathrm{m}^2} = 100\ \mu\Omega.$$

3. Resistance and Resistivity

(b) What is the resistance of the block if the two parallel sides are the two rectangular sides (with dimensions $1.2\ \mathrm{cm} \times 15\ \mathrm{cm}$)?

$$A = 1.2\ \mathrm{cm} \times 15\ \mathrm{cm} = 1.8 \times 10^{-4}\ \mathrm{m}^2.$$

$$R = \rho \frac{L}{A} = (9.68 \times 10^{-8}\ \Omega \cdot \mathrm{m}) \frac{0.012\ \mathrm{m}}{1.8 \times 10^{-3}\ \mathrm{m}^2} = 0.65\ \mu\Omega.$$

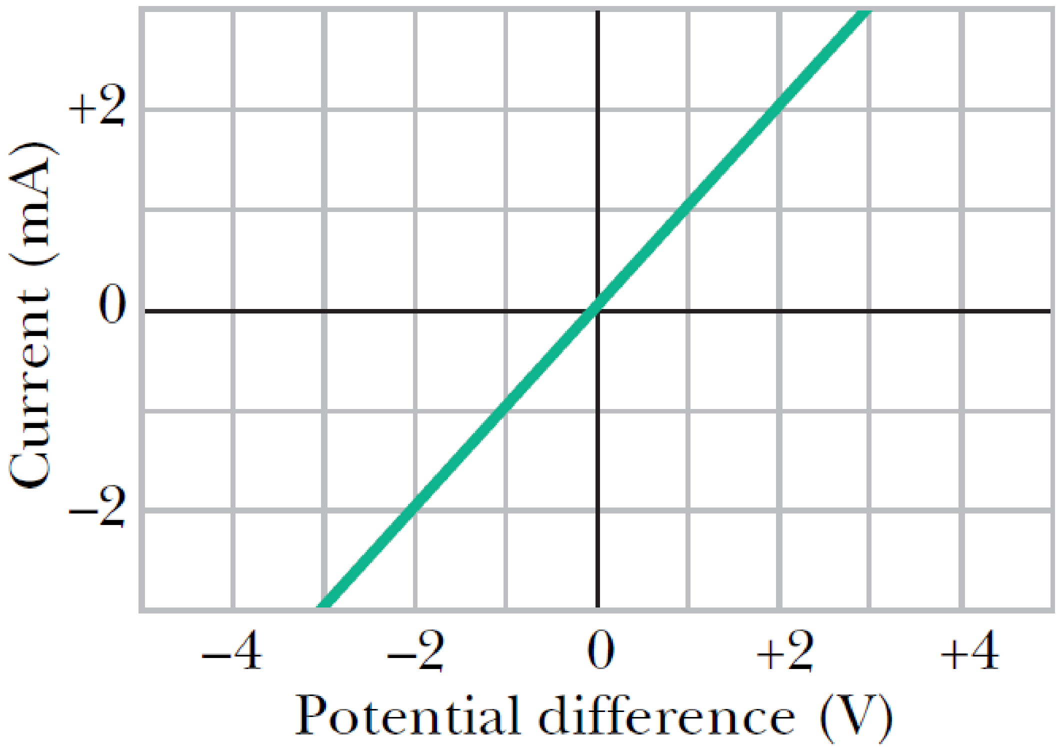
4. Ohm's Law

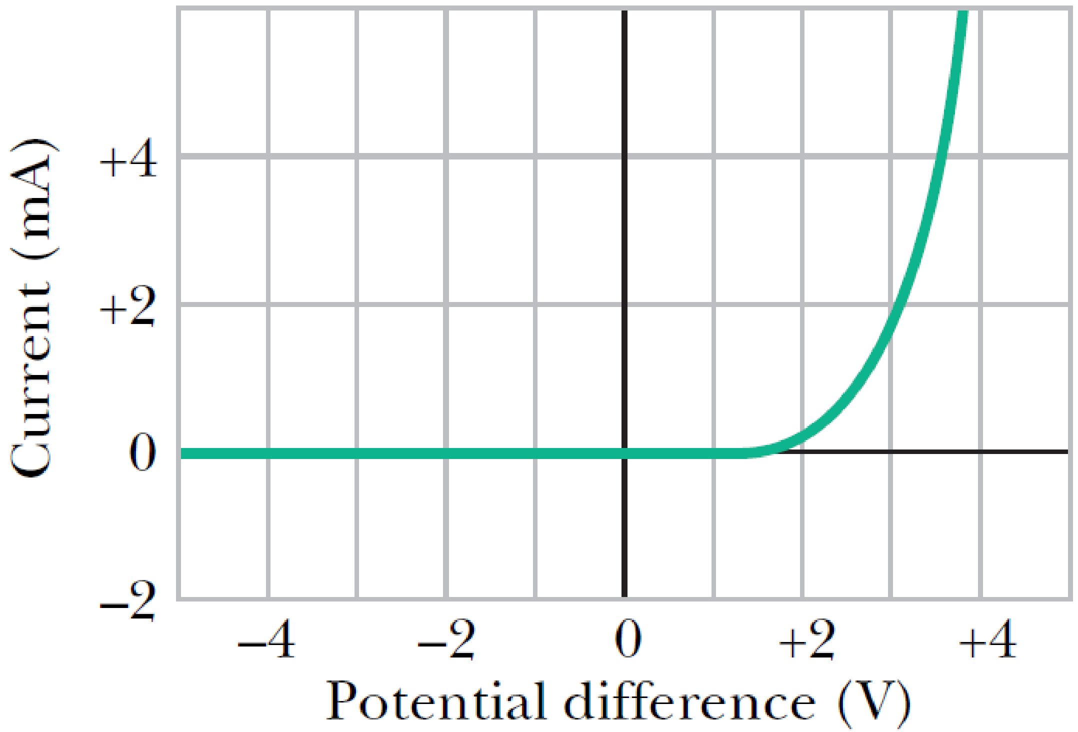
Ohm's law is an assertion that the current through a device is *always* directly proportional to the potential difference applied to the device.

