Chapter 24

Electric Potential

1. Electric Potential Energy

When an electrostatic force acts between two or more charged particles within a system of particle, we can assign an **electric potential energy** U to the system. If the system changes its configuration from state i to state f, the electrostatic force does work W on the particles. The resulting change ΔU in potential energy of the system is

$$\Delta U = U_f - U_i = -W.$$

The work done by the electrostatic force is *path independent*, since the electrostatic force is *conservative*.

We take the reference configuration of a system of charged particles to be that in which the particles are all infinitely separated from each other. The energy of this reference configuration is set to be zero.

1. Electric Potential Energy

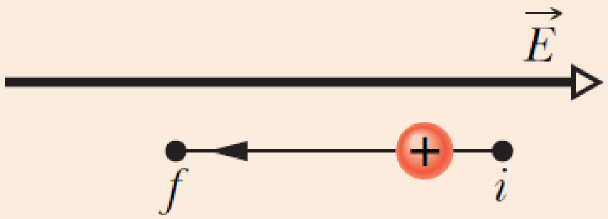
Suppose that several charged particles come together from initially infinite separations (state i) to form a system of neighboring particles (state f). Let the initial potential energy U_i be zero and let W_∞ represents the work done by the electrostatic force between the particles during the process. The final potential energy U of the system is

$$U = -W_\infty.$$

1. Electric Potential Energy

CHECKPOINT 1

In the figure, a proton moves from point i to point f in a uniform electric field directed as shown. (a) Does the electric field do positive or negative work on the proton? (b) Does the electric potential energy of the proton increase or decrease?



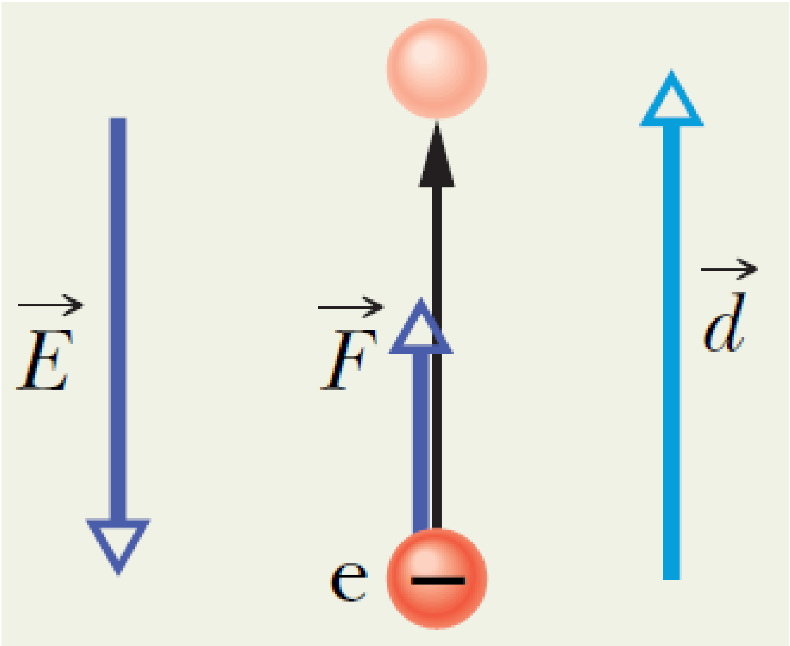
(a) Negative work.

(b) Increases.

$$W = -\Delta U$$

1. Electric Potential Energy

Example 1: Electrons are continually being knocked out of air molecules in the atmosphere by cosmic-ray particles coming in from space. Once released, each electron experiences an electrostatic force $\vec{F}$ due to the electric field $\vec{E}$ that is produced in the atmosphere by charged particles already on Earth. Near Earth's surface the electric field has the magnitude $E = 150$ N/C and is directed downward. What is the change ΔU in the electric potential energy of a released electron when the electrostatic force causes it to move vertically upward through a distance $d = 520$ m?

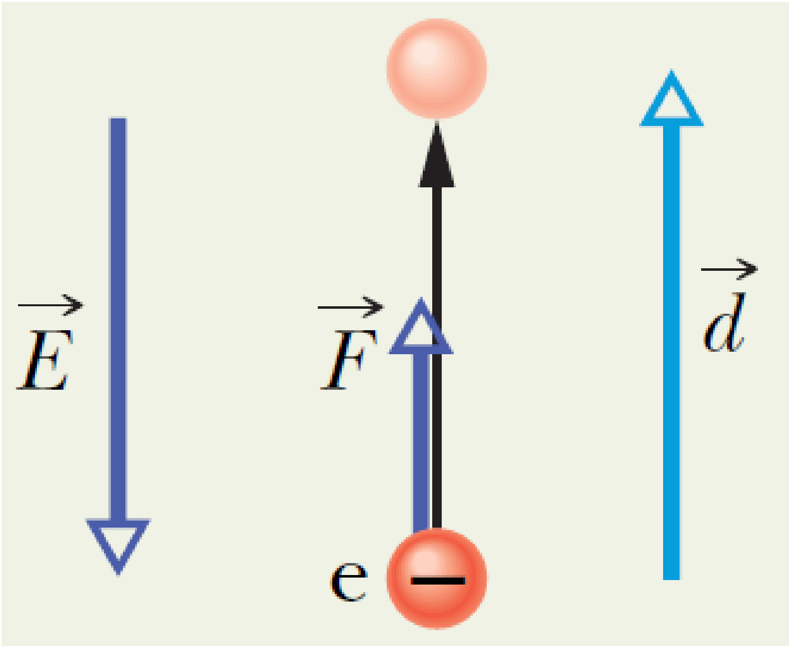


1. Electric Potential Energy

The work W done on the electron by the electrostatic force is

$$W = \vec{F} \cdot \vec{d} = Fd\cos 0 = Eed$$
$$= (150\ \mathrm{N/C})(1.60\ \times 10^{-19}\ \mathrm{C})(520\ \mathrm{m})$$
$$= 1.2\ \times 10^{-14}\ \mathrm{J}.$$

$$\Delta U = -W = -1.2\ \times 10^{-14}\ \mathrm{J}.$$



2. Electric Potential

The potential energy of a charged particle in an electric field depends on the charge magnitude. The potential energy *per unit charge*, however, has a unique value at any point in an electric field.

For example, a proton that has potential energy of $2.40 \times 10^{-17}\ \mathrm{J}$ in an electric field has potential per unit charge of

$$\frac{2.40 \times 10^{-17}\ \mathrm{J}}{1.60 \times 10^{-19}\ \mathrm{C}} = 150\ \mathrm{J/C}.$$

Suppose we next replace the proton with an alpha particle (charge $+2e$). The alpha particle would have electric potential energy of $4.80 \times 10^{-17}\ \mathrm{J}$. However, the potential energy per unit charge would be the same $150\ \mathrm{J/C}$.

2. Electric Potential

The potential energy per unit charge U/q is a characteristic only of the electric field. The potential energy per unit charge at a point in an electric field is called the **electric potential** V (or simply **potential**) at that point. Thus,

$$V = \frac{U}{q}.$$

Note that V is a scalar, not a vector.

The electric potential difference ΔV between any two points i and f in an electric field is equal to the difference in potential energy per unit charge between the two points:

$$\Delta V = V_f - V_i = \frac{U_f}{q} - \frac{U_i}{q} = \frac{\Delta U}{q}.$$

2. Electric Potential

Substituting $-W$ for ΔU we find that

$$\Delta V = V_f - V_i = -\frac{W}{q}.$$

The potential difference between two points is the negative of the work done by the electrostatic force on a unit charge as it moved from one point to the other.

If we set $U_i = 0$ at infinity as our reference potential energy, V must be zero there too. The electric potential at any point in an electric field is thus

$$V = \frac{U}{q} = -\frac{W_\infty}{q}.$$

The SI unit for potential V is joule per coulomb or volt (V):

$$1 \text{ volt} = 1 \text{ joul per coulomb}.$$

2. Electric Potential

In terms of this unit, we can write the electric field in a more convenient unit:

$$1\ \frac{\mathrm{N}}{\mathrm{C}} = \left(1\ \frac{\mathrm{N}}{\mathrm{C}}\right)\left(\frac{1\ \mathrm{V}}{1\ \mathrm{J/C}}\right)\left(\frac{1\ \mathrm{J}}{1\ \mathrm{N}\cdot\mathrm{m}}\right) = 1\ \frac{\mathrm{V}}{\mathrm{m}}.$$

We can also define an energy unit that is convenient for energy scale of the atomic physics; the electron-volt (eV). One electron-volt is the energy equal to the work required to move a single elementary charge e through a potential difference of one volt ($q\Delta V$). Thus

$$1\ \mathrm{eV} = e(1\ \mathrm{V}) = (1.60 \times 10^{-19}\ \mathrm{C})(1\ \mathrm{J/C}) = 1.60 \times 10^{-19}\ \mathrm{J}.$$

2. Electric Potential

<u>Work Done by an Applied Force</u>

Suppose we move a particle of charge q from point i to point f in an electric field by applying a force to it. By the work-kinetic energy theorem,

$$\Delta K = W_{\mathrm{app}} + W,$$

where W_{app} is the work done by the applied force and W is the work done by the electric field.

If $\Delta K = 0$, we obtain that

$$W_{\mathrm{app}} = -W.$$

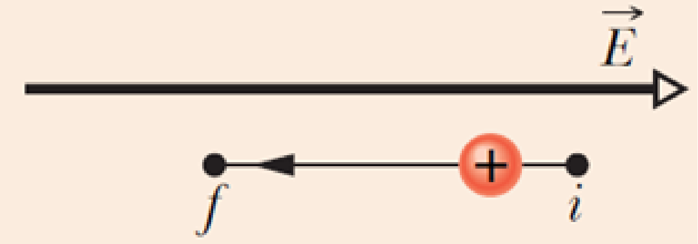
Using that $\Delta U = -W$, we obtain that

$$W_{\mathrm{app}} = \Delta U = q\Delta V.$$

2. Electric Potential

CHECKPOINT 2

In the figure of Checkpoint 1, we move the proton from point *i* to point *f* in a uniform electric field directed as shown. (a) Does our force do positive or negative work? (b) Does the proton move to a point of higher or lower potential?



(a) Positive work.

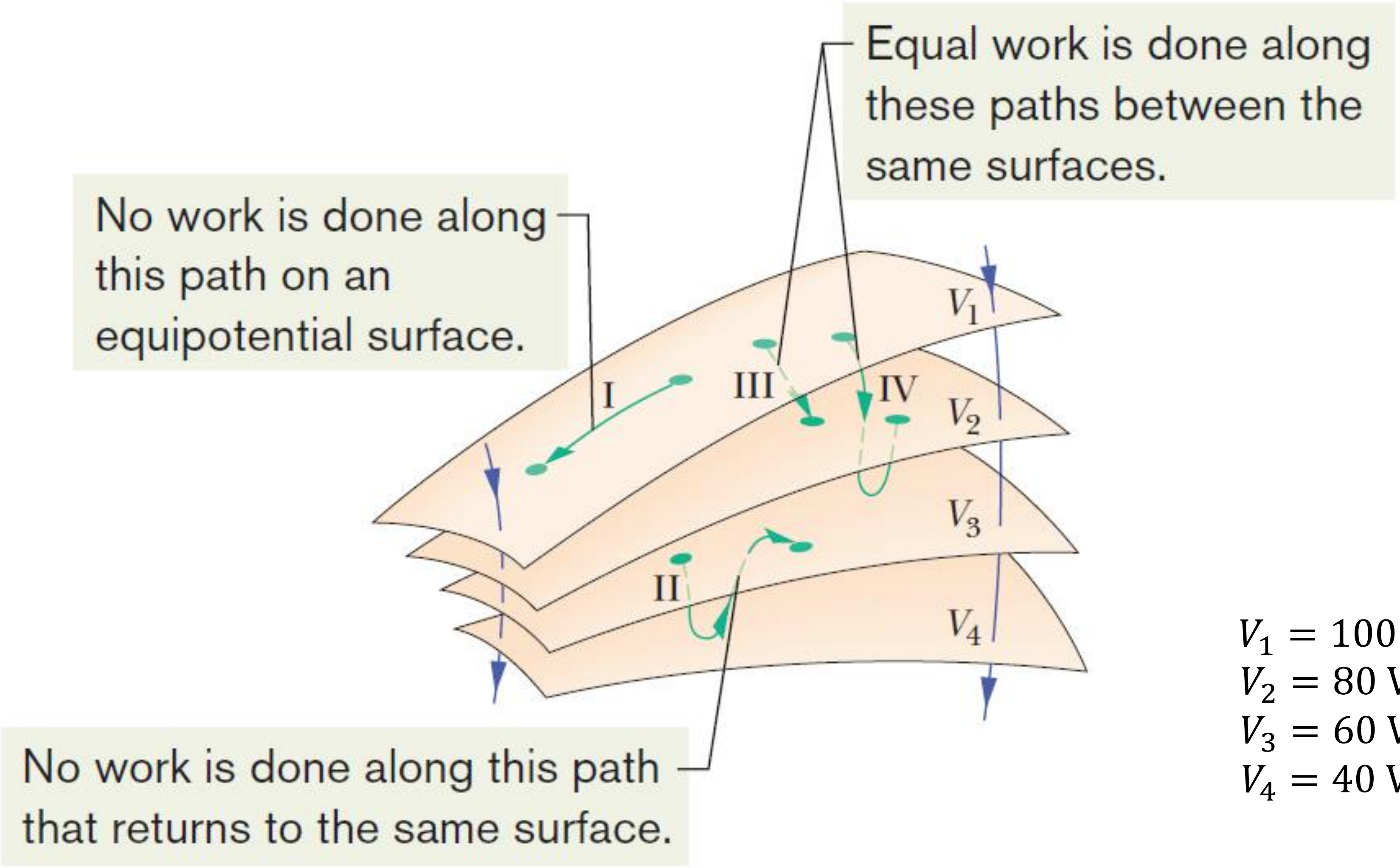
(b) Higher potential.

