Chapter 25

Capacitance

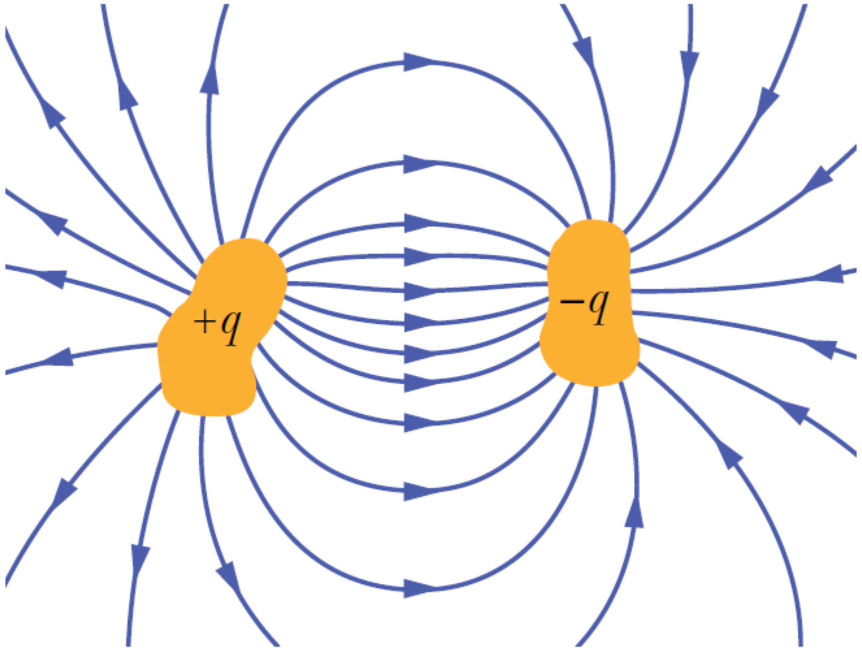
1. Capacitors

A capacitor is a two-terminal device that stores electric energy.

2. Capacitance

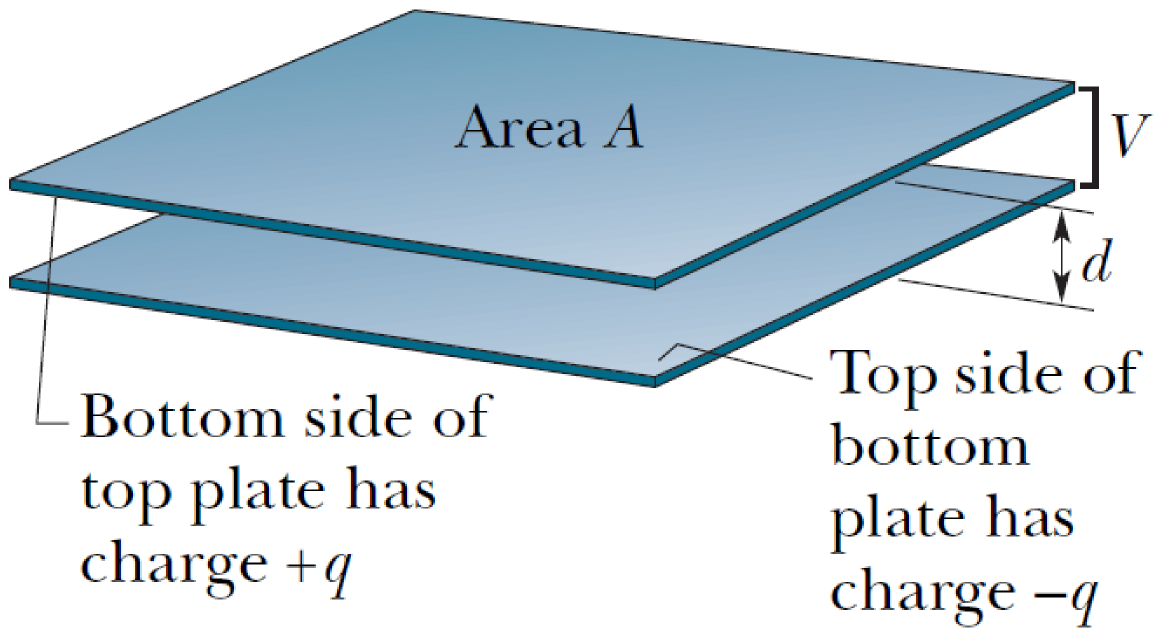
The figure shows the basic elements of *any* capacitor—two isolated conductors of any shape.

These conductors are called plates, regardless of their geometry.



2. Capacitance

The figure shows a less general but more conventional arrangement, called a **parallel-plate capacitor**, consisting of two parallel conducting plates of area A separated by a distance d.



2. Capacitance

When a capacitor is *charged,* its plates have charges of equal magnitudes but opposite signs: $+q$ and $-q$. However, we refer to the *charge of a capacitor* as being q, the absolute value of these charges on the plates.

The plates are equipotential surfaces. Moreover, there is a potential difference between the two plates. For historical reasons, we represent the absolute value of this potential difference with V rather than with the ΔV.

The charge q and the potential difference V are related by

$$q = CV,$$

where C is called the **capacitance** of the capacitor. The capacitance is a measure of how much charge must be put on the plates to produce a certain potential difference between them: The greater the capacitance, the more charge is required. The capacitance depends only on the geometry of a capacitor.

2. Capacitance

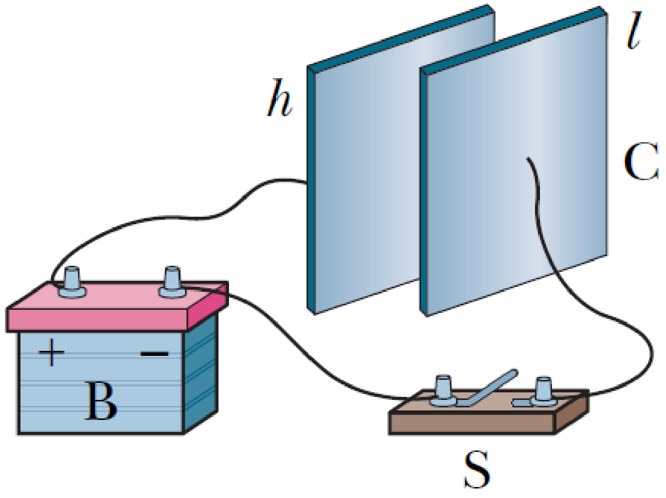
The SI unit of capacitance is the coulomb per volt and has the special name **farad** (F):

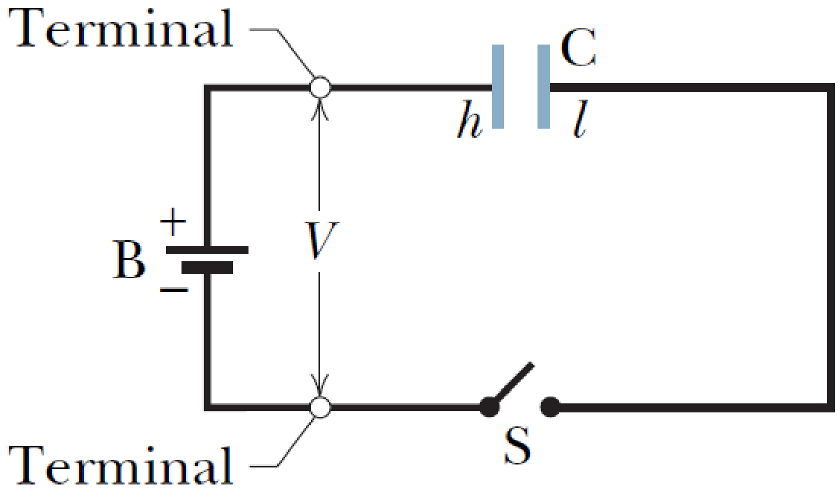
$$1 \text{ farad} = 1 \text{ F} = 1 \text{ coulomb per volt} = 1 \text{ C/V}.$$

2. Capacitance

<u>Charging a Capacitor</u>

To charge a capacitor we place it in an *electric circuit* with a *battery*. An **electric circuit** is a path through which charge can flow. A **battery** is a device that maintains a potential difference between its *terminals* by means of internal electrochemical reactions in which electric force can move internal charge. **Terminals** are points at which charge can enter or leave the battery.





2. Capacitance

CHECKPOINT 1

Does the capacitance C of a capacitor increase, decrease, or remain the same (a) when the charge q on it is doubled and (b) when the potential difference V across it is tripled?

$$q = CV$$

(a) The same.

(b) The same.

3. Calculating the Capacitance

We want here to calculate the capacitance of a capacitor once we know its geometry. To do that, we will

(1) assume a charge of magnitude q lies on the plates,

(2) calculate the electric field $\vec{E}$ between the plates in terms of q using Gauss' law,

(3) calculate the potential difference V between the plates and

(4) calculate C using $q = CV$.

3. Calculating the Capacitance

Calculating the Electric Field

To evaluate the electric field between the plates of a capacitor, we will take a Gaussian surface enclosing the positive charge q. Additionally, we will approximate the electric field $\vec{E}$ to be constant over the Gaussian surface. The charge q and the electric field E are then related by

$$EA = \frac{q}{\varepsilon_0},$$

where A here is the area of the Gaussian surface.

3. Calculating the Capacitance

Calculating the Potential Difference

The potential difference between the plates of a capacitor is related to the electric field $\vec{E}$ by

$$V_f - V_i = -\int_i^f \vec{E} \cdot d\vec{s}\,.$$

We will choose a path that follows an electric field line from the negative plate to the positive plate. Letting V represent $V_f - V_i$, we can write

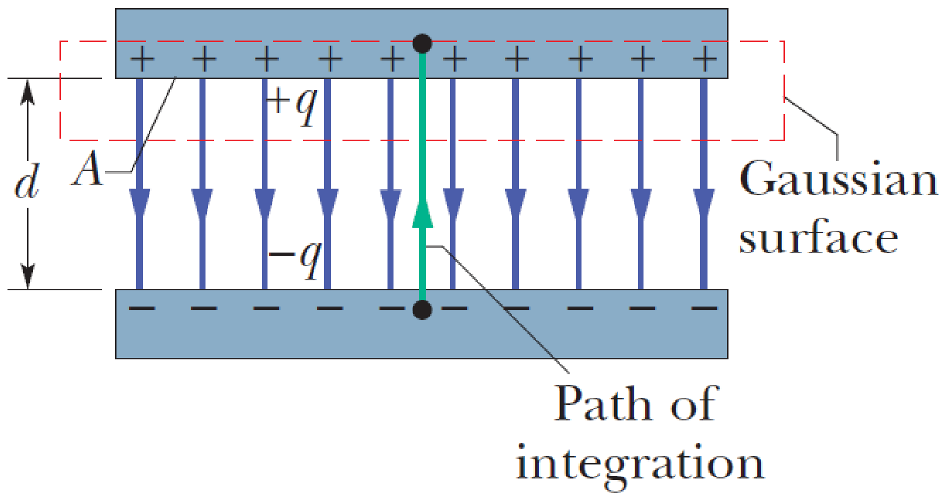
$$V_f - V_i = -\int_-^+ \cos 180^\circ\, E ds = \int_-^+ E ds\,.$$

Let us now find the capacitance of certain charge configuration.

3. Calculating the Capacitance

<u>A Parallel-Plate Capacitor</u>

We assume that the plates of the capacitors are so large and so close together that we can neglect the fringing of $\vec{E}$ near the edges. The electric field is then $E = q/\varepsilon_0 A$.



The potential V is

$$V = \int_{-}^{+} E ds = E \int_{-}^{+} ds = \frac{q}{\varepsilon_0 A} \int_{0}^{d} ds = \frac{qd}{\varepsilon_0 A}.$$

Using $q = CV$ we find that

$$C = \frac{q}{V} = \frac{\varepsilon_0 A}{d}.$$

3. Calculating the Capacitance

A Parallel-Plate Capacitor

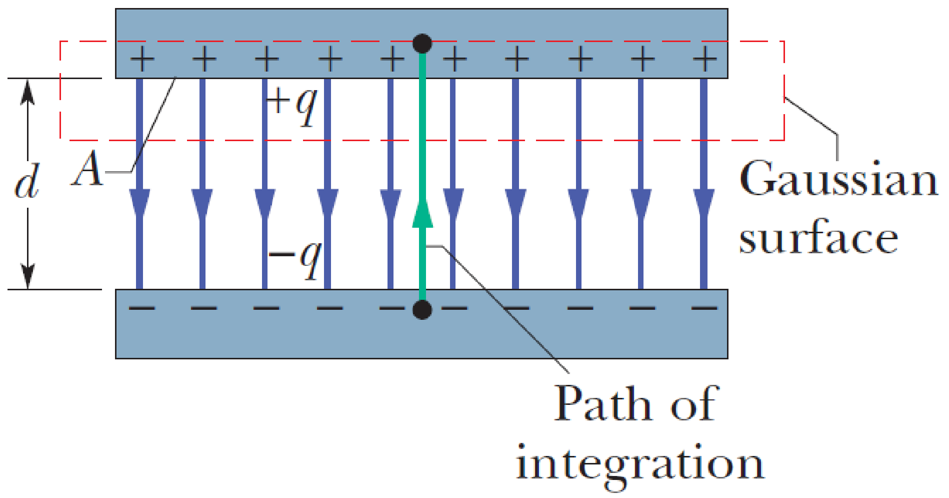
$$C = \frac{\varepsilon_0 A}{d}.$$

A here is the area of the either plate.

