Chapter 22

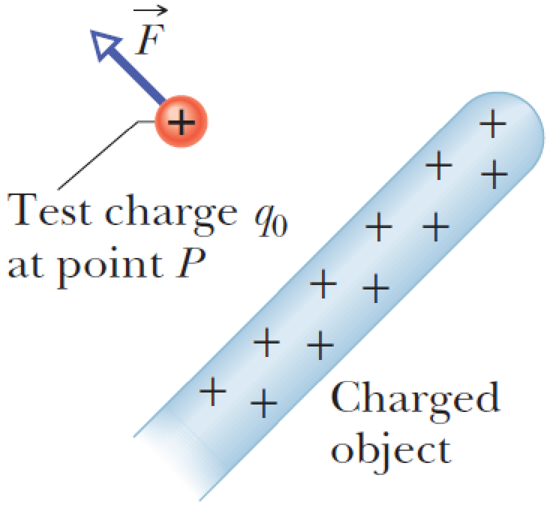
Electric Field

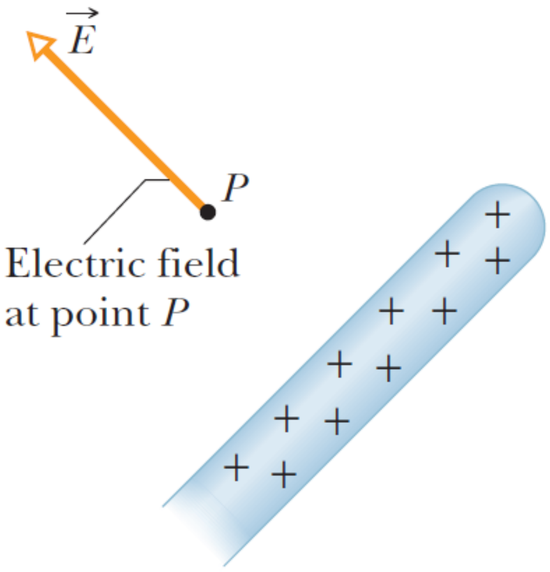
1. The Electric Field

The **electric field** $\vec{E}$ at a point due to a charged object is defined as

$$\vec{E} = \frac{\vec{F}}{q_0},$$

where q_0 is a positive test charge at that point and $\vec{F}$ is the electrostatic force that acts on it.





1. The Electric Field

The SI unit of electric field is Newton per Coulomb (N/C).

To examine the role of an electric field in the interaction between charged objects, we need to:

(1) Calculate the electric field produced by a given distribution of charge.

(2) Calculate the force that a given field exerts on a charge placed in it.

Some Electric Fields

Field Location or Situation	Value (N/C)
At the surface of a uranium nucleus	3×10^{21}
Within a hydrogen atom, at a radius of 5.29×10^{-11} m	5×10^{11}
Electric breakdown occurs in air	3×10^{6}
Near the charged drum of a photocopier	10^{5}
Near a charged comb	10^{3}
In the lower atmosphere	10^{2}
Inside the copper wire of household circuits	10^{-2}

2. Electric Field Lines

The space around a charged body can be visualized as filled with *lines of force*; the **electric field lines**.

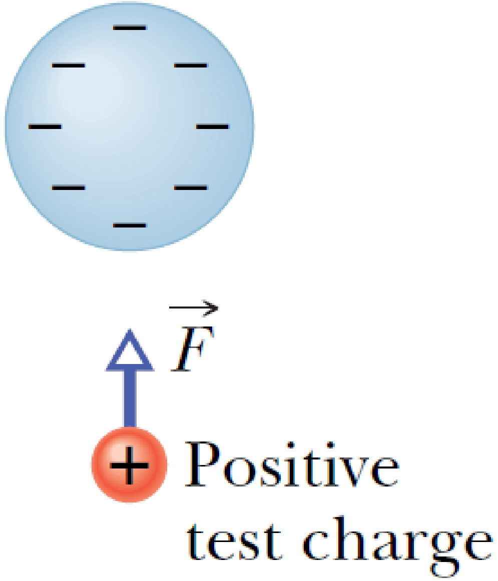
The relation between the field lines and the electric field vectors is:

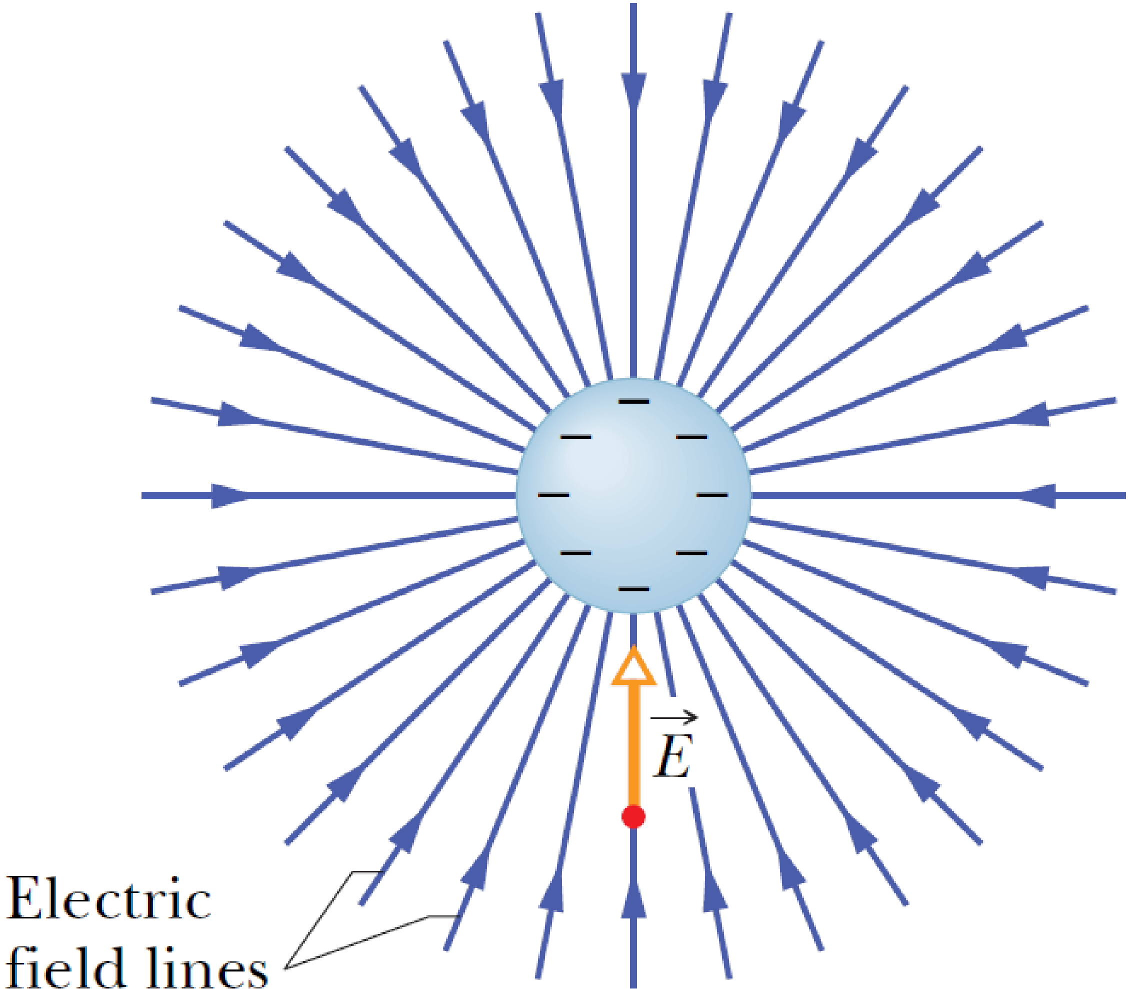
(1) The direction of $\vec{E}$ at any point is given by the direction of a straight field line or the direction of the tangent to a curved field line at that point.

(2) The field lines density is proportional to the magnitude of $\vec{E}$.

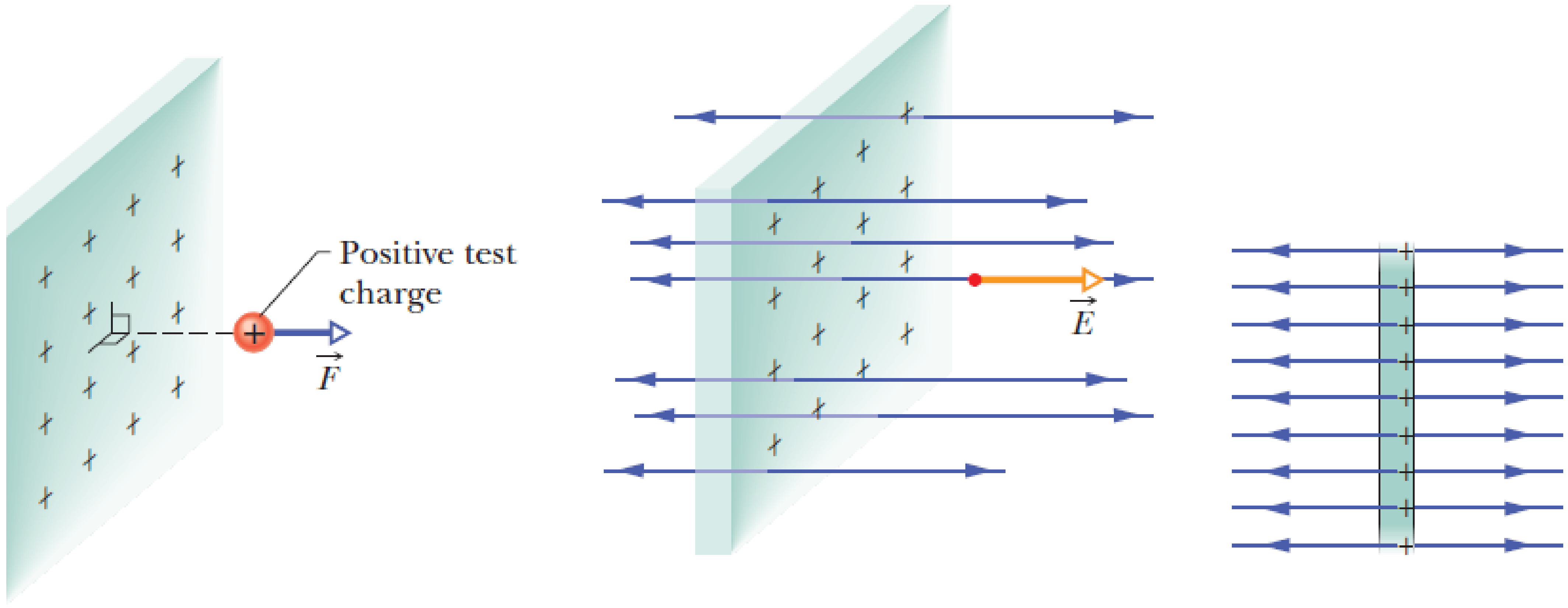
Electric field lines extend away from positive charge (where they originate) and toward negative charge (where they terminate).

