Chapter 21

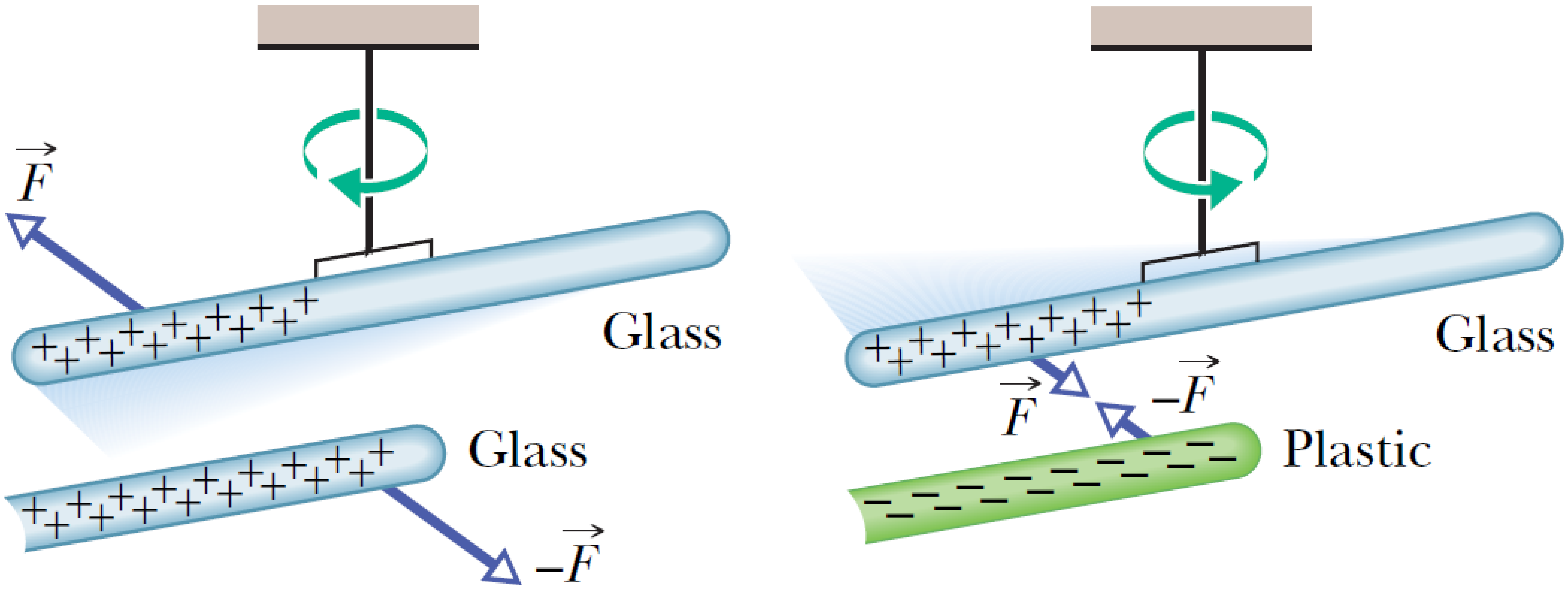
Electric Charge

1. Electric Charge

Electric charge is an intrinsic characteristic of the fundamental particles making up objects. It is a property that comes automatically with those particles whenever they exist.

The vast amount of charge in an everyday object is hidden because the object contains equal amounts of **positive charge** and **negative charge**. Such an object is said to be **electrically neutral**; it contains no **net charge**.

1. Electric Charge



Charges with the same electrical sign repel each other, and charges with opposite electrical signs attract each other.

2. Conductors and Insulators

Conductors are materials through which charge can move rather feely. Examples of conductors include metals, tap water and human body.

Nonconductors or **insulators** are martials though which charge cannot move freely. Examples of insulators include rubber, glass and distilled water.

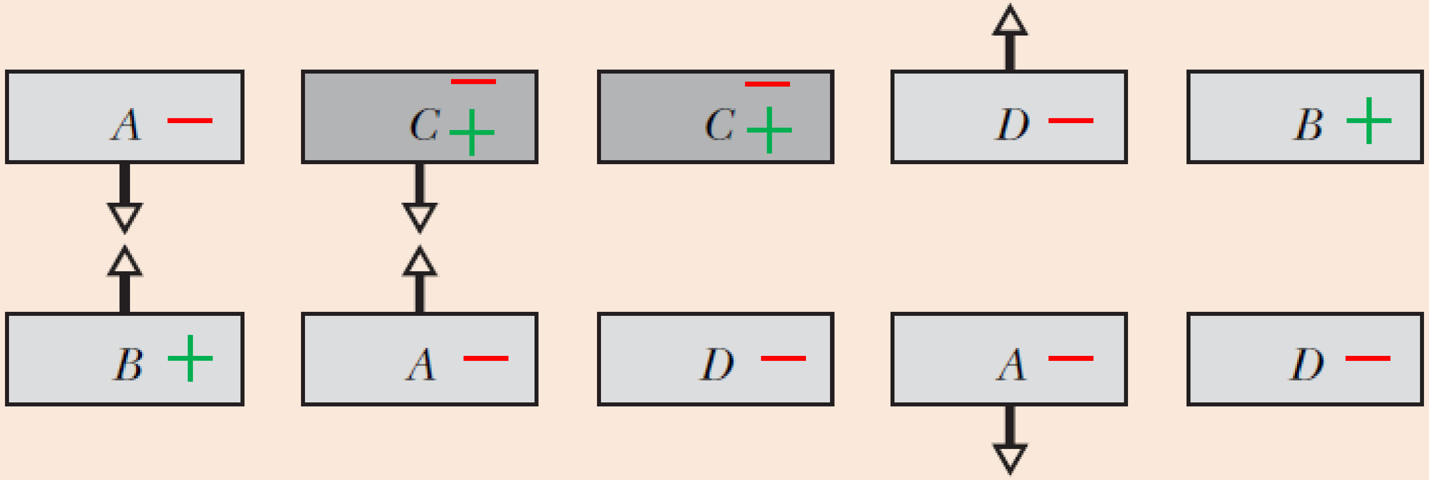
Semiconductors are materials that are intermediate between conductors and insulators. Examples of semiconductors include silicon and germanium.