Note that the capacitance C depends only on the geometry of the capacitor.

We can express the permittivity constant ε_0 in a unit more appropriate for problems involving capacitors:

$$\varepsilon_0 = 8.85 \times 10^{-12}\ \frac{F}{m} = 8.85\ \frac{pF}{m}.$$



3. Calculating the Capacitance

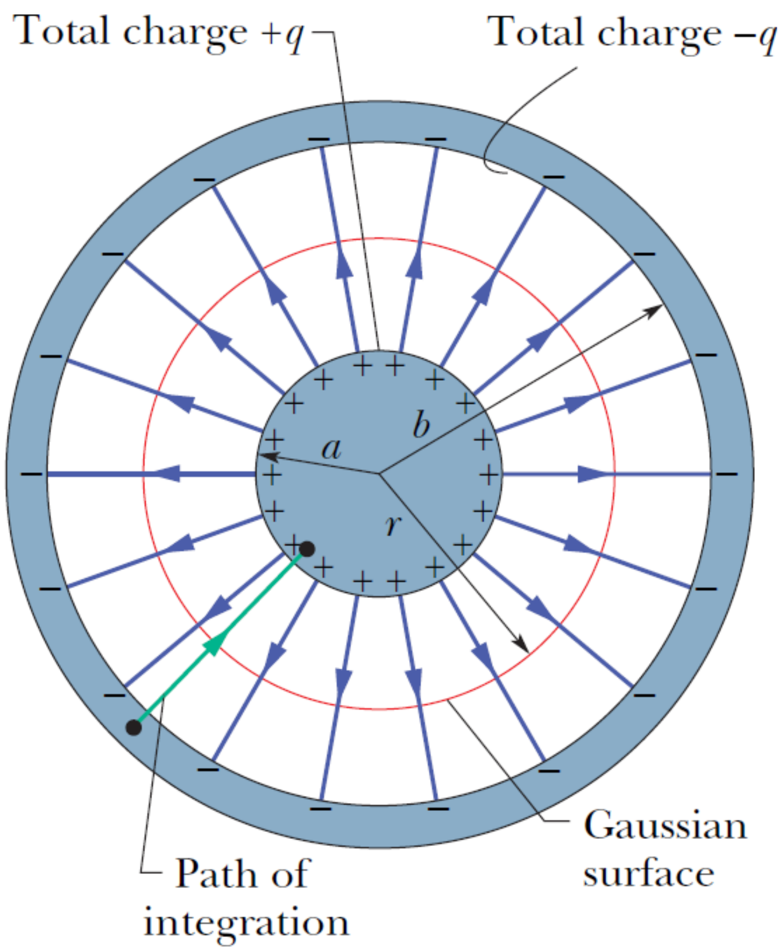
<u>A Cylindrical Capacitor</u>

The figure shows a cross section of a cylindrical capacitor of length L formed by two coaxial cylinders of radii a and b. We assume that $L \gg b$ so that we can neglect the fringing of the electric field near the two edges.

The charge q and the electric field E are related by (using Gauss' law)

$$EA = E(2\pi rL) = \frac{q}{\varepsilon_0},$$

where L and r are the length and radius of the Gaussian surface, respectively.



3. Calculating the Capacitance

<u>A Cylindrical Capacitor</u>

Solving for E we find that

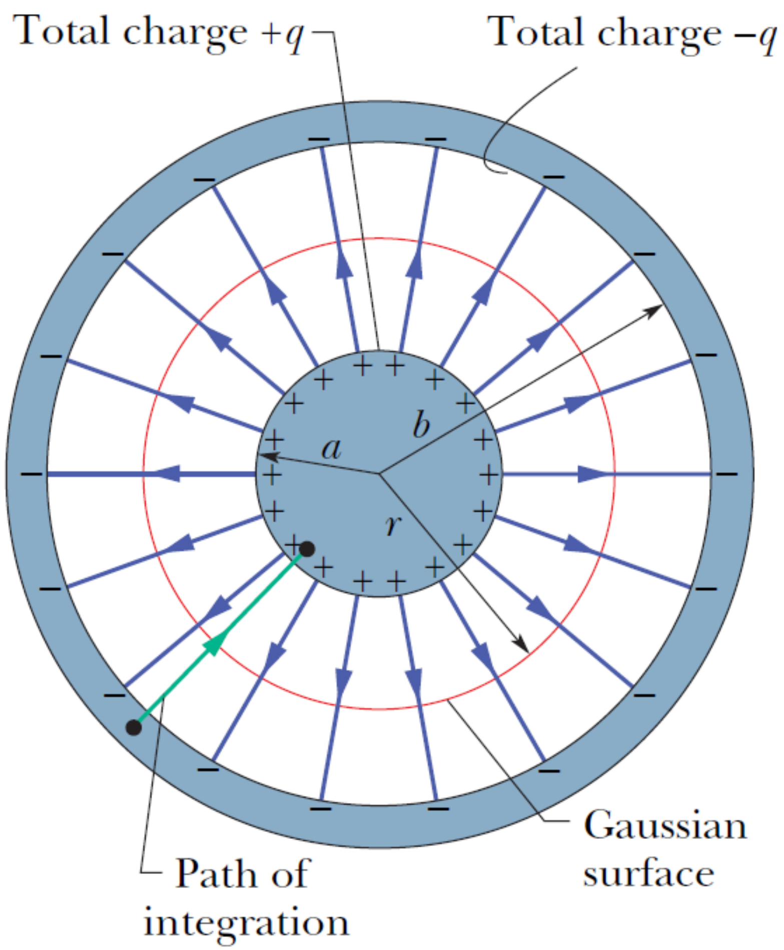
$$E = \frac{q}{2\pi\varepsilon_0 Lr}.$$

The potential difference becomes

$$V = \int_{-}^{+} E ds = \int_{b}^{a} E(-dr) = \frac{-q}{2\pi\varepsilon_0 L}\int_{b}^{a}\frac{dr}{r}$$

$$= \frac{q}{2\pi\varepsilon_0 L}\ln\frac{b}{a}.$$

We used that $ds = -dr$.

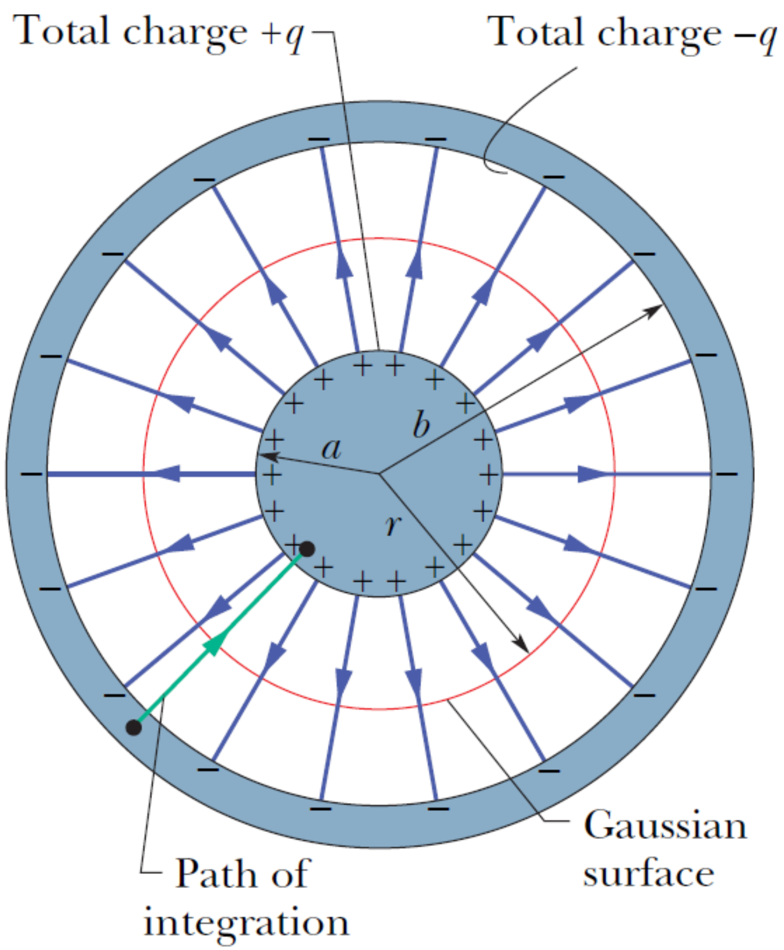


3. Calculating the Capacitance

<u>A Cylindrical Capacitor</u>

Using $q = CV$ we find that

$$C = \frac{q}{V} = 2\pi\varepsilon_0 \frac{L}{\ln b/a}.$$



3. Calculating the Capacitance

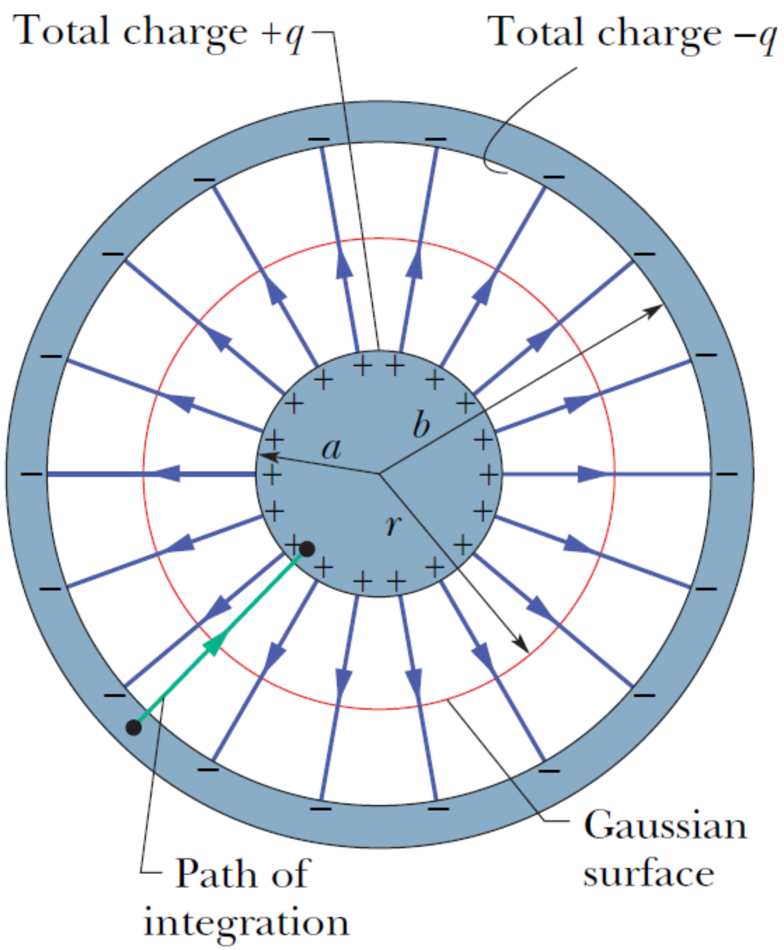
<u>A Spherical Capacitor</u>

The figure is a cross-section of a capacitor that consists of two concentric spherical shells, of radii a and b. Gauss' law gives

