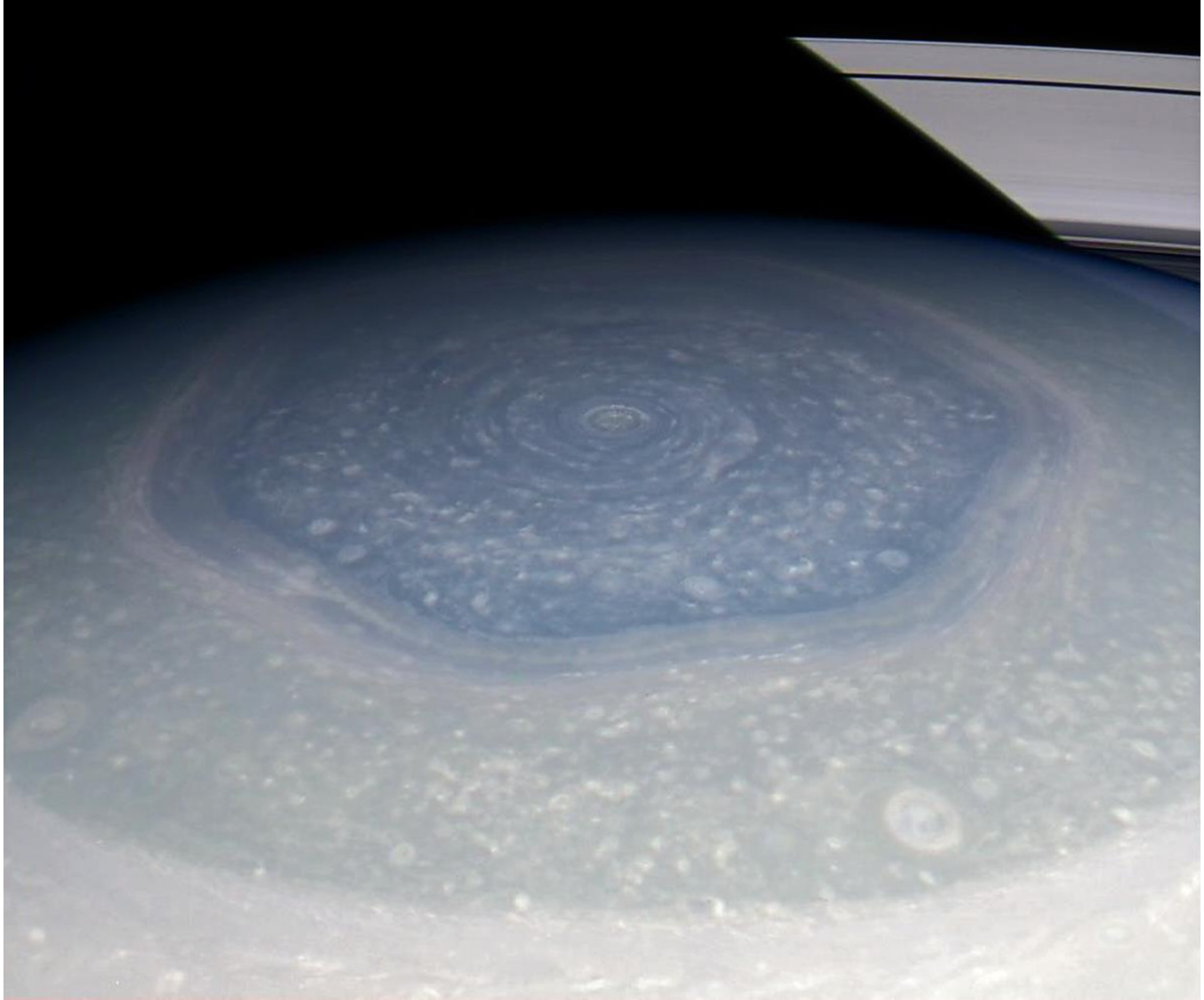


Chapter 17

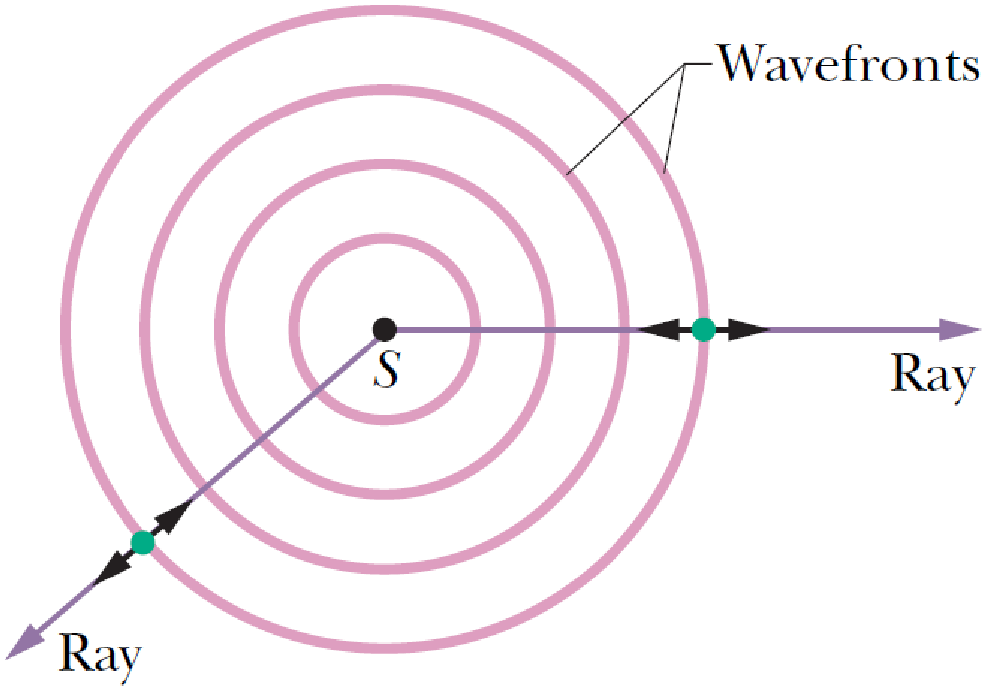
WAVES II

1. Sound Waves

Sound waves are longitudinal mechanical waves that can travel through solids, liquids and gases. We focus in this chapter on sound waves that travel through air and that are audible to people.

In the figure, point S represents a tiny sound source, or a **point source**, that emits sound in all directions.

Wavefronts are surfaces over which the oscillations (or displacements) due to the sound wave have the same value. Wavefronts are represented by whole or partial circles.

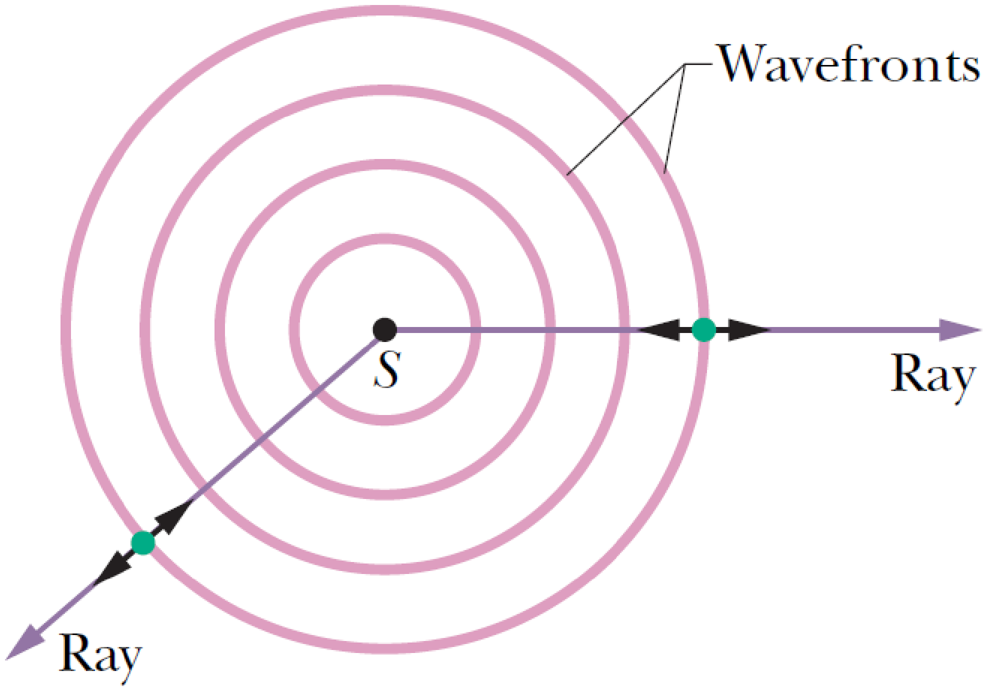


1. Sound Waves

Rays are directed lines perpendicular to the wavefronts that indicate the direction of travel of the wavefronts.