$$W_{\text{app}} = \Delta U = q\Delta V$$

3. Equipotential Surfaces

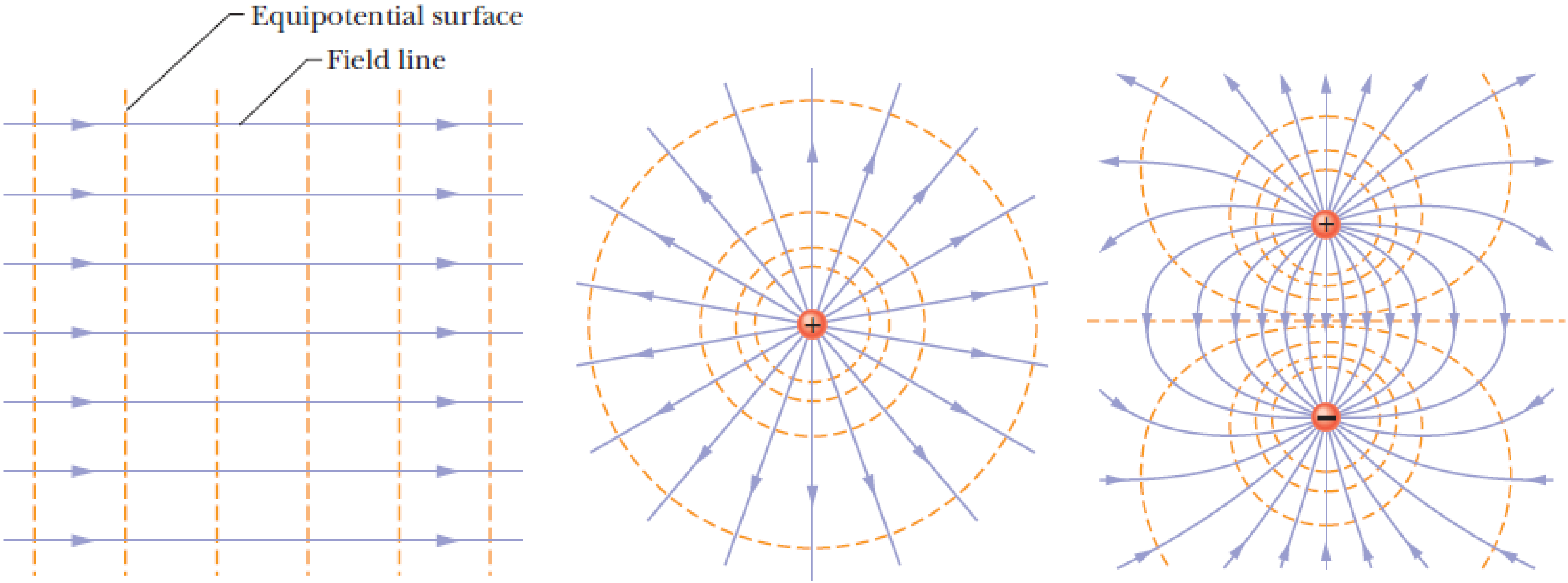
An **equipotential surface** is composed of adjacent points that have the same electric potential. No net work W is done on a charged particle by an electric field when the particle's initial point i and final point f lie on the same equipotential surface.

3. Equipotential Surfaces



$V_1 = 100$ V
$V_2 = 80$ V
$V_3 = 60$ V
$V_4 = 40$ V

3. Equipotential Surfaces



3. Equipotential Surfaces

Equipotential surfaces are perpendicular to the electric field $\vec{E}$.

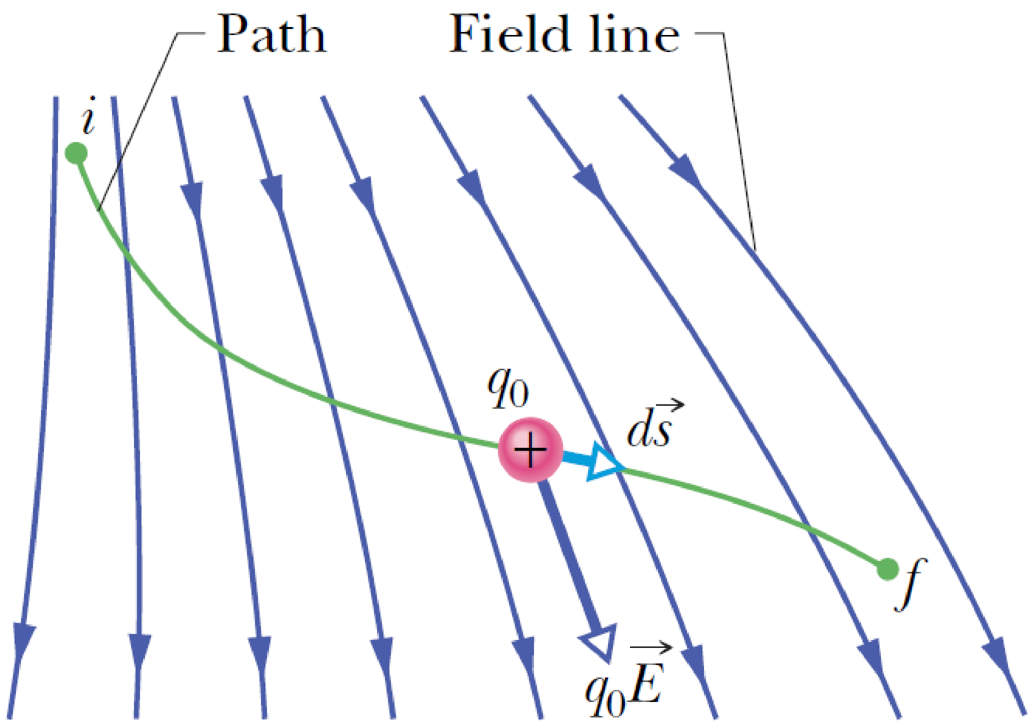
If $\vec{E}$ were not perpendicular to an equipotential surface, it would have a component along the surface. This component would then do work on a charged particle as it moved along the surface.

4. Calculating the Potential from the Field

We can calculate the potential difference between any two points i and f if we know the electric field vector $\vec{E}$ along any path connecting those points.

Consider the situation shown in the figure. The differential work dW done on the particle by the electric field as it moves through a differential displacement $d\vec{s}$ is

$$dW = \vec{F} \cdot d\vec{s} = q_0 \vec{E} \cdot d\vec{s}.$$



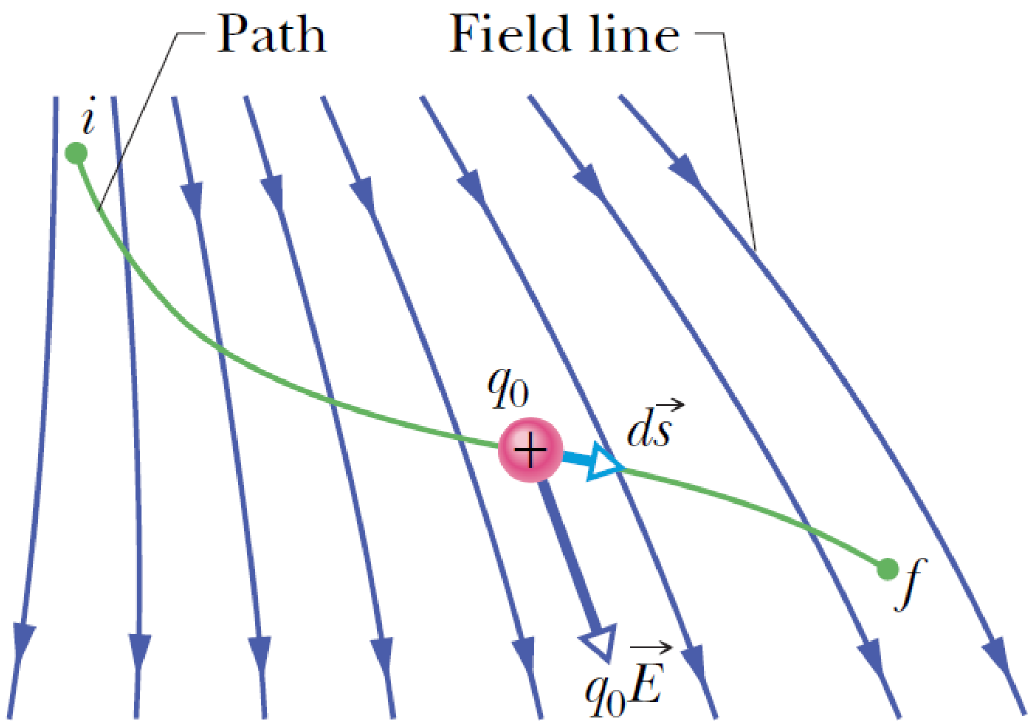
4. Calculating the Potential from the Field

The total work W done between points i and f is

$$W = q_0 \int_i^f \vec{E} \cdot d\vec{s}\,.$$

Using that $W = -\Delta U = -q_0 \Delta V$ we obtain that

$$V_f - V_i = -\int_i^f \vec{E} \cdot d\vec{s}\,.$$

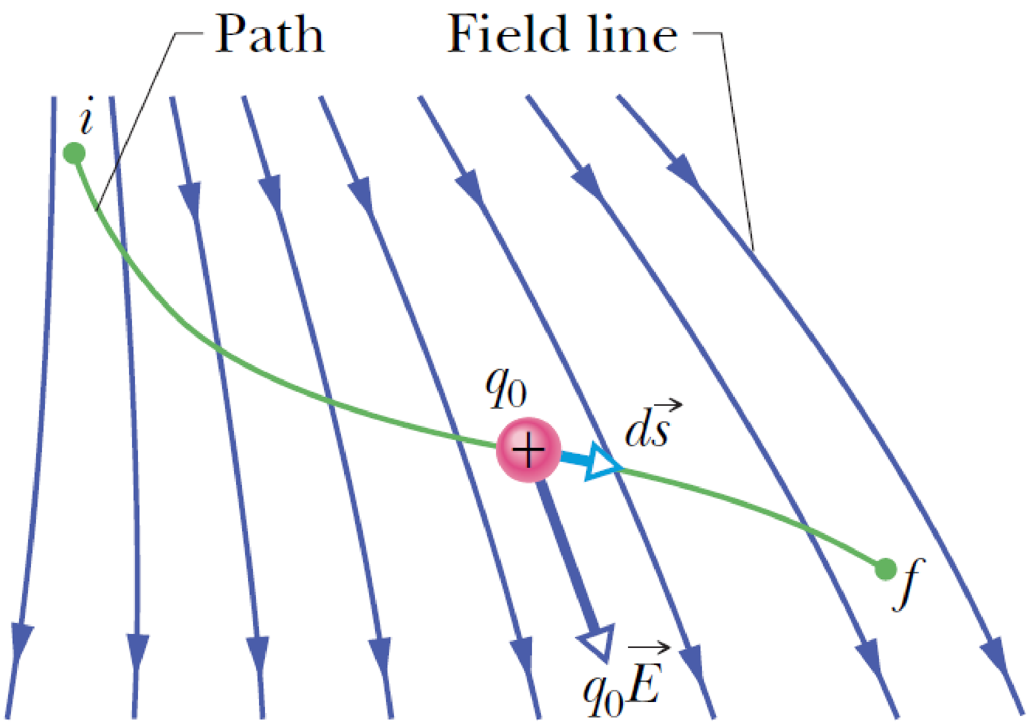


4. Calculating the Potential from the Field

If we set $V_i = 0$ and rewrite V_f as V we get

$$V = -\int_i^f \vec{E} \cdot d\vec{s}\,.$$

This equation gives us the potential V at any point f in an electric field *relative to zero potential* at point i. If we let point i be at infinity, this equation gives us the potential V at any point f relative to zero potential at infinity.