$$EA = E(4\pi r^2) = \frac{q}{\varepsilon_0},$$

where r is the radius of the Gaussian surface. Solving for E gives

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}.$$



3. Calculating the Capacitance

<u>A Spherical Capacitor</u>

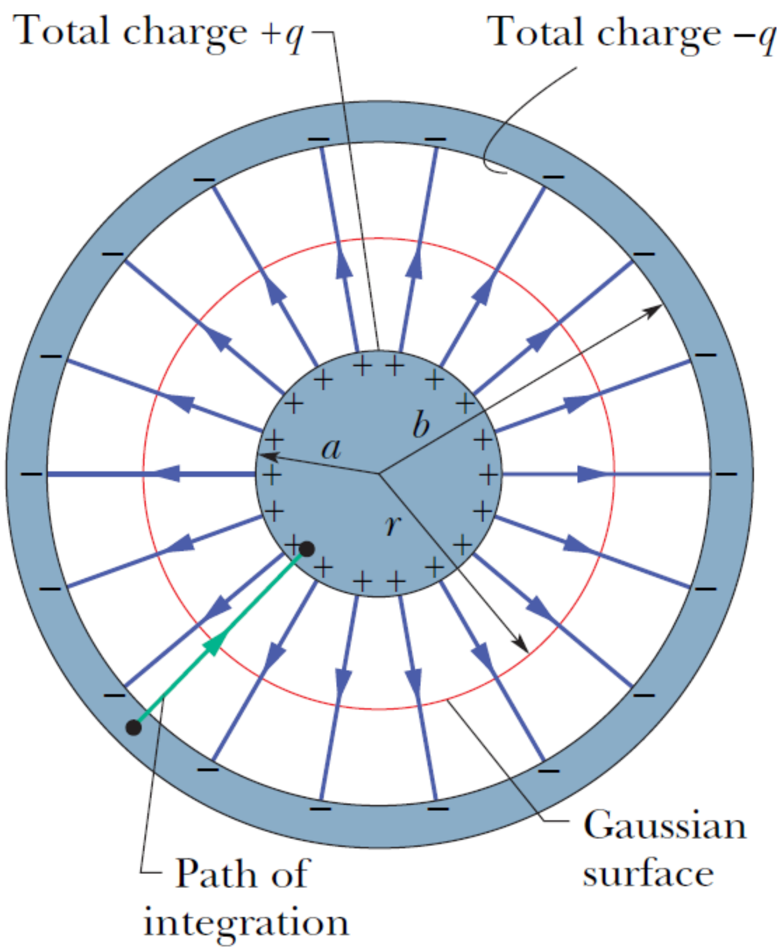
The electric potential is then

$$V = \int_{-}^{+} E ds = \int_{b}^{a} E(-dr) = \frac{-q}{4\pi\varepsilon_0} \int_{b}^{a} \frac{dr}{r^2}$$

$$= \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{a} - \frac{1}{b}\right) = \frac{q}{4\pi\varepsilon_0} \frac{b-a}{ab}.$$

Using $q = CV$ we find that

$$C = \frac{q}{V} = 4\pi\varepsilon_0 \frac{ab}{b-a}.$$



3. Calculating the Capacitance

<u>An Isolated Sphere</u>

We can assign capacitance to a single isolated sphere of radius R by assuming that the "missing plate" is a conducting sphere of infinite radius.

To find an expression for the capacitance we rewrite the capacitance of a spherical capacitor as

$$C = 4\pi\varepsilon_0 \frac{a}{1 - a/b}.$$

Replacing a with R and taking the limit that $b \to \infty$ gives that

$$C = 4\pi\varepsilon_0 R.$$

3. Calculating the Capacitance

CHECKPOINT 2

For capacitors charged by the same battery, does the charge stored by the capacitor increase, decrease, or remain the same in each of the following situations? (a) The plate separation of a parallel-plate capacitor is increased. (b) The radius of the inner cylinder of a cylindrical capacitor is increased. (c) The radius of the outer spherical shell of a spherical capacitor is increased.

(a) Decreases.

(b) Increases.

(c) Decreases.

$$q = CV$$

$$C = \frac{\varepsilon_0 A}{d}$$

$$C = 2\pi\varepsilon_0 \frac{L}{\ln b/a}$$

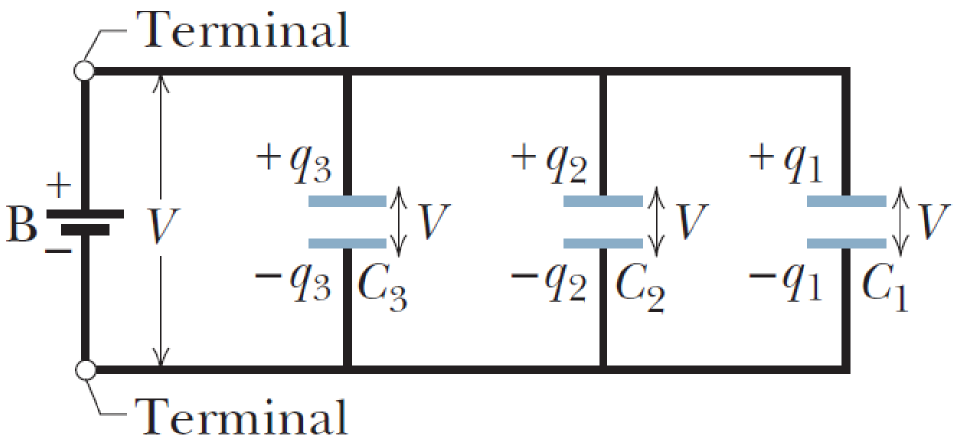
$$C = 4\pi\varepsilon_0 \frac{ab}{b-a}$$

4. Capacitors in Parallel and in Series

When there are several capacitors in a circuit, we can sometimes replace them with a single **equivalent capacitor**. Such a replacement can simplify circuit analysis. There are two basic ways in which capacitors are combined:

Capacitors in Parallel

When a potential difference V is applied across several capacitors connected in parallel, that potential difference V is applied across each capacitor. The total charge q stored on the capacitors is the sum of the charges stored on all the capacitors.



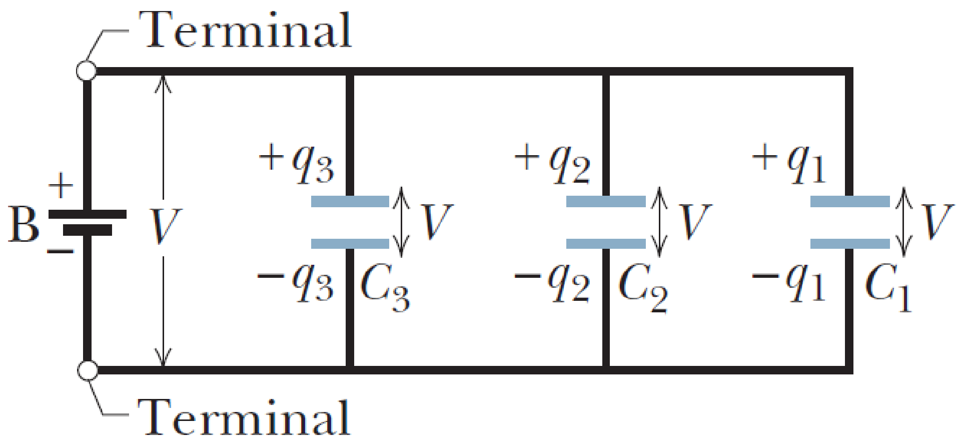
4. Capacitors in Parallel and in Series

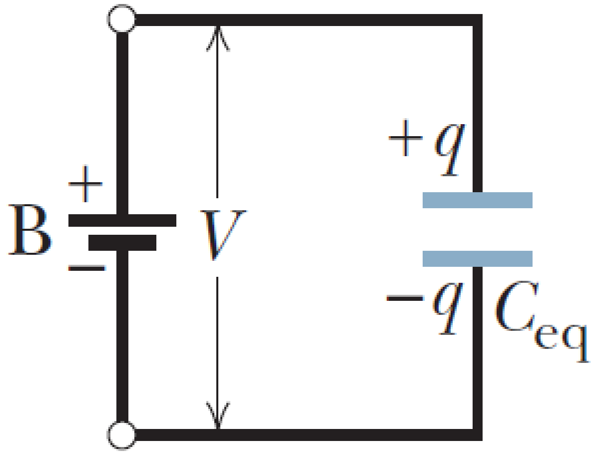
<u>Capacitors in Parallel</u>

Capacitors connected in parallel can be replaced with an equivalent capacitor that has the same *total* charge q and the same potential difference V as the actual capacitors.

Let us derive an expression for the equivalent capacitance C_{eq}. The charge on each actual capacitor is

$$q_1 = C_1 V, \qquad q_2 = C_2 V, \qquad q_3 = C_3 V.$$





4. Capacitors in Parallel and in Series

Capacitors in Parallel

The total charge q on the parallel combination is then

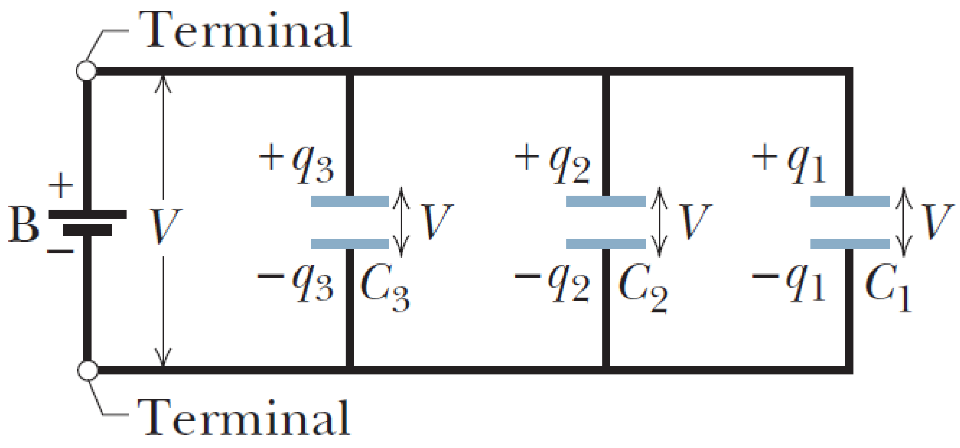
$$q = q_1 + q_2 + q_3 = (C_1 + C_2 + C_3)V.$$

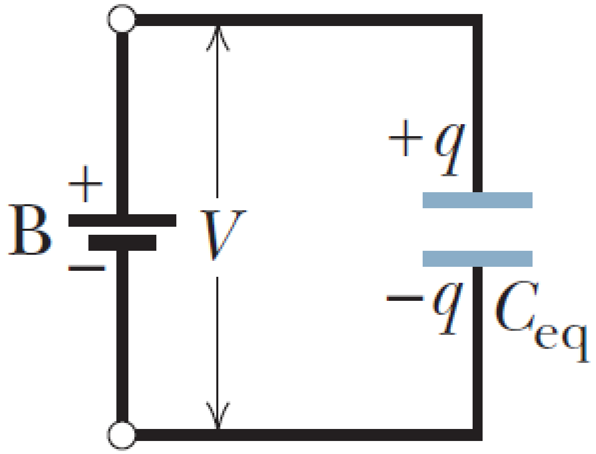
Thus,

$$C_{\text{eq}} = \frac{q}{V} = C_1 + C_2 + C_3.$$

Generally, for n capacitors connected in parallel

$$C_{\text{eq}} = \sum_{i=1}^{n} C_i\,.$$





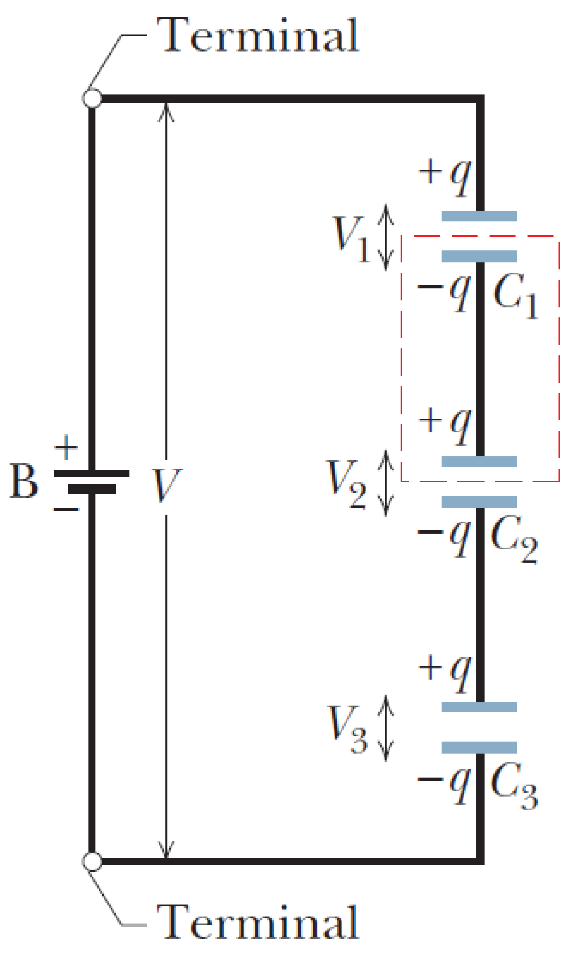
4. Capacitors in Parallel and in Series

<u>Capacitors in Series</u>

When a potential difference V is applied across several capacitors connected in series, the capacitors have identical charge q. The sum of the potential differences across all the capacitors is equal to the applied potential difference V.