The wavefronts in the figure are spheres, and therefore the waves are said to be **spherical**.

Far from the source, the wavefronts can be approximated as planes, and the waves are said to be **planar**.



2. The Speed of Sound

The speed of any mechanical wave depends on both an inertial property of the medium (to store kinetic energy) and an elastic property of the medium (to store potential energy).

We can thus generalize the equation for the speed of a wave along a string by writing

$$v = \sqrt{\frac{\tau}{\mu}} = \sqrt{\frac{\text{elastic propery}}{\text{inertial property}}}.$$

Because the medium of a sound wave is air, we replace μ with ρ, the density of air.

In a string, the potential energy is associated with the stretching of the string elements. In air, potential energy is associated with compressions and expansions of volume elements of the air.

2. The Speed of Sound

The property that determines the extent to which an element of a medium changes in volume when the pressure on it changes is the **bulk modulus** B:

$$B = -\frac{\Delta p}{\Delta V / V}.$$

Replacing τ with B yields the expression for the speed of sound:

$$v = \sqrt{\frac{B}{\rho}}.$$

The Speed of Sound

Medium	Speed (m/s)
Gases	
Air (0°C)	331
Air (20°C)	343
Helium	965
Hydrogen	1284
Liquids	
Water (0°C)	1402
Water (20°C)	1482
Seawater	1522
Solids	
Aluminum	6420
Steel	5941
Granite	6000

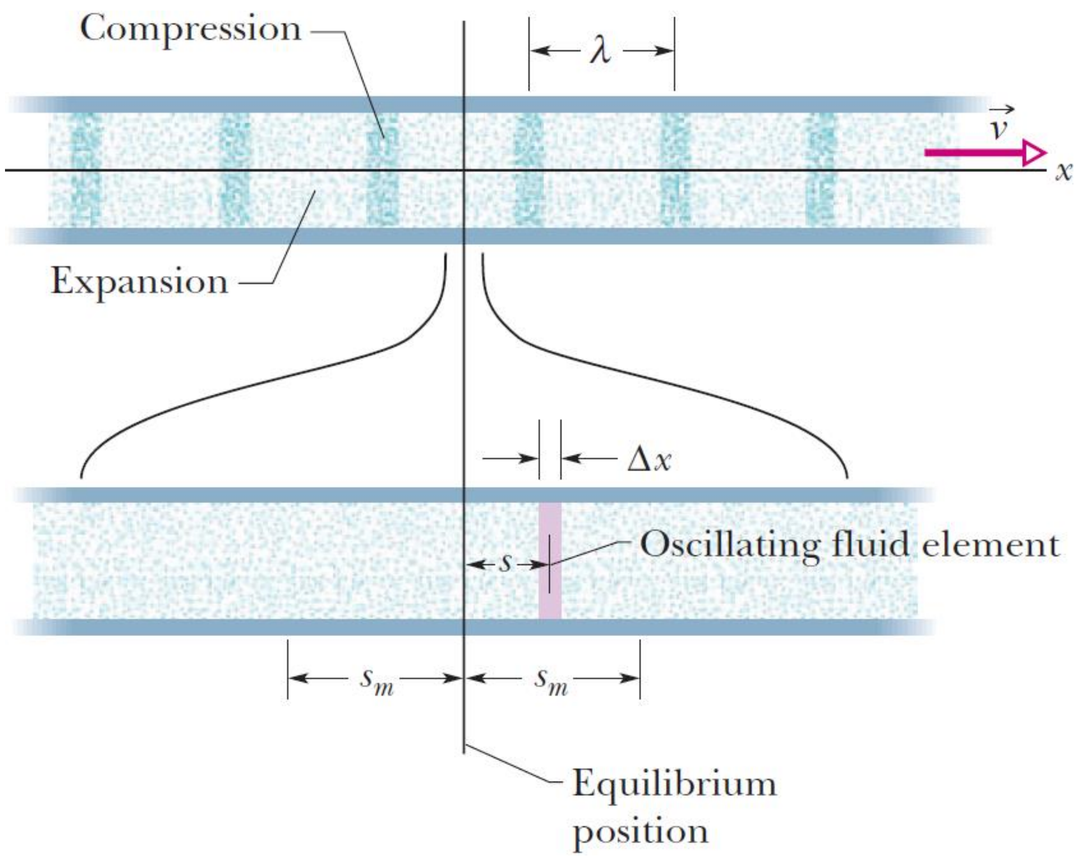
3. Traveling Sound Waves

The oscillations of air elements due to a traveling sound wave are similar to those of a string's elements due to a transverse wave. However, air elements oscillate longitudinally rather than transversely.

The displacements $s(x, t)$ of air elements are given by

$$s(x, t) = s_m \cos(kx - \omega t),$$

where s_m is the **displacement amplitude**.



3. Traveling Sound Waves

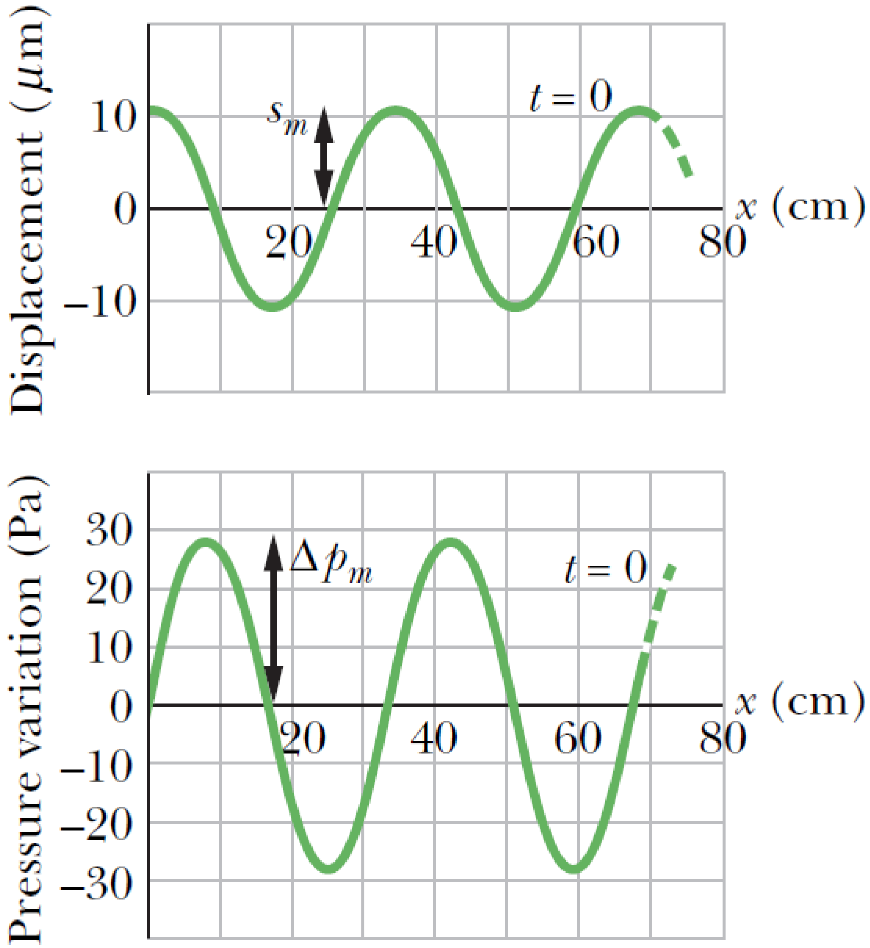
The air pressure varies sinusoidally as the wave moves:

$$\Delta p(x,t) = \Delta p_m \sin(kx - \omega t),$$

where Δp_m is the **pressure amplitude**; the maximum increase or decrease in pressure. It is given by

$$\Delta p_m = (v\rho\omega)s_m.$$

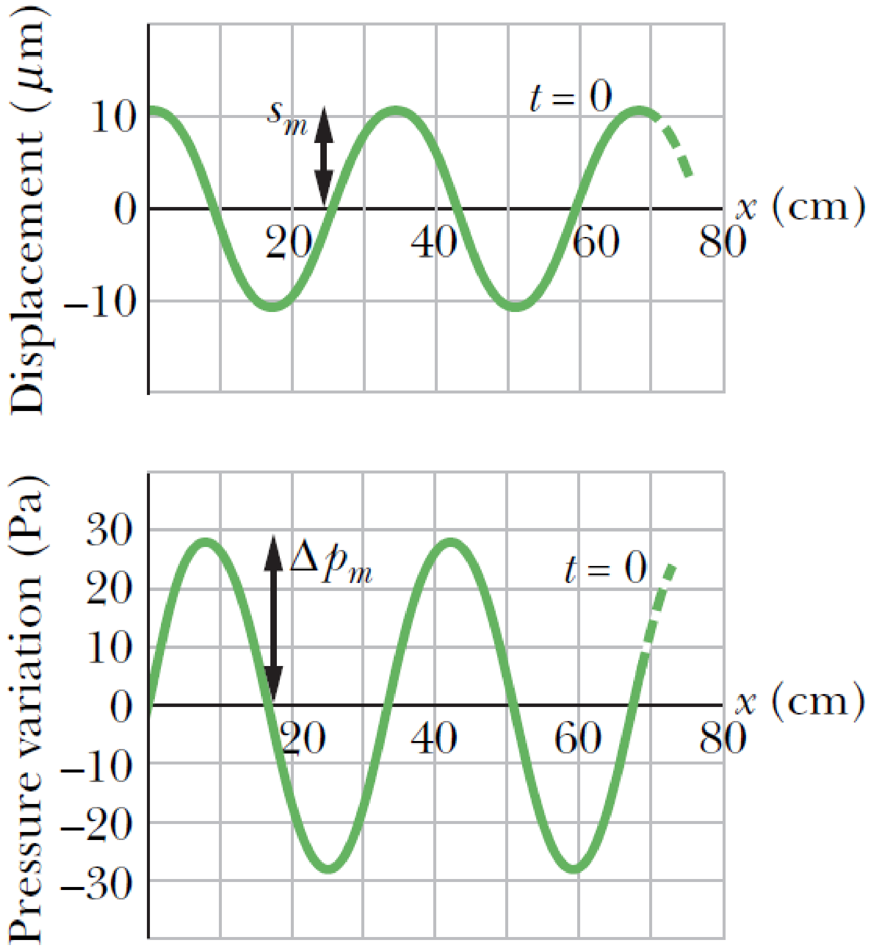
Usually, Δp_m is very much less than the pressure where there is no wave.



3. Traveling Sound Waves

Note that negative Δp corresponds to an expansion and positive Δp corresponds to compression.

s and Δp are $\frac{\pi}{2}$ out of phase, as shown in the figure.



3. Traveling Sound Waves

Example 1: The maximum pressure amplitude Δp_m that the human ear can tolerate in loud sounds is about 28 Pa (which is very much less than the normal air pressure of about 10^5 Pa). What is the displacement amplitude s_m for such a sound in air of density $\rho = 1.21 \text{ kg/m}^3$, at a frequency of 1000 Hz and a speed of 343 m/s?

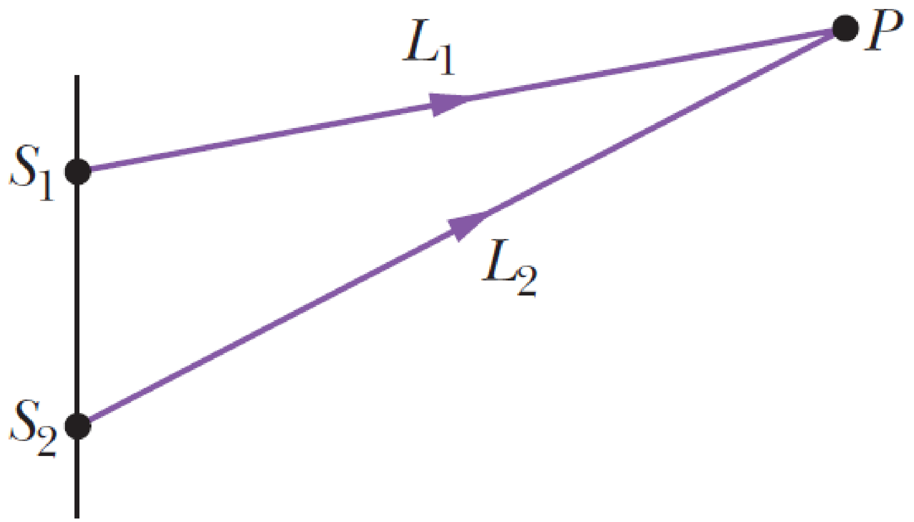
We know that $\Delta p_m = (v\rho\omega)s_m$, which gives

$$s_m = \frac{\Delta p_m}{v\rho\omega} = \frac{28 \text{ Pa}}{(343 \text{ m/s})(1.21 \text{ kg/m}^3)[2\pi(1000 \text{ Hz})]} = 1.1 \times 10^{-5} \text{ m} = 11 \text{ µm}.$$

4. Interference

Consider the situation shown in the figure. Two point sources S_1 and S_2 emit sound waves that are in phase and of the same wavelength λ. We assume that the distances L_1 and L_2 are much larger than that between the sources. Thus we can approximate the waves at P to be in the same direction.

If $L_1 = L_2$, the two waves would be in phase at P and their interference there would be fully constructive. However, L_1 is different from L_2 in general. The phase difference ϕ at P depends on the **path length difference** $\Delta L = |L_2 - L_1|$.



4. Interference

We use the fact that a phase difference of 2π corresponds to one wavelength. Thus, we write

$$\frac{\phi}{2\pi} = \frac{\Delta L}{\lambda},$$

or

$$\phi = 2\pi \frac{\Delta L}{\lambda}.$$

Fully constructive interference occurs when

$$\phi = 2\pi m\,, \qquad m = 0, 1, 2, \ldots$$

which corresponds to

$$\frac{\Delta L}{\lambda} = 0, 1, 2, \ldots$$

4. Interference

Fully destructive interference occurs when

$$\phi = \pi(2m+1) \qquad\qquad m = 0, 1, 2, \ldots$$

which corresponds to

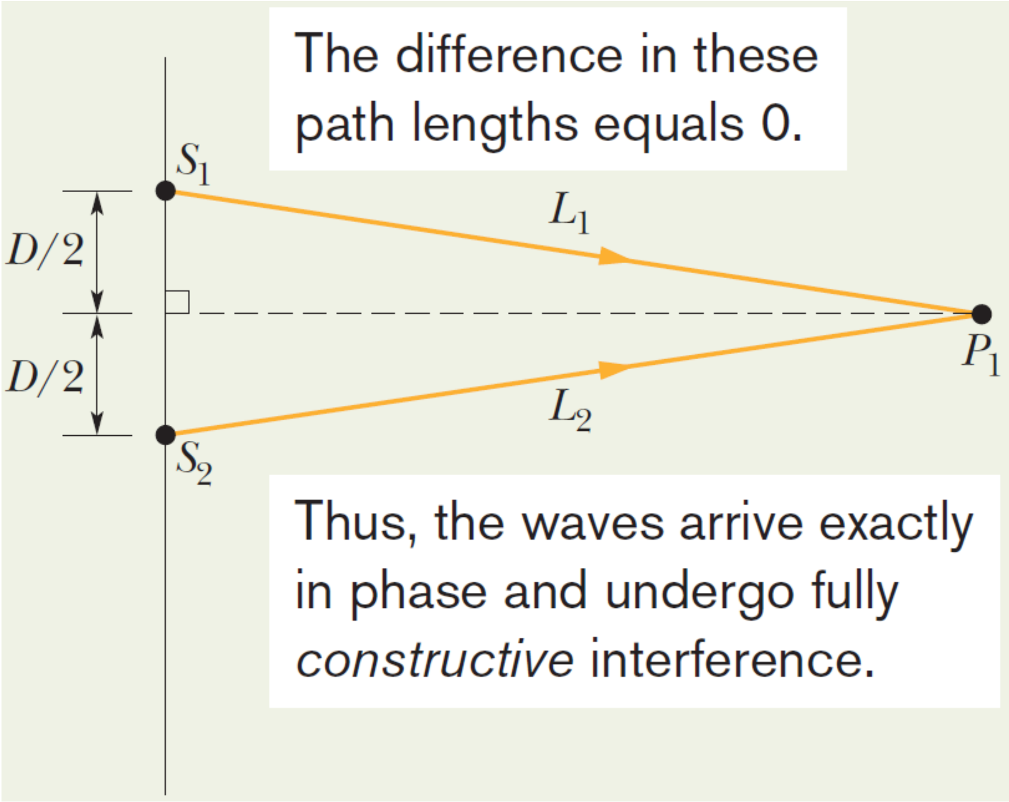
$$\frac{\Delta L}{\lambda} = \frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \ldots \qquad\qquad \phi = 2\pi\frac{\Delta L}{\lambda}.$$

The interference is intermediate for other values of $\Delta L/\lambda$.