2. Electric Field Lines

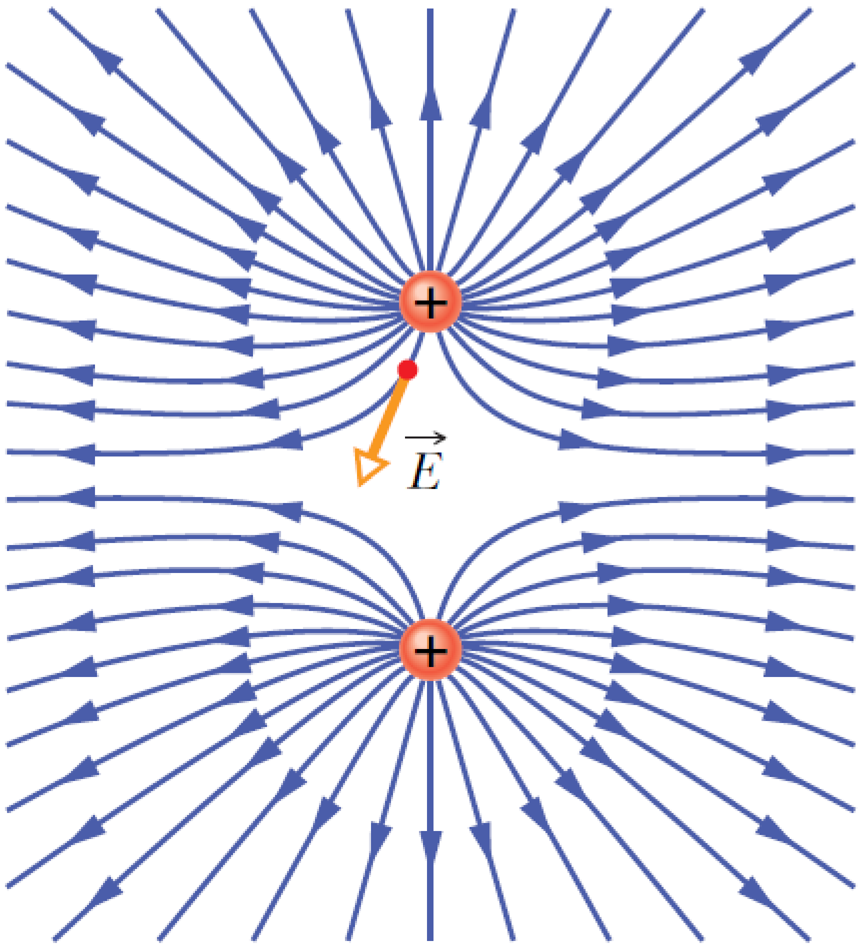


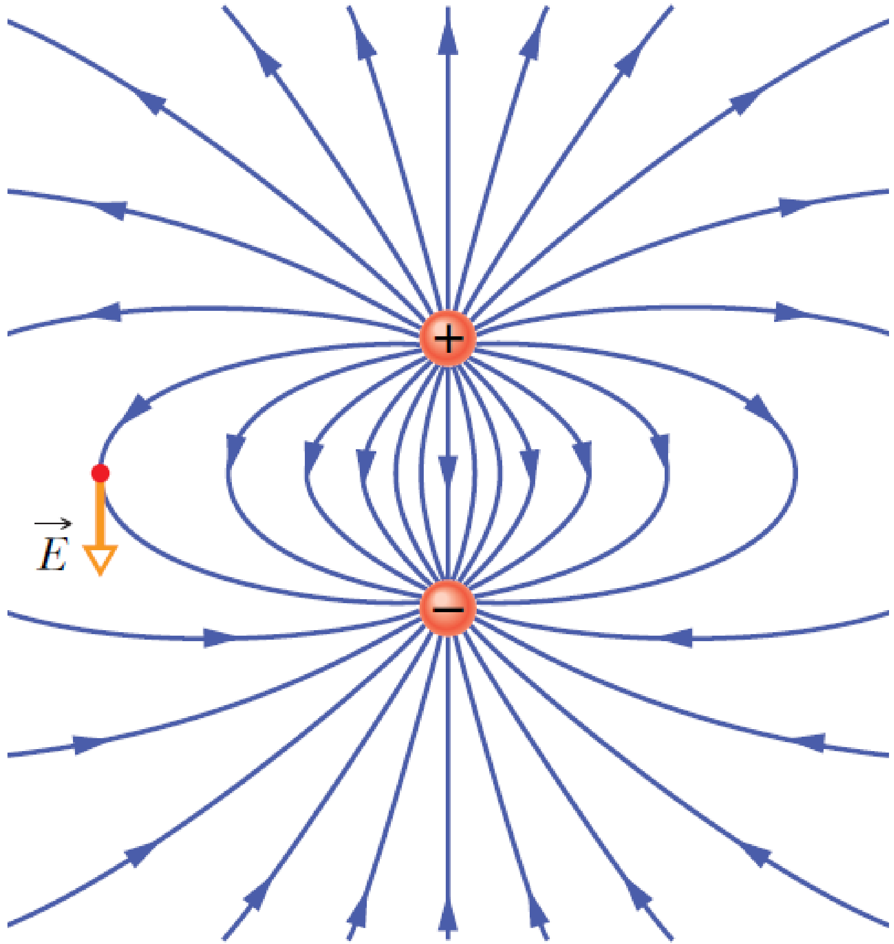


2. Electric Field Lines



2. Electric Field Lines





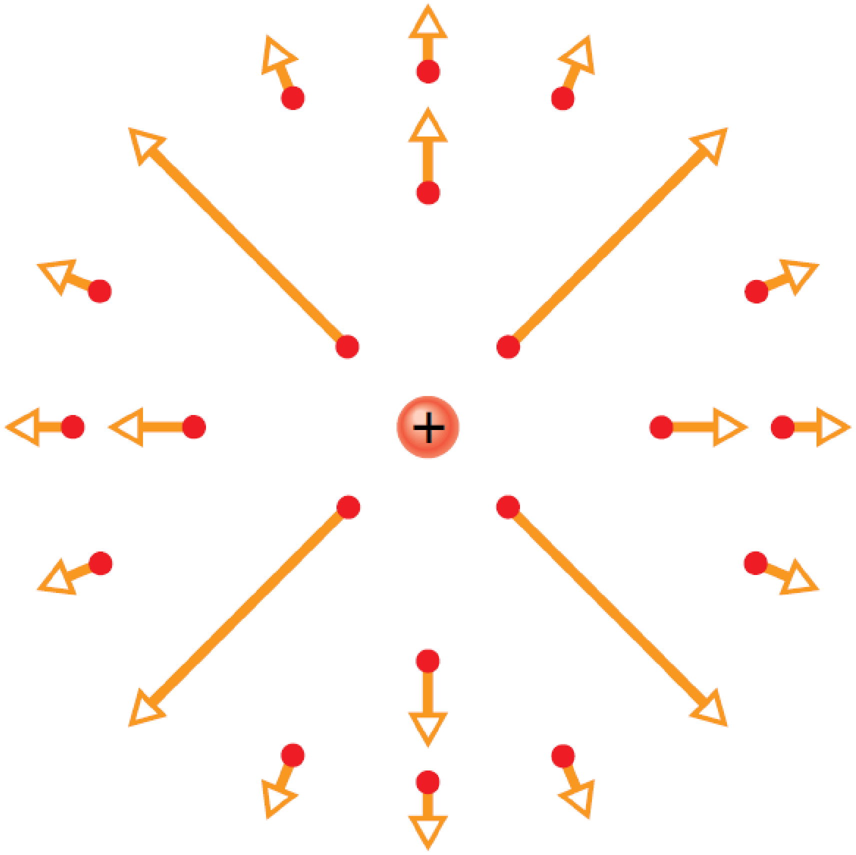
3. The Electric Field Due to a Point Charge

The electrostatic force between a charge q and a test charge q_0 at a distance r is

$$\vec{F} = \frac{1}{4\pi\varepsilon_0}\frac{qq_0}{r^2}\hat{\mathrm{r}}.$$

The electric field vector is then

$$\vec{E} = \frac{\vec{F}}{q_0} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{\mathrm{r}}.$$



3. The Electric Field Due to a Point Charge

We can find the net, or resultant, electric field due to more than one point charge. The net force $\vec{F}_0$ from n point charges acting on the test charge is

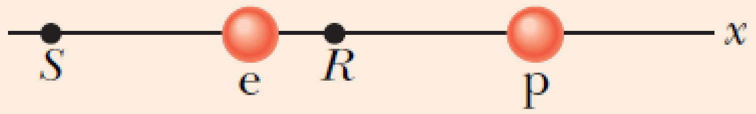
$$\vec{F}_0 = \vec{F}_{01} + \vec{F}_{02} + \vec{F}_{03} + \cdots + \vec{F}_{0n}.$$

The net electric field at the position of the test charge is therefore

$$\vec{E} = \frac{\vec{F}_0}{q_0} = \frac{\vec{F}_{01}}{q_0} + \frac{\vec{F}_{02}}{q_0} + \frac{\vec{F}_{03}}{q_0} + \cdots + \frac{\vec{F}_{0n}}{q_0}$$
$$= \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \cdots + \vec{E}_n.$$

3. The Electric Field Due to a Point Charge

CHECKPOINT 1

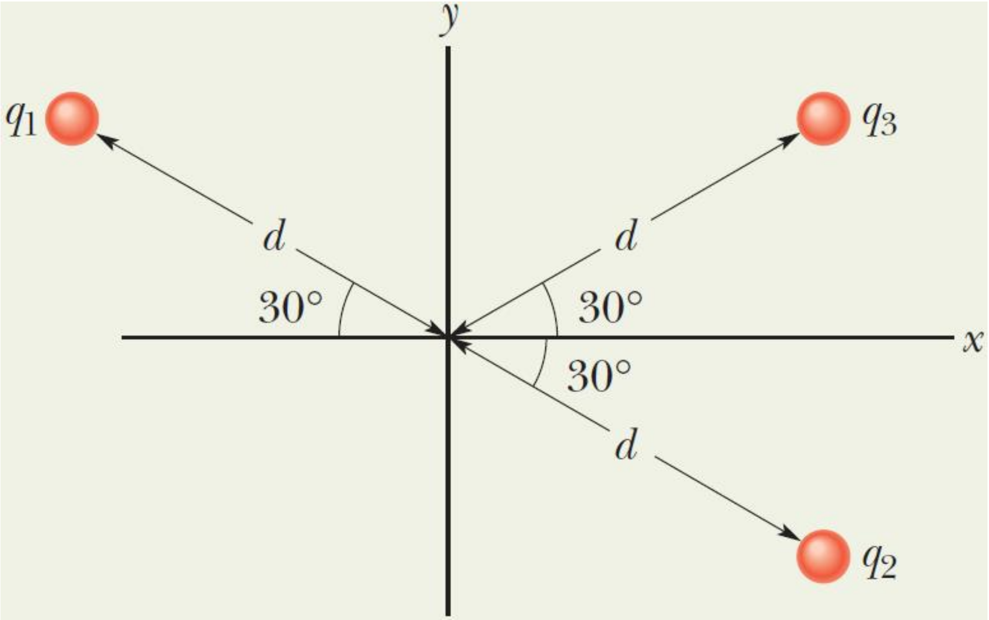


The figure here shows a proton p and an electron e on an x axis. What is the direction of the electric field due to the electron at (a) point S and (b) point R? What is the direction of the net electric field at (c) point R and (d) point S?

(a) Rightward.
(b) Leftward.
(c) Leftward.
(d) Rightward

3. The Electric Field Due to a Point Charge

Example 1: The figure shows three particles with charges $q_1 = +2Q$, $q_2 = -2Q$, and $q_3 = -4Q$, each a distance d from the origin. What net electric field $\vec{E}$ is produced at the origin?



The three charges produce three fields $\vec{E}_1$, $\vec{E}_2$ and $\vec{E}_3$. Their magnitudes are

$$E_1 = \frac{1}{4\pi\varepsilon_0}\frac{2Q}{d^2},$$

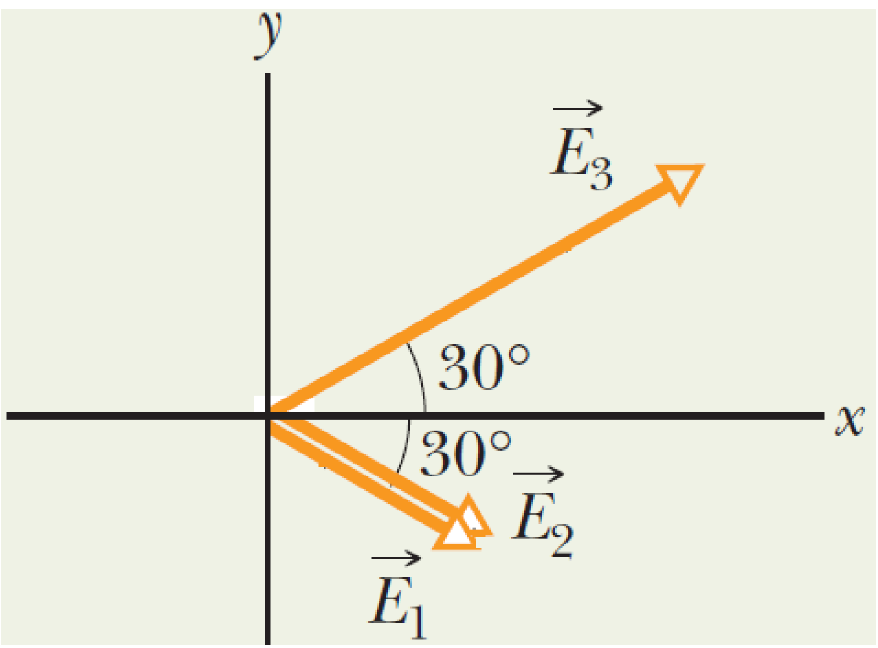
$$E_2 = \frac{1}{4\pi\varepsilon_0}\frac{2Q}{d^2},$$

$$E_3 = \frac{1}{4\pi\varepsilon_0}\frac{4Q}{d^2},$$

3. The Electric Field Due to a Point Charge

$\vec{E}_1$ and $\vec{E}_2$ are in the same direction. Their resultant electric field $\vec{E}_{12}$ has the same direction and has the magnitude

$$E_{12} = E_1 + E_2 = \frac{1}{4\pi\varepsilon_0}\frac{2Q}{d^2} + \frac{1}{4\pi\varepsilon_0}\frac{2Q}{d^2}$$
$$= \frac{1}{4\pi\varepsilon_0}\frac{4Q}{d^2}.$$