Superconductors are perfect conductors.

2. Conductors and Insulators

CHECKPOINT 1

The figure shows five pairs of plates: A, B, and D are charged plastic plates and C is an electrically neutral copper plate. The electrostatic forces between the pairs of plates are shown for three of the pairs. For the remaining two pairs, do the plates repel or attract each other?



C and D attract.

B and D attract.

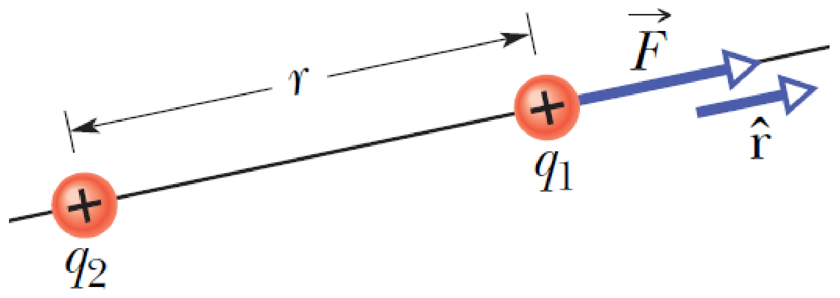
3. Coulomb’s Law

The repulsion or attraction force between two charges, due to their charges, is called an **electrostatic force**. The electrostatic force is given by **Coulombs' law**.

The force on particle 1 in the figure is given by

$$\vec{F} = k\frac{q_1 q_2}{r^2}\vec{r}.$$

where $\vec{r}$ is a unit vector along the axis extending through the two particles, r is the distance between them, and k is the **electrostatic constant**.



3. Coulomb's Law

The SI unit of charge is the **coulomb** (C). The coulomb unit is derived from the SI base unit **ampere** for the electric current i. Current is the rate at which charge moves past a point or through a region:

$$i = \frac{dq}{dt}.$$

This relation tells us that

$$1\ \mathrm{C} = (1\ \mathrm{A})(1\ \mathrm{s}).$$

The electrostatic constant k is usually written $1/4\pi\varepsilon_0$. The magnitude of the force in Coulomb's law becomes

$$F = \frac{1}{4\pi\varepsilon_0}\frac{|q_1 q_2|}{r^2}.$$

3. Coulomb's Law

The electrostatic constant k has the value

$$k = \frac{1}{4\pi\varepsilon_0} = 8.99 \times 10^9 \ \frac{\mathrm{N \cdot m^2}}{\mathrm{C^2}}.$$

The constant ε_0 is called the **permittivity constant** and has the value

$$\varepsilon_0 = 8.85 \times 10^{-12} \ \frac{\mathrm{C^2}}{\mathrm{N \cdot m^2}}.$$

3. Coulomb's Law

If we have n charged particles, they interact independently in pairs and the force on any one of them (say particle 1), is given by the vector sum

$$\vec{F}_{1,\mathrm{net}} = \vec{F}_{12} + \vec{F}_{13} + \vec{F}_{14} + \cdots + \vec{F}_{1n}.$$

<u>The shell theorem for electrostatics</u>:

- A shell of uniform charge attracts or repels a charged particle that is outside the shell as if all the shell's charge were concentrated at its center.
- If a charged particle is located inside a shell of uniform charge, there is no net electrostatic force on the particle from the shell.

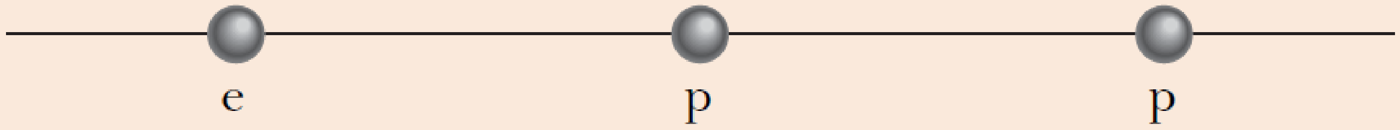
3. Coulomb's Law

Spherical Conductors:

When excess charge exists on a spherical shell, it spreads uniformly over the external surface of the sphere. This is due to the fact that charges of the same sign repel each other and try to redistribute themselves to maximize the distances between them.

3. Coulomb's Law

CHECKPOINT 2



The figure shows two protons (symbol p) and one electron (symbol e) on an axis. What is the direction of (a) the electrostatic force on the central proton due to the electron, (b) the electrostatic force on the central proton due to the other proton, and (c) the net electrostatic force on the central proton?

(a) Leftward.

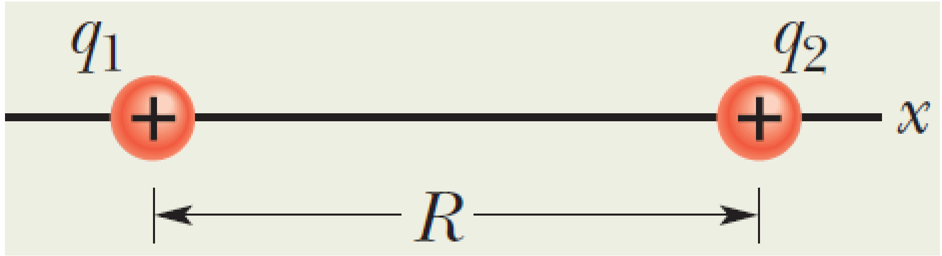
(b) Leftward.

(c) Leftward.

3. Coulomb's Law

Example 1:

(a) The figure shows two positively charged particles fixed in place on an x axis. The charges are $q_1 = 1.60 \times 10^{-19}\,\mathrm{C}$ and $q_2 = 3.20 \times 10^{-19}\,\mathrm{C}$, and the particle separation is $R_0 = 0.0200\,\mathrm{m}$. What are the magnitude and direction of the electrostatic force on particle 1 from particle 2?