Capacitors that are connected in series can be replaced with an equivalent capacitor that has the same charge q and the same *total* potential difference V as the actual series capacitors.

Let us derive an expression for the equivalent capacitance $C_{\mathbf{eq}}$



4. Capacitors in Parallel and in Series

<u>Capacitors in Series</u>

The potential difference of each capacitor is

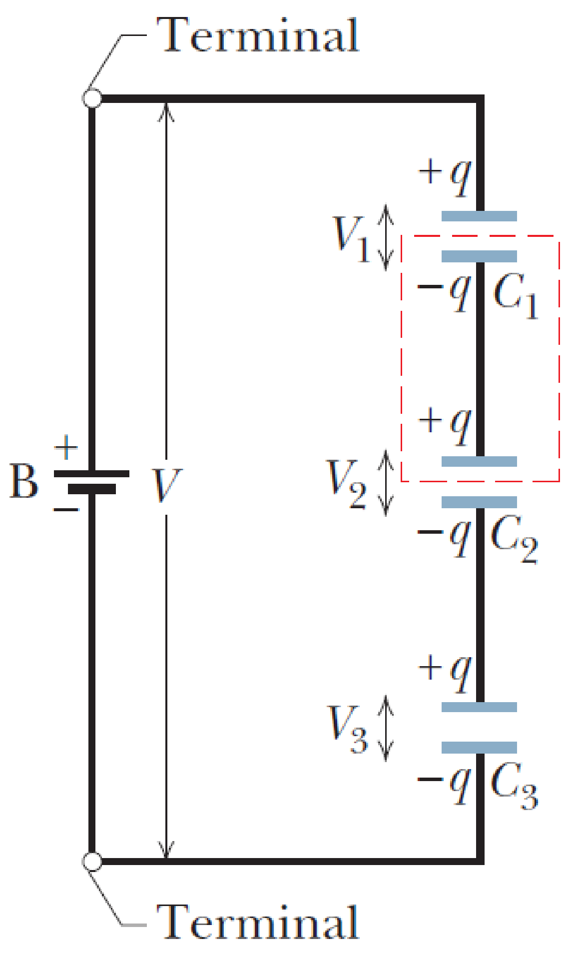
$$V_1 = \frac{q}{C_1}, \qquad V_2 = \frac{q}{C_2}, \qquad V_3 = \frac{q}{C_3}.$$

The total potential difference V due to the battery is the sum of these three potential differences:

$$V = V_1 + V_2 + V_3 = q\left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right).$$

Thus,

$$C_{\mathrm{eq}} = \frac{q}{V} = \frac{1}{1/C_1 + 1/C_2 + 1/C_3}.$$



4. Capacitors in Parallel and in Series

<u>Capacitors in Series</u>

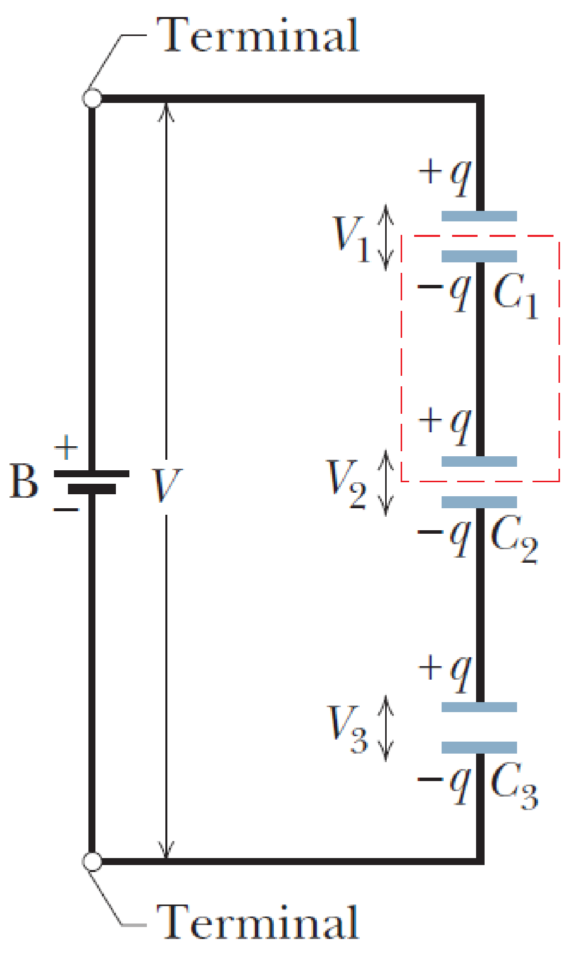
or

$$\frac{1}{C_{\mathrm{eq}}}=\frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3}.$$

Generally, for n capacitors connected in series

$$\frac{1}{C_{\mathrm{eq}}}=\sum_{i=1}^{n}\frac{1}{C_i}.$$

Note that C_{eq} is always less that the smallest capacitance in the series.



4. Capacitors in Parallel and in Series

CHECKPOINT 3

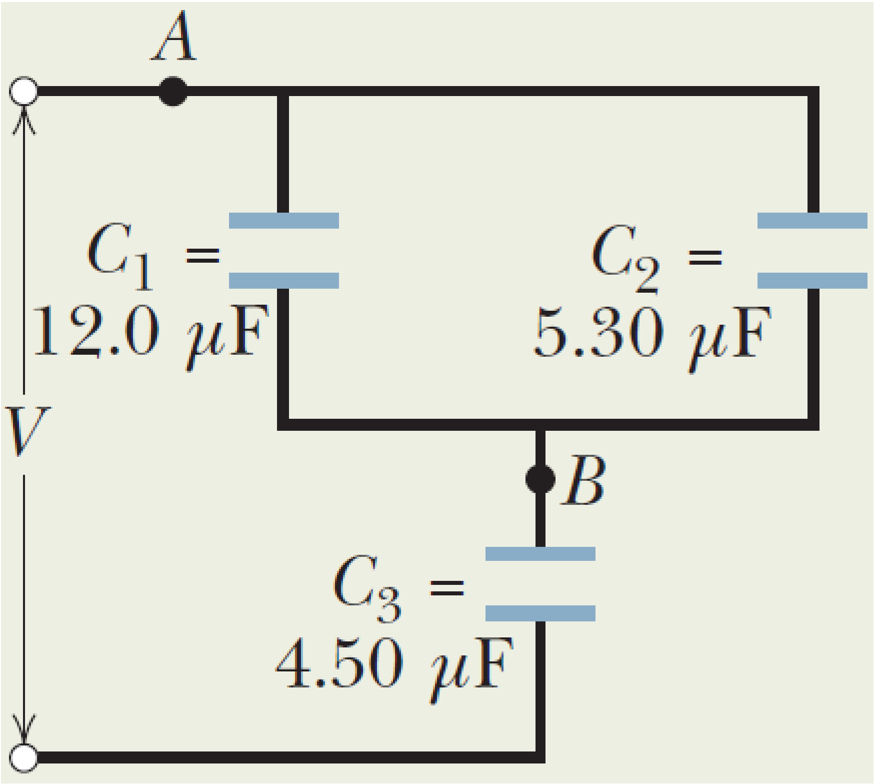
A battery of potential V stores charge q on a combination of two identical capacitors. What are the potential difference across and the charge on either capacitor if the capacitors are (a) in parallel and (b) in series?

(a) $q/2, V$.

(b) $q, V/2$.

4. Capacitors in Parallel and in Series

Example 1: (a) Find the equivalent capacitance for the combination of capacitances shown in the figure, across which potential difference V is applied. Assume $C_1 = 12.0\,\mu\text{F}$, $C_2 = 5.30\,\mu\text{F}$ and $C_3 = 4.50\,\mu\text{F}$.



We first replace capacitors 1 and 2 that are connected in parallel by their equivalent capacitor. The capacitance C_{12} of the equivalent capacitor is

$$C_{12} = C_1 + C_2 = 12.0\,\mu\text{F} + 5.30\,\mu\text{F} = 17.3\,\mu\text{F}.$$

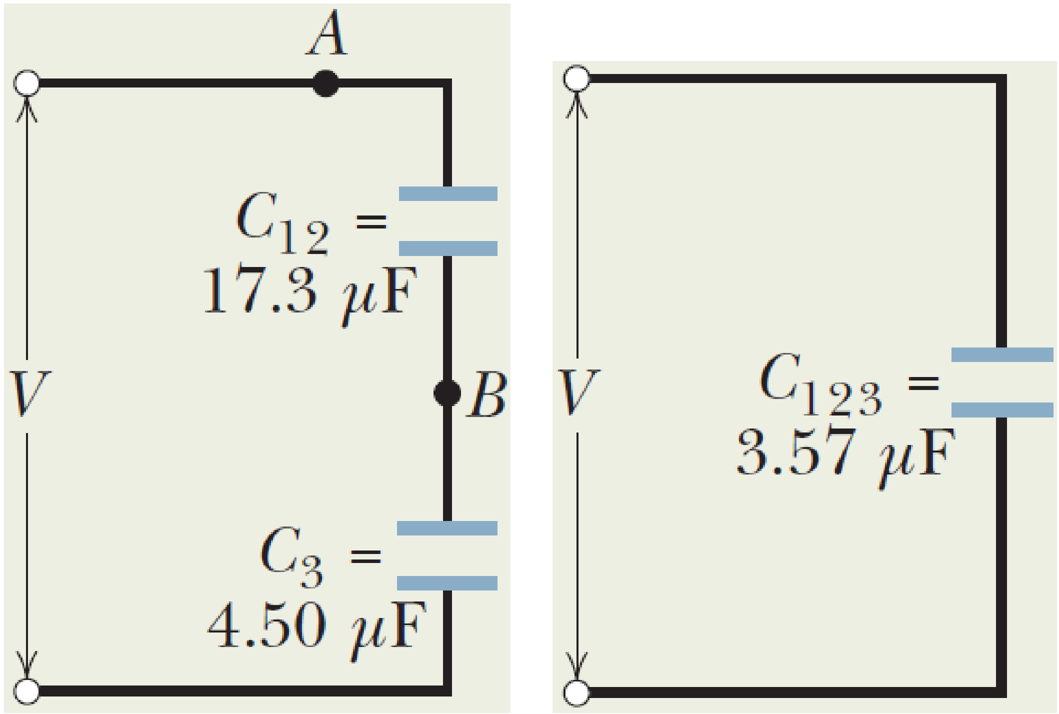
4. Capacitors in Parallel and in Series

Now capacitors C_{12} and C_3 are in series. Their equivalent capacitance C_{123} is

$$\frac{1}{C_{123}} = \frac{1}{C_{12}} + \frac{1}{C_3} = \frac{1}{17.3\ \mu\text{F}} + \frac{1}{4.50\ \mu\text{F}}$$
$$= 0.280\ 1/\mu\text{F}.$$

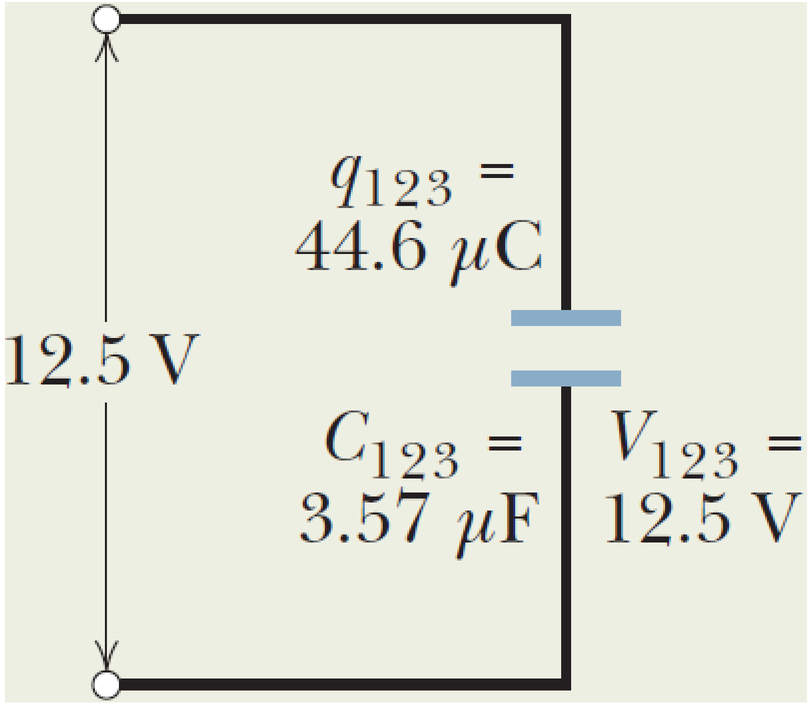
Therefore,

$$C_{123} = 3.57\ \mu\text{F}.$$



4. Capacitors in Parallel and in Series

(b) The potential difference applied to the input terminals in the figure is $V = 12.5\ \mathrm{V}$. What is the charge on C_1?