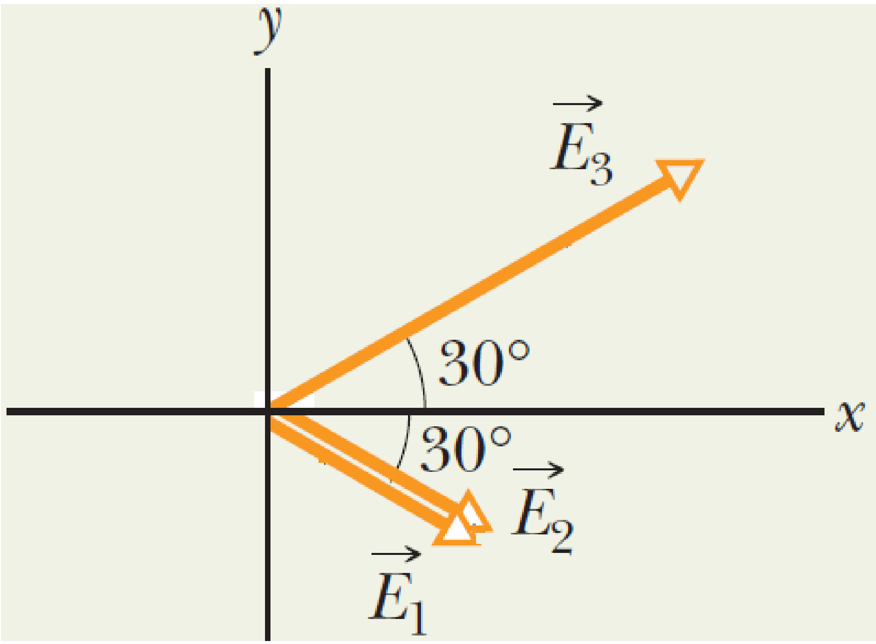


3. The Electric Field Due to a Point Charge

The net electric field $\vec{E} = \vec{E}_3 + \vec{E}_{12}$ is in the positive x direction. Its magnitude is

$$E = 2E_{12,x} = 2E_{3,x} = 2E_3 \cos 30°$$

$$= 2\frac{1}{4\pi\varepsilon_0}\frac{4Q}{d^2}\left(\frac{\sqrt{3}}{2}\right) = \frac{1}{4\pi\varepsilon_0}\frac{4\sqrt{3}Q}{d^2}.$$



4. The Electric Field Due to an Electric Dipole

An **electric dipole** is shown in the figure.

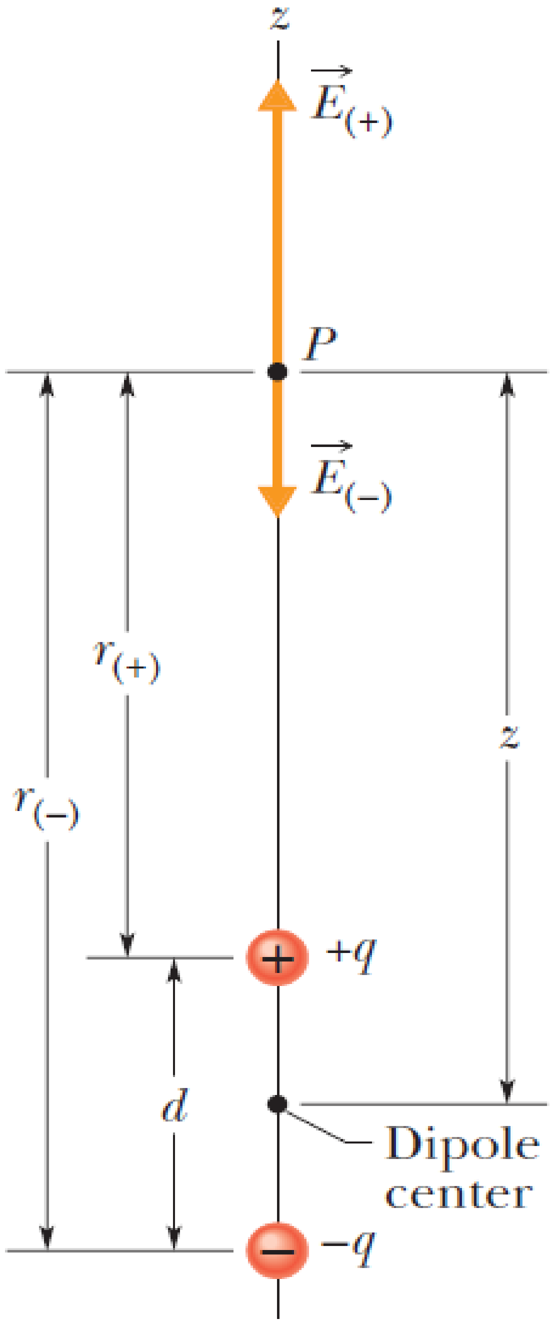
We want to find the electric field due to the dipole at point P, a distance z from the *dipole center* and on the *dipole axis*.

According to the superposition principle for electric fields, the magnitude E of the electric field at P is

$$E = E_+ - E_- = \frac{1}{4\pi\varepsilon_0}\frac{q}{r_+^2} - \frac{1}{4\pi\varepsilon_0}\frac{q}{r_-^2}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{q}{(z - d/2)^2} - \frac{1}{4\pi\varepsilon_0}\frac{q}{(z + d/2)^2}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{q}{z^2}\left[\frac{1}{(1 - d/2z)^2} - \frac{1}{(1 + d/2z)^2}\right].$$



4. The Electric Field Due to an Electric Dipole

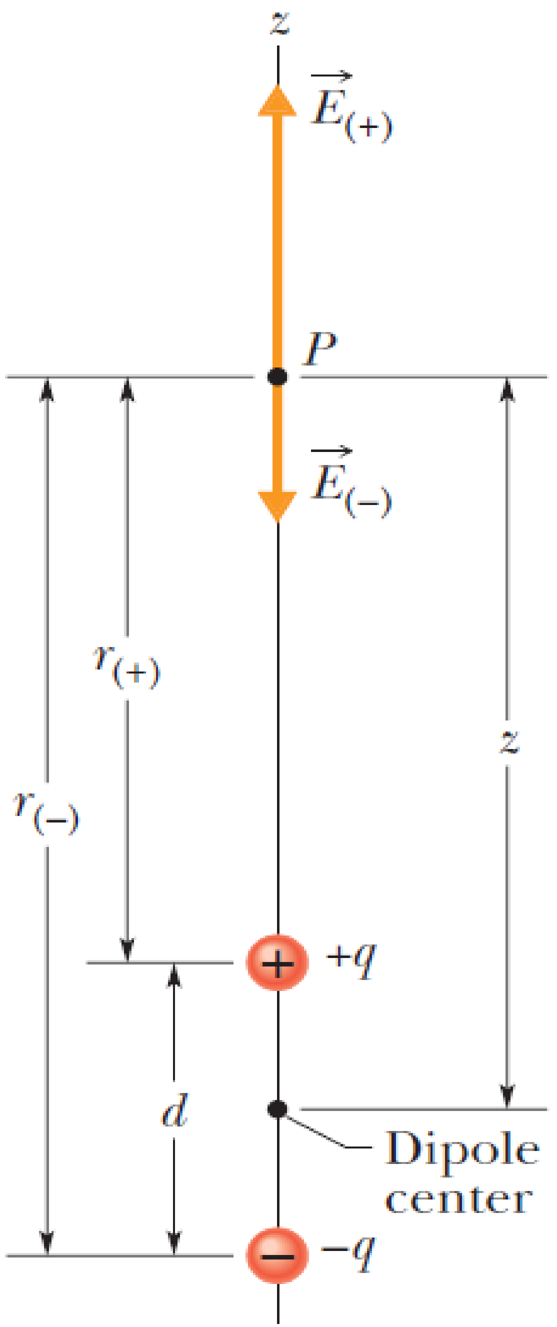
Using the binomial approximation $(1+x)^n \approx 1 + nx$ for $nx \ll 1$, we obtain

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{z^2}\left[\frac{1}{(1-d/2z)^2} - \frac{1}{(1+d/2z)^2}\right]$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{q}{z^2}\left[1+\frac{d}{z} - \left(1-\frac{d}{z}\right)\right] = \frac{1}{2\pi\varepsilon_0}\frac{qd}{z^3}.$$

In terms of the **electric dipole moment** $\vec{p} \equiv q\vec{d}$, we write

$$E = \frac{1}{2\pi\varepsilon_0}\frac{p}{z^3}$$



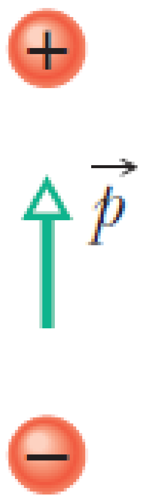
4. The Electric Field Due to an Electric Dipole

$$\vec{p} = q\vec{d}$$

The direction of $\vec{p}$ (or $\vec{d}$) is taken from the negative to the positive end of the dipole.

