The electrostatic potential can be found numerically by the relaxation method. From the Laplace's equation, the value of the electric potential at a point is equal to the average value of the potentials at the points around it. You need to discretize the continuous space into a grid. The potentials at the boundary grid points are always kept at the given values. The relaxation method is an iterative method. You start with a guess for the values for the potential at the inner grid points. During one iteration, you update the value at the inner grid points by calculating the average value of its nearest neighbors. You iterate repeatedly until the change of potential values is negligible. You will apply the relaxation method to a one-dimensional problem.

- Find the analytical solution for the potential in the range $0\ m\ \leq x \leq 1\ m$ if there is no charge within the range and V = 2.0 V at x = 0 m and V= 12.0 V at x = 1 m.
- Using the analytical solution show that the electric potential at $x = x_0$ can be written as

$$V(x_0) = \frac{V(x_0 - \Delta x) + V(x_0 + \Delta x)}{2}.$$

Assume you have 11 equidistance grid points in the range $0\ m\ \leq x \leq 1\ m$ with the first grid point at x=0 m fixed at 2 V and the last grid point at x =1 m fixed at 12 V. Start with a guess that the potentials at all inner gird points is zero. There are more than one method to update the values at the inner points. In the Jacobi method, the values are updated using only the old values:

$$V[i]_{new} = \frac{V[i-1]_{old} + V[i+1]_{old}}{2}.$$

In the Gauss–Seidel method, the values are updated using the old values and the new values once they become available. Assuming that you are starting updating from grid 1, the updated values at grid i is then given by

$$V[i]_{new} = \frac{V[i-1]_{new} + V[i+1]_{old}}{2}.$$

- Use Mathematica to show on the same plot your analytical solution, your initial guess, and your numerical solution using Jacobi method after 10, 20, 30, 40, 50, and 60 iterations. Use: PlotRange -> {0, 12} and in the ListPlot: Joined -> True.
- Repeat the above using Gauss–Seidel method. Which method is faster?
- Repeat the above step using Gauss–Seidel method and the initial condition V = 12 V at all the inner grid points.