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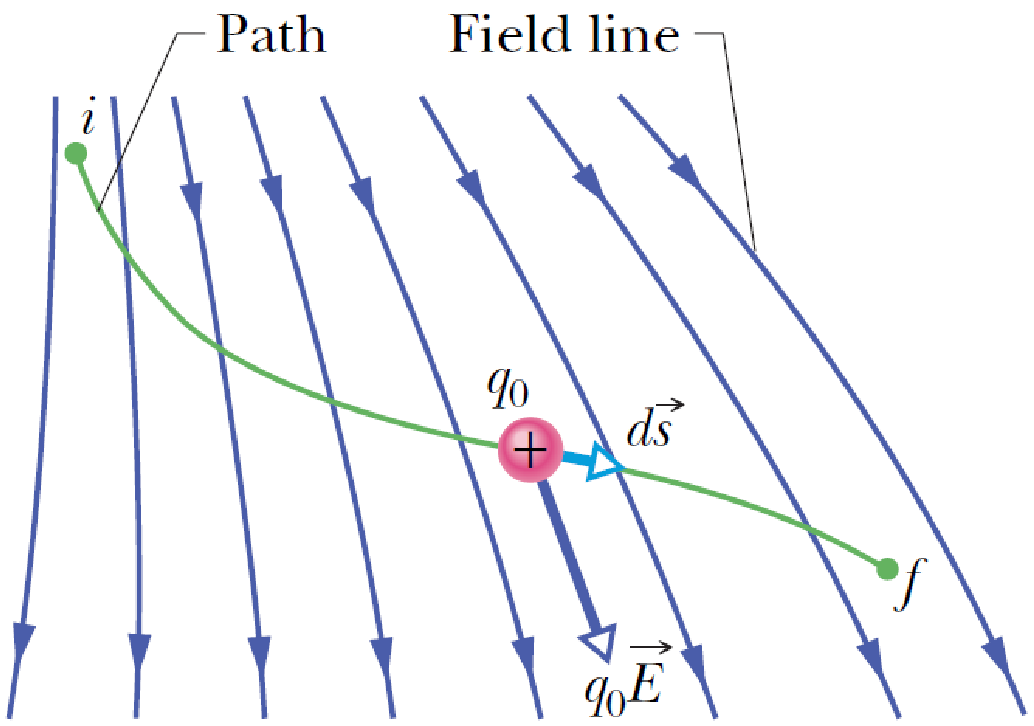
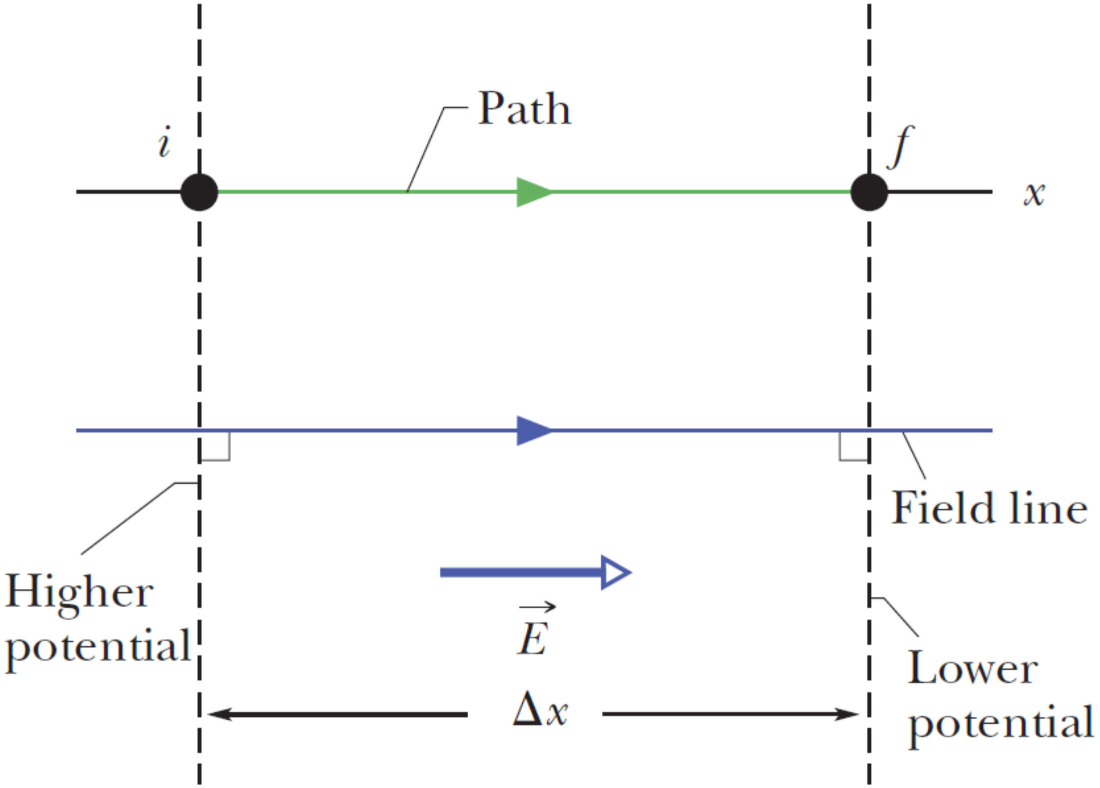


4. Calculating the Potential from the Field

Uniform Field: Let's find the potential difference between points i and f in the case of a uniform electric field. We obtain

$$V_f - V_i = -\int_i^f \vec{E} \cdot d\vec{s} = -\int_i^f E ds \cos 0$$

$$= -E \int_i^f ds = -E\Delta x.$$

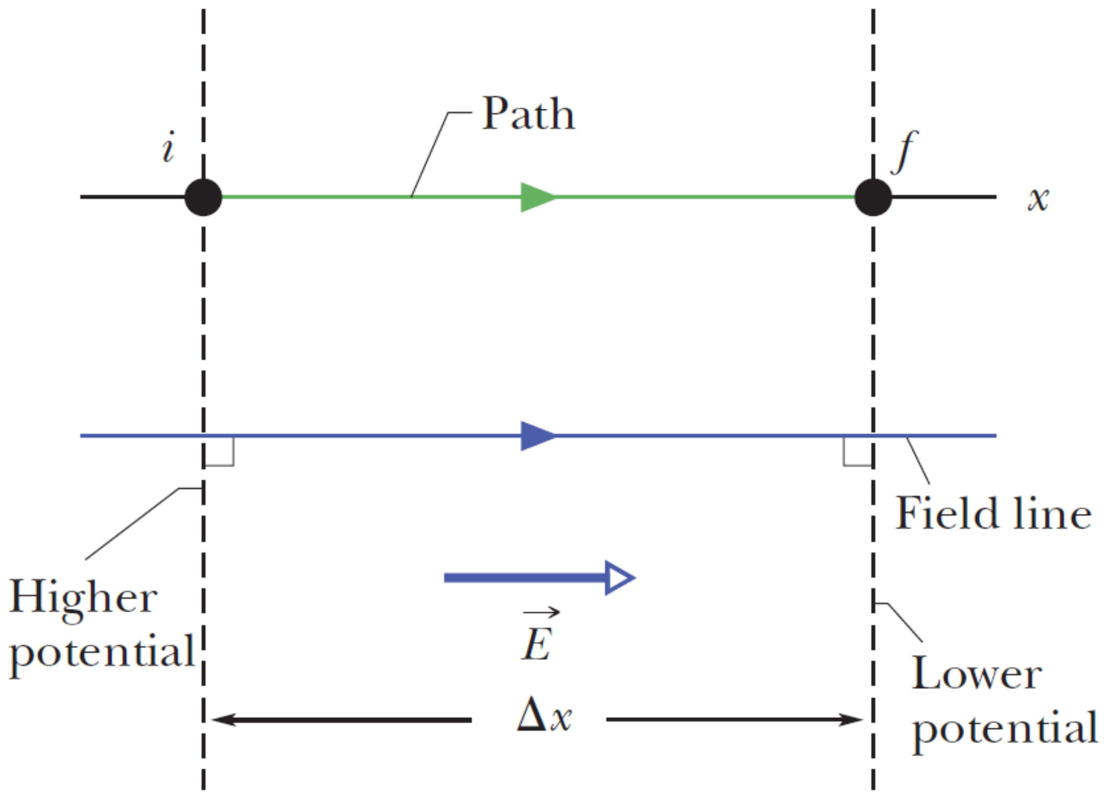


The is the change in voltage ΔV between two equipotential lines in a uniform field of magnitude E, separated by distance Δx

$$\Delta V = -E\Delta x.$$

4. Calculating the Potential from the Field

The electric field vector points from higher potential toward lower potential. This is true for any electric field.

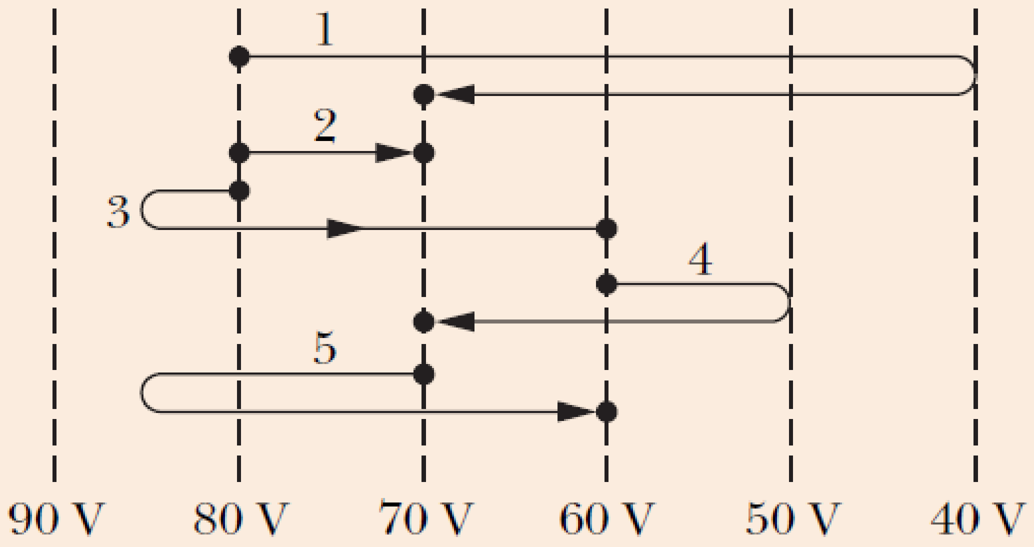


4. Calculating the Potential from the Field

CHECKPOINT 3

$$W_{\text{app}} = q\Delta V = -e\Delta V$$

The figure here shows a family of parallel equipotential surfaces (in cross section) and five paths along which we shall move an electron from one surface to another. (a) What is the direction of the electric field associated with the surfaces? (b) For each path, is the work we do positive, negative, or zero? (c) Rank the paths according to the work we do, greatest first.

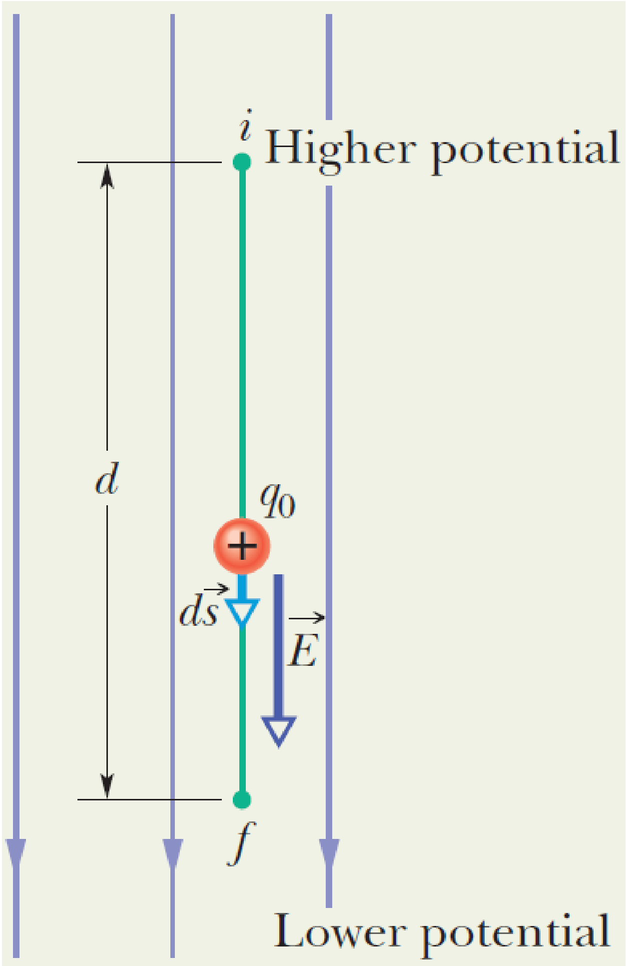


(a) Rightward. (b) Positive: 1,2,3,5. Negative: 4. (c) 3 then 1,2,5 tie, then 4.

4. Calculating the Potential from the Field

Example 2:

(a) The figure shows two points i and f in a uniform electric field $\vec{E}$. The points lie on the same electric field line (not shown) and are separated by a distance d. Find the potential difference $V_f - V_i$ by moving a positive test charge q_0 from i to f along the path shown, which is parallel to the field direction.



$$\vec{E} \cdot d\vec{s} = E\,ds\cos 0 = E\,ds.$$

$$V_f - V_i = -\int_i^f \vec{E} \cdot d\vec{s} = -E\int_i^f ds = -Ed.$$

The potential always decreases along a path that extends in the direction of the electric field lines.

4. Calculating the Potential from the Field

Example 2:

(b) Now find the potential difference $V_f - V_i$ by moving the positive test charge q_0 from i to f along the path icf shown in the figure.