In general, E for a dipole varies as $1/r^3$ for all distant points. Here r is the distance between the point in question and the center of the dipole.

The direction $\vec{E}$ for a distant point on the dipole axis is always in the direction of $\vec{p}$.



4. The Electric Field Due to an Electric Dipole

Example 2: An electron and proton are located on the y axis. The electron is at $y = 5.00 \times 10^{-6}$ m and the proton is at $y = -5.00 \times 10^{-6}$ m.

(a) What is the electric dipole moment of this configuration?

$$\vec{p} = q\vec{d} = (1.602 \times 10^{-19}\ \mathrm{C})(-1.00 \times 10^{-5}\ \mathrm{m})\,\hat{\jmath} = (-1.60 \times 10^{-24}\ \mathrm{C \cdot m})\,\hat{\jmath}.$$

(b) What is the electric field along the dipole axis at $y = 1.00 \times 10^{-3}$ m?

$$\vec{E} = \frac{1}{2\pi\varepsilon_0}\frac{\vec{p}}{z^3} = \frac{1}{2\pi\left(8.85 \times 10^{-12}\,\frac{\mathrm{C}^2}{\mathrm{N \cdot m^2}}\right)}\frac{-1.602 \times 10^{-24}\ \mathrm{C \cdot m}}{(1.00 \times 10^{-3}\ \mathrm{m})^3}\,\hat{\jmath}$$

$$= \left(-2.88 \times 10^{-5}\,\frac{\mathrm{N}}{\mathrm{C}}\right)\hat{\jmath}.$$

5. A Point Charge in an Electric Field

When a charged particle of charge q is in an external electric field $\vec{E}$, an electrostatic force $\vec{F}$ acts on the particle, where

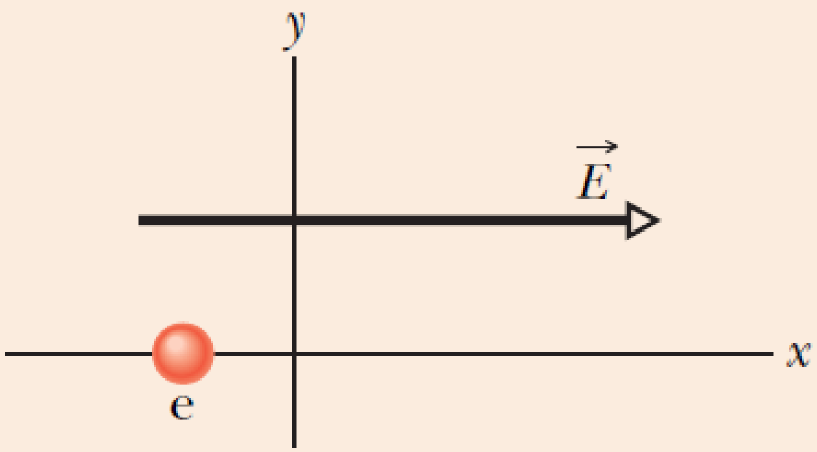
$$\vec{F} = q\vec{E}.$$

This equation tells us that the electrostatic force $\vec{F}$ acting on a charged particle located in an external electric field $\vec{E}$ has the direction of $\vec{E}$ if the charge q of the particle is positive and has the opposite direction if q is negative.

5. A Point Charge in an Electric Field

CHECKPOINT 3

(a) In the figure, what is the direction of the electrostatic force on the electron due to the external electric field shown? (b) In which direction will the electron accelerate if it is moving parallel to the y axis before it encounters the external field? (c) If, instead, the electron is initially moving rightward, will its speed increase, decrease, or remain constant?



(a) Leftward.

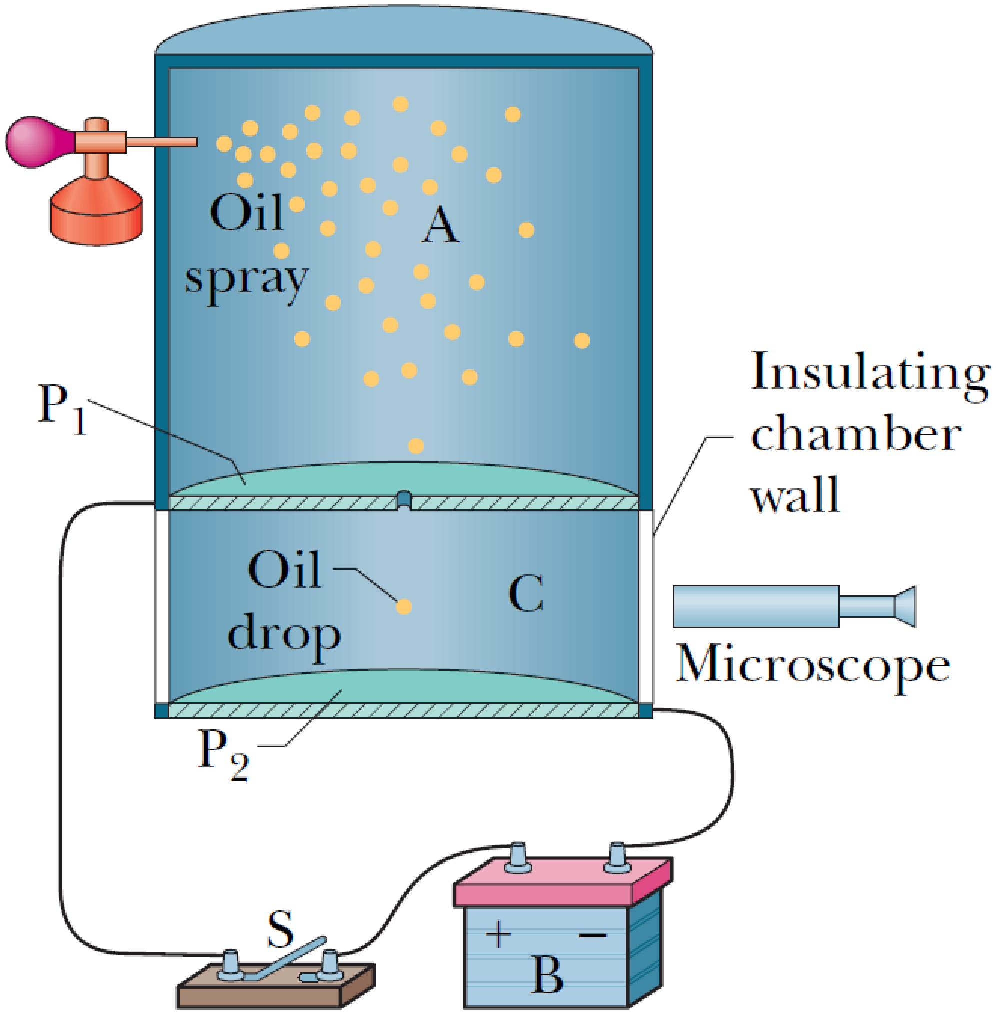
(b) Leftward.

(c) Decrease.