4. Interference

Example 2: In the figure, two point sources S_1 and S_2, which are in phase and separated by distance $D = 1.5\lambda$, emit identical sound waves of wavelength λ.

(a) What is the path length difference of the waves from S_1 and S_2 at point P_1, which lies on the perpendicular bisector of distance D, at a distance greater than D from the sources? What type of interference occurs at P_1?



$$\Delta L = 0.$$

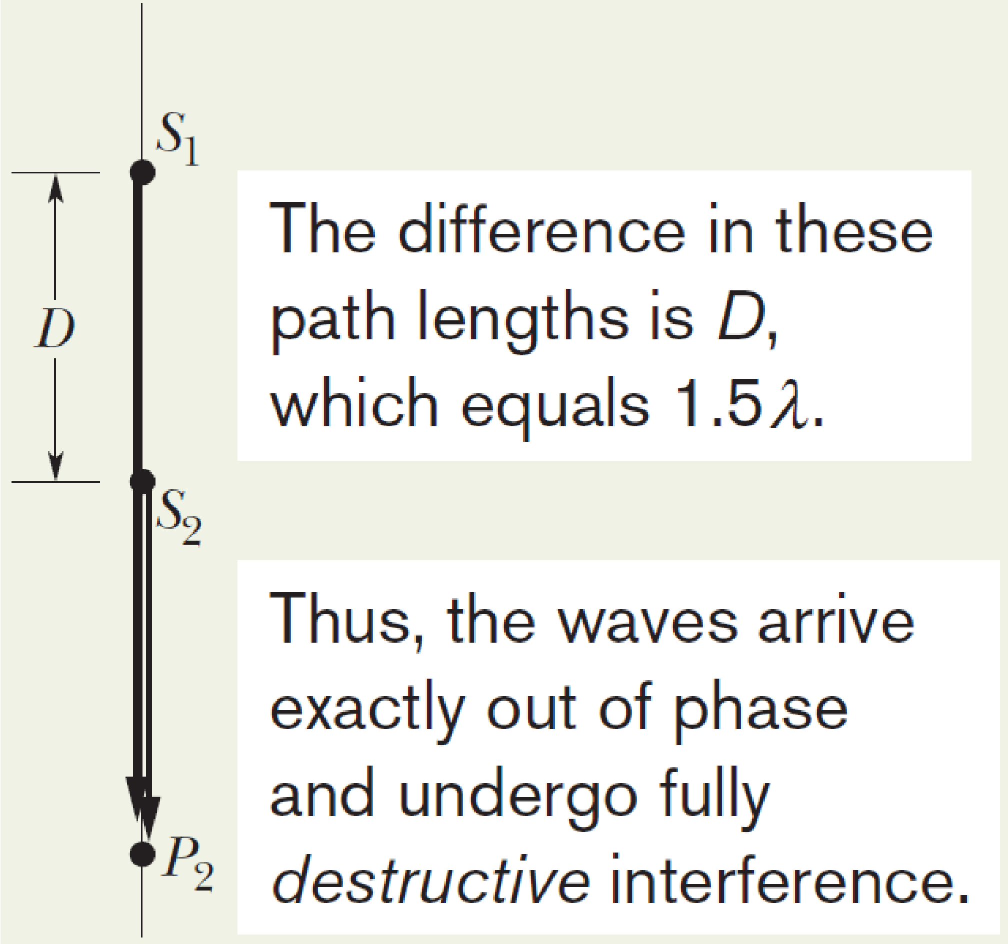
The interference is therefore fully constructive.

4. Interference

(b) What are the path length difference and type of interference at point P_2?

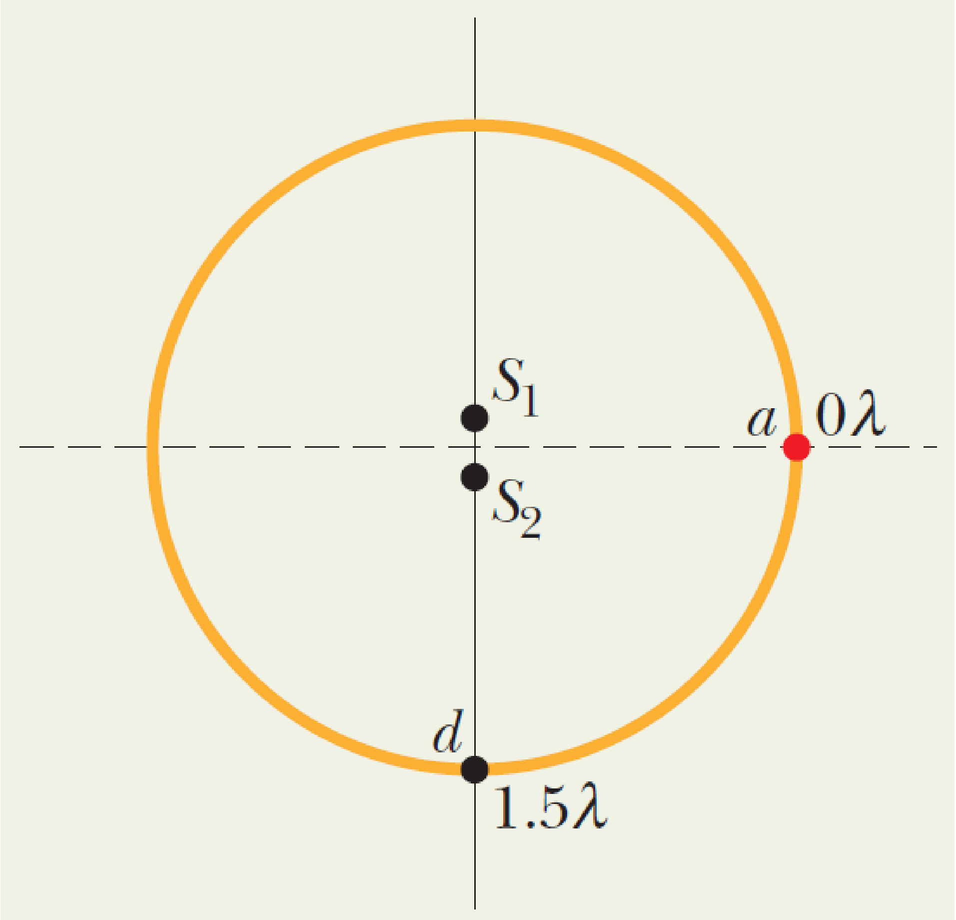
$$\Delta L = 1.5\ \lambda.$$

The interference is therefore fully destructive.



4. Interference

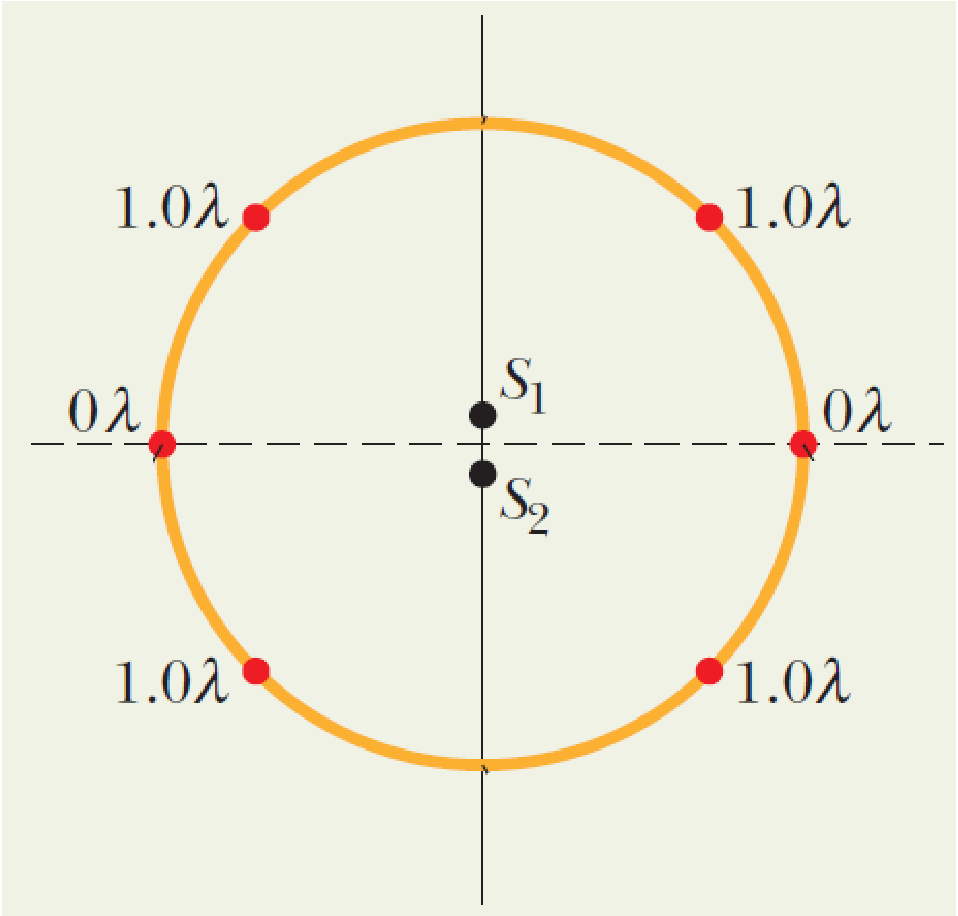
(c) The figure shows a circle with a radius much greater than D, centered on the midpoint between sources S_1 and S_2. What is the number of points N around this circle at which the interference is fully constructive? (That is, at how many points do the waves arrive exactly in phase?)