The charge on capacitor C_{123} is

$$q_{123} = C_{123}V = (3.57\ \mu\mathrm{F})(12.5\ \mathrm{V}) = 44.6\ \mu\mathrm{C}.$$

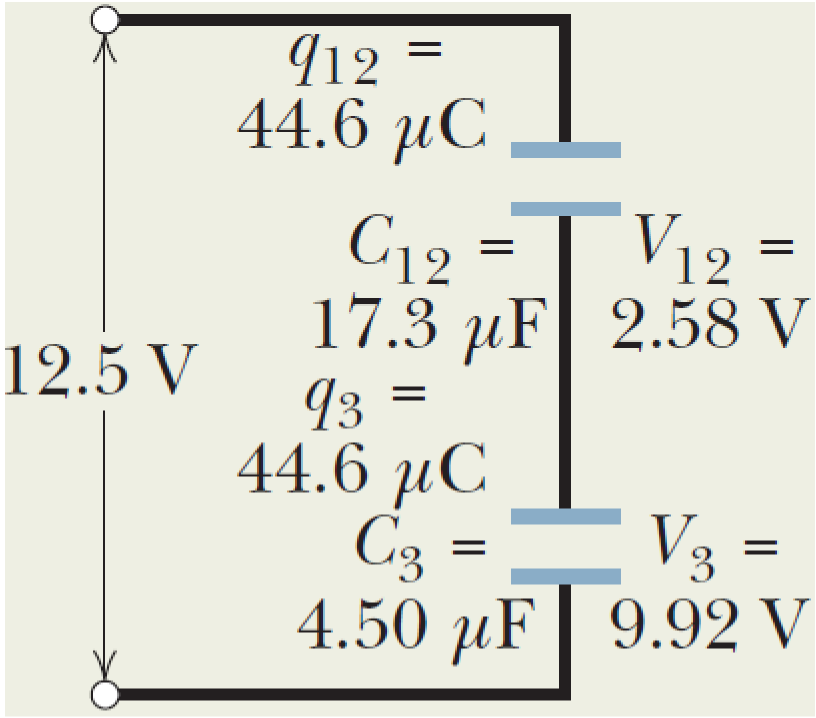
4. Capacitors in Parallel and in Series

The charge on capacitors C_{12} and C_3 is the same as that on capacitor C_{123}:

$$q_{12} = q_3 = q_{123} = 44.6\ \mu\text{C}.$$

The potential difference across C_{12} is

$$V_{12} = \frac{q_{12}}{C_{12}} = \frac{44.6\ \mu\text{C}}{17.3\ \mu\text{F}} = 2.58\ \text{V}.$$



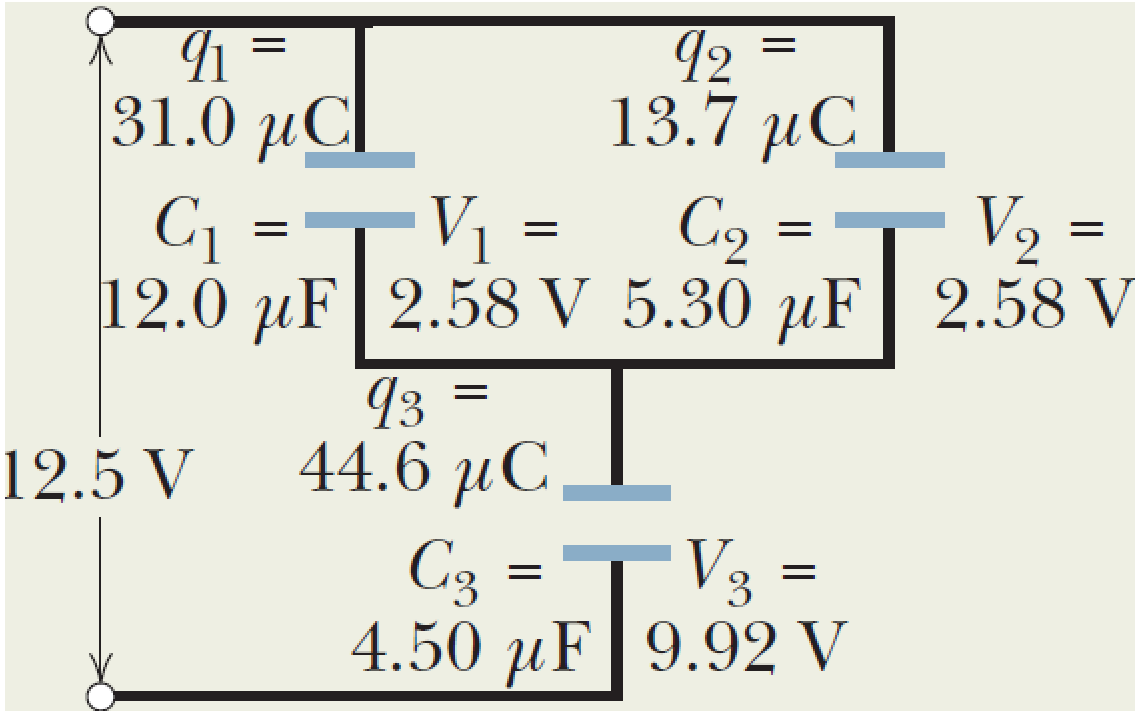
4. Capacitors in Parallel and in Series

The potential difference across C_1 is the same as that across C_{12}. Thus,

$$V_1 = V_2 = V_{12} = 2.58\text{ V}.$$

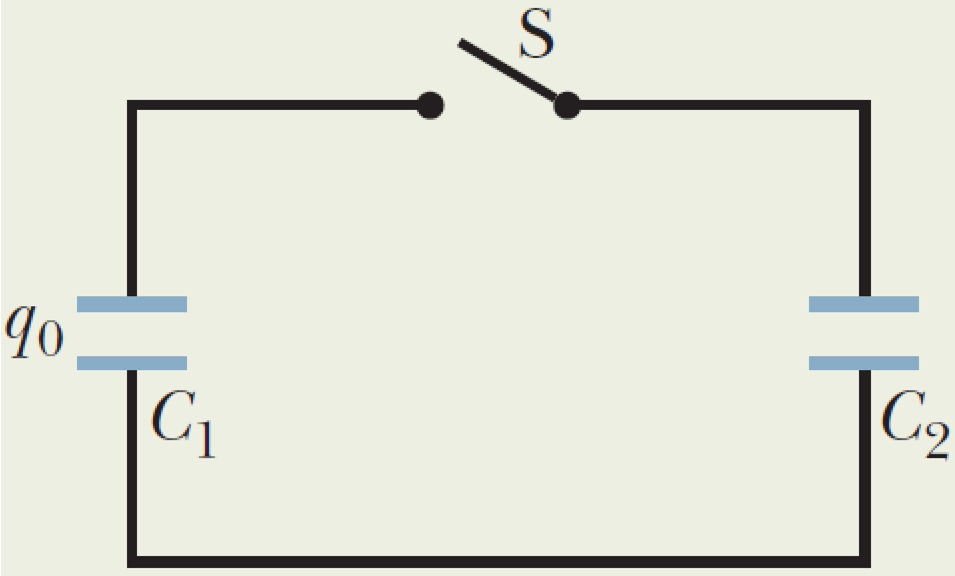
Thus,

$$q_1 = C_1V_1 = (12.0\ \mu\text{F})(2.58\text{ V})$$
$$= 31.0\ \mu\text{C}.$$



4. Capacitors in Parallel and in Series

Example 2: Capacitor 1, with $C_1 = 3.55\ \mu\mathrm{F}$, is charged to a potential difference $V_0 = 6.30\ \mathrm{V}$, using a $6.30\ \mathrm{V}$ battery. The battery is then removed, and the capacitor is connected as in the figure to an uncharged capacitor 2, with $C_2 = 8.95\ \mu\mathrm{F}$. When switch S is closed, charge flows between the capacitors. Find the charge on each capacitor when equilibrium is reached.



At equilibrium, capacitor 2 is charged until the potentials V_1 and V_2 across the two capacitors become equal.

4. Capacitors in Parallel and in Series

Thus,

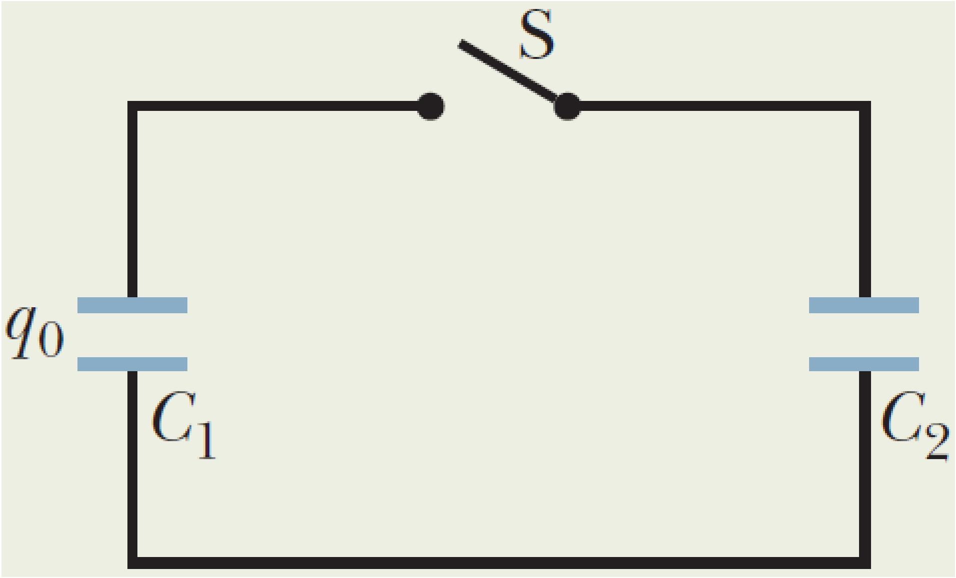
$$V_1 = V_2,$$

implies that

$$\frac{q_1}{C_1} = \frac{q_2}{C_2}.$$

Charge conservation requires that

$$q_1 + q_2 = q_0 = V_0 C_1.$$



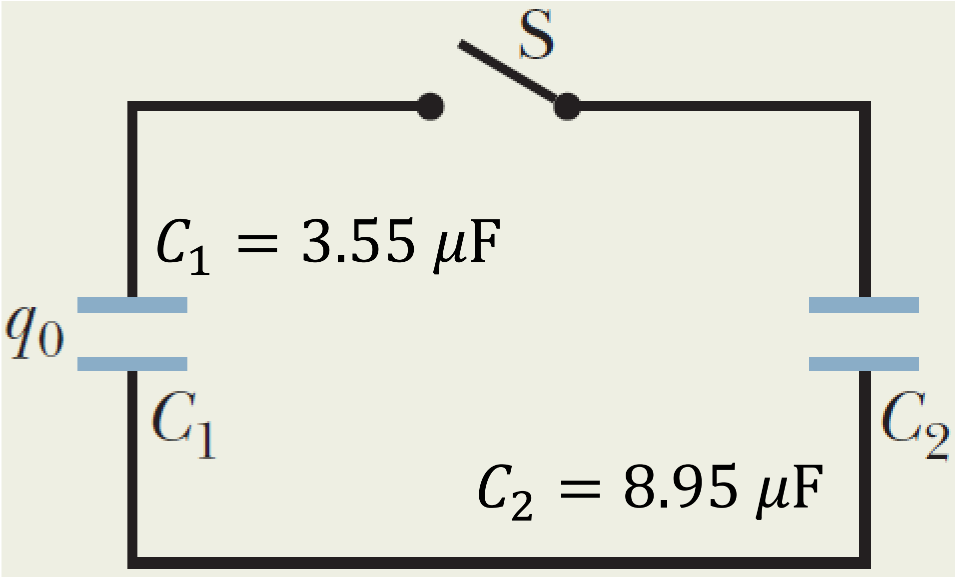
4. Capacitors in Parallel and in Series

Solving for q_1 and q_2 yields

$$q_1 = \frac{C_1^2}{C_1 + C_2} V_0 = 6.35\ \mu\text{C},$$

and

$$q_2 = \frac{C_1 C_2}{C_1 + C_2} V_0 = 16.0\ \mu\text{C}.$$



5. Energy Stored in an Electric Field

To charge a capacitor, work must be done by an external agent. Starting with an uncharged capacitor, work must be done to transfer electrons from one plate to another. This work is practically done by a battery.