5. A Point Charge in an Electric Field

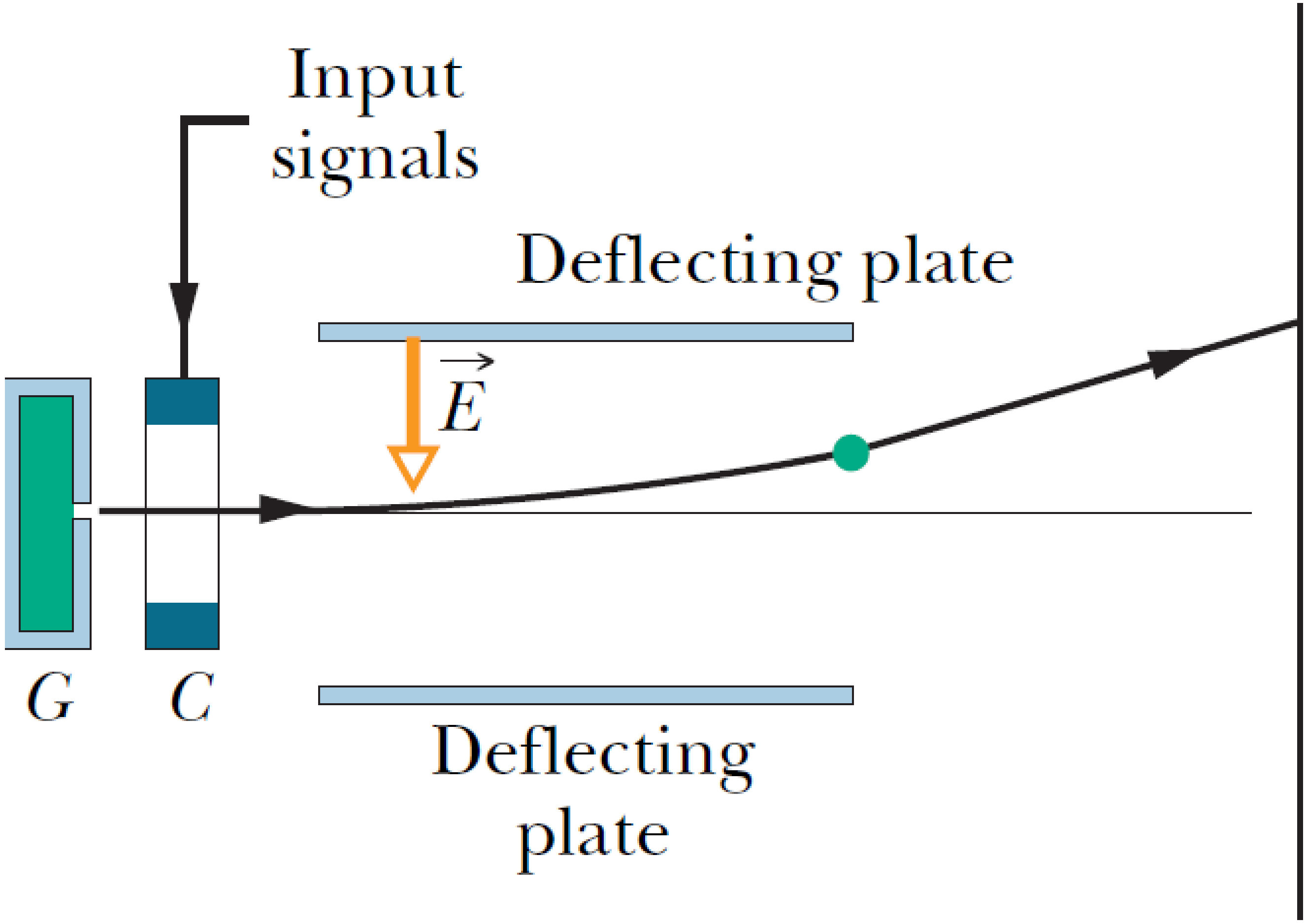
Measuring the Elementary Charge:

Millikan oil droplet experiment.



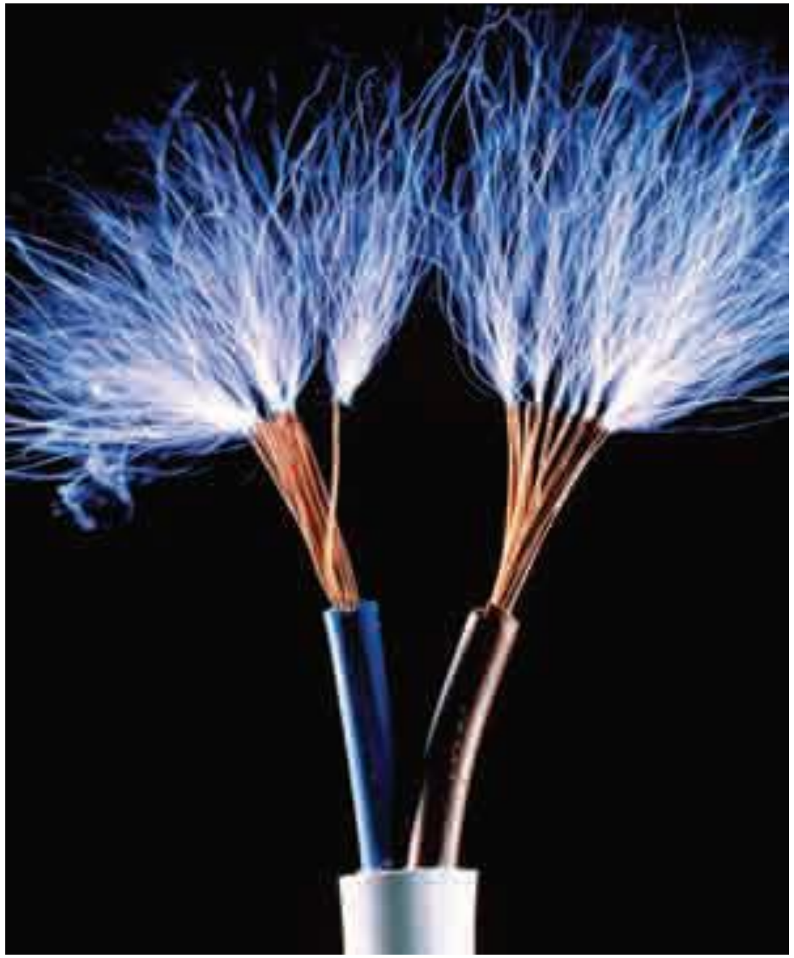
5. A Point Charge in an Electric Field

<u>Ink-Jet Printing:</u>



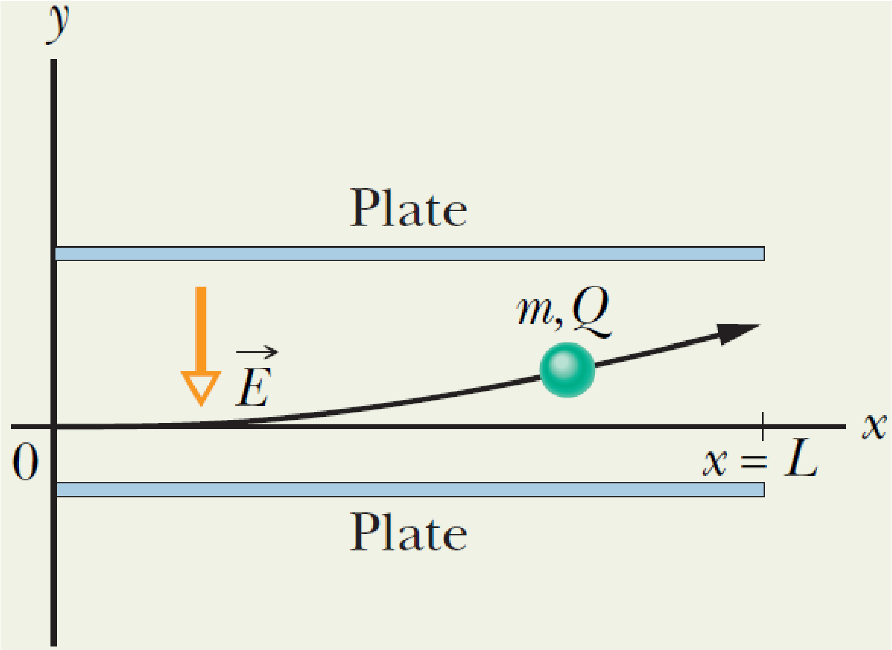
5. A Point Charge in an Electric Field

<u>Electrical Breakdown and Sparking:</u>



5. A Point Charge in an Electric Field

Example 3: The figure shows the deflecting plates of an ink-jet printer, with superimposed coordinate axes. An ink drop with a mass $m = 1.3 \times 10^{-10}$ kg and a negative charge of magnitude $Q = 1.5 \times 10^{-13}$ C enters the region between the plates, initially moving along the x axis with speed $v_x = 18$ m/s. The length L of each plate is 1.6 cm. The plates are charged and thus produce an electric field at all points between them. Assume that field $\vec{E}$ is downward directed, is uniform, and has a magnitude of 1.4×10^6 N/C. What is the vertical deflection of the drop at the far edge of the plates? (The gravitational force on the drop can be neglected.)