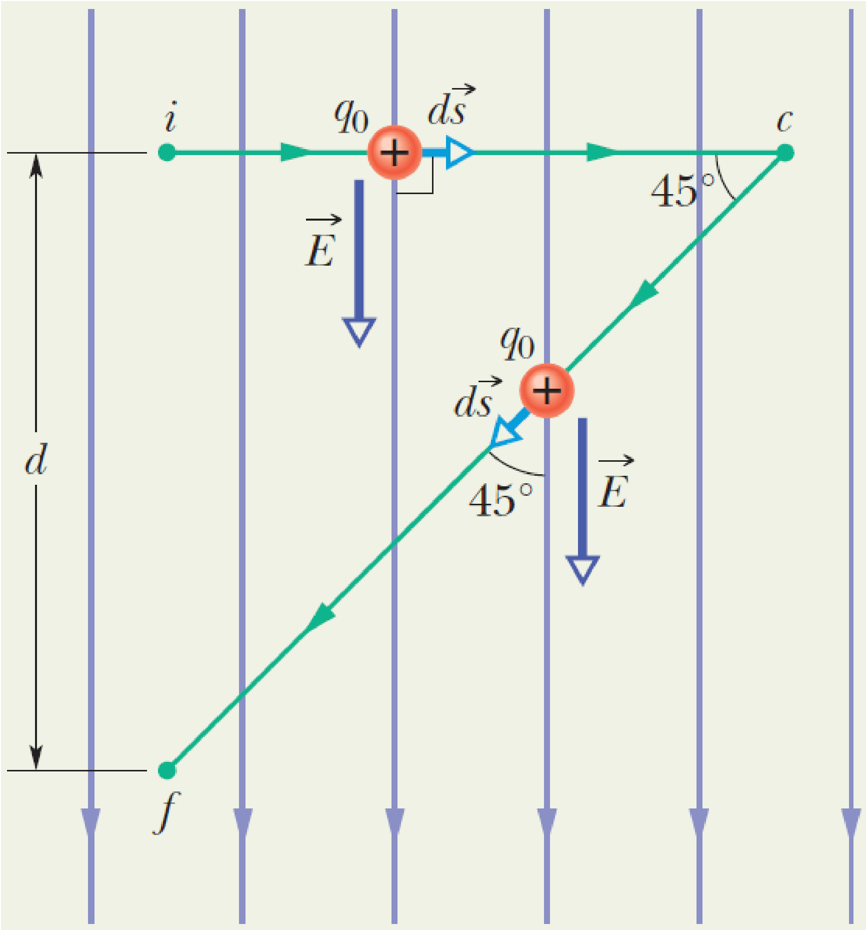
$$V_f - V_i = V_f - V_c.$$

Along path cf

$$\vec{E} \cdot d\vec{s} = E ds \cos 45°.$$

$$V_f - V_i = -\int_c^f \vec{E} \cdot d\vec{s} = -E \cos 45° \int_c^f ds$$

$$= -E \cos 45° \left(\sqrt{2}d\right) = -Ed.$$



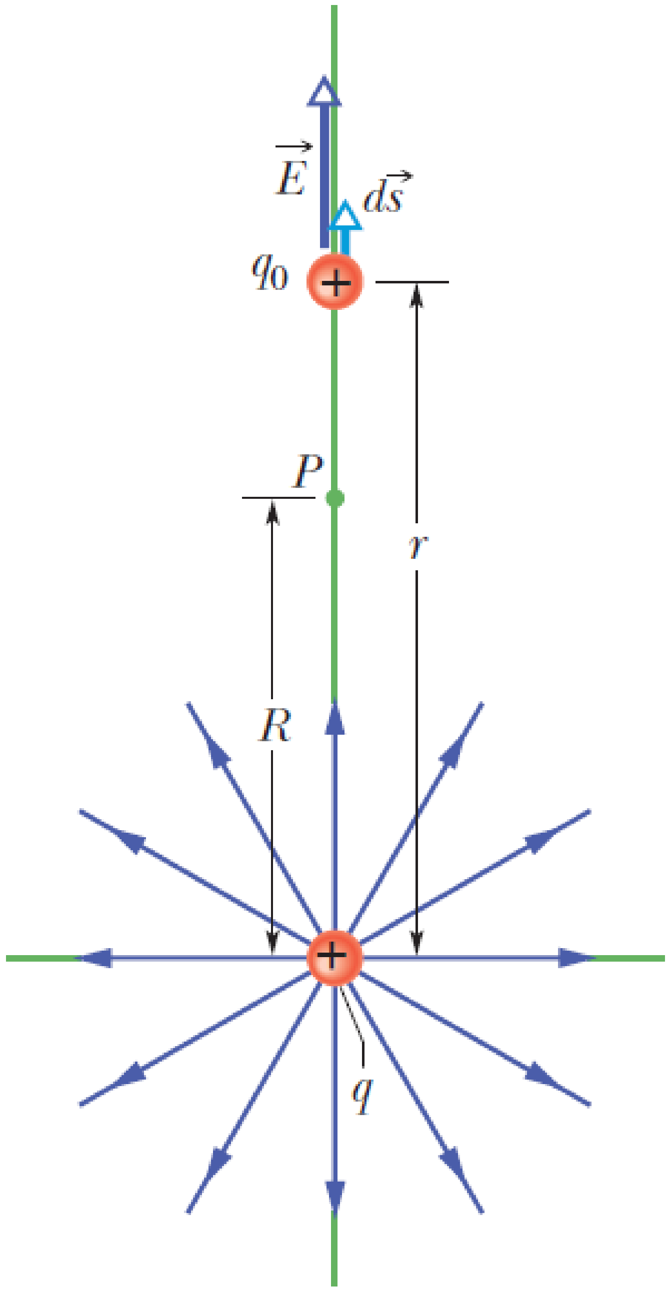
5. Potential Due to a Point Charge

We here want to derive an expression for the electric potential V for the space around a charged particle, relative to zero potential at infinity.

To do that, we move a positive test charge q_0 from point P to infinity.

We choose the simplest path connecting the two points—a radial straight line. Evaluating the dot product gives

$$\vec{E} \cdot d\vec{s} = E ds \cos 0 = E\ dr = \frac{q}{4\pi\varepsilon_0}\frac{dr}{r^2}.$$



5. Potential Due to a Point Charge

The line integral becomes

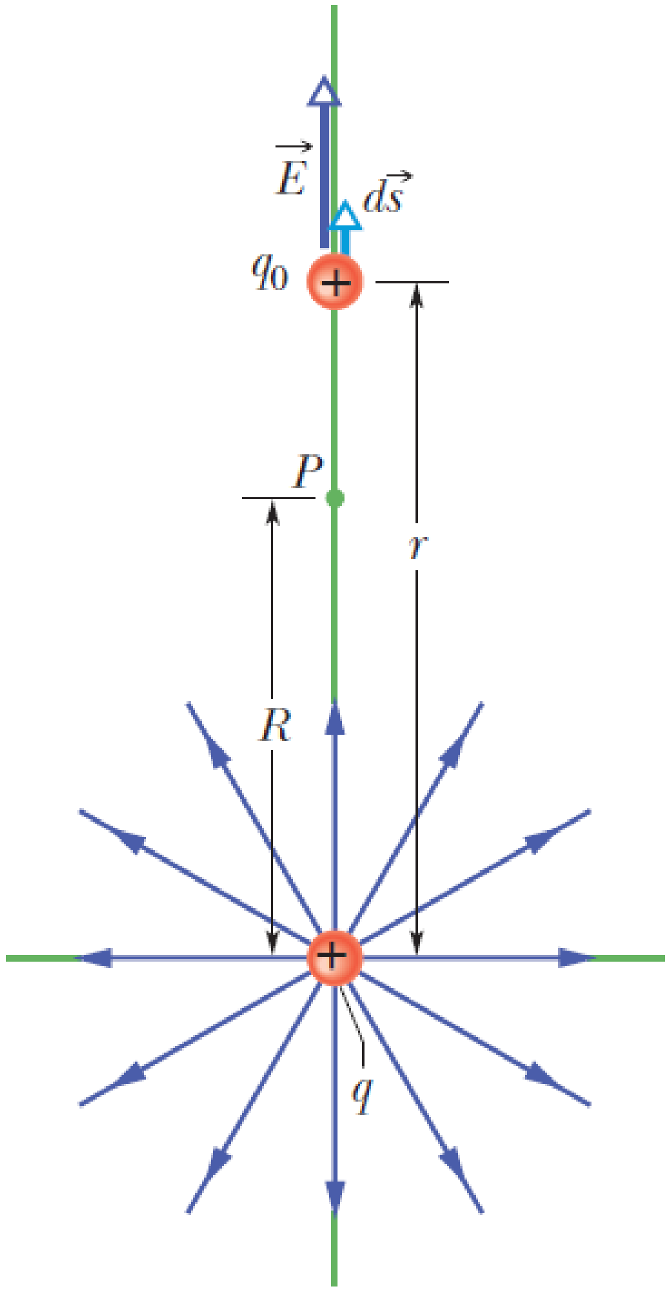
$$V_f - V_i = -\int_i^f \frac{q}{4\pi\varepsilon_0}\frac{dr}{r^2} = -\frac{q}{4\pi\varepsilon_0}\int_R^\infty \frac{dr}{r^2}$$

$$= -\frac{q}{4\pi\varepsilon_0}\left[-\frac{1}{r}\right]_R^\infty = -\frac{q}{4\pi\varepsilon_0}\frac{1}{R}.$$

Setting $V_f = 0$ (at ∞), $V_i = V$ and switching R to r we get

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}.$$

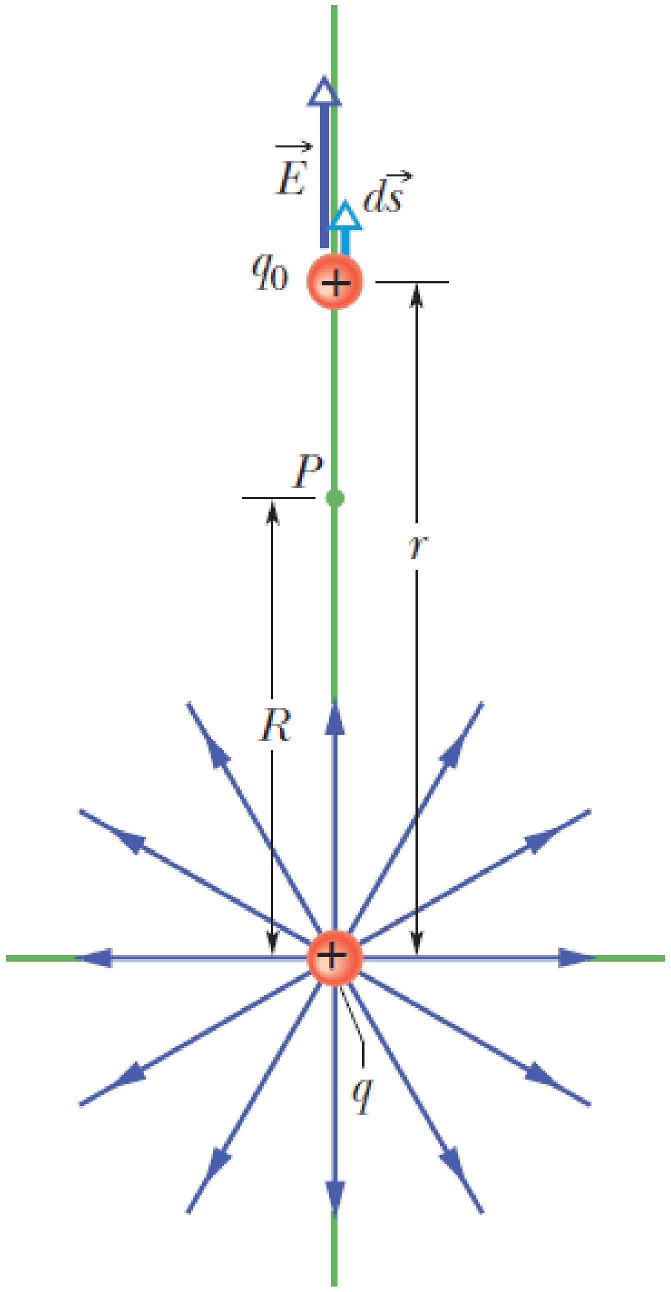
Note that q can be positive or negative and that V has the same sign as q.



5. Potential Due to a Point Charge

A positively charged particle produces a positive electric potential. A negatively charged particle produces a negative electric potential.

Also, note that the expression for V holds outside or on the surface of a spherically symmetric charge distribution.