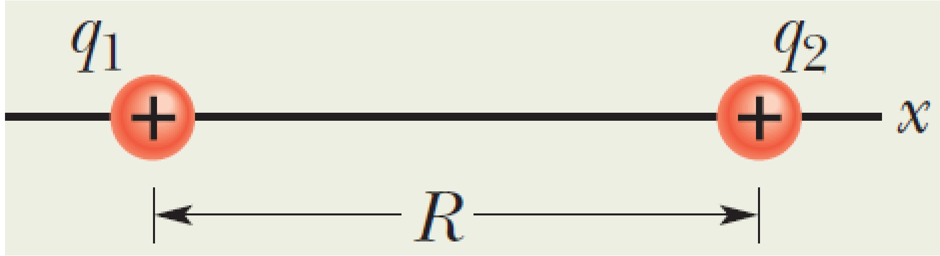


$$F_{12} = \frac{1}{4\pi\varepsilon_0}\frac{|q_1||q_2|}{R^2}.$$

3. Coulomb's Law

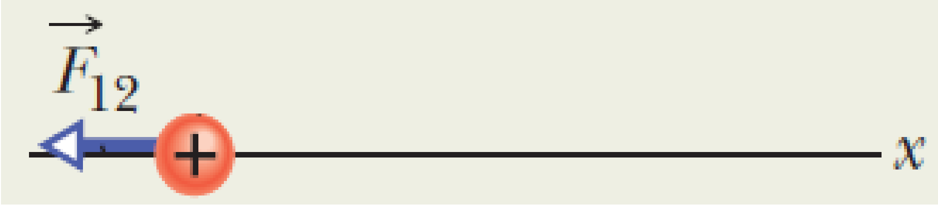
$$F_{12} = \frac{(1.60 \times 10^{-19}\ \mathrm{C})(3.20 \times 10^{-19}\ \mathrm{C})}{4\pi\left(8.85 \times 10^{-12}\ \cdot \frac{\mathrm{C}^2}{\mathrm{N}\cdot\mathrm{m}^2}\right)(0.0200\ \mathrm{m})^2}$$

$$= 1.15 \times 10^{-24}\ \mathrm{N}.$$



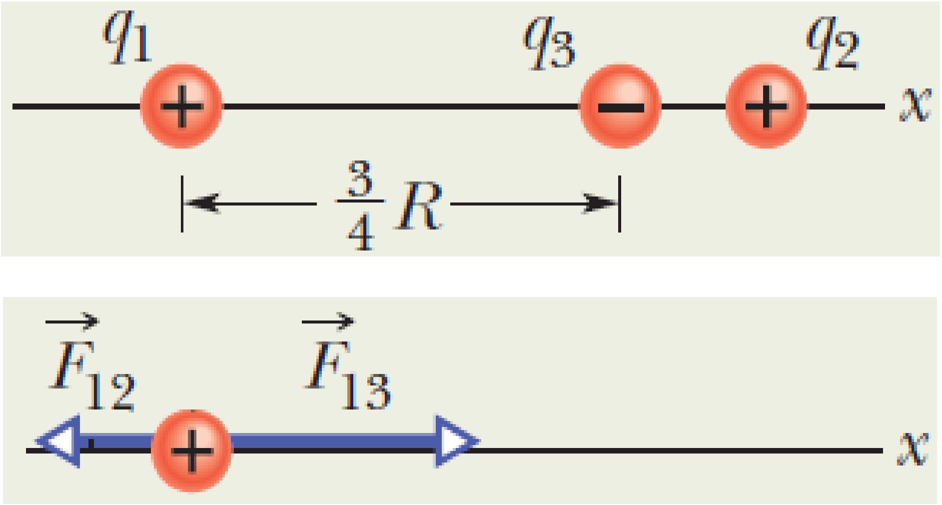
$\vec{F}_{12}$ is along the negative x axis. In unit-vector notation

$$\vec{F}_{12} = -(1.15 \times 10^{-24}\ \mathrm{N})\hat{\imath}.$$



3. Coulomb's Law

(b) The figure is identical to previous except that particle 3 now lies on the x axis between particles 1 and 2. Particle 3 has charge $q_3 = -3.20 \times 10^{-19}$ C and is at a distance $3/4R$ from particle 1. What is the net electrostatic force $\vec{F}_{1,\text{net}}$ on particle 1 due to particles 2 and 3?



$$\vec{F}_{1,\text{net}} = \vec{F}_{1,2} + \vec{F}_{1,3}$$

$$F_{13} = \frac{1}{4\pi\varepsilon_0}\frac{|q_1||q_3|}{(3R/4)^2}$$

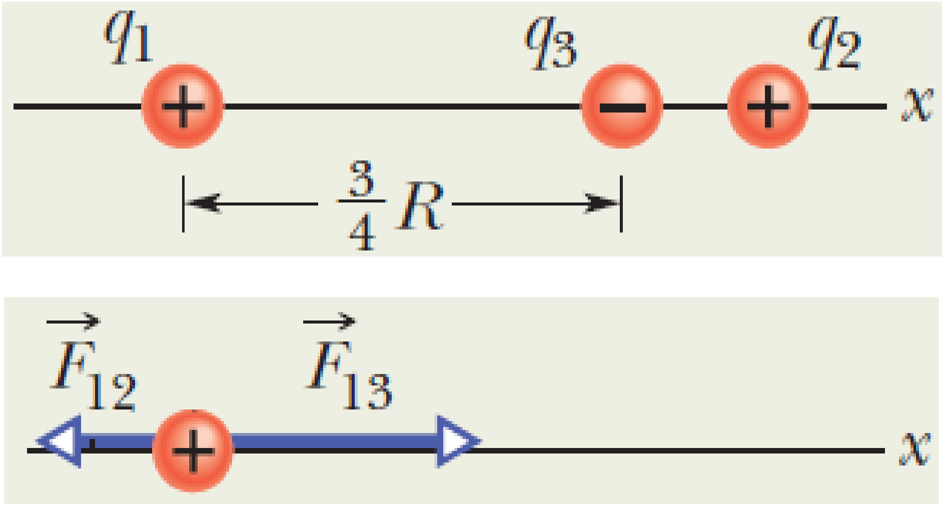
3. Coulomb's Law

$$F_{13} = \frac{1}{4\pi\varepsilon_0}\frac{|q_1||q_3|}{(3R/4)^2}$$

$$F_{13} = \frac{(1.60 \times 10^{-19}\ \mathrm{C})(3.20 \times 10^{-19}\ \mathrm{C})}{4\pi\left(8.85 \times 10^{-12}\ \cdot \frac{\mathrm{C}^2}{\mathrm{N}\cdot\mathrm{m}^2}\right)\left(\frac{3}{4}\right)^2 (0.0200\ \mathrm{m})^2}$$