A conducting device obeys Ohm's law when the resistance of the device is independent of the magnitude and polarity of the applied potential difference.

A conducting material obeys Ohm's law when the resistivity of the material is independent of the magnitude and direction of the applied electric field.

4. Ohm's Law





$V = iR$

$V = iR(V)$

4. Ohm’s Law

CHECKPOINT 4

The following table gives the current i (in amperes) through two devices for several values of potential difference V (in volts). From these data, determine which device does not obey Ohm’s law.

Device 1		Device 2	
V	i	V	i
2.00	4.50	2.00	1.50
3.00	6.75	3.00	2.20
4.00	9.00	4.00	2.80

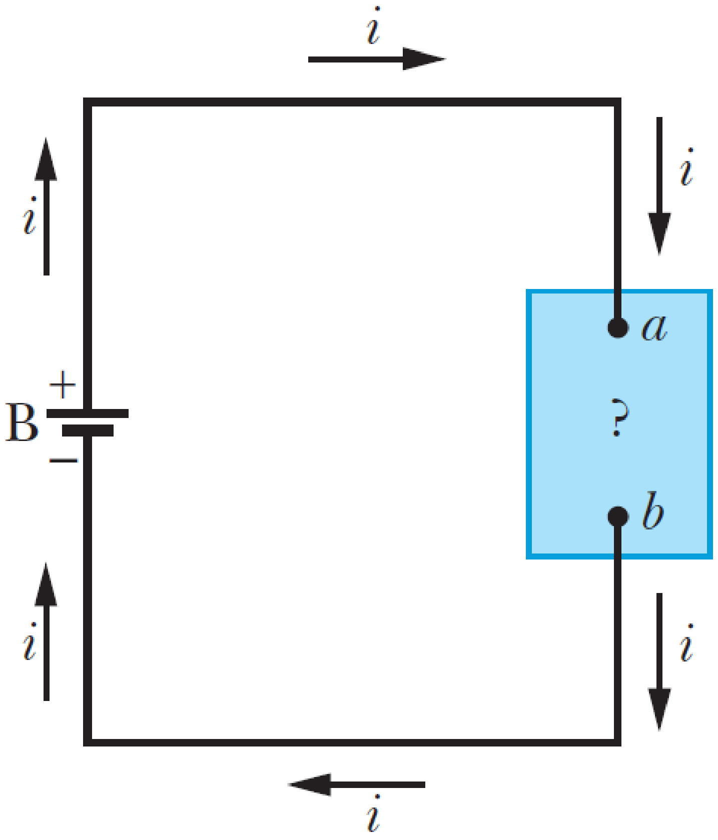
Device 2

5. Power in Electric Circuits

The figure shows an electric circuit consisting of a battery B connected by wires of negligible resistance to a conducting device. The battery maintains a potential difference V across its own terminals and thus across the terminals of the device. A steady current i is produced in the circuit, directed from terminal a to terminal b.

The amount of charge dq that moves between the terminals in time dt is idt. This charge moves through a decrease in potential of magnitude V, and thus its electric potential energy decreases in magnitude by the amount

$$dU = dq\,V = i\,dt\,V.$$



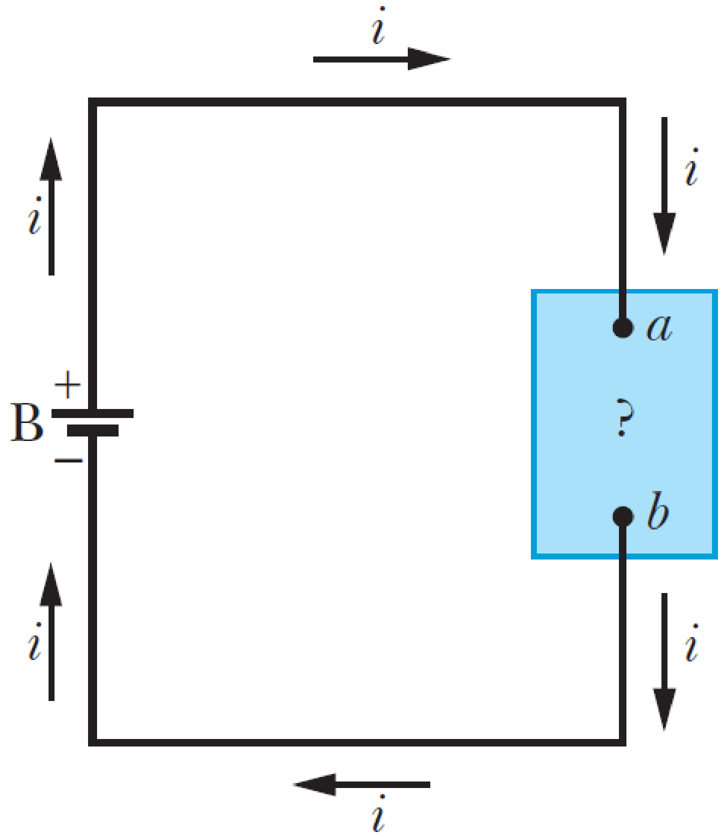
5. Power in Electric Circuits

The energy is transferred in the device to some other form. The power P associated with the energy transfer is the rate of transfer dU/dt. Thus,

$$P = iV.$$

This power P is also the rate at which energy is transferred from the battery to the device.

In resistors or devices with resistance, electrons' potential energy is transferred to the resistor or device as heat via collisions.



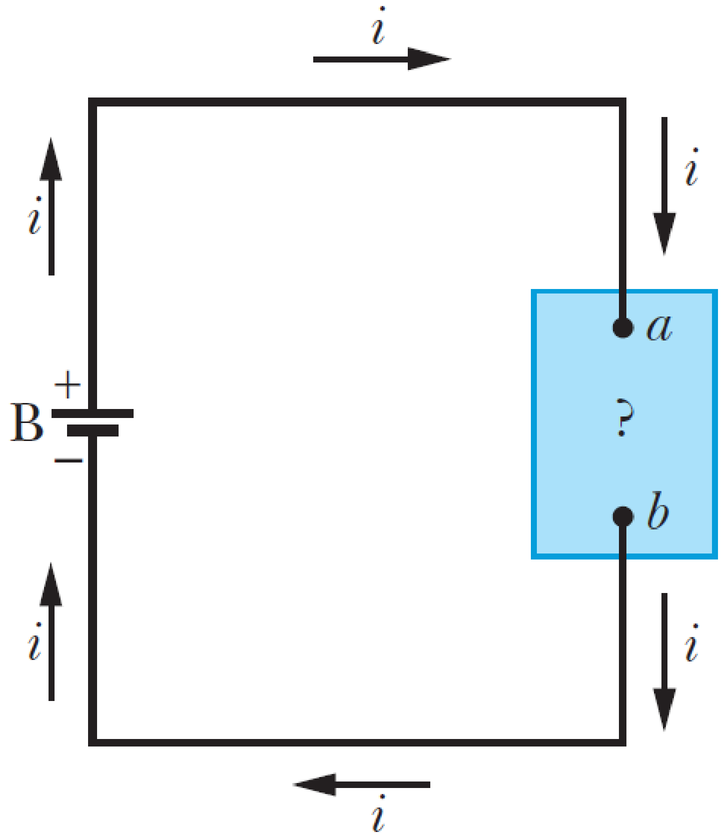
5. Power in Electric Circuits

For a resistor or some other device with resistance R, the rate of electrical energy dissipation due to the resistance is

$$P = i^2 R,$$

or

$$P = V^2/R.$$



5. Power in Electric Circuits

CHECKPOINT 5

A potential difference V is connected across a device with resistance R, causing current i through the device. Rank the following variations according to the change in the rate at which electrical energy is converted to thermal energy due to the resistance, greatest change first: (a) V is doubled with R unchanged, (b) i is doubled with R unchanged, (c) R is doubled with V unchanged, (d) R is doubled with i unchanged.

(a) and (b), then (d), then (c).

$$P = i^2 R$$

$$P = V^2/R$$

5. Power in Electric Circuits

Example 4: You are given a length of uniform heating wire made of a nickel–chromium–iron alloy called Nichrome; it has a resistance R of $72\ \Omega$. At what rate is energy dissipated in each of the following situations? (a) A potential difference of $120\ \mathrm{V}$ is applied across the full length of the wire. (b) The wire is cut in half, and a potential difference of $120\ \mathrm{V}$ is applied across the length of each half.

(a)

$$P = \frac{V^2}{R} = \frac{(120\ \mathrm{V})^2}{72\ \Omega} = 200\ \mathrm{W}.$$

(b) The resistance of each half is $R/2$. For each half,

$$P' = \frac{V^2}{R/2} = 400\ \mathrm{W}.$$

The power dissipated in both halves is $2P' = 800\ \mathrm{W}$.