At point a, $\Delta L = 0$. It increases as you go down along the circle until it becomes $1.5\,\lambda$ at point d. Therefore, there is point between a and d at which $\Delta L = \lambda$, where a fully constructive interference occurs.

4. Interference

Thus, there are six points ($N = 6$) at which the interference is fully constructive.



5. Intensity and Sound Level

The intensity I of a sound wave at a surface is the average rate per unit area at which energy is transferred by the wave through or onto the surface:

$$I = \frac{P}{A},$$

where P is the average power (time rate of energy transfer) of the wave and A is the area of the surface intercepting the sound. The intensity I of a sound wave is given by

$$I = \frac{1}{2} v\rho\omega^2 s_m^2.$$

5. Intensity and Sound Level

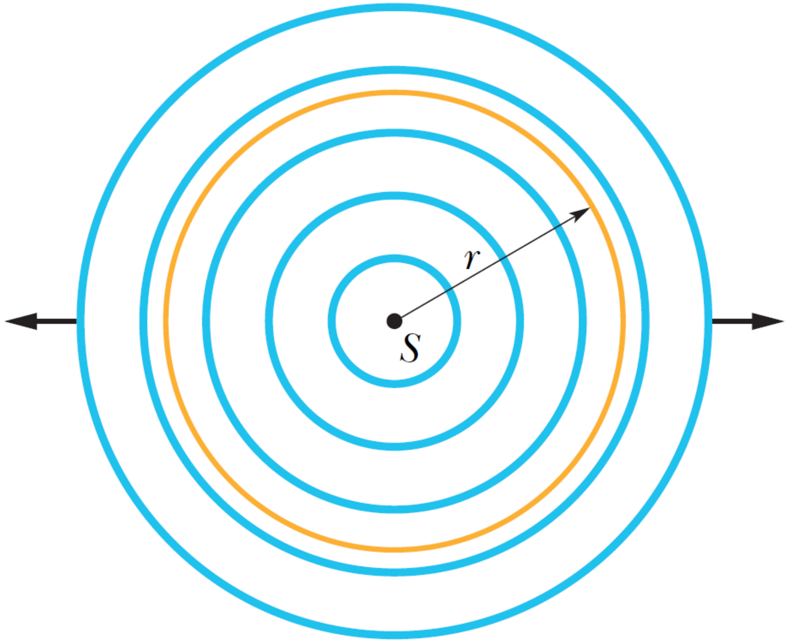
5. Intensity and Sound Level

<u>Variation of Intensity with Distance</u>

Consider a sound source S that emits sound isotropically (with equal intensity in all directions), as shown in the figure.

Imagine a sphere of radius r, centered on the source. All the energy emitted by the source must pass through the surface of the sphere.

Therefore, the time rate at which energy is transferred through the surface must equal the time rate at which energy is emitted by the source (the power P_s of the source) .

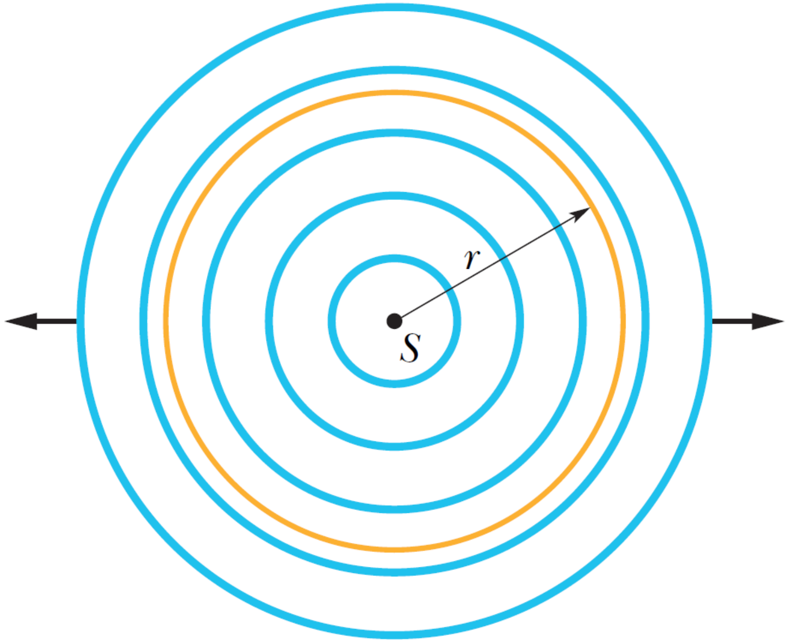


5. Intensity and Sound Level

<u>Variation of Intensity with Distance</u>

The intensity I at a sphere of radius r must then by

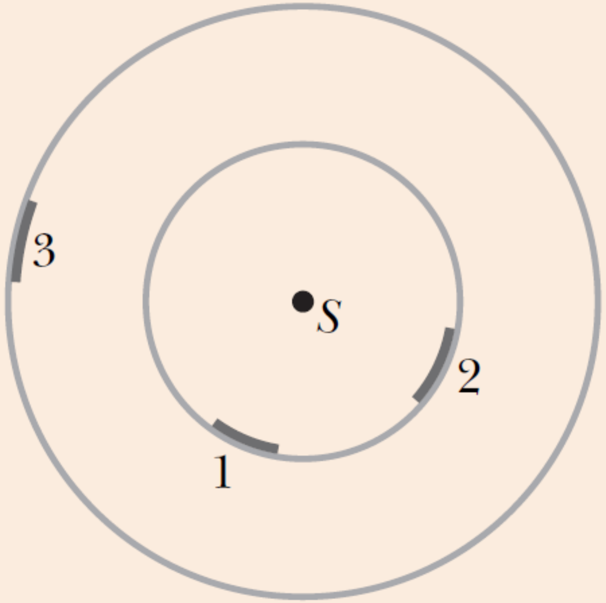
$$I = \frac{P_s}{A_{\text{sphere}}} = \frac{P_s}{4\pi r^2}.$$



5. Intensity and Sound Level

CHECKPOINT 2

The figure indicates three small patches 1, 2, and 3 that lie on the surfaces of two imaginary spheres; the spheres are centered on an isotropic point source *S* of sound. The rates at which energy is transmitted through the three patches by the sound waves are equal. Rank the patches according to (a) the intensity of the sound on them and (b) their area, greatest first.



(a) 1 & 2 tie, 3.

(b) 3, 1& 2 tie.

$$I = \frac{P_s}{4\pi r^2}$$

$$P = IA$$

5. Intensity and Sound Level

Example 3: A firecracker, emitting a pulse of sound that travels radially outward from the explosion center. The power of this acoustic emission is $P_s = 3.5 \times 10^4$ W.