$$= 2.05 \times 10^{-24}\ \mathrm{N}.$$

$$\vec{F}_{13} = (2.05 \times 10^{-24}\ \mathrm{N})\hat{\imath}.$$



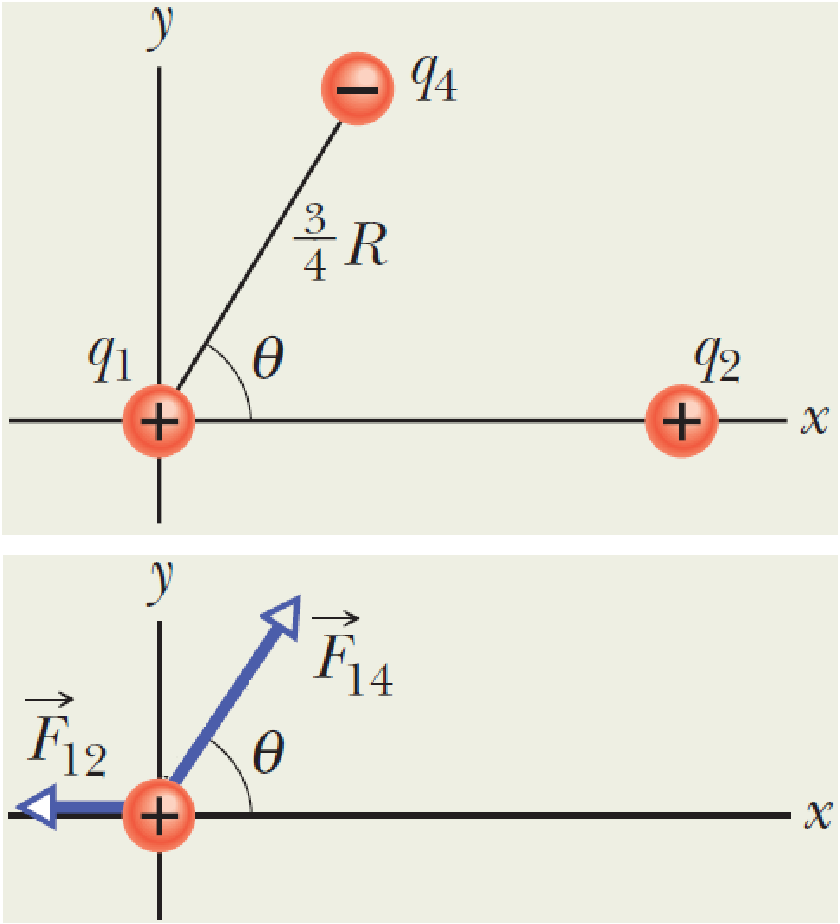
$$\vec{F}_{1,\mathrm{net}} = \vec{F}_{12} + \vec{F}_{13}$$

$$= -(1.15 \times 10^{-24}\ \mathrm{N})\hat{\imath} + (2.05 \times 10^{-24}\ \mathrm{N})\hat{\imath}$$

$$= (9.0 \times 10^{-25}\ \mathrm{N})\hat{\imath}.$$

3. Coulomb’s Law

(c) The figure is identical to that in part (a) except that particle 4 is now included. It has charge $q_4 = -3.20 \times 10^{-19}$ C, is at a distance 3 /4R from particle 1, and lies on a line that makes an angle $\theta = 60°$ with the x axis. What is the net electrostatic force on particle 1 due to particles 2 and 4?



$$\vec{F}_{1,\text{net}} = \vec{F}_{1,2} + \vec{F}_{1,4}$$

$$F_{14} = \frac{1}{4\pi\varepsilon_0}\frac{|q_1||q_4|}{(3R/4)^2} = 2.05 \times 10^{-24}\ \text{N}.$$

$$\vec{F}_{1,4} = (F_{14}\cos 60°)\hat{\imath} + (F_{14}\sin 60°)\hat{\jmath}$$

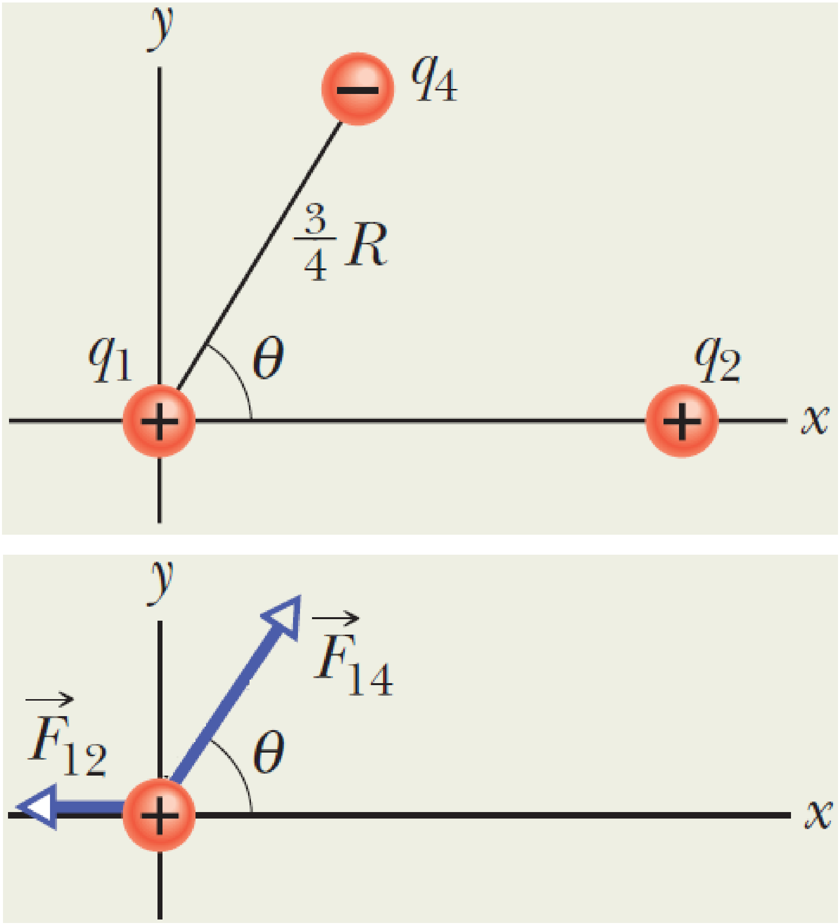
3. Coulomb's Law

$$\vec{F}_{1,4} = (1.025 \times 10^{-24}\ \text{N})\hat{\imath} + (1.775 \times 10^{-24}\ \text{N})\hat{\jmath}.$$