5. A Point Charge in an Electric Field

The vertical distance travelled is

$$y = \frac{1}{2}a_y t^2 = \frac{1}{2}\left(\frac{F}{m}\right) t^2 = \frac{1}{2}\left(\frac{QE}{m}\right) t^2.$$

The drop spends time t between the plates where

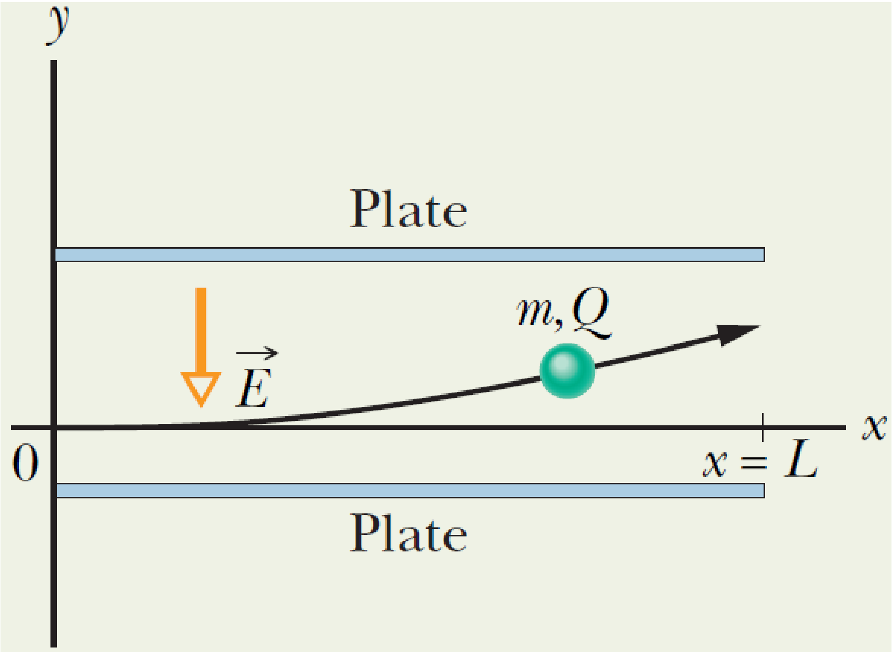
$$t = L/v_x.$$

This gives

$$y = \frac{QEL^2}{2mv_x^2}$$

$$= \frac{(1.5 \times 10^{-13}\ \mathrm{C})(1.4 \times 10^{6}\ \mathrm{N/C})(0.016\ \mathrm{m})^2}{2(1.3 \times 10^{-10}\ \mathrm{kg})(18\ \mathrm{m/s})^2}$$

$$= 0.64\ \mathrm{mm}$$



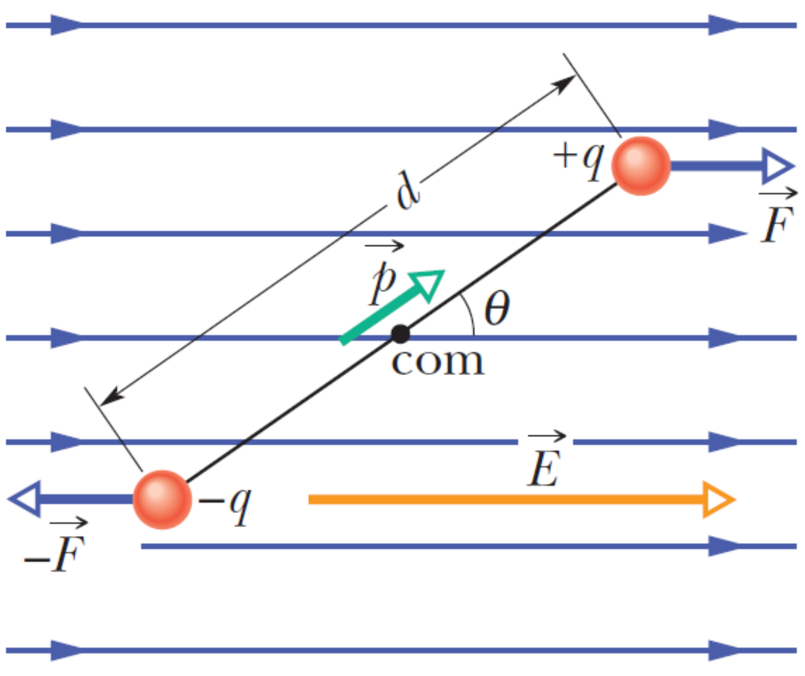
6. A Dipole in an Electric Field

The behavior of a dipole in a uniform external electric field $\vec{E}$ can be described completely in terms of the two vectors $\vec{E}$ and $\vec{p}$ only.

Consider the dipole shown in the figure. The dipole moment $\vec{p}$ makes an angle θ with a uniform electric field $\vec{E}$.

The forces on the positive and negative charges act in opposite directions. The net force on the dipole is zero and its center of mass does not accelerate.

The forces, however, produce torque on the dipole about its center of mass.



6. A Dipole in an Electric Field

The center of mass is at some distance x from one end and thus a distance $d - x$ from the other end.

The net torque τ on the dipole is

$$\tau = xF\sin\theta + (d - x)F\sin\theta = Fd\sin\theta\,.$$

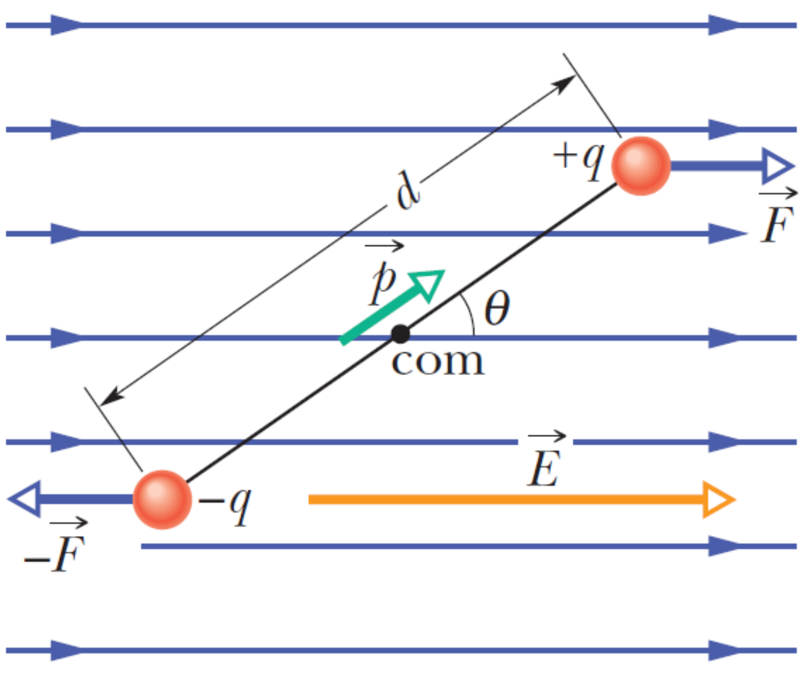
In terms of p,

$$\tau = qEd\sin\theta = pE\sin\theta\,.$$

We can generalize this equation to vector form as

$$\vec{\tau} = \vec{p} \times \vec{E}.$$

The torque tends to rotate the dipole into the direction of the field $\vec{E}$, thereby reducing θ.

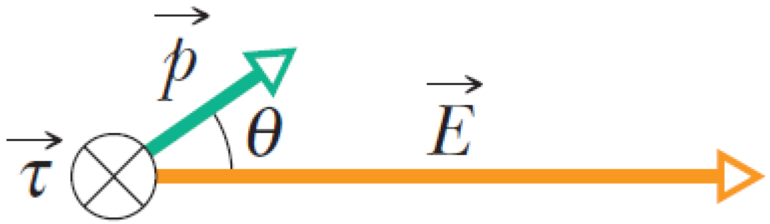


6. A Dipole in an Electric Field

The direction of $\vec{\tau}$ is into the page, as represented by the symbol $\otimes$.

We can represent a clockwise torque by including a minus sign with the magnitude of torque:

$$\tau = -pE\sin\theta\,.$$



6. A Dipole in an Electric Field

Potential Energy of an Electric Dipole

Potential energy can be associated with the orientation of an electric dipole in an electric field.

When the dipole moment $\vec{p}$ is lined up with the field ($\vec{\tau} = \vec{p} \times \vec{E} = 0$) the dipole is in its equilibrium orientation at which the dipole's potential energy is minimum. Work is required to rotate the dipole to any other orientation.