6. Potential Due to a Group of Point Charges

The *net* potential due to n point charges at a given point is

$$V = \sum_{i=1}^{n} V_i = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{n}\frac{q_i}{r_i}.$$

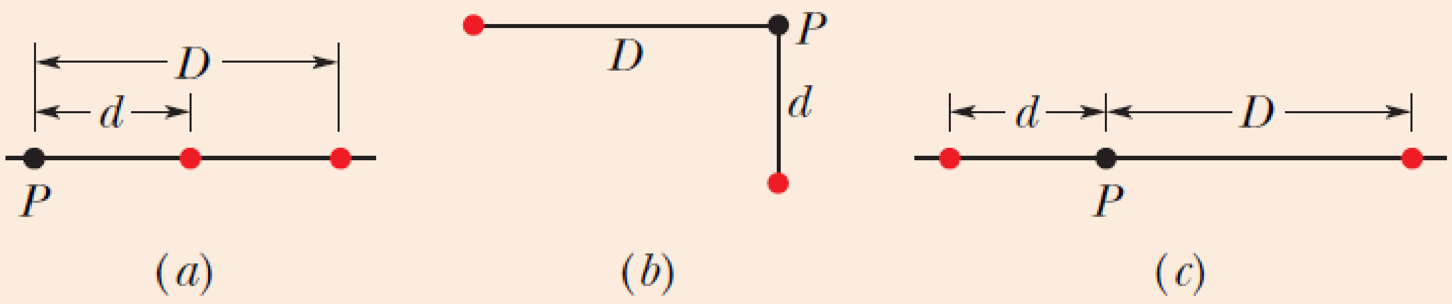
Here q_i is the charge (positive or negative) of the ith particle, and r_i is the radial distance of the given point from the ith charge.

An important advantage of potential over electric field is that we sum scalar quantities instead of vector quantities.

6. Potential Due to a Group of Point Charges

CHECKPOINT 4

The figure here shows three arrangements of two protons. Rank the arrangements according to the net electric potential produced at point P by the protons, greatest first.

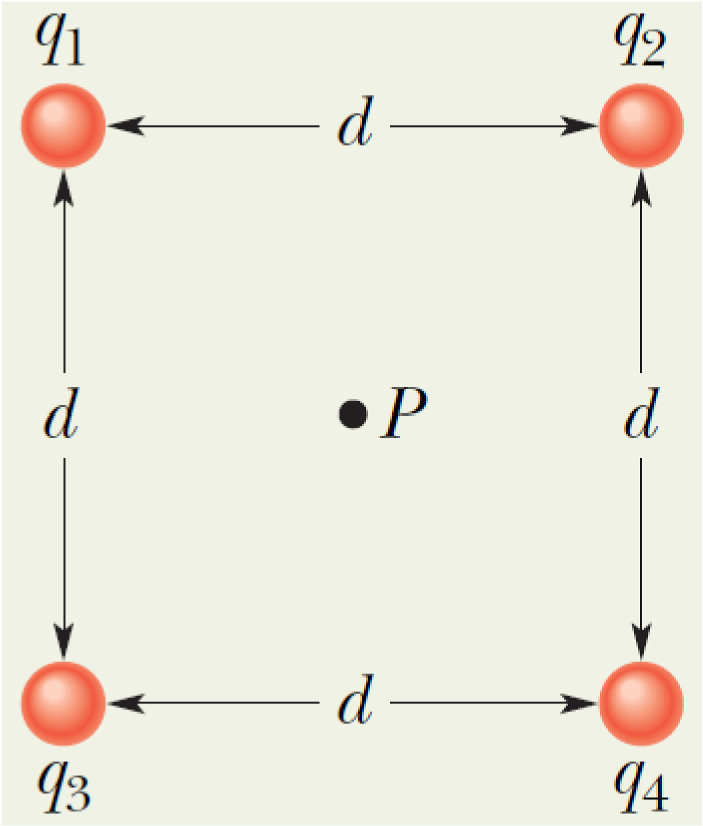


All the same.

6. Potential Due to a Group of Point Charges

Example 3: What is the electric potential at point *P*, located at the center of the square of point charges shown in the figure? The distance $d = 1.3\ \text{m}$, and the charges are

$$q_1 = 12\ \text{nC}, \qquad q_2 = 31\ \text{nC},$$
$$q_3 = -24\ \text{nC}, \qquad q_4 = 17\ \text{nC}.$$



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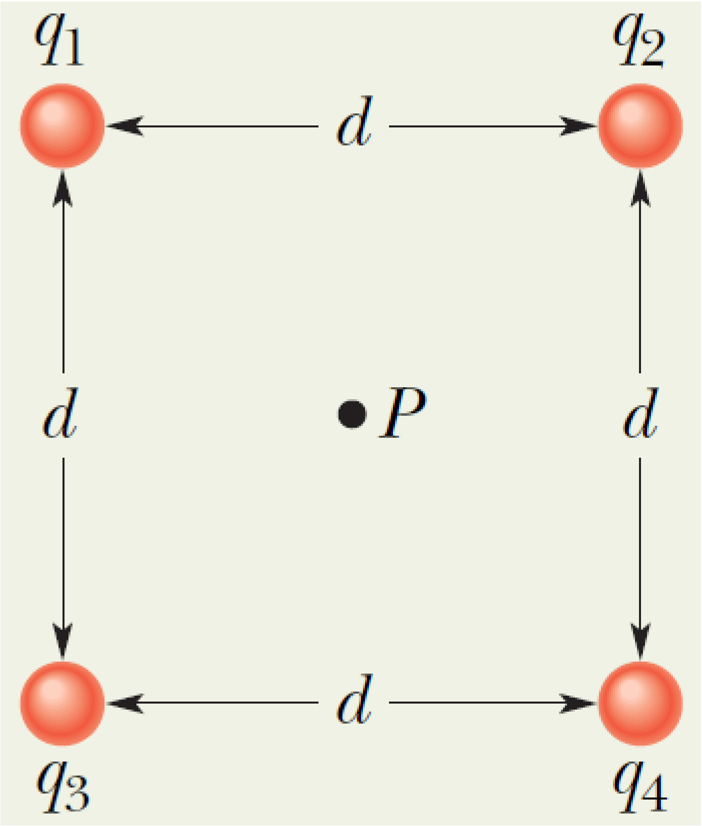


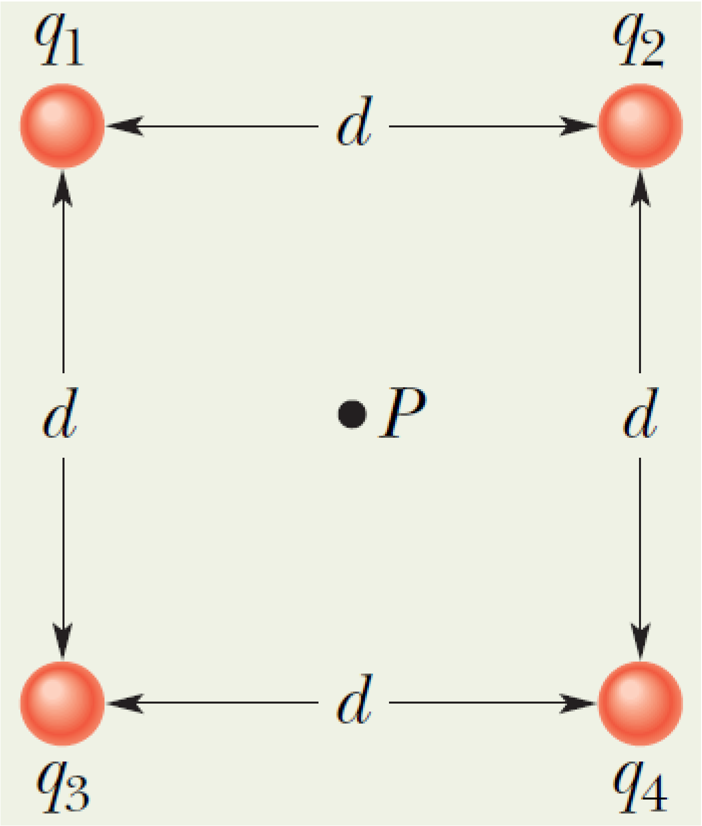
$$V = \sum_{i=1}^{4} V_i = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{r} + \frac{q_2}{r} + \frac{q_3}{r} + \frac{q_4}{r}\right)$$
$$= \frac{1}{4\pi\varepsilon_0}\frac{q_1 + q_2 + q_3 + q_4}{r}$$

6. Potential Due to a Group of Point Charges

$$q_1 + q_2 + q_3 + q_4 = (12 + 31 - 24 + 17) \times 10^{-9}\ \text{C}$$
$$= 36 \times 10^{-9}\ \text{C}.$$
$$r = \frac{\sqrt{2}}{2} d.$$

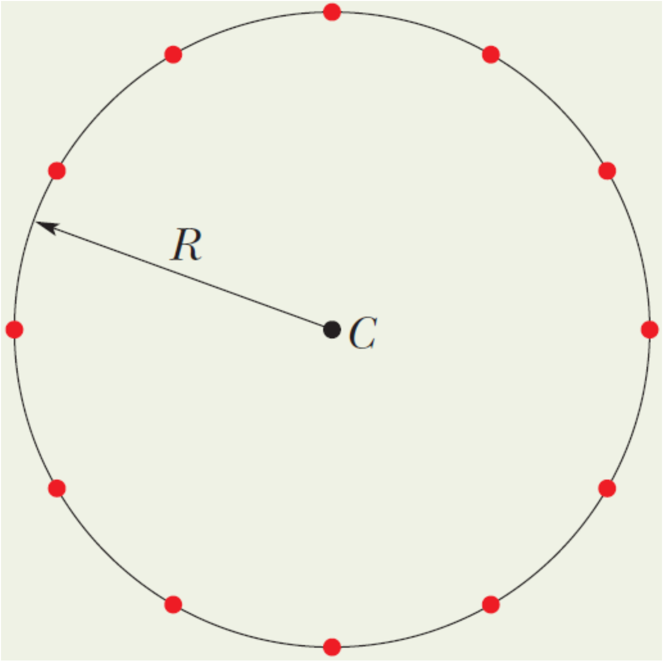
Substituting gives

$$V = \left(8.99 \times \frac{10^9 \text{N} \cdot \text{m}^2}{\text{C}^2}\right)\left(\frac{3.6 \times 10^{-8}\ \text{C}}{1.3/\sqrt{2}\text{m}}\right)$$
$$= 350\ \text{V}.$$



6. Potential Due to a Group of Point Charges

Example 4: (a) In the figure electrons (of charge $-e$) are equally spaced and fixed around a circle of radius R. Relative to $V = 0$ at infinity, what are the electric potential and electric field at the center C of the circle due to these electrons?



$$V = \sum_{i=1}^{12} V_i = -\frac{1}{4\pi\varepsilon_0}\frac{12e}{R}$$

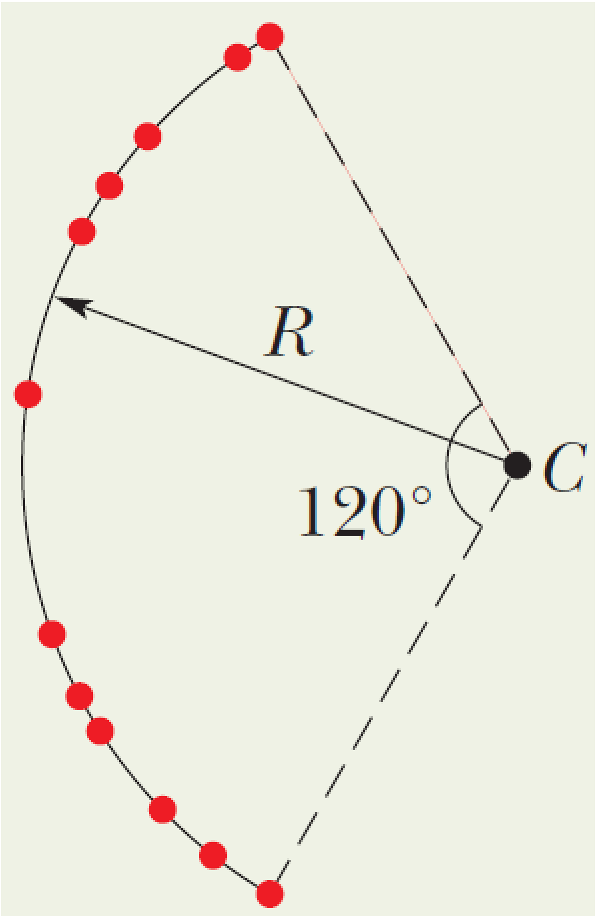
By the symmetry of the problem, $E = 0$.

6. Potential Due to a Group of Point Charges

Example 4: (b) If the electrons are moved along the circle until they are nonuniformly spaced over a 120° arc (see the figure), what then is the potential at C? How does the electric field at C change (if at all)?

The potential is not changed because the distances between C and each charge are not changed.

The electric field is no longer zero, however, because the arrangement is no longer symmetric. A net field is now directed toward the charge distribution.



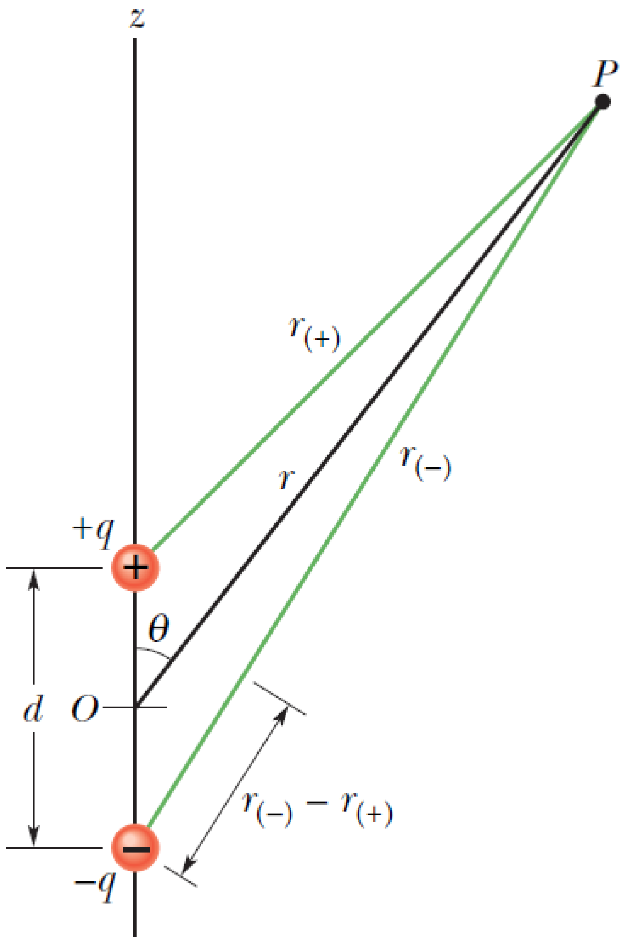
7. Potential Due to an Electric Dipole

The potential due to an electric dipole is the net potential due to the positive and negative charges.

At point P, the net potential is

$$V = \sum_{i=1}^{2} V_i = V_{(+)} + V_{(-)} = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{r_{(+)}} - \frac{q}{r_{(-)}}\right)$$

$$= \frac{q}{4\pi\varepsilon_0}\left(\frac{r_{(-)} - r_{(+)}}{r_{(+)}r_{(-)}}\right).$$



7. Potential Due to an Electric Dipole

For $r \gg d$, we can make the following approximation (see the figure):

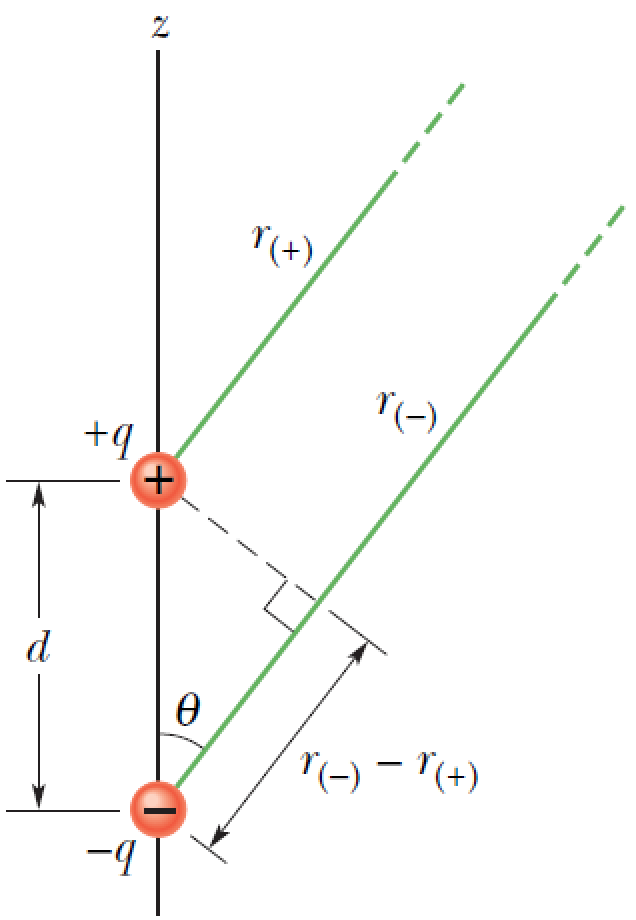
$$r_{(-)} - r_{(+)} \approx d\cos\theta\,,$$

and

$$r_{(-)} r_{(+)} \approx r^2.$$

Substituting in the expression for V gives

$$V = \frac{q}{4\pi\varepsilon_0}\frac{d\cos\theta}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{p\cos\theta}{r^2}.$$

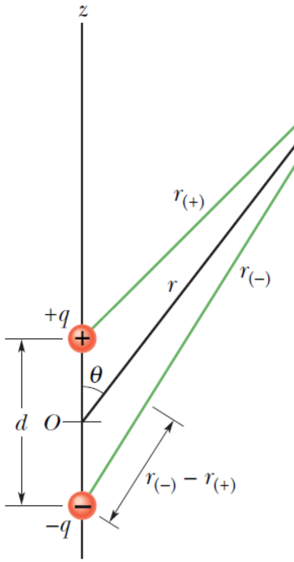


7. Potential Due to an Electric Dipole

CHECKPOINT 5

Suppose that three points are set at equal (large) distances r from the center of the dipole in Fig. 24-10: Point a is on the dipole axis above the positive charge, point b is on the axis below the negative charge, and point c is on a perpendicular bisector through the line connecting the two charges. Rank the points according to the electric potential of the dipole there, greatest (most positive) first.

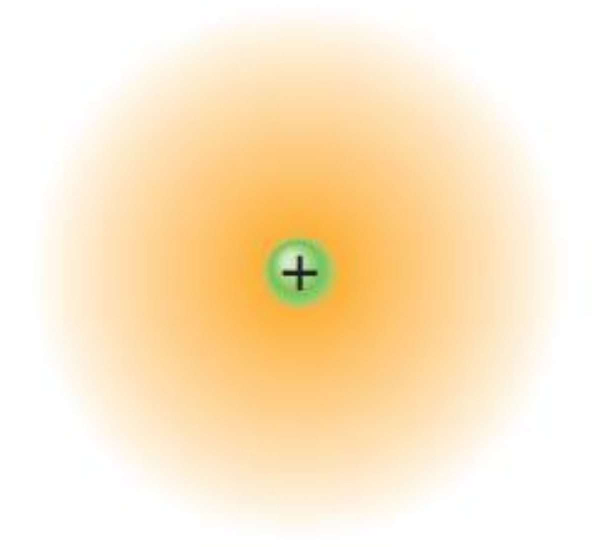
Point a, c then b.



$$V = \frac{1}{4\pi\varepsilon_0}\frac{p\cos\theta}{r^2}$$

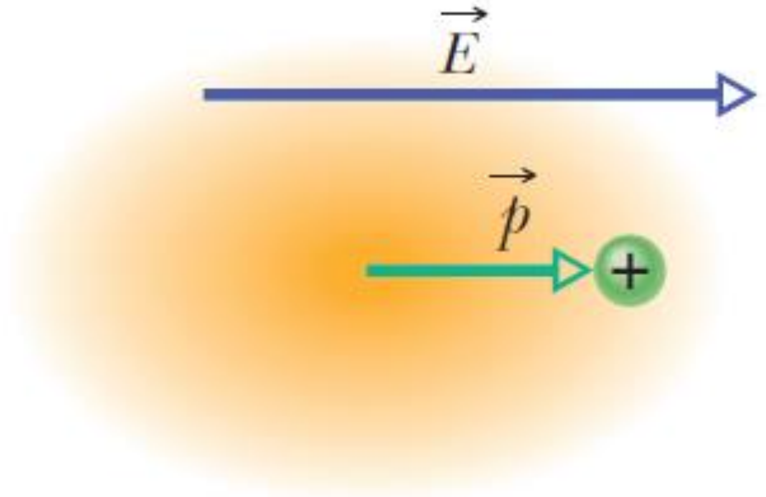
7. Potential Due to an Electric Dipole

Induced Dipole Moment



7. Potential Due to an Electric Dipole

Induced Dipole Moment



8. Calculating the Field from the Potential

Let us see graphically how we can calculate the field from the potential. If we know the potential V at all points near a charge distribution, we can draw a family of equipotential surfaces. The electric field lines, sketched perpendicular to those surfaces, reveal the variation of $\vec{E}$.

We want now to write the mathematical equivalence of this graphical procedure.

8. Calculating the Field from the Potential

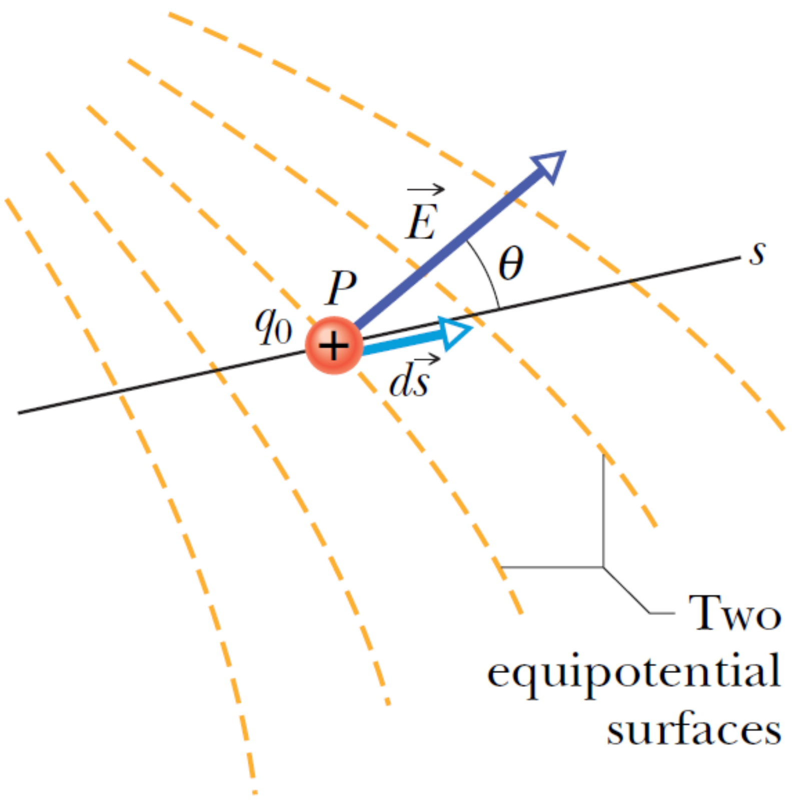
The figure shows cross sections of a family of closely spaced equipotential surfaces.

The potential difference between each pair of adjacent surfaces is dV. The electric field $\vec{E}$ at any point P is perpendicular to the equipotential surface through P.

Suppose that a positive test charge q_0 moves through a displacement $d\vec{s}$ from an equipotential surface to the adjacent surface.

The work dW done by the electric field on the test charge is

$$dW = -q_0 dV.$$



8. Calculating the Field from the Potential

The work dW can also be written as

$$dW = \vec{F} \cdot d\vec{s} = q_0 \vec{E} \cdot d\vec{s} = q_0 E ds \cos\theta \,.$$

Equating the two expression for dW yields

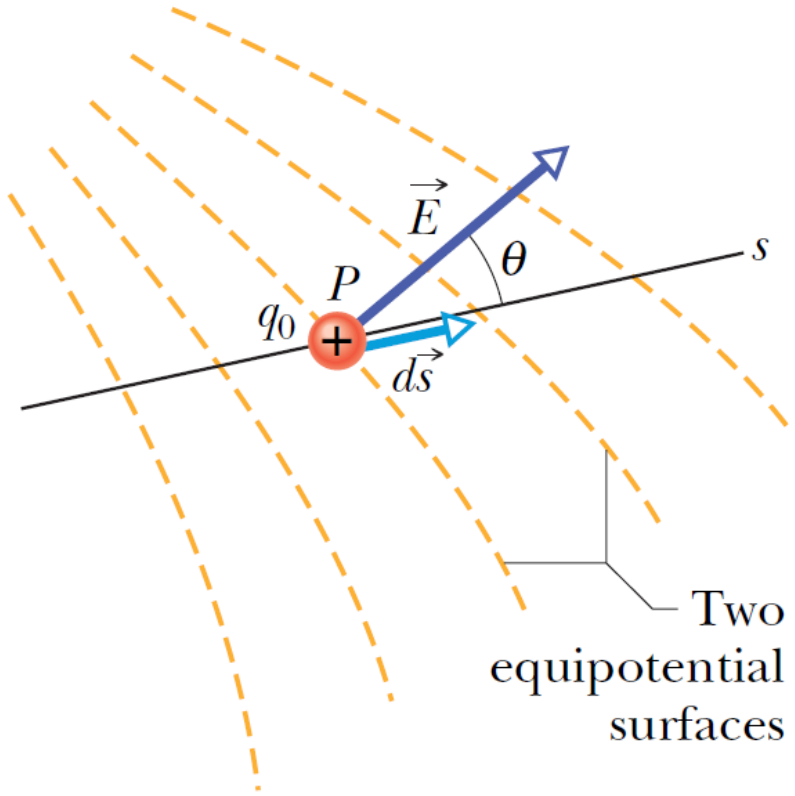
$$-q_0 dV = q_0 E ds \cos\theta \,,$$

or

$$E \cos\theta = -\frac{dV}{ds}.$$

$E \cos\theta$ is the component of $\vec{E}$ in the direction of $d\vec{s}$. Thus,