We visualize the work required to charge a capacitor as being stored in the form of electric potential energy U *in the electric field between the plates*. We recover this energy by discharging the capacitor.

Suppose that a charge q' has been transferred from one plate to the other. The potential difference between the plates at that instant will be q'/C. If an extra increment of charge dq' is then transferred, the increment of work required will be

$$dW = V'dq' = \frac{q'}{C}dq'.$$

5. Energy Stored in an Electric Field

The work required to charge up the capacitor to a final charge q is

$$W = \int dW = \frac{1}{C}\int_0^q q'dq' = \frac{q^2}{2C}.$$

This work is stored as potential energy U in the capacitor. Thus,

$$U = \frac{q^2}{2C}.$$

Using $q = CV$, we can rewrite U as

$$U = \frac{1}{2}CV^2.$$

5. Energy Stored in an Electric Field

Consider a parallel-plate capacitor of charge q and plate separation d. The volume of the space between the plates is $\mathrm{Vol} = Ad$. The stored potential energy in the capacitor is $U = q^2/2C$.

If the plate separation is doubled ($d_2 = 2d$) the space volume is doubled too. Because the capacitance is halved ($C_2 = C/2$), the stored potential energy in the capacitor is doubled: $U_2 = 2U$.

This argument is a demonstration of our earlier assumption; the potential energy of a charged capacitor may be viewed as being stored in the electric field between its plates.

5. Energy Stored in an Electric Field

Energy Density:

The **energy density** u, or the potential energy per unit volume between the plates of a parallel-plate capacitor, where the field is uniform, is given by

$$u = \frac{U}{V} = \frac{U}{Ad} = \frac{CV^2}{2Ad}.$$

Using $C = \varepsilon_0 A/d$, u become

$$u = \frac{1}{2}\varepsilon_0 E^2.$$

Although we have here derived this result for the electric field between the plates of a parallel-plate capacitor, it holds for any electric field.

5. Energy Stored in an Electric Field

Example 3: An isolated conducting sphere whose radius $R = 6.85\ \mathrm{cm}$ has a charge $q = 1.25\ n\mathrm{C}$.

(a) How much potential energy is stored in the electric field of this charged conductor?

$$U = \frac{q^2}{2C} = \frac{q^2}{2(4\pi\varepsilon_0 R)} = \frac{(1.25\ n\mathrm{C})^2}{8\pi\left(8.85\,\frac{p\mathrm{F}}{\mathrm{m}}\right)(0.0685\ \mathrm{m})} = 103\ n\mathrm{J}.$$

(b) What is the energy density at the surface of the sphere?

The electric field at the surface of the sphere is

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{R^2}$$

5. Energy Stored in an Electric Field

The energy density is

$$u = \frac{1}{2}\varepsilon_0 E^2 = \frac{1}{2}\varepsilon_0 \left(\frac{1}{4\pi\varepsilon_0}\frac{q}{R^2}\right)^2 = \frac{q^2}{32\pi^2\varepsilon_0 R^4} = \frac{(1.25\ n\mathrm{C})^2}{32\pi^2\left(8.85\,\frac{p\mathrm{F}}{\mathrm{m}}\right)(0.0685\ \mathrm{m})^4}$$

$$= 25.4\ \frac{\mu\mathrm{J}}{\mathrm{m}^3}.$$

6. Capacitor with a Dielectric

A **dielectric** is an insulating material such as plastic, glass or waxed paper. If you fill the space between the plates of a capacitor with a dielectric, the capacitance increases by a factor κ, called the **dielectric constant**.

Moreover, the introduction of a dielectric fixes the maximum potential difference that can be applied between the plates to a certain value $V_{\max}$, called the **breakdown potential**. The dielectric material breaks down and conducts electricity when $V_{\max}$ is exceeded. The maximum value of electric field that a dielectric can tolerate without breakdown is called the **dielectric strength**.

Some Properties of Dielectrics[a]

Material	Dielectric Constant κ	Dielectric Strength (kV/mm)
Air (1 atm)	1.00054	3
Polystyrene	2.6	24
Paper	3.5	16
Transformer oil	4.5	
Pyrex	4.7	14
Ruby mica	5.4	
Porcelain	6.5	
Silicon	12	
Germanium	16	
Ethanol	25	
Water (20°C)	80.4	
Water (25°C)	78.5	
Titania ceramic	130	
Strontium titanate	310	8

6. Capacitor with a Dielectric

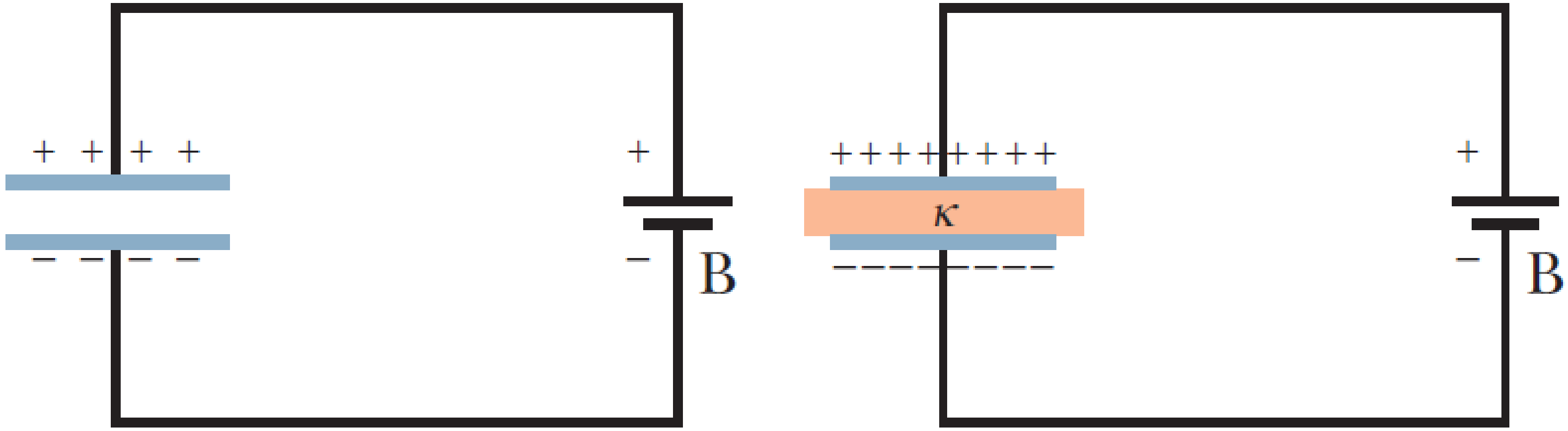
The capacitance of any capacitor can be written in the general form

$$C_0 = \varepsilon_0 \mathcal{L}, \qquad \text{(in vaccum)}$$

where $\mathcal{L}$ has the dimension of length. For parallel-plate capacito $\mathcal{L} = A/d$. With a dielectric completely filling the space between the plates, the capacitance becomes

$$C = \varepsilon_0 \kappa \mathcal{L} = \kappa C_0 \approx \kappa C_{\mathrm{air}}.$$

6. Capacitor with a Dielectric

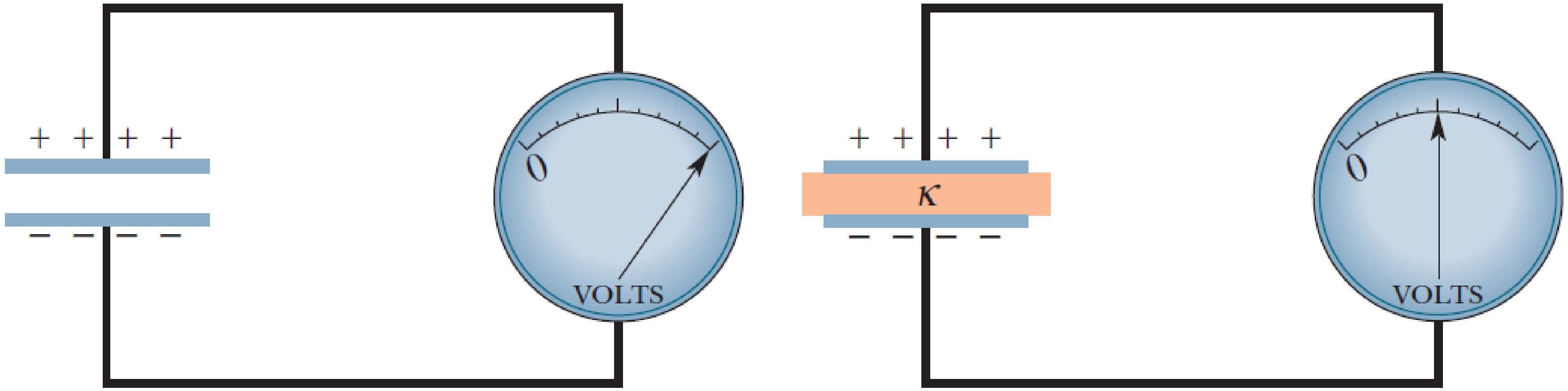


V = a constant

What happens when we insert a dielectric between the plates of a capacitor that is connected to a battery?

The capacitance will increase by a factor of κ. Because $q = CV$, the capacitor will get charge to $q_2 = (\kappa C)V = \kappa q$.

6. Capacitor with a Dielectric



$q =$ a constant

What happens when we insert a dielectric between the plates of a charged capacitor?

The capacitance will increase by a factor of κ. Because $V = q/C$, the potential across the capacitor will drop to $V_2 = q/(\kappa C) = V/\kappa$.

6. Capacitor with a Dielectric

In a region completely filled by a dielectric material of dielectric constant κ, all electrostatic equations containing the permittivity constant ε_0 are to be modified by replacing ε_0 with $\kappa\varepsilon_0$.

For example, the magnitude of the electric field due to a point charge inside a dielectric is

$$E = \frac{1}{4\pi\varepsilon_0\kappa}\frac{q}{r^2}.$$

Also, the expression for the electric field just outside an isolated conductor immersed in a dielectric becomes

$$E = \frac{\sigma}{\varepsilon_0\kappa}.$$

Since $\kappa > 1$, the effect of a dielectric is to weaken the electric field that would be otherwise present.

6. Capacitor with a Dielectric

Example 4: A parallel-plate capacitor whose capacitance C is $13.5\ \mathrm{pF}$ is charged by a battery to a potential difference $V = 12.5\ \mathrm{V}$ between its plates. The charging battery is now disconnected, and a porcelain slab ($\kappa = 6.50$) is slipped between the plates.

(a) What is the potential energy of the capacitor before the slab is inserted?

$$U_i = \frac{1}{2}CV^2 = \frac{1}{2}(13.5 \times 10^{-12}\ \mathrm{F})(12.5\ \mathrm{V})^2 = 1055 \times 10^{-12}\ \mathrm{J} \approx 1100\ \mathrm{pJ}.$$

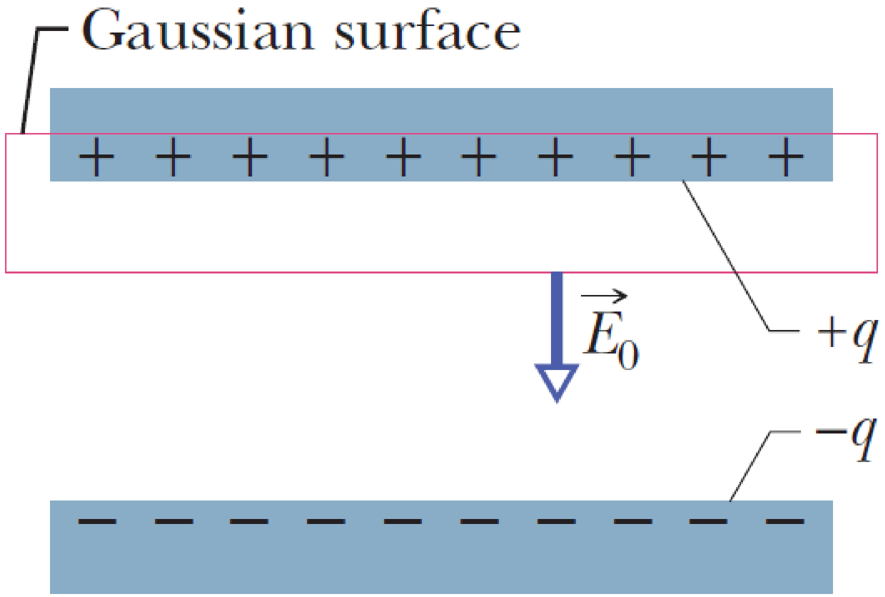
(b) What is the potential energy of the capacitor–slab device after the slab is inserted?

