This is not a replacement for your textbook. It is essential to study from your textbook!

Objectives

Ch29: