KFUPM
Department of Physics
Phys530

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Homework #3

Due on Wednesday, Nov. 22, 2006

Problem 1

For a small system in contact with a huge reservoir of temperature T, we derive the probability of finding the small system in a state characterized by the energy value E_r by expanding the logarithm of the number of the states of the reservoir, $\ln \Omega(E'_r)$, instead of $\Omega(E'_r)$ itself, around $E'_r = E^{(0)}$, where $E^{(0)}$ is the total energy of the system and reservoir.

(a) Why do not we expand $\Omega(E'_r)$, instead of $\ln \Omega(E'_r)$, around $E'_r = E^{(0)}$?

(b) Support your argument with the case of an ideal gas by comparing the third and second terms in the expansion of $\ln \Omega(E'_r)$, and $\Omega(E'_r)$.

Problem 2

In this problem, we shall use a simple example to examine different assumptions and approximations of section 3.2 of your textbook. Consider the following "canonical ensemble" of 20 identical systems with the total energy of 200 energy units. Each system has 101 energy levels separated by one energy unit, $E_r = r$ energy units with $r = 0, 1, \ldots, 100$.

(a) Plot the real and imaginary parts of the integrand of Eq. 19 along the positive real axis. Does it have a steep minimum?

(b) Find the x coordinate at which the integrand is minimum by numerically solving Eq. 23. Call this coordinate x_0. Does the value of x_0 agree with your plot in (a)?

(c) Plot the real and imaginary parts of the integrand along the axis which is parallel to imaginary axis (y-axis) and passing through $z = x_0$. From your plots, can you say that your integrand has a saddle shape at $z = x_0$? Caution for "large values" of y, the integrand is oscillating and diverging. Stay away from these regions.

(d) Take a contour of a circular shape that passes through $z = x_0$ and whose center coincides with the origin.. Plot the real and imaginary parts of the integrand along this contour. Comment on the shape of your plot.

(e) Numerically evaluate the integration of Eq. 19 using the contour of (d).

(f) Numerically evaluate the integration of Eq. 19 along a line passing through x_0 and parallel to y-axis. Choose your integration limits carefully to stay away from the regions where the integrand is diverging and at the same time you achieve a good approximation to integration over the contour of (d). Compare your result to that of (e).

(g) Plot the approximated integrand of Eq. 26 and compare it to that of (d) by overlapping the two plots.

(h) Numerically evaluate Eq. 27 and compare it to that obtained in (e).

Problem 3

Problem 3.3 from your textbook.

Problem 4

Problem 3.18 from your textbook.

Problem 5

Problem 3.28 from your textbook.