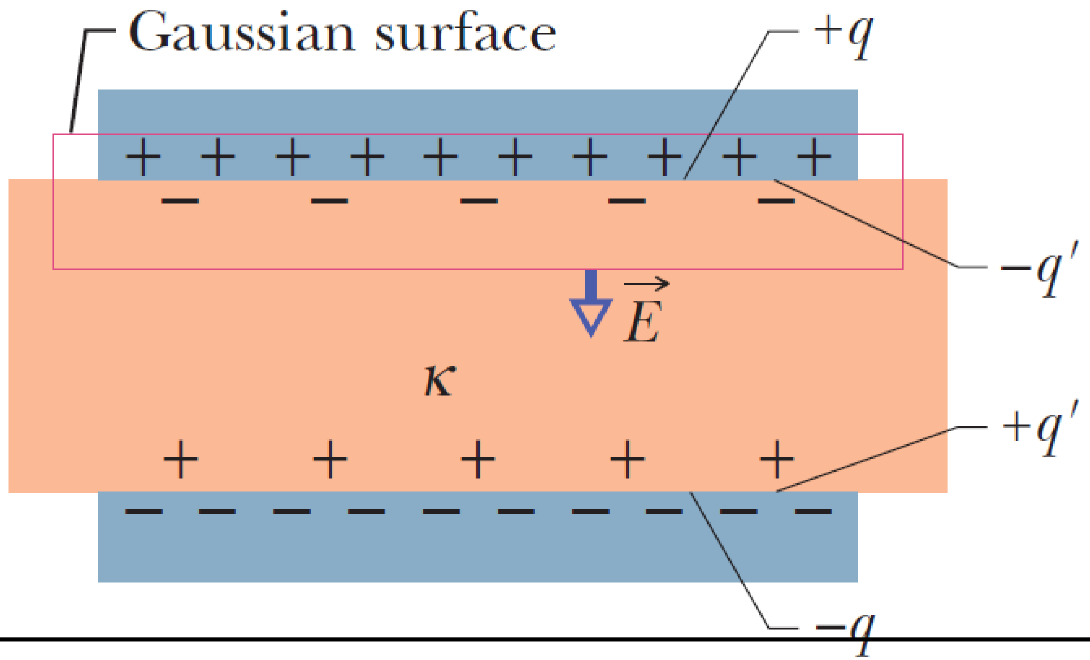
$$U_f = \frac{1}{2}C_f V_f^2 = \frac{1}{2}(\kappa C)\left(\frac{V}{\kappa}\right)^2 = \frac{U_i}{\kappa} = \frac{1055 \times 10^{-12}\ \mathrm{J}}{6.50} = 160 \times 10^{-12}\ \mathrm{J} = 160\ \mathrm{pJ}.$$

7. Dielectrics and Gauss’ Law

The figures show a parallel-plate capacitor with and without a dielectric. The field between the plates induces charges on the faces of the dielectric.

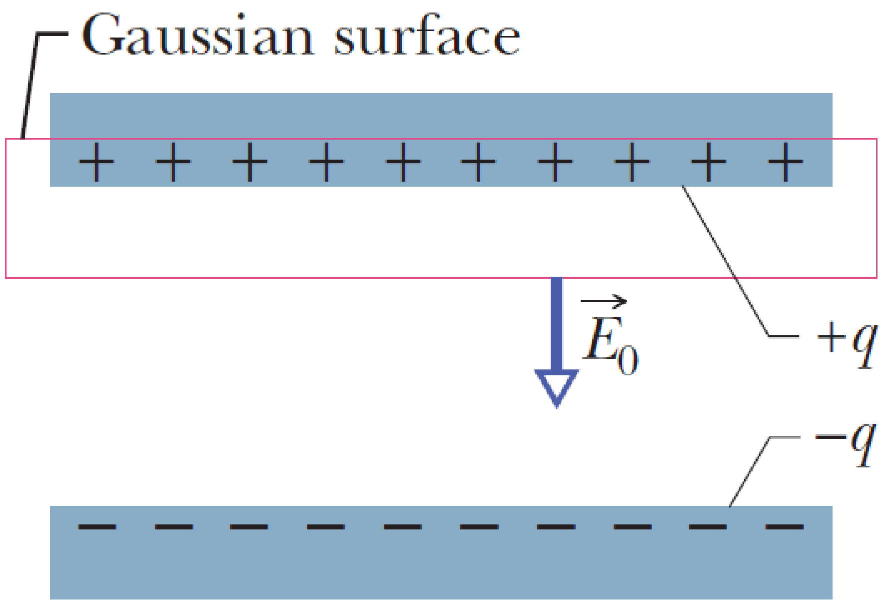
The charge on a conducting plate is said to be **free charge** because it can move if we change the electric potential of the plate; the **induced charge** on the surface of the dielectric is not free charge because it cannot move from that surface.





7. Dielectrics and Gauss' Law

To find the electric field $\vec{E}_0$ between the plates, without the dielectric, we enclose the charge $+q$ on the top plate with a Gaussian surface and apply Gauss' law:



$$\oint \vec{E} \cdot d\vec{A} = AE_0 = \frac{q}{\varepsilon_0},$$

or

$$E_0 = \frac{q}{\varepsilon_0 A}.$$

7. Dielectrics and Gauss' Law

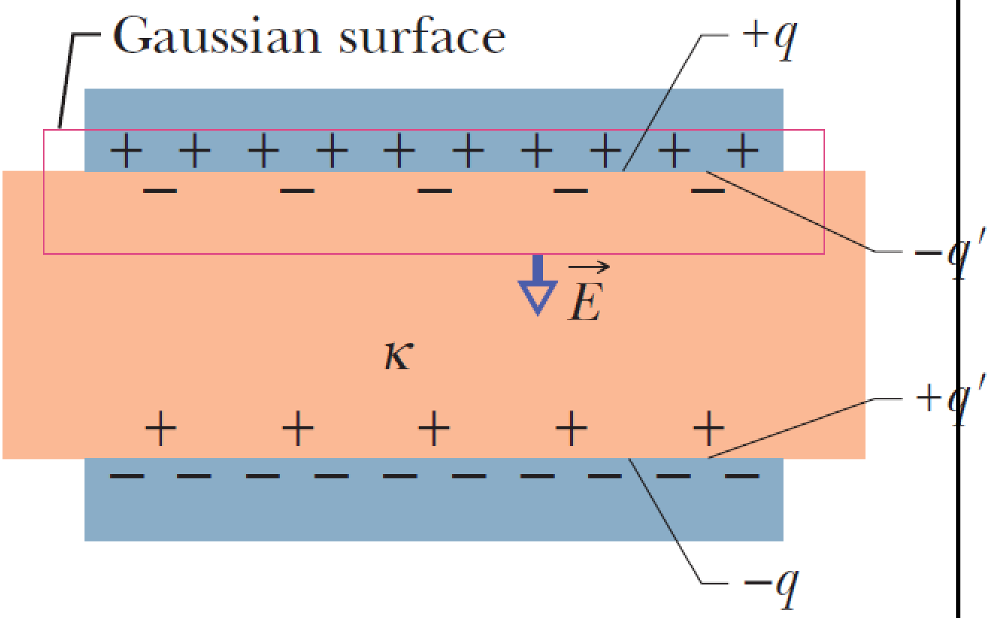
With the dielectric present, the Gaussian surface encloses charge $+q$ on the top plate as well as the induced charge $-q'$ on the top face of the dielectric.

Gauss' law now reads

$$\oint \vec{E} \cdot d\vec{A} = AE = \frac{q - q'}{\varepsilon_0},$$

or

$$E = \frac{q - q'}{\varepsilon_0 A}.$$



7. Dielectrics and Gauss' Law

$$E = \frac{q - q'}{\varepsilon_0 A}.$$

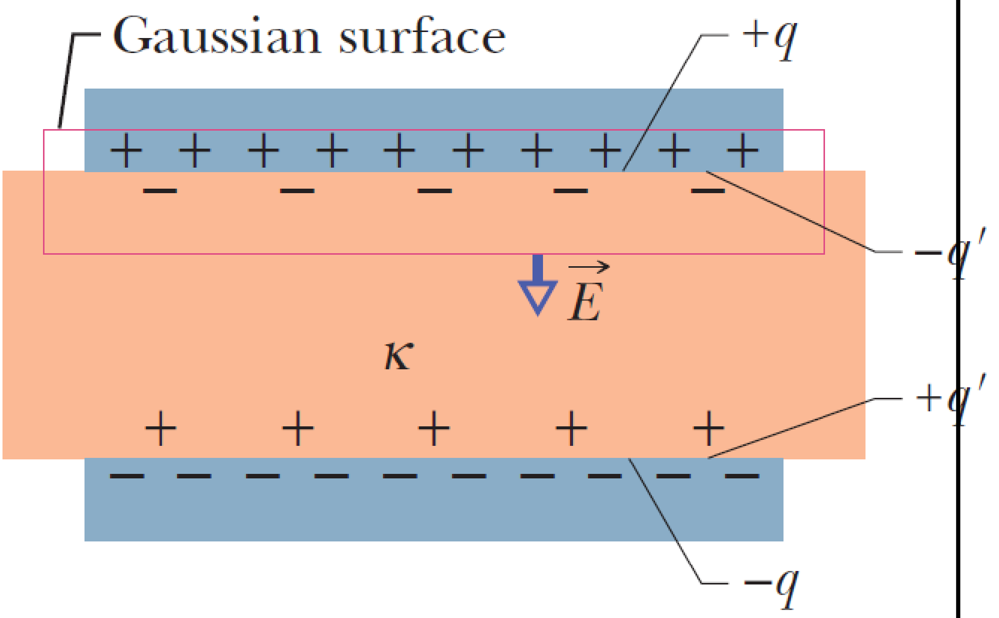
The effect of the dielectric is to weaken the original field E_0 by a factor of κ. Thus,

$$E = \frac{E_0}{\kappa} = \frac{q}{\kappa \varepsilon_0 A}.$$

We can conclude that

$$q - q' = \frac{q}{\kappa}.$$

The magnitude of the induced charge q' is less than that of the free charge q and is zero when no dielectric is present.



$$q' = q\left(1 - \frac{1}{\kappa}\right)$$

7. Dielectrics and Gauss' Law

Therefore, Gauss's law in the presence of a dielectric becomes

$$\oint \kappa \vec{E} \cdot d\vec{A} = \frac{q}{\varepsilon_0}. \qquad (q \text{ is the free charge only})$$

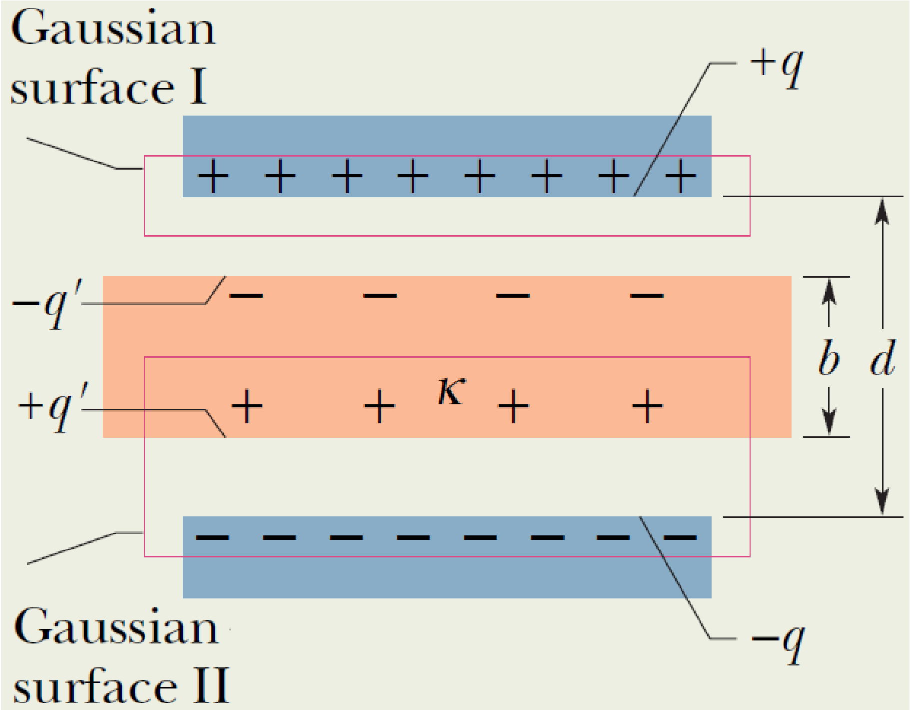
Although we derived this equation for a parallel-plate capacitor, it is generally true.