$$E_s = -\frac{\partial V}{\partial s}.$$



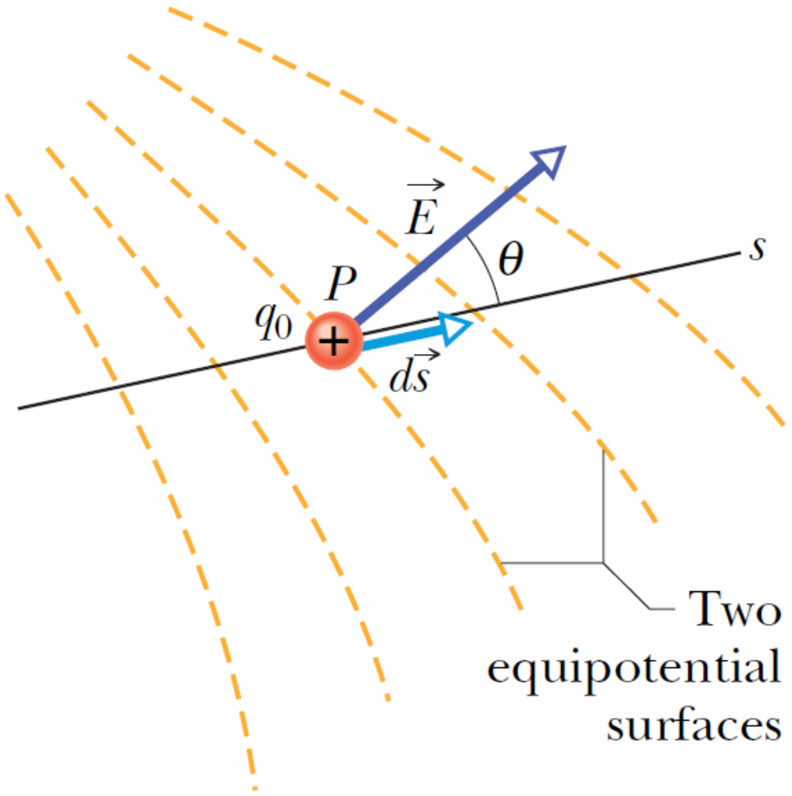
8. Calculating the Field from the Potential

The component of $\vec{E}$ in any direction is the negative of the rate at which the electric potential changes with distance in that direction.

If we take ds to be along the x, y and z axes, we obtain the x, y and z components of $\vec{E}$ at any point:

$$E_x = -\frac{\partial V}{\partial x}; \quad E_y = -\frac{\partial V}{\partial y}; \quad E_z = -\frac{\partial V}{\partial z}.$$

If we know the function $V(x, y, z)$, we can find the components of $\vec{E}$ at any point.



8. Calculating the Field from the Potential

When the electric field $\vec{E}$ is uniform, we write that

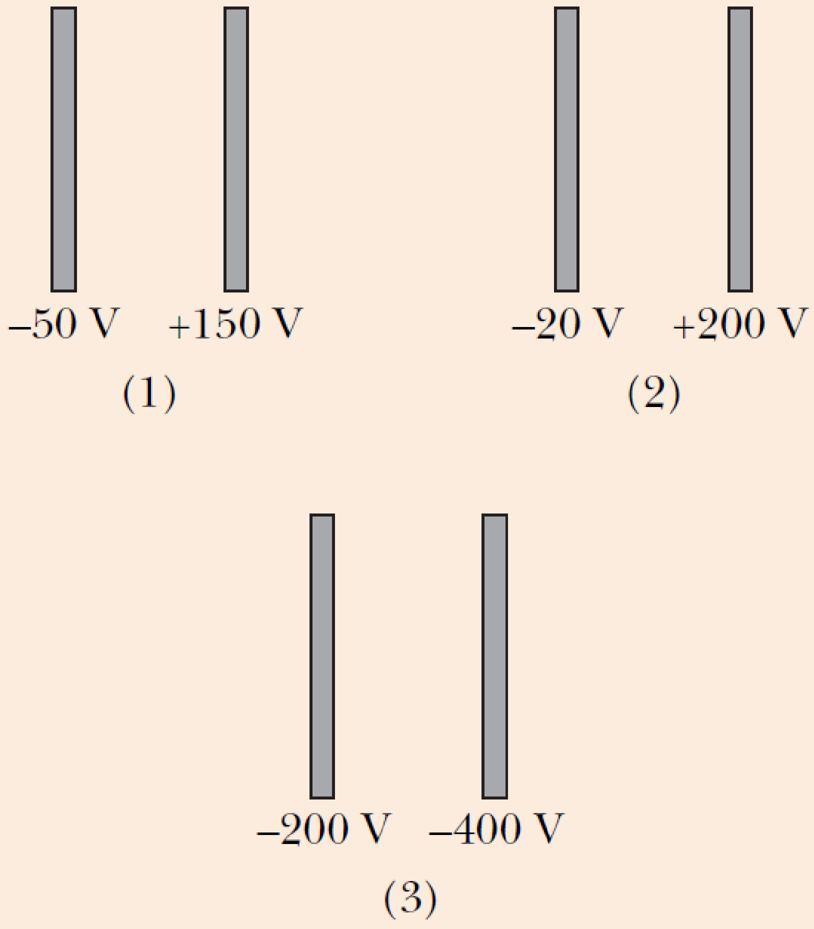
$$E = -\frac{\Delta V}{\Delta s}.$$

where s is perpendicular to the equipotential surfaces. The component of the electric field is zero in any direction parallel to the equipotential surfaces because there is no change in potential along an equipotential surface.

8. Calculating the Field from the Potential

CHECKPOINT 6

The figure shows three pairs of parallel plates with the same separation, and the electric potential of each plate. The electric field between the plates is uniform and perpendicular to the plates. (a) Rank the pairs according to the magnitude of the electric field between the plates, greatest first. (b) For which pair is the electric field pointing rightward? (c) If an electron is released midway between the third pair of plates, does it remain there, move rightward at constant speed, move leftward at constant speed, accelerate rightward, or accelerate leftward?



$$E = -\frac{\Delta V}{\Delta s}$$

(a) 2, then 3 & 1.

(b) 3.

(c) Accelerates leftward.

8. Calculating the Field from the Potential

Example 5: Starting with the expression for the potential due to an electric dipole,

$$V = \frac{1}{4\pi\varepsilon_0}\frac{p\cos\theta}{r^2}.$$

derive an expression for the radial component of electric field due to the electric dipole.

$$E_r = -\frac{\partial V}{\partial r} = -\frac{p\cos\theta}{4\pi\varepsilon_0}\frac{d}{dr}\frac{1}{r^2}$$

$$= -\frac{p\cos\theta}{4\pi\varepsilon_0}\left(-\frac{2}{r^3}\right) = \frac{1}{2\pi\varepsilon_0}\frac{p\cos\theta}{r^3}.$$

8. Calculating the Field from the Potential

Example 6: The electric potential at any point on the central axis of a uniformly charged disk is given by

$$V = \frac{\sigma}{2\varepsilon_0}\left(\sqrt{z^2 + R^2} - z\right).$$

Starting with this expression, derive an expression for the electric field at any point on the axis of the disk.

$$E_z = -\frac{\partial V}{\partial z} = \frac{\sigma}{2\varepsilon_0}\frac{d}{dz}\left(z - \sqrt{z^2 + R^2}\right)$$

$$= \frac{\sigma}{2\varepsilon_0}\left(1 - \frac{z}{\sqrt{z^2 + R^2}}\right).$$

9. Electric Potential Energy of a System of Charges

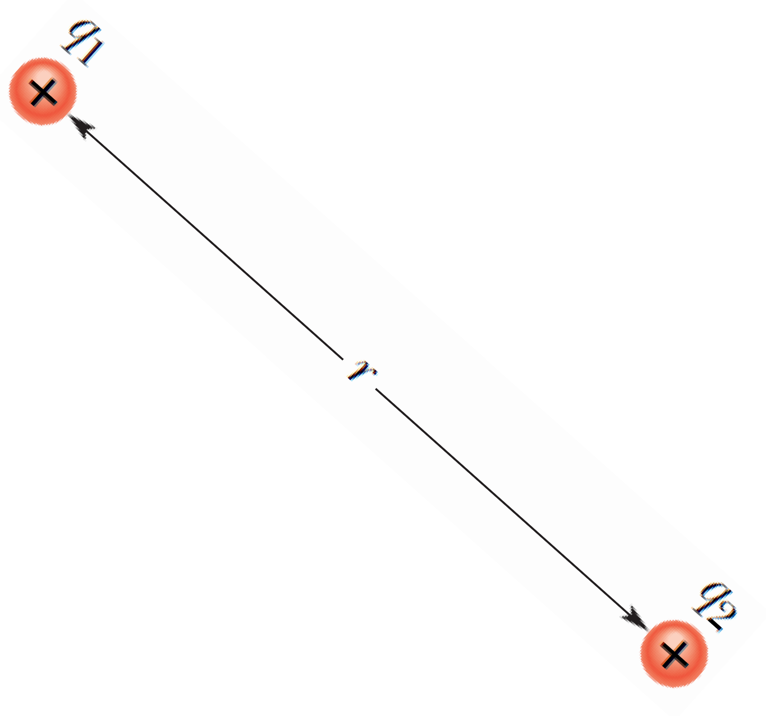
The electric potential energy of a system of fixed point charges is equal to the work that must be done by an external agent to assemble the system, bringing each charge in from an infinite distance.

We assume that the charges are stationary at both the initial and final states.

9. Electric Potential Energy of a System of Charges

Let us find the electric potential energy of the system shown in the figure. We can build the system by bringing q_2 from infinity and put it at distance r from q_1. The work we do to bring q_2 near q_1 is q_2V, where V is the potential that has been set up by q_1. Thus, the electric potential of the pair of point charges shown in the figure is

$$U = W_{\text{app}} = q_2V = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r}.$$



U has the same sign as that of q_1q_2.

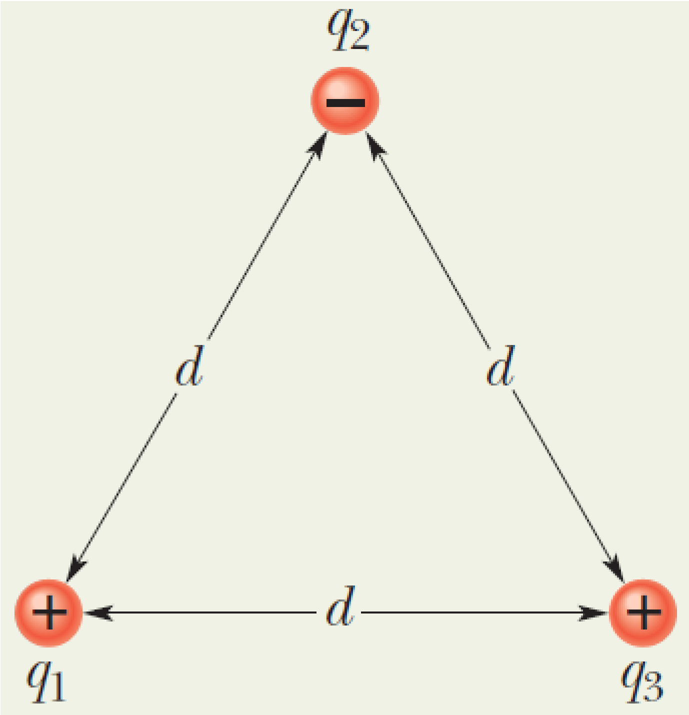
The total potential energy of a system of several particles is the sum of the potential energies for every pair of particles in the system.

9. Electric Potential Energy of a System of Charges

Example 7: The figure shows three point charges held in fixed positions by forces that are not shown. What is the electric potential energy U of this system of charges? Assume that $d = 12\ \text{cm}$ and that

$$q_1 = +q, \qquad q_2 = -4q, \qquad q_3 = +2q,$$

and $q = 150\ \text{nC}$.



