Examples: Kepler's third law PHYS 215 (Introduction to Astronomy)

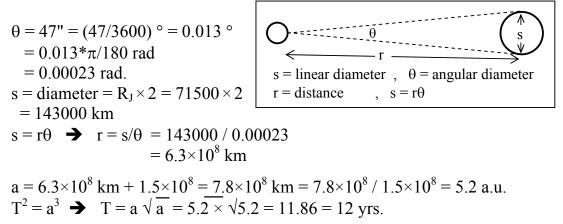
1. Find the orbital period (period of revolution) of Jupiter if its distance from the Sun is about 7.8×10^8 km. ($a_e = 1.5 \times 10^8$ km).

a = 7.8×10⁸ km = 7.8×10⁸ / 1.5×10⁸ = 5.2 a.u. T² = a³ → T = a \sqrt{a} = 5.2 × $\sqrt{5.2}$ = 11.86 = 12 yrs.

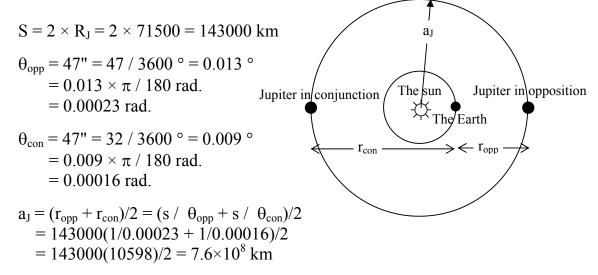
2. Find the orbital period of Jupiter if its distance from the Earth is about 6.3×10^8 km when it is in opposition with the Sun. ($a_e = 1.5 \times 10^8$ km).

a = 6.3×10⁸ km + 1.5×10⁸ = 7.8×10⁸ km = 7.8×10⁸ / 1.5×10⁸ = 5.2 a.u. T² = a³ → T = a \sqrt{a} = 5.2 × $\sqrt{5.2}$ = 11.86 = 12 yrs.

3. Find the orbital period of Jupiter if its angular diameter is about 47" as seen from the Earth when it is in opposition with the Sun. ($a_e = 1.5 \times 10^8$ km, $R_J = 71500$ km)



4. It is found that the angular diameter of Jupiter at opposition is $\theta_{opp} = 47$ " and it is $\theta_{con} = 32$ " at conjunction with the Sun as seen from the Earth. Find the distance of Jupiter from the Sun. (R_J = 71500 km)



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5. In order to find the mass of the Moon a space craft is made to orbit around it. Find the mass of the Moon if the space craft is orbiting at a height of 127 km above its surface with a period of 2 hrs. Also find the orbital speed of the space craft.($R_m = 1738$ km, $G = 6.67 \times 10^{-11}$ m³/kg.s²)

$$T^{2} = (4\pi^{2}/GM_{m}) a^{3} \Rightarrow M_{m} = 4\pi^{2} \times a^{3}/G \times T^{2}$$

= $4\pi^{2} \times (127000 + 1738000)^{3}/G \times (2 \times 3600)^{2} = 7.4 \times 10^{22} \text{ kg}$
v = $2\pi \times a/T = 2\pi \times (127 + 1738)/2 = 5859 \text{ km/hr} = 1.63 \text{ km/s}$
v = $\sqrt{G \times M/a} = \sqrt{6.67 \times 10^{-11} \times 7.4 \times 10^{22} / (127000 + 1738000)} = 1627 \text{ m/s}$
= 1.63 km/s

6. How long Hubble Space Telescope (HST) takes (in minutes) to circle once around the Earth if it is at a height of about 600 km above the Earth surface? ($R_{\oplus} = 6400$ km, $G = 6.67 \times 10^{-11}$ m³/(kg.s²), $M_{\oplus} = 6 \times 10^{24}$ kg)

$$P^{2} = (4\pi^{2} / G M) a^{3}$$

$$P = \sqrt{4\pi^2} (600000 + 6400000)^3 / 6.67 \times 10^{-11} \times 6 \times 10^{24}$$

= 5817 seconds = 97 minutes

7. How fast Hubble Space Telescope (HST) is moving (in km / s) as it circles the Earth at a height of about 600 km above its surface? ($R_{\oplus} = 6400 \text{ km}, \text{ G} = 6.67 \times 10^{-11} \text{ m}^3/(\text{kg.s}^2), M_{\oplus} = 6 \times 10^{24} \text{ kg}$)

P² = (4π²/G M) a³ → P²/(4π² a²) = (1/G M) a = 1/v²
v =
$$\sqrt{G M / a}$$
 = $\sqrt{6.67 \times 10^{-11} \times 6 \times 10^{24} / (600000 + 6400000)}$
= 7561 m/s = 7.6 km/s

or
$$v = 2\pi a / P = 2\pi (600000 + 6400000) / 5817 = 7561 m/s = 7.6 km / s$$

8. How fast Hubble Space Telescope (HST) is moving (in degrees per minute) as it circles the Earth at a height of about 600 km above its surface? (P = 97 min.)

360 ° in P sec. $\Rightarrow \theta$ ° in 1 sec $\theta = 360/P = 360/97$ = 3.7 °/min. = 223 °/hr. = 14.85 revolutions per day

Phys 215: alshukri@kfupm.edu.sa

Introduction to Astronomy Physics Department KFUPM , Dhahran Saudi Arabia

= about 15 rev. per day