<u>Notes</u>:

1. The quantity $\varepsilon_0 \kappa \vec{E}$ is sometimes called the **dielectric displacement field** $\vec{D}$, so that Gauss' law can be rewritten as $\oint \vec{D} \cdot d\vec{A} = q$.
2. The charge enclosed by the Gaussian surface is taken to be the <u>free charge only</u>.
3. We keep κ inside the integral to allow cases in which κ is not constant.

7. Dielectrics and Gauss' Law

Example 6: The figure shows a parallel-plate capacitor of plate area A and plate separation d. A potential difference V_0 is applied between the plates by connecting a battery between them. The battery is then disconnected, and a dielectric slab of thickness b and dielectric constant κ is placed between the plates as shown. Assume $A = 115\ \text{cm}^2$, $d = 1.24\ \text{cm}$, $V_0 = 85.5\ \text{V}$, $b = 0.780\ \text{cm}$, and $\kappa = 2.61$.



7. Dielectrics and Gauss' Law

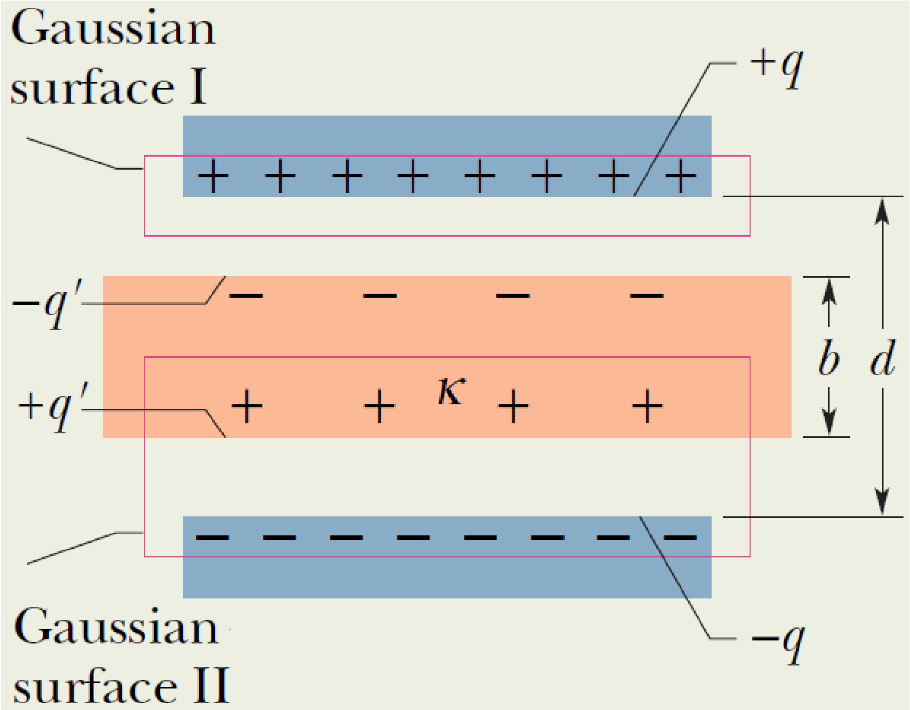
(a) What is the capacitance C_0 before the dielectric slab is inserted?

$$C_0 = \frac{\varepsilon_0 A}{d} = \frac{(8.85\ \mathrm{pF})(0.0115\ \mathrm{m}^2)}{0.0124\ \mathrm{m}} = 8.21\ \mathrm{pF}.$$

(b) What free charge appears on the plates?

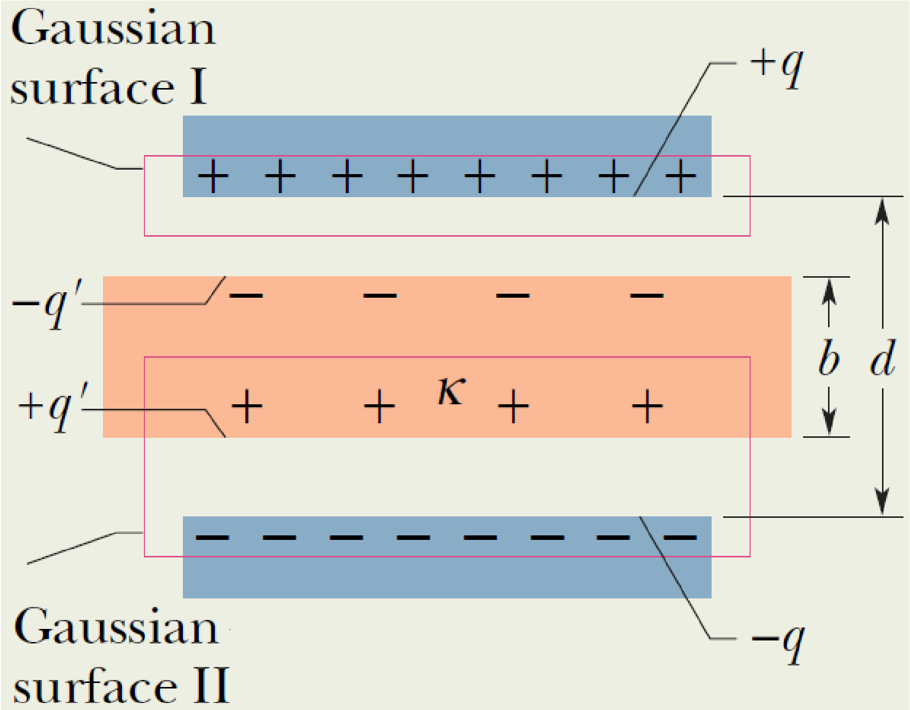
$$q = C_0 V_0 = (8.21\ \mathrm{pF})(85.5\ \mathrm{V}) = 702\ \mathrm{pC}.$$

The free charge is not changed by inserting the slab because the battery was disconnected.



7. Dielectrics and Gauss' Law

(c) What is the electric field E_0 in the gaps between the plates and the dielectric slab?



Applying Gauss' law to Gaussian surface I yields

$$\oint \kappa \vec{E} \cdot d\vec{A} = \kappa E_0 A = (1) E_0 A = \frac{q}{\varepsilon_0},$$

or

$$E_0 = \frac{q}{\varepsilon_0 A} = \frac{702 \text{ pC}}{(8.85 \text{ pF})(0.0115 \text{ m}^2)} = 6.90 \frac{\text{kV}}{\text{m}}.$$

Note that E_0 is note affected by inserting the slab.

7. Dielectrics and Gauss' Law

(d) What is the electric field E_1 in the dielectric slab?

Applying Gauss' law to Gaussian surface II yields

$$\oint \kappa \vec{E} \cdot d\vec{A} = -\kappa E_1 A = -\frac{q}{\varepsilon_0},$$

or

$$E_1 = \frac{q}{\kappa \varepsilon_0 A} = \frac{E_0}{\kappa} = \frac{6.90\ \mathrm{kV/m}}{2.61} = 2.64\ \mathrm{kV/m}.$$

Recall that we take the free charge only (here $-q$) as the enclosed charge.

(d) What is the electric field E_1 in the dielectric slab?

Applying Gauss' law to Gaussian surface II yields

$$\oint \kappa \vec{E} \cdot d\vec{A} = -\kappa E_1 A = -\frac{q}{\varepsilon_0},$$

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$$E_1 = \frac{q}{\kappa \varepsilon_0 A} = \frac{E_0}{\kappa} = \frac{6.90\ \mathrm{kV/m}}{2.61} = 2.64\ \mathrm{kV/m}.$$

Recall that we take the free charge only (here $-q$) as the enclosed charge.

7. Dielectrics and Gauss' Law

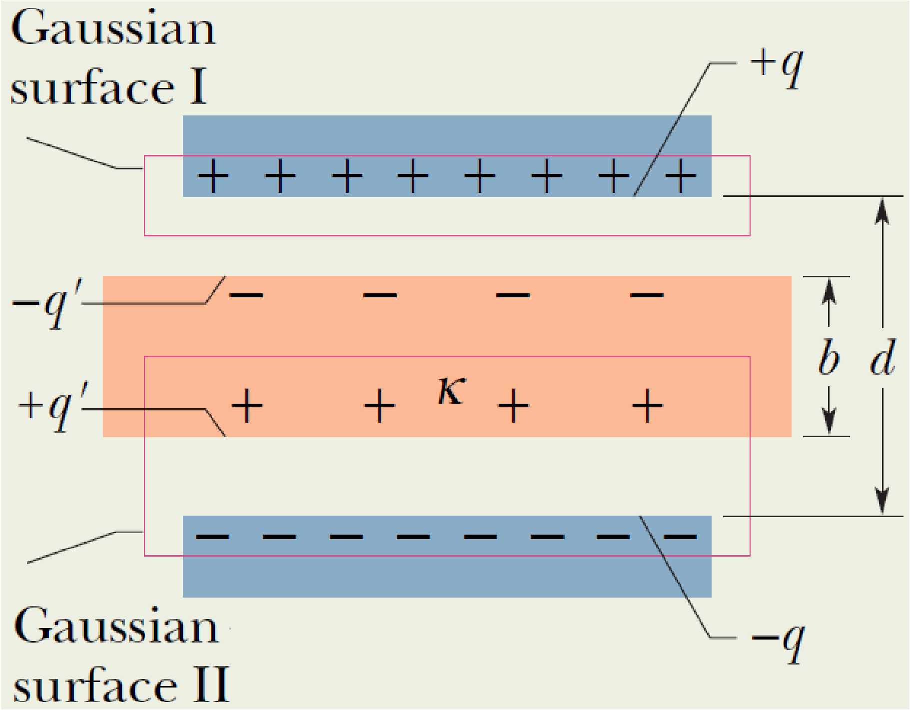
(e) What is the potential difference V between the plates after the slab has been introduced?

We integrate along a line from the bottom plate to the top plate

$$V = \int_{-}^{+} E ds = E_0(d - b) + E_1 b$$

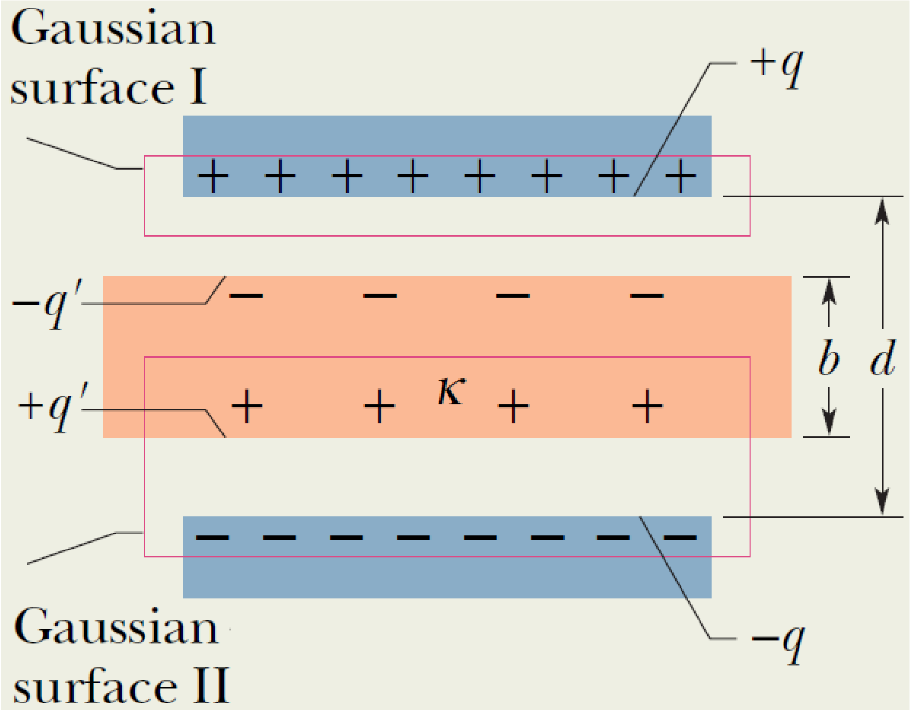
$$= 6.90 \text{ kVm } (0.0124 \text{ m} - 0.00780 \text{ m})$$

$$+ 2.64 \text{ kVm } (0.00780 \text{ m}) = 52.3 \text{ V}.$$



7. Dielectrics and Gauss' Law

(f) What is the capacitance with the slab in place between the plates of the capacitor?



From parts (c) and (e)

$$C = \frac{q}{V} = \frac{702\ \text{pC}}{52.3\ \text{V}} = 13.2\ \text{pF}.$$