We take the potential energy to be zero at $\theta = 90°$. We can then find the potential energy U at any other configuration using $\Delta U = -W = -\int \tau \, d\theta$:

$$U - 0 = -W = -\int_{90°}^{\theta} \tau \, d\theta = pE \int_{90°}^{\theta} \sin\theta \; d\theta = -pE[\cos\theta]_{90°}^{\theta}$$

$$= -pE\cos\theta \, .$$

6. A Dipole in an Electric Field

Potential Energy of an Electric Dipole

We can generalize this equation to vector form:

$$U = -\vec{p} \cdot \vec{E}.$$

The potential energy is least when $\theta = 0$ ($U = -pE$), when $\vec{p}$ and $\vec{E}$ are lined-up.

The potential energy is greatest when $\theta = 180°$ ($U = pE$), when $\vec{p}$ and $\vec{E}$ are in opposite direction.

When the dipole rotates from θ_i to θ_f, the work W done on the dipole by the electric field is

$$W = -\Delta U = -\left(U_f - U_i\right).$$

6. A Dipole in an Electric Field

<u>Potential Energy of an Electric Dipole</u>

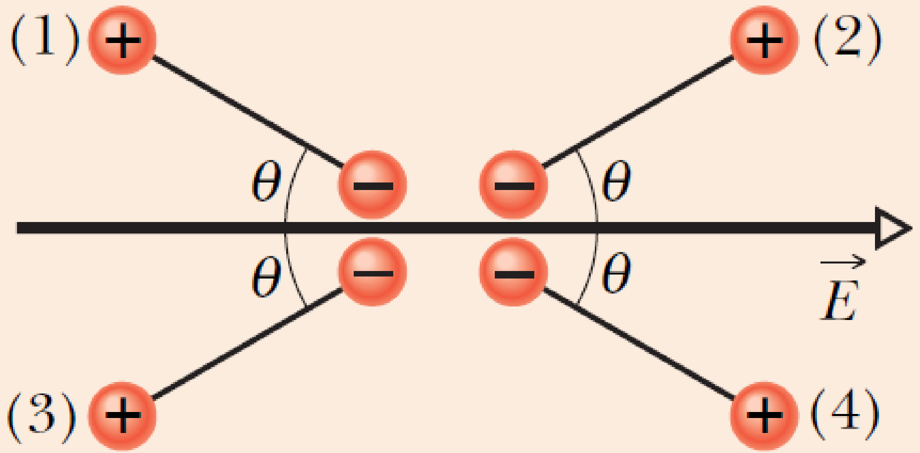
If θ is changed by an applied torque, the work W_a done on the dipole by the applied torque is the negative of the work W done on the dipole by the field:

$$W_a = -W = U_f - U_i.$$

6. A Dipole in an Electric Field

CHECKPOINT 4

The figure shows four orientations of an electric dipole in an external electric field. Rank the orientations according to (a) the magnitude of the torque on the dipole and (b) the potential energy of the dipole, greatest first.



(a) All tie.

(b) 1 & 3, then 2 & 4.

(See the next slide!)

$$\tau = pE\sin\theta$$

$$U = -pE\cos\theta$$

6. A Dipole in an Electric Field

Let's take $\theta = 30°$

$$|\tau_1| = pE \sin 150° = \frac{1}{2}pE$$

$$|\tau_2| = pE \sin 30° = \frac{1}{2}pE$$

$$|\tau_3| = pE \sin 150° = \frac{1}{2}pE$$

$$|\tau_4| = pE \sin 30° = \frac{1}{2}pE$$

$$U_1 = -pE \cos 150° = \frac{\sqrt{3}}{2}pE$$

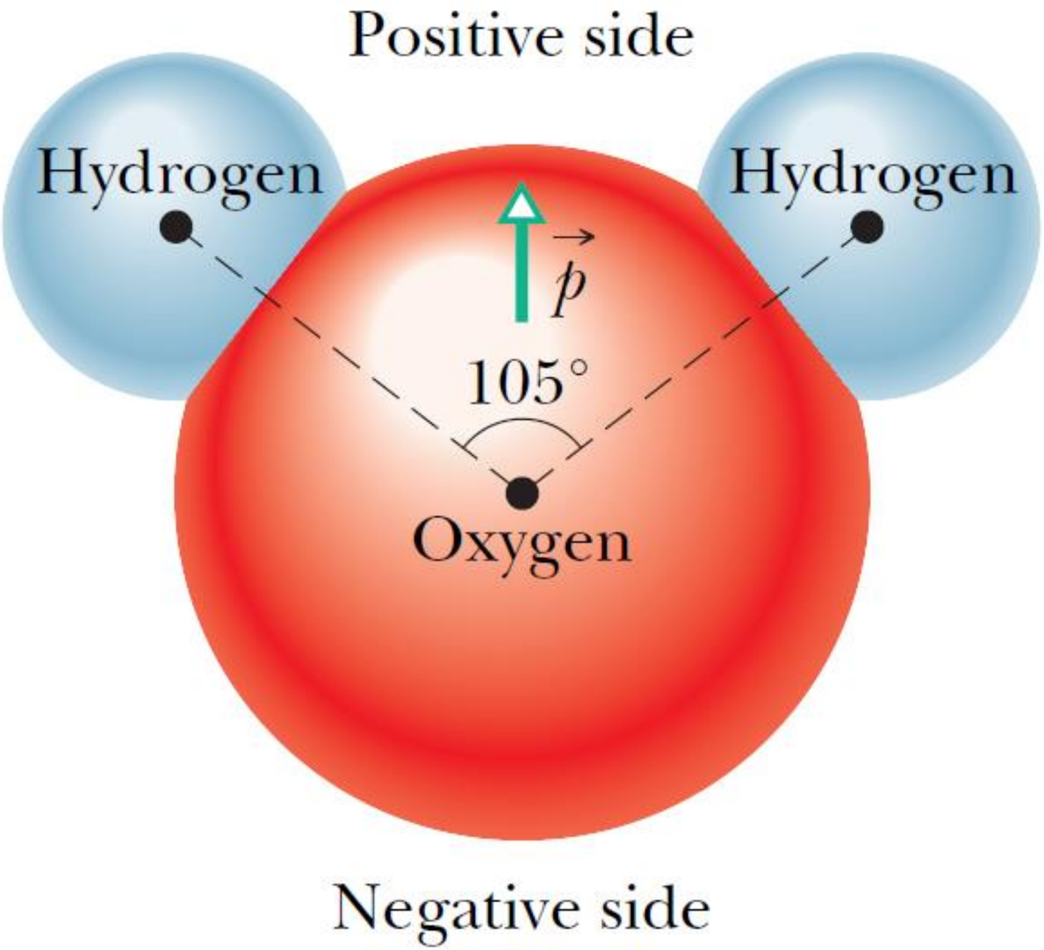
$$U_2 = -pE \cos 30° = -\frac{\sqrt{3}}{2}pE$$

$$U_3 = -pE \cos 150° = \frac{\sqrt{3}}{2}pE$$

$$U_4 = -pE \cos 30° = -\frac{\sqrt{3}}{2}pE$$

6. A Dipole in an Electric Field

The water molecule is an example of electric dipole.



6. A Dipole in an Electric Field

Example 4: A neutral water molecule ($\mathrm{H_2O}$) in its vapor state has an electric dipole moment of magnitude $6.2 \times 10^{-30}\ \mathrm{C \cdot m}$.

(a) How far apart are the molecule's centers of positive and negative charge?

There are 10 electrons and 10 protons in a water molecule.

$$d = \frac{p}{q} = \frac{p}{10e} = \frac{6.2 \times 10^{-30}\ \mathrm{C \cdot m}}{10(1.60 \times 10^{-19}\ \mathrm{C})} = 3.9\ \mathrm{pm}.$$

(b) If the molecule is placed in an electric field of $1.5 \times 10^{4}\ \mathrm{N/C}$, what maximum torque can the field exert on it?

$$\tau = pE \sin 90° = (6.2 \times 10^{-30}\ \mathrm{C \cdot m})(1.5 \times 10^{4}\ \mathrm{N/C}) = 9.3 \times 10^{-26}\ \mathrm{N \cdot m}.$$

6. A Dipole in an Electric Field

(c) How much work must an *external agent* do to rotate this molecule by $180°$ in this field, starting from its fully aligned position, for which $\theta = 0$?

$$W_a = \Delta U = U_{180} - U_0 = (-pE\cos 180°) - (-pE\cos 0) = 2pE$$
$$= 2(6.2 \times 10^{-30}\ \mathrm{C\cdot m})(1.5 \times 10^4\ \mathrm{N/C}) = 1.9 \times 10^{-25}\ \mathrm{J}.$$