(a) What is the intensity I of the sound when it reaches a distance $r = 100$ m from the firecracker?

$$I = \frac{P_s}{4\pi r^2} = \frac{3.5 \times 10^4 \text{ W}}{4\pi (100 \text{ m})^2} = 0.279 \text{ W/m}^2.$$

5. Intensity and Sound Level

(b) At what time rate P_{he} is sound energy intercepted by a human ear, aimed at the firecracker and located a distance $r = 100\ \mathrm{m}$ from the firecracker? The radius of the human ear canal is $0.35\ \mathrm{mm}$.

$$A_{he} = \pi r^2 = \pi(0.35 \times 10^{-3}\ \mathrm{m})^2 = 3.85 \times 10^{-7}\ \mathrm{m}^2.$$

$$P_{he} = IA_{he} = (0.279\ \mathrm{W/m^2})(3.85 \times 10^{-7}\ \mathrm{m}^2)$$
$$= 1.1 \times 10^{-7}\ \mathrm{W}.$$

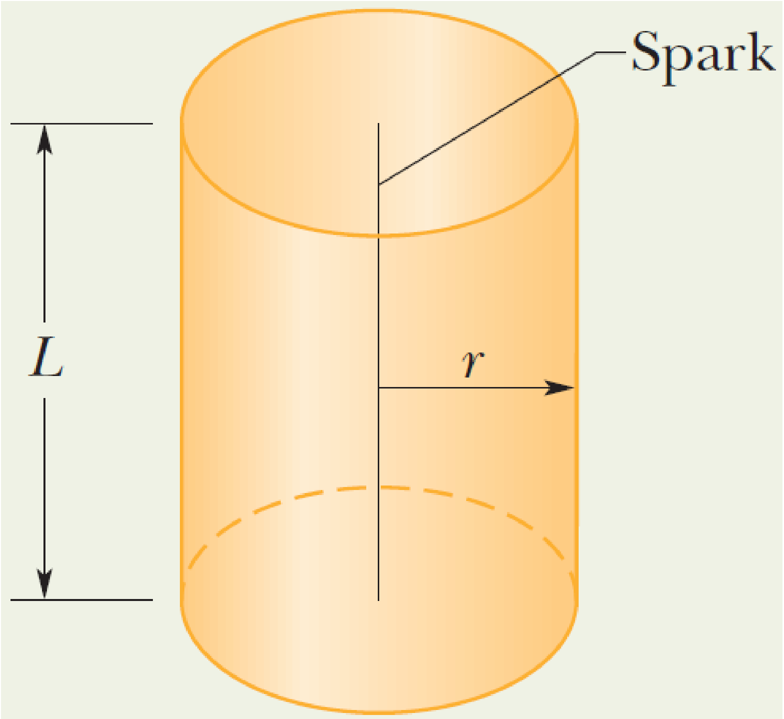
5. Intensity and Sound Level

Example 4: An electric spark jumps along a straight line of length $L = 10$ m, emitting a pulse of sound that travels radially outward from the spark. (The spark is said to be a line source of sound.) The power of this acoustic emission is $P_s = 1.6 \times 10^4$ W.

(a) What is the intensity I of the sound when it reaches a distance $r = 12$ m from the spark?

The radial sound pulse passes through a cylindrical surface of area $A = 2\pi r L$. The intensity is then

$$I = \frac{P_s}{2\pi r L} = \frac{1.6 \times 10^4 \text{ W}}{2\pi(12 \text{ m})(10 \text{ m})} = 21 \text{ W/m}^2.$$



5. Intensity and Sound Level

(b) At what time rate P_d is sound energy intercepted by an acoustic detector of area $A_d = 2.0\ \text{cm}^2$, aimed at the spark and located a distance $r = 12$ m from the spark?

$$P_d = IA_d = (21\ \text{W/m}^2)(2.0 \times 10^{-4}\ \text{m}^2) = 4.2\ \text{mW}.$$

(b) At what time rate P_d is sound energy intercepted by an acoustic detector of area $A_d = 2.0\ \text{cm}^2$, aimed at the spark and located a distance $r = 12$ m from the spark?

$$P_d = IA_d = (21\ \text{W/m}^2)(2.0 \times 10^{-4}\ \text{m}^2) = 4.2\ \text{mW}.$$

5. Intensity and Sound Level

The **anechoic chamber** at Orfield Laboratories. It holds the Guinness World Record for the world's quietest place.

$\beta = -9.4\ \text{dB}$

5. Intensity and Sound Level

<u>The Decibel Scale</u>

It is much more convenient to speak of **sound level** β instead of sound intensity I. The sound level is defined as

$$\beta = (10\ \text{dB}) \log \frac{I}{I_0}.$$

Here dB is the abbreviation of **decibel** and I_0 is a standard reference intensity ($= 10^{-12}\ \text{W}/\text{m}^2$), chosen near the lowest limit of human range of hearing.

$\beta = 60\ \text{dB}$ corresponds to an intensity that is 10^6 times the standard reference level.

Some Sound Levels (dB)

Hearing threshold	0
Rustle of leaves	10
Conversation	60
Rock concert	110
Pain threshold	120
Jet engine	130

5. Intensity and Sound Level

Example 4: If earplugs decreases the sound level of some source by $20\ \mathrm{dB}$, what is the ratio of the final intensity I_f of the sound waves to their initial intensity I_i?

$$\beta_f - \beta_i = (10\ \mathrm{dB})\left(\log\frac{I_f}{I_0} - \log\frac{I_i}{I_0}\right) = (10\ \mathrm{dB})\log\frac{I_f}{I_0}\frac{I_0}{I_i} = (10\ \mathrm{dB})\log\frac{I_f}{I_i}.$$

Solving for I_f/I_i we obtain

$$\frac{I_f}{I_i} = 10^{\frac{\beta_f-\beta_i}{10\ \mathrm{dB}}}.$$

Using $\beta_f - \beta_i = -20\ \mathrm{dB}$ yields

$$\frac{I_f}{I_i} = 10^{\frac{-20\ \mathrm{dB}}{10\ \mathrm{dB}}} = 10^{-2}.$$

6. Sources of Musical Sound

Musical sounds can be set by oscillating strings, membranes, air columns and so on.

We have seen in Ch. 16 that setting up standing waves on a string make it oscillate with a large sustained amplitude.

We can set up standing sound waves in an air-filled pipe, with one or two open ends. The waves get reflected at both ends of the pipe, whether closed or open! If the wavelength is suitably matched to the length of the pipe, the superposition of the waves in the pipe sets up a standing wave pattern.

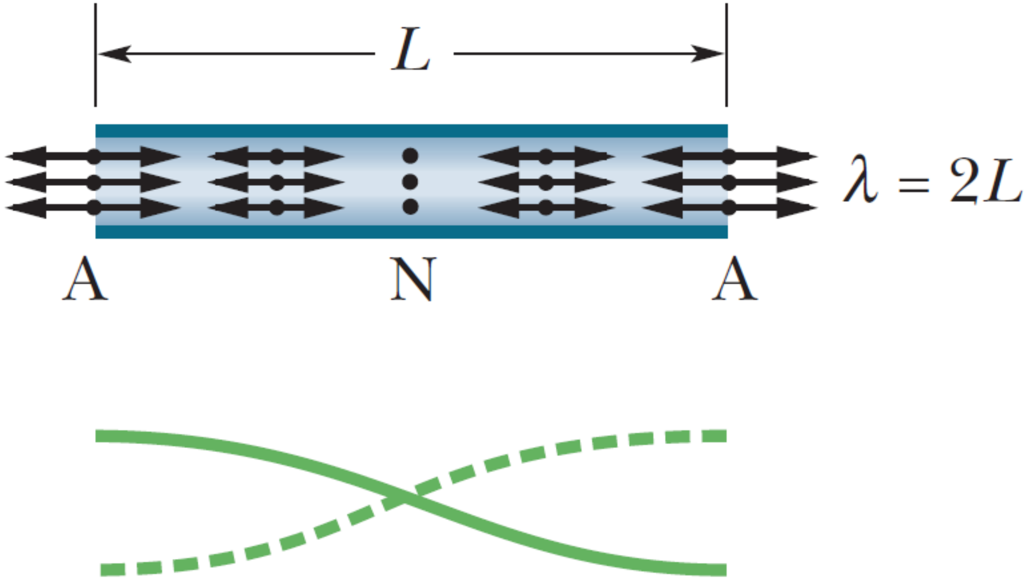
The advantage of such a standing wave is that the air in the pipe oscillates with large, sustained amplitude, emitting sound at any open end at the same frequency of the standing wave in the pipe.

6. Sources of Musical Sound

Standing waves in a pipe are similar to those in a string. There is an antinode at an open end of a pipe and a node at a closed end.

The simplest standing wave pattern (the fundamental mode or first harmonic), that can be set in a pipe with **two open ends**, is shown in the figure.

The wavelength λ is twice the pipe's length L, or $\lambda = 2L$.