$$\vec{F}_{1,\text{net}} = \vec{F}_{1,2} + \vec{F}_{1,4} = -(1.15 \times 10^{-24}\ \text{N})\hat{\imath}$$
$$+(1.025 \times 10^{-24}\ \text{N})\hat{\imath} + (1.775 \times 10^{-24}\ \text{N})\hat{\jmath}$$
$$= -(1.25 \times 10^{-25}\ \text{N})\hat{\imath} + (1.775 \times 10^{-24}\ \text{N})\hat{\jmath}.$$

$$F_{1,\text{net}} = \sqrt{F_{1,\text{net},x}^2 + F_{1,\text{net},y}^2}$$
$$= 1.025 \times 10^{-24}\ \text{N}.$$

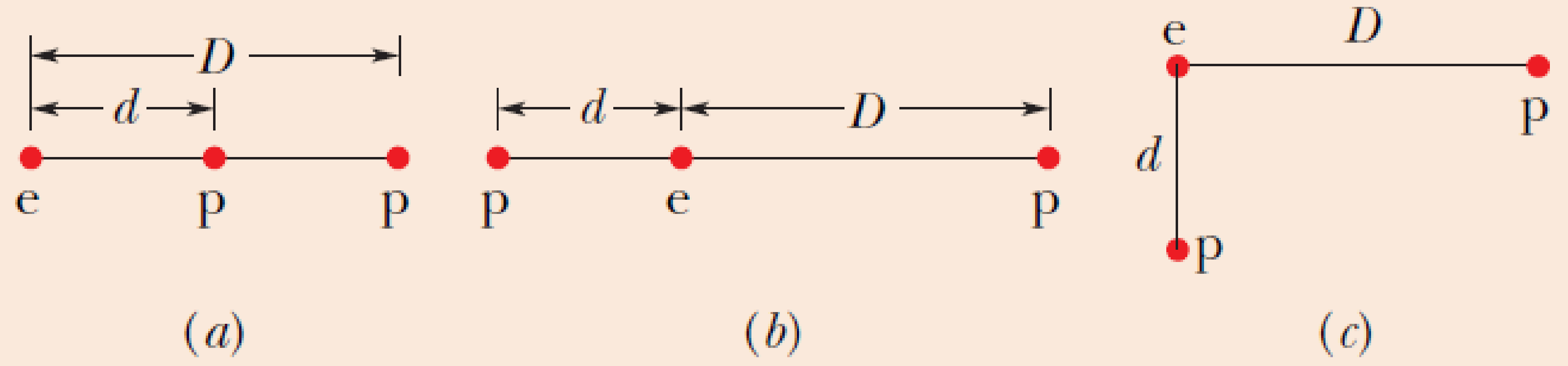
$$\phi = \tan^{-1}\frac{F_{1,\text{net},y}}{F_{1,\text{net},x}} = -86^\circ + 180^\circ = 94^\circ.$$



3. Coulomb's Law

CHECKPOINT 3

The figure here shows three arrangements of an electron e and two protons p. (a) Rank the arrangements according to the magnitude of the net electrostatic force on the electron due to the protons, largest first. (b) In situation *c*, is the angle between the net force on the electron and the line labeled *d* less than or more than 45°?

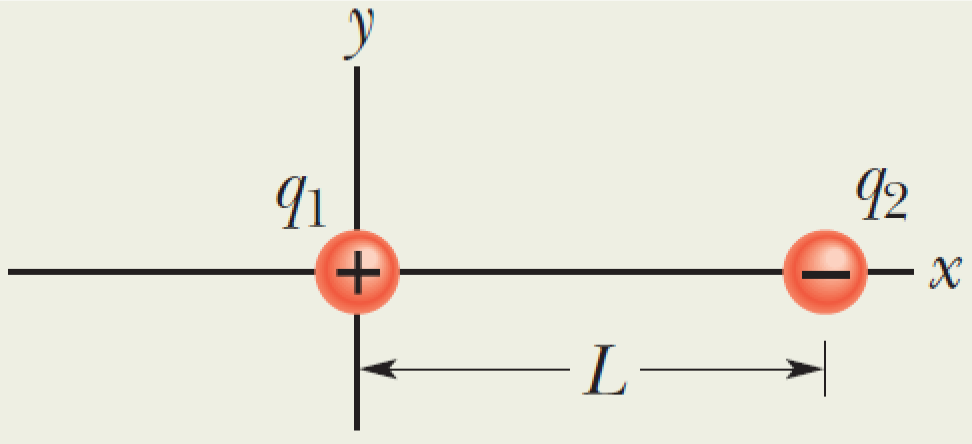


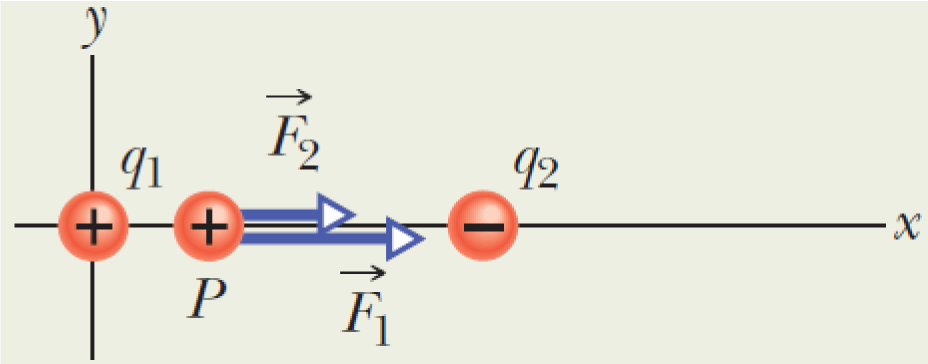
(A) a, c, then b.

