Chapter 3 (Kepler and Newton laws)

Phys215: Introduction to Astronomy
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I) Properties of Ellipses:

Equation of an ellipse: $\mathbf{r} = \mathbf{a} \times (\mathbf{1} - \mathbf{e}^2) / (\mathbf{1} + \mathbf{e} \times \cos(\theta))$, or $\mathbf{x}^2 / \mathbf{a}^2 + \mathbf{y}^2 / \mathbf{b}^2 = \mathbf{1}$

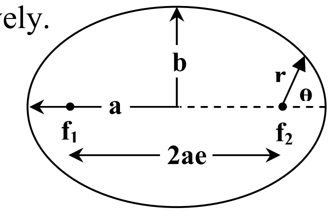
where: **e** is the eccentricity, $\mathbf{f_1}$ & $\mathbf{f_2}$ are the locations of the two foci

a & **b** are the semi-major axis and semi-minor axis, respectively.

Perihelion & perigee : $\theta = 0$ ➔ $r_{min} = a(1-e)$

Aphahelion & aphagee: $\theta = 180$ ➔ $r_{max} = a(1+e)$

$a = (r_{max} + r_{min})/2$, $e = \sqrt{1 - (b/a)^2}$,



II) Kepler's Three Laws (equations)

1. First Law (1609): Planets orbit the Sun on elliptical orbits with the Sun at one focus.
2. Second Law (1609): The (imaginary) line joining the Sun and a planet sweeps equal areas in equal time intervals.
3. Third Law (1618): The square of the period of revolution is directly proportional to the cube of the semi-major axis of the elliptical orbit.

III) Newton's Three Laws (equations)

a. Newton's 1st law (law of inertia) : if $\mathbf{F} = 0$ ➔ $\mathbf{v} = \text{const}$

b. Newton's 2nd law : ($\mathbf{F} \propto \mathbf{a}$ and $F \propto m$ ➔ $\mathbf{F} = m\,\mathbf{a}$)

c. Newton's 3rd law (Action and reaction forces) : $\mathbf{F}_{12} = -\,\mathbf{F}_{21}$.

1. Gravitational Force

$\mathbf{F} = \mathbf{G}\ \mathbf{m_1}\ \mathbf{m_2} / \mathbf{r}^2$, **G** = Universal Gravitational Constant = $6.67\times10^{-11}\ m^3/kg\ s^2$

2. Calculus: Newton invented calculus to explain Physics

3. $\mathbf{F} = m\,\mathbf{a} = m\, d^2\mathbf{r} / dt^2$, $\mathbf{r} = \mathbf{r}(x, y, z) = \mathbf{r}(r, \theta, \varphi)$

If $\mathbf{F} = 0$ ➔ $\mathbf{a} = d\mathbf{v}/dt = d^2\mathbf{r}/dt^2 = 0$

4. Proof of Kepler's 1st law: Conservation of Energy (E) and angular momentum (L)

$\mathbf{r} = \mathbf{a} \times (\mathbf{1} - \mathbf{e}^2) / (\mathbf{1} + \mathbf{e} \times \cos(\theta))$, $a = -\,GM/eE$ and $e = \sqrt{1 - 2\,E\,L^2/G^2\,m^2\,M^2}$

5. Proof of Kepler's 3rd law:

Circular Motion & Gravitational force: $\mathbf{F} = \mathbf{m\,v^2/r} = \mathbf{G\,m\,M/r^2}$

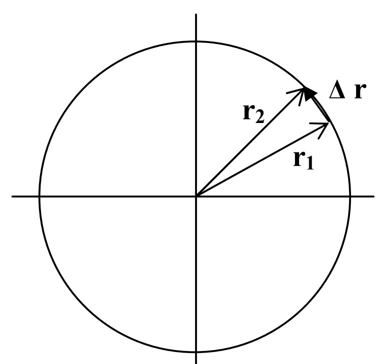
$\mathbf{F} = \mathbf{m\,v^2/r} = \mathbf{G\,m\,M/r^2}$ ➔ $\mathbf{v^2} = \mathbf{G\,M/r}$

➔ $\mathbf{v} = \sqrt{\mathbf{G\,M/r}}$

$\mathbf{T} = \mathbf{C/v} = \mathbf{2\,\pi\,r/v}$ ➔ $\mathbf{T^2} = \mathbf{4\,\pi^2\,r^2/v^2}$

$\mathbf{T^2} = \mathbf{4\,\pi^2\,r^2/(G\,M/r)}$ ➔ $\mathbf{T^2} = \mathbf{4\,\pi^2\,r^3/G\,M}$

$\mathbf{T^2} = \mathbf{(4\,\pi^2/G\,M)\ r^3}$ ➔ $\mathbf{T^2 \propto r^3}$



6. Proof of Kepler's 2nd law:

conservation of angular momentum

$\mathbf{\Delta A/\Delta t} = \mathbf{½\,r\,\Delta r/\Delta t} = \mathbf{½\,r\,v} = \mathbf{m\,r\,v/2m} = \mathbf{L/2m} = \mathbf{constant.}$

($\boldsymbol{\tau} = \mathbf{r}\times\mathbf{F} = \mathbf{r}\times m\, d\mathbf{r}/dt = 0 = d(\mathbf{r}\times m\mathbf{v})/dt = d\mathbf{L}/dt = 0$)