6. Sources of Musical Sound

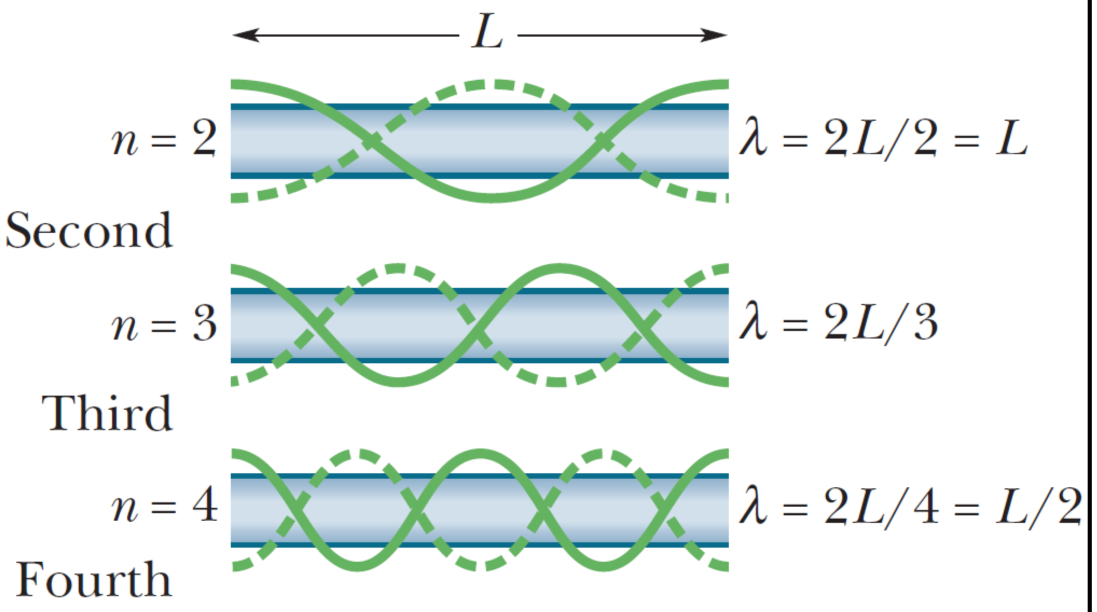
The second harmonic requires that $\lambda = L$. The third harmonic requires that $\lambda = 2L/3$.

Generally, the resonant frequency for a pipe of length L with two open ends corresponds to wavelengths

$$\lambda = \frac{2L}{n}, \qquad n = 1, 2, 3, \ldots$$

The corresponding resonant frequencies are

$$f = \frac{v}{\lambda} = n\frac{v}{2L}, \qquad n = 1, 2, 3, \ldots$$

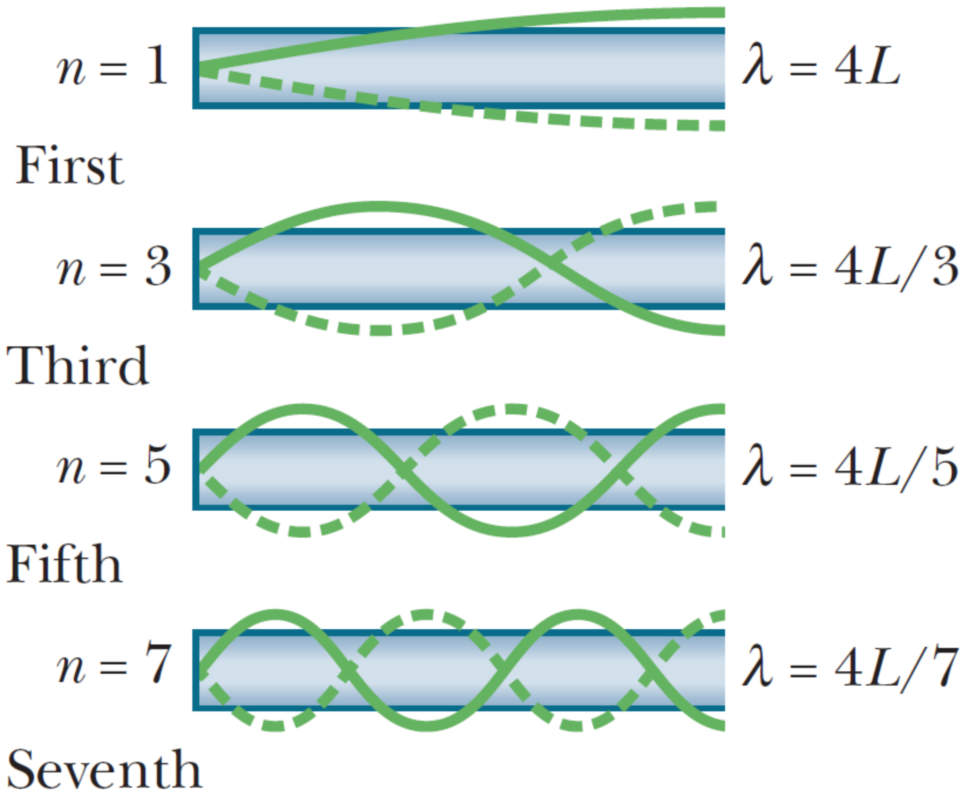


6. Sources of Musical Sound

For a pipe with **one open end**, there must be an antinode at the open end and a node at the closed end.

The simplest standing wave pattern requires that $L = \lambda/4$, or $\lambda = 4L$.

The next simplest pattern requires that $L = 3\lambda/4$ or $\lambda = 4L/3$, and so on.



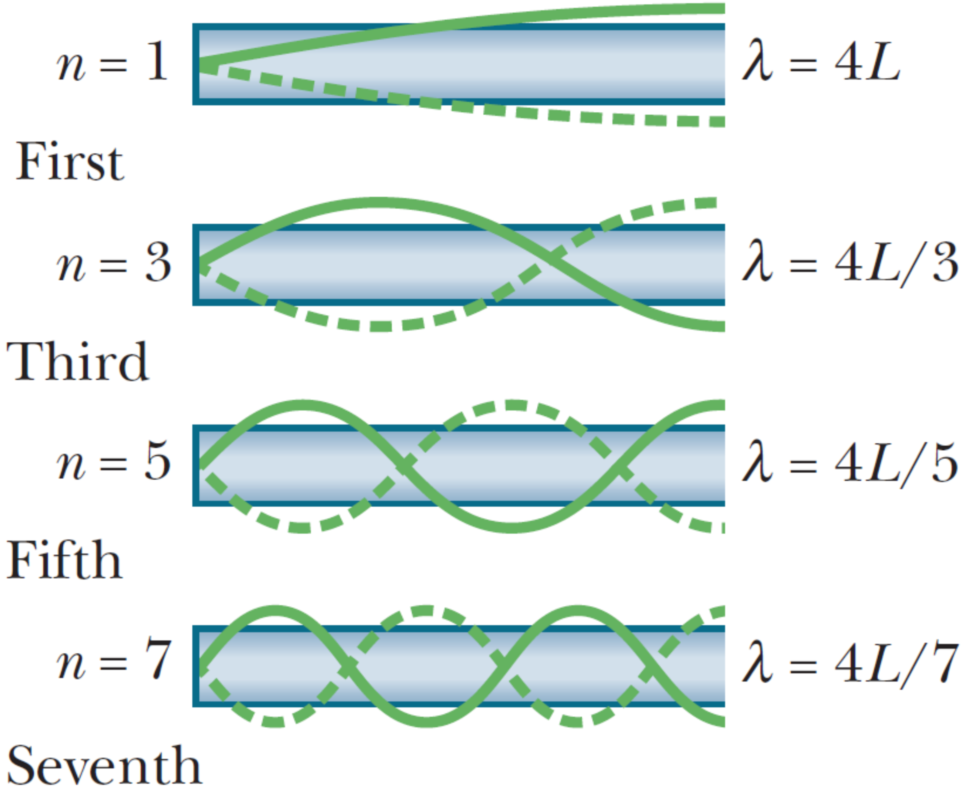
6. Sources of Musical Sound

Generally, the resonant frequency for a pipe of length L with one open end corresponds to wavelengths

$$\lambda = \frac{4L}{n}, \qquad n = 1, 3, 5, \ldots \quad \text{(odd n)}$$

The corresponding resonant frequencies are

$$f = \frac{v}{\lambda} = n\frac{v}{4L}, \qquad n = 1, 3, 5, \ldots \quad \text{(odd n)}$$



6. Sources of Musical Sound

CHECKPOINT 3

Pipe A, with length L, and pipe B, with length $2L$, both have two open ends. Which harmonic of pipe B has the same frequency as the fundamental of pipe A?

The second harmonic

$$f = n\frac{v}{2L}$$

$$f_A = n_A\frac{v}{2L}; \qquad f_{A1} = \frac{v}{2L}$$

$$f_B = n\frac{v}{2(2L)} = n_B\frac{v}{4L}; \qquad f_{B2} = 2\frac{v}{4L} = \frac{v}{2L}$$

6. Sources of Musical Sound

Example 5: Pipe A is open at both ends and has length $L_A = 0.343\text{ m}$. We want to place it near three other pipes in which standing waves have been set up, so that the sound can set up a standing wave in pipe A. Those other three pipes are each closed at one end and have lengths $L_B = 0.500L_A$, $L_C = 0.250L_A$, and $L_D = 2.00L_A$. For each of these three pipes, which of their harmonics can excite a harmonic in pipe A? What frequency do you hear from the tube?