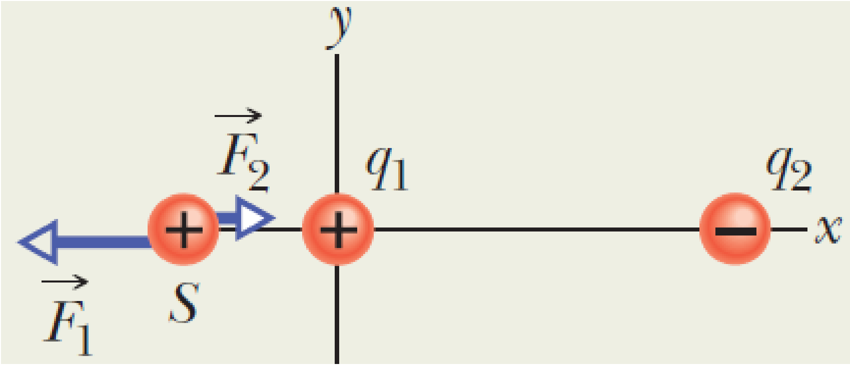
(B) Less.

3. Coulomb's Law

Example 2: The figure shows two particles fixed in place: a particle of charge $q_1 = +8q$ at the origin and a particle of charge $q_2 = -2q$ at $x = L$. At what point (other than infinitely far away) can a proton be placed so that it is in *equilibrium* (the net force on it is zero)?







3. Coulomb's Law

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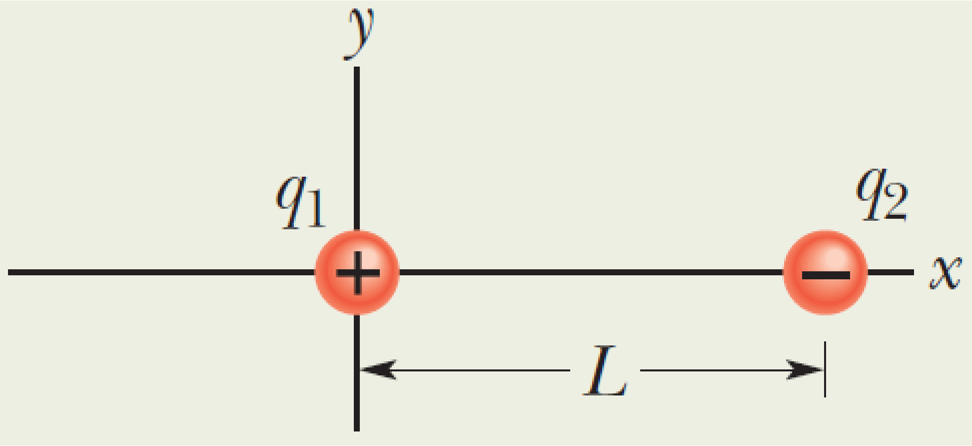
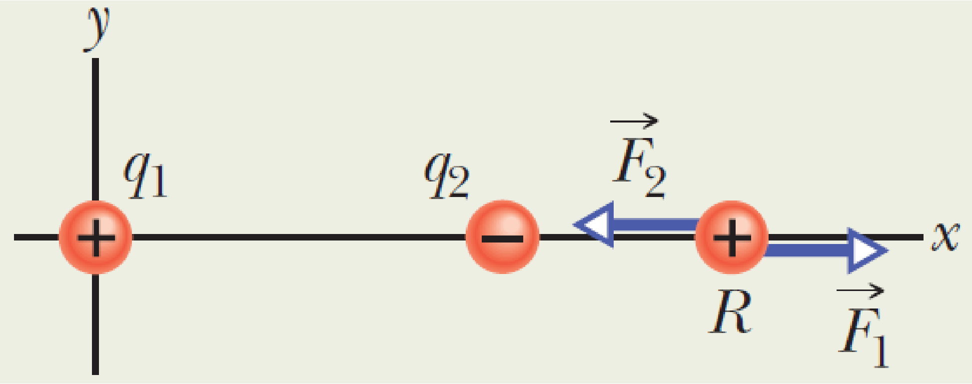


The proton must be placed to right of q_2.

We want to have that

$$F_1 = F_2.$$

Coulomb's law gives



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3. Coulomb's Law

Coulomb's law gives

$$\frac{1}{4\pi\varepsilon_0}\frac{|q_1 q_p|}{x^2} = \frac{1}{4\pi\varepsilon_0}\frac{|q_2 q_p|}{(x-L)^2},$$

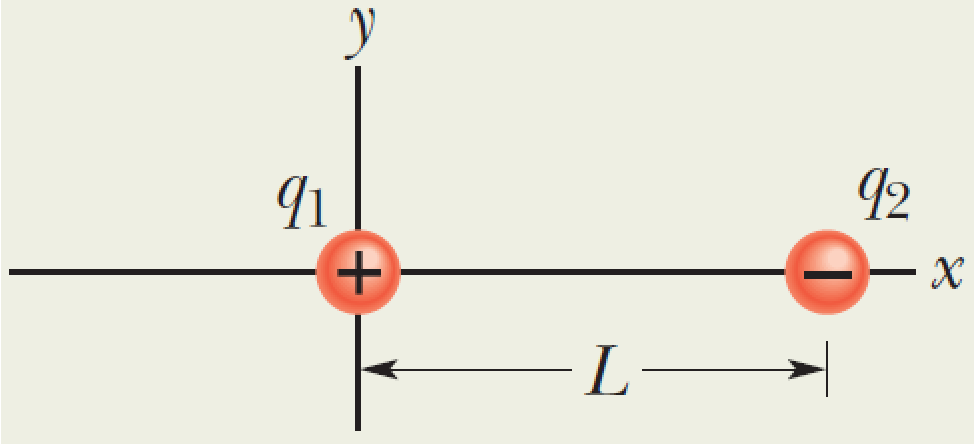
or

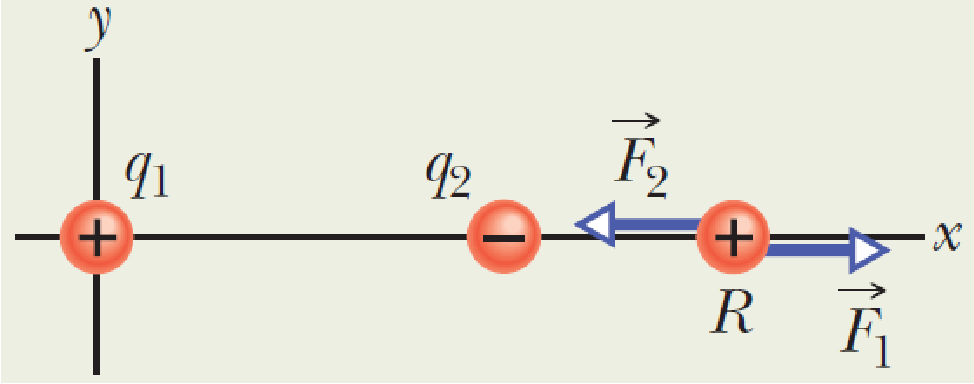
$$\frac{1}{4\pi\varepsilon_0}\frac{8q q_p}{x^2} = \frac{1}{4\pi\varepsilon_0}\frac{2q q_p}{(x-L)^2}.$$

Simplifying gives

$$\frac{x^2}{(x-L)^2} = 4,$$

which gives $x = 2L$.

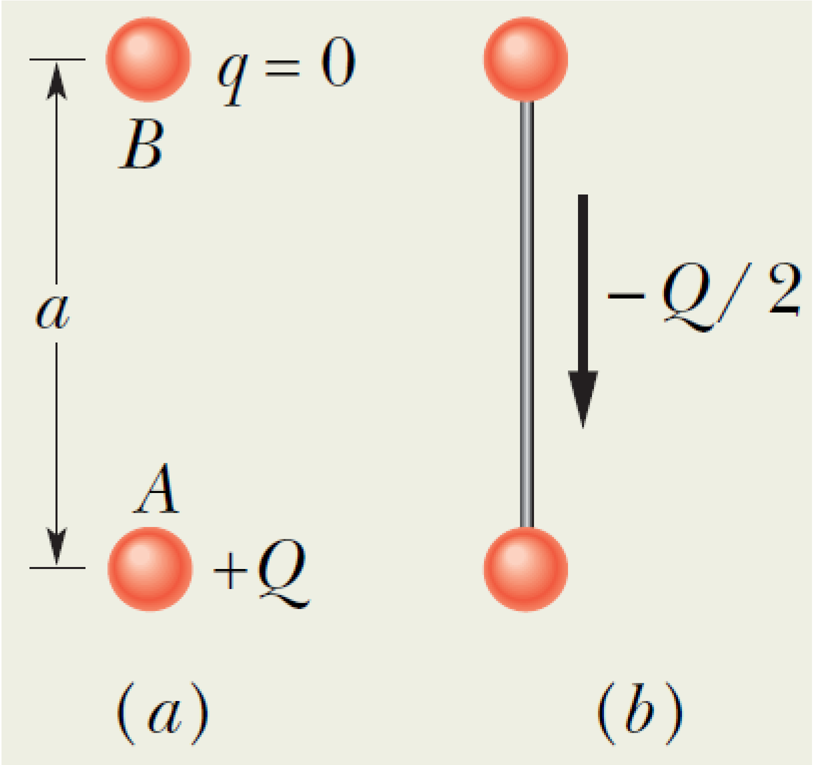




3. Coulomb's Law

Example 3: In figure (a) two identical, electrically isolated conducting spheres A and B are separated by a (center-to-center) distance a that is large compared to the spheres. Sphere A has a positive charge of Q, and sphere B is electrically neutral. Initially, there is no electrostatic force between the spheres.

(a) Suppose the spheres are connected for a moment by a conducting wire. The wire is thin enough so that any net charge on it is negligible. What is the electrostatic force between the spheres after the wire is removed?

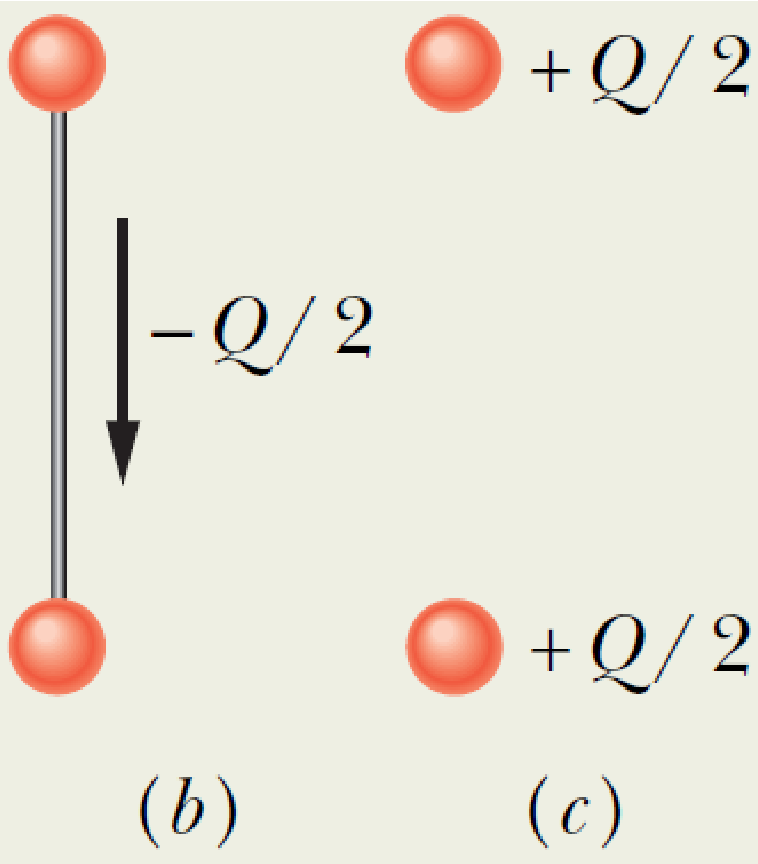


3. Coulomb's Law

Because the spheres are identical, connecting them means that they end up with identical charges. Coulomb's law gives

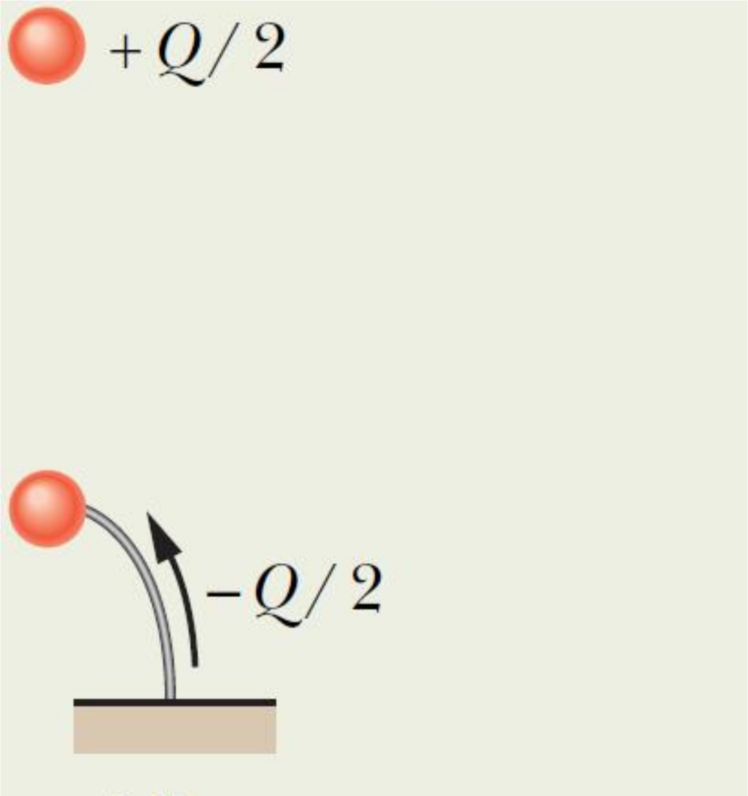
$$F = \frac{1}{4\pi\varepsilon_0}\frac{|q_A||q_B|}{R^2} = \frac{1}{4\pi\varepsilon_0}\frac{(Q/2)(Q/2)}{a^2}$$

$$= \frac{Q^2}{16\pi\varepsilon_0 a^2}.$$



3. Coulomb's Law

(b) Next, suppose sphere A is grounded momentarily, and then the ground connection is removed. What now is the electrostatic force between the spheres?

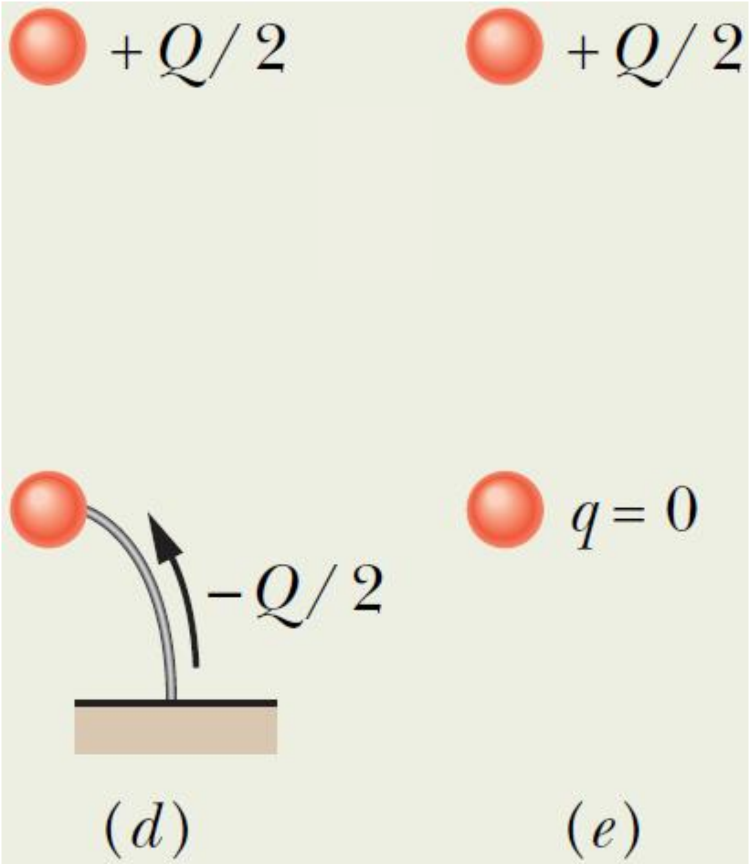


(d)

3. Coulomb's Law

(b) Next, suppose sphere A is grounded momentarily, and then the ground connection is removed. What now is the electrostatic force between the spheres?

The electrostatic force between the spheres is now zero.



4. Charge is Quantized and Conserved

Any charge q (positive or negative) can be written as

$$q = ne, \qquad n = \pm 1, \pm 2, \pm 3, \dots,$$

where e is the **elementary charge**, which has the approximate value

$$e = 1.602 \times 10^{-19}\ \mathrm{C}.$$

We say that a physical quantity is **quantized** if it can have only discrete values. An object can have charge of $+10e$, $-32e$, but not $4.32e$.

<u>Charge Conservation</u>

The net charge of any isolated system cannot change.

4. Charge is Quantized and Conserved

CHECKPOINT 4

Initially, sphere A has a charge of $-50e$ and sphere B has a charge of $+20e$. The spheres are made of conducting material and are identical in size. If the spheres then touch, what is the resulting charge on sphere A?

After the spheres touch, their net charge is $-30e$ and the charge of each sphere is $-15e$.

4. Charge is Quantized and Conserved

Example 4: The nucleus in an iron atom has a radius of about 4.0×10^{-15} m and contains 26 protons.

(a) What is the magnitude of the repulsive electrostatic force between two of the protons that are separated by 4.0×10^{-15} m?

$$F = k\frac{|q_A||q_B|}{R^2} = (8.99 \times 10^9\ \mathrm{N \cdot m^2/C^2})\frac{(1.602 \times 10^{-19}\ \mathrm{C})^2}{(4.0 \times 10^{-15}\ \mathrm{m})^2} = 14\ \mathrm{N}.$$

(b) What is the magnitude of the gravitational force between those same two protons?

$$F_g = G\frac{m_p^2}{R^2} = (6.67 \times 10^{-11}\ \mathrm{N \cdot m^2/kg^2})\frac{(1.67 \times 10^{-27}\ \mathrm{kg})^2}{(4.0 \times 10^{-15}\ \mathrm{m})^2} = 1.2 \times 10^{-35}\ \mathrm{N}.$$