The electric potential energy of the system (energy required to assemble the system) is

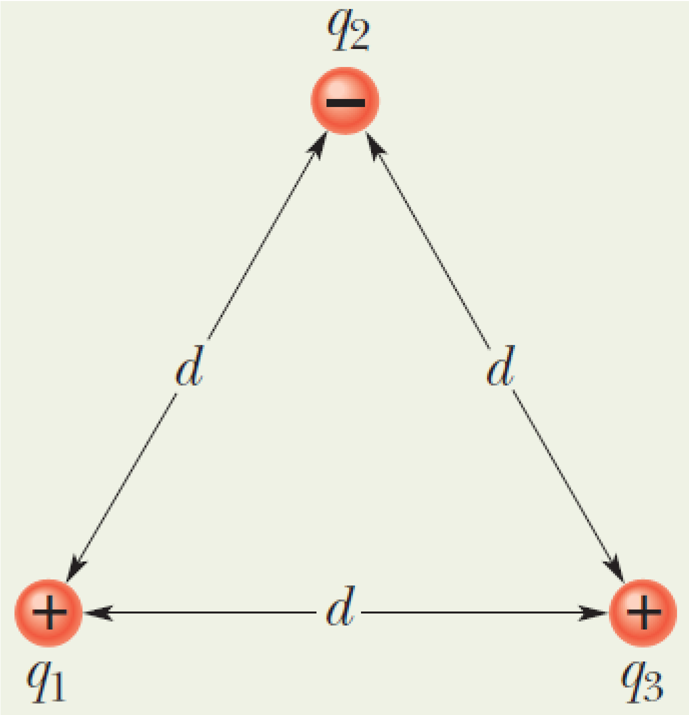
$$U = U_{12} + U_{13} + U_{23}$$
$$= \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{d} + \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_3}{d} + \frac{1}{4\pi\varepsilon_0}\frac{q_2 q_3}{d}.$$

9. Electric Potential Energy of a System of Charges

$$U = \frac{1}{4\pi\varepsilon_0}\frac{(q)(-4q)}{d} + \frac{1}{4\pi\varepsilon_0}\frac{(q)(2q)}{d} + \frac{1}{4\pi\varepsilon_0}\frac{(-4q)(2q)}{d}$$

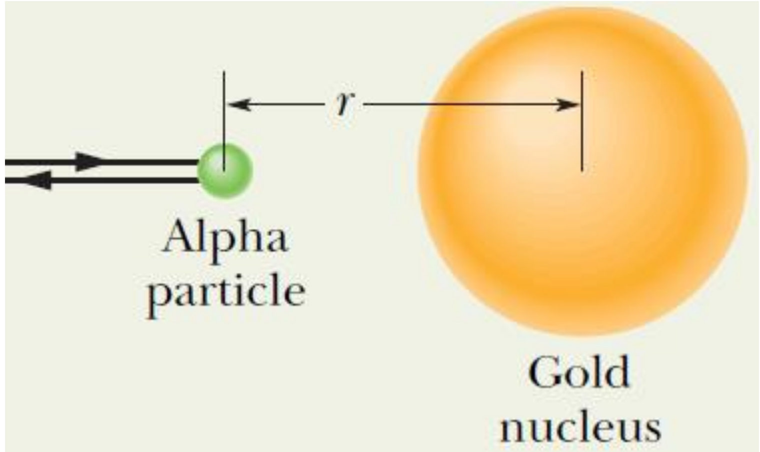
$$= -\frac{10q^2}{4\pi\varepsilon_0 d} = \left(8.99\times10^{12}\ \frac{\mathrm{N\cdot m^2}}{\mathrm{C^2}}\right)\frac{10(150\times10^{-9}\mathrm{C})}{0.12\ \mathrm{m}}$$

$$= -17\ \mathrm{mJ}.$$



9. Electric Potential Energy of a System of Charges

Example 8: An alpha particle (two protons, two neutrons) moves into a stationary gold atom (79 protons, 118 neutrons), passing through the electron region that surrounds the gold nucleus like a shell and headed directly toward the nucleus. The alpha particle slows until it momentarily stops when its center is at radial distance $r = 9.23\ \text{fm}$ from the nuclear center. Then it moves back along its incoming path. (Because the gold nucleus is much more massive than the alpha particle, we can assume the gold nucleus does not move.) What was the kinetic energy K_i of the alpha particle when it was initially far away (hence external to the gold atom)? Assume that the only force acting between the alpha particle and the gold nucleus is the (electrostatic) Coulomb force.

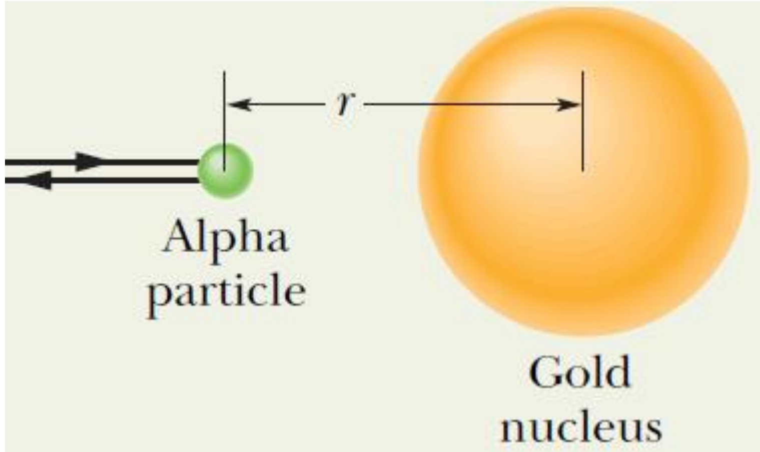


9. Electric Potential Energy of a System of Charges

During the entire process, the mechanical energy of the alpha particle-gold atom system is conserved.

$$K_i + U_i = K_f + U_f.$$

U_i is zero because the atom is neutral. U_f is the electric potential energy of the *alpha particle-nucleus* system. The electron shell produces zero electric field inside it. K_f is zero.



$$K_i = U_f = \frac{1}{4\pi\varepsilon_0}\frac{(2e)(79e)}{9.23\times 10^{-15}\ \text{m}} = 3.94\times 10^{-9}\ \text{J}$$

$$= 24.6\ \text{MeV}.$$

10. Potential of a Charged Isolated Conductor

In Ch. 23, we concluded that $\vec{E} = 0$ within an isolated conductor. We then used Gauss' law to prove that all excess charge placed on a conductor lies entirely on its surface. Here we use the first of these facts to prove an extension of the second:

'An excess charge placed on an isolated conductor will distribute itself on the surface of that conductor so that all points of the conductor—whether on the surface or inside—come to the same potential. This is true even if the conductor has an internal cavity and even if that cavity contains a net charge.'

To prove this fact we use that

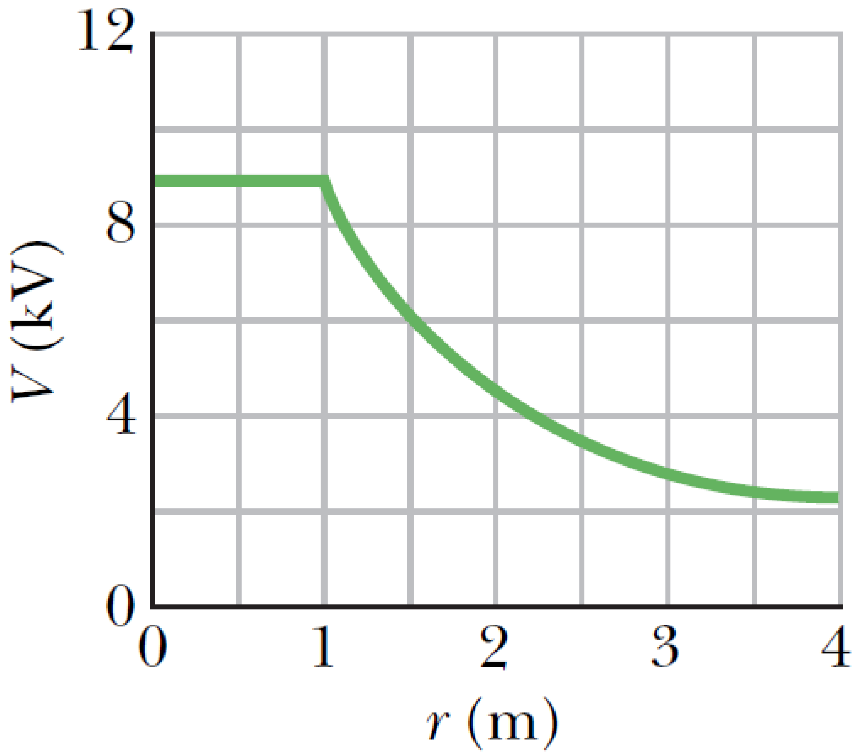
$$V_f - V_i = -\int_i^f \vec{E} \cdot d\vec{s}\,.$$

Since $\vec{E} = 0$ for all point within a conductor, $V_f = V_i$ for all possible pairs of points i and f in the conductor.

10. Potential of a Charged Isolated Conductor

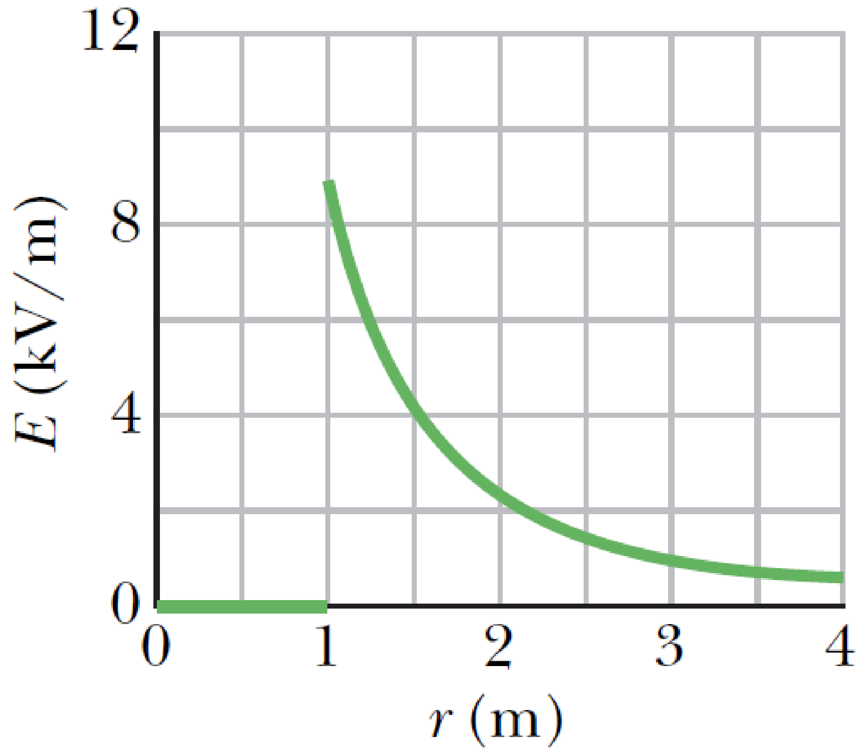
The figure shows the potential against radial distance r from the center from an isolated spherical conducting shell of radius $R = 1.0 \text{ m}$ and charge $q = 0.10\ \mu\text{C}$.

For points outside the shell up to its surface, $V(r) = k\ q/r$. Now assume that we push a test charge though a hole in the shell. No extra work is required to do this because $\vec{E}$ becomes zero once the test charge gets inside the shell. Thus, the potential at all points inside the shell has the same value as that of its surface.



10. Potential of a Charged Isolated Conductor

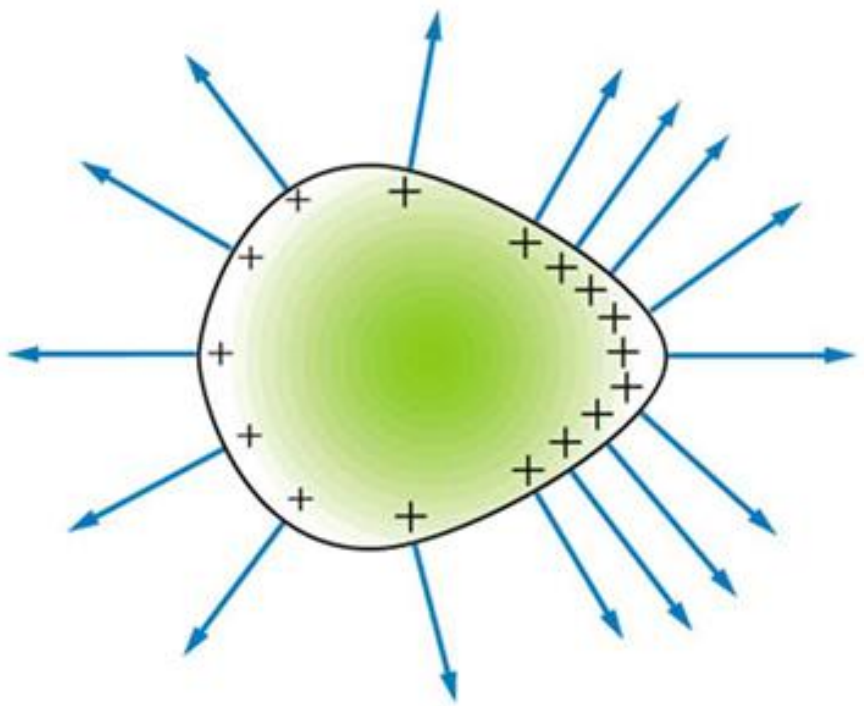
The figure shows the variation of the electric field with radial distance for the same shell. Everywhere inside the shell $E = 0$. Outside the shell $E = k\,q/r^2$. The electric field can be obtained from the potential using $E = -dV/dr$.



10. Potential of a Charged Isolated Conductor

Spark Discharge from a Charged Conductor

On nonspherical conductors, a surface charge does not distribute itself uniformly over the surface of the conductor. At sharp points or sharp edges, the surface charge density—and thus the external electric field, which is proportional to it—may reach very high values. The air around such sharp points or edges may become ionized, producing the corona discharge.



10. Potential of a Charged Isolated Conductor

Isolated Conductor in an External Electric Field

If an isolated conductor is placed in an external electric field, all points of the conductor still come to a single potential regardless of whether the conductor has an excess charge or not. The free conduction electrons distribute themselves on the surface in such a way that the electric field they produce at interior points cancels the external electric field. Furthermore, the electron distribution causes the net electric field at all points on the surface to be perpendicular to the surface.