Pipe A:

$$f_A = \frac{n_A v}{2L_A} = n_A \frac{343\text{ m/s}}{2(0.3430\text{ m})} = n_A(500\text{ Hz}), \qquad n_A = 1,2,3,\ldots$$

6. Sources of Musical Sound

Pipe *B*:

$$f_B = \frac{n_B v}{4L_B} = n_B \frac{343\ \text{m/s}}{4(0.500 L_A)} = n_B \frac{343\ \text{m/s}}{4(0.500)(0.3430\ \text{m})} = n_B(500\ \text{Hz}),$$

$$n_B = 1,3,5, \ldots$$

Thus, $f_A = f_B$ for $n_A = n_B$, with $n_B = 1,3,5, \ldots$.

Pipe *C*:

$$f_C = \frac{n_C v}{4L_C} = n_C \frac{343\ \text{m/s}}{4(0.250 L_A)} = n_C \frac{343\ \text{m/s}}{4(0.250)(0.3430\ \text{m})} = n_C(1000\ \text{Hz}),$$

$$n_C = 1,3,5, \ldots$$

Thus, $f_A = f_C$ for $n_A = 2n_C$, with $n_C = 1,3,5, \ldots$.

6. Sources of Musical Sound

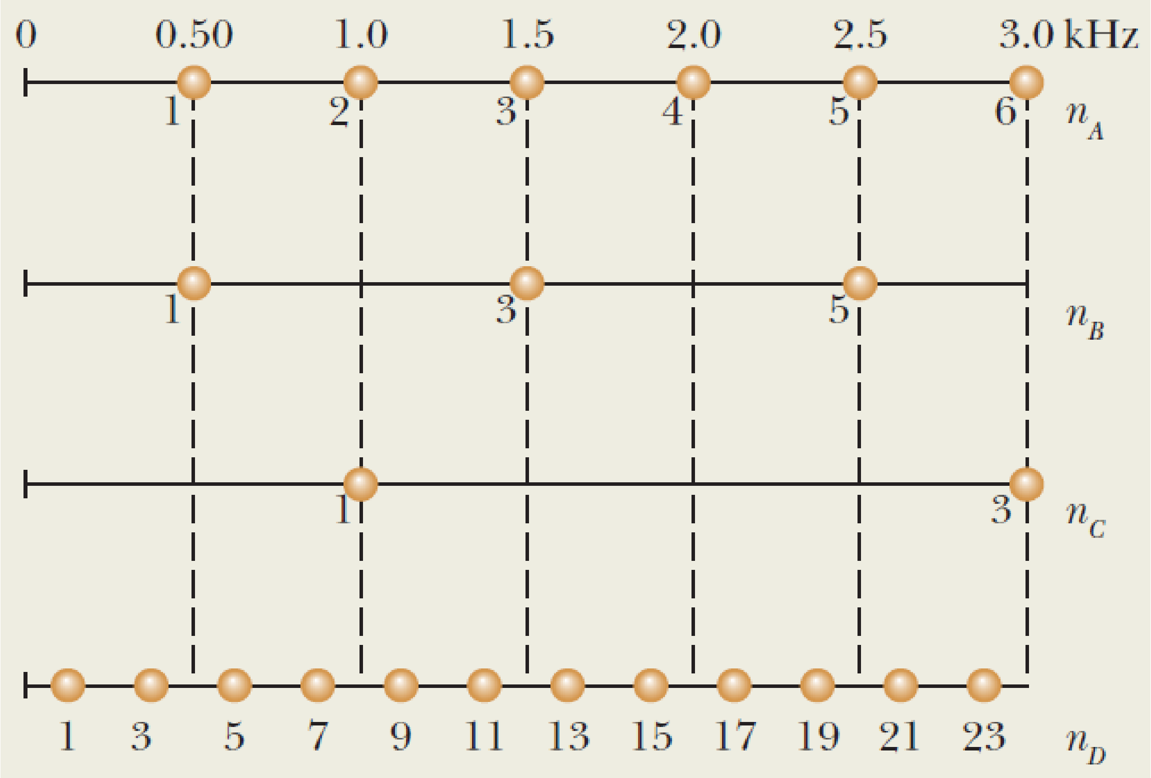
Pipe *D*:

$$f_D = \frac{n_D v}{4L_D} = n_D \frac{343 \text{ m/s}}{4(2L_A)}$$

$$= n_D \frac{343 \text{ m/s}}{4(2)(0.3430 \text{ m})}$$

$$= n_D(125 \text{ Hz}) \qquad n_D = 1,3,5, \ldots$$

Thus, $f_A = f_D$ for $n_A = \frac{1}{2} n_D$, which is not possible since n_D is odd.



6. Sources of Musical Sound

Example 6: Weak background noises from a room set up the fundamental standing wave in a cardboard tube of length $L = 67.0 \text{ cm}$ with two open ends. Assume that the speed of sound in the air within the tube is 343 m/s.

(a) What frequency do you hear from the tube?

$$f_1 = \frac{v}{2L} = \frac{343 \text{ m/s}}{2(0.670 \text{ m})} = 256 \text{ Hz}.$$

(b) If you jam your ear against one end of the tube, what fundamental frequency do you hear from the tube?

The tube now has one open end. Thus

$$f_1 = \frac{v}{4L} = \frac{343 \text{ m/s}}{4(0.670 \text{ m})} = 128 \text{ Hz}.$$

https://www.youtube.com/watch?v=a3RfULw7aAY

6. The Doppler Effect

Consider a source S of sound and a detector D that are in motion relative to air, with speeds v_S and v_D, respectively. The emitted frequency f and detected frequency f' are related by

$$f' = f\,\frac{v \pm v_D}{v \pm v_S},$$

where v is the speed of sound through air.

<u>Sign Rule:</u>

- When the source or detector moves toward the other, the sign on its speed must give an upward shift in frequency.
- When the source or detector moves away from the other, the sign on its speed must give a downward shift in frequency.

6. The Doppler Effect

CHECKPOINT 4

The figure indicates the directions of motion of a sound source and a detector for six situations in stationary air. For each situation, is the detected frequency greater than or less than the emitted frequency, or can't we tell without more information about the actual speeds?

	Source	Detector
(*a*)	→	• 0 speed
(*b*)	←	• 0 speed
(*c*)	→	→
(*d*)	←	←
(*e*)	→	←
(*f*)	←	→

(a) $f' = f\frac{v}{v-v_S}$, greater

(b) $f' = f\frac{v}{v+v_S}$, less

(c) $f' = f\frac{v-v_D}{v-v_S}$, can't tell

(d) $f' = f\frac{v+v_D}{v+v_S}$, can't tell

(e) $f' = f\frac{v+v_D}{v-v_S}$, greater

(f) $f' = f\frac{v-v_D}{v+v_S}$, less

6. The Doppler Effect

Example 5: Bats navigate and search out prey by emitting, and then detecting reflections of, ultrasonic waves, which are sound waves with frequencies greater than can be heard by a human. Suppose a bat emits ultrasound at frequency $f_{be} = 82.52\ \mathrm{kHz}$ while flying with velocity $\vec{v}_b = (9.00\ \mathrm{m/s})\hat{\imath}$ as it chases a moth that flies with velocity $\vec{v}_m = (8.00\ \mathrm{m/s})\hat{\imath}$. What frequency f_{md} does the moth detect?

The source (bat) is moving *toward* the detector (moth). The detector (moth) is moving *away* from the source (bat). Thus,

$$f_{md} = f_{be}\frac{v - v_D}{v - v_S} = (82.52\ \mathrm{kHz})\frac{343\ \mathrm{m/s} - 8.00\ \mathrm{m/s}}{343\ \mathrm{m/s} - 9.00\ \mathrm{m/s}} = 82.8\ \mathrm{kHz}.$$

6. The Doppler Effect

What frequency f_{bd} does the bat detect in the returning echo from the moth?

The source (moth) is moving *away* from the detector (bat). The detector (bat) is moving *toward* the source (moth). Thus,

$$f_{bd} = f_{md}\frac{v + v_D}{v + v_S} = (82.77\ \mathrm{kHz})\frac{343\ \mathrm{m/s} + 9.00\ \mathrm{m/s}}{343\ \mathrm{m/s} + 8.00\ \mathrm{m/s}} = 83.0\ \mathrm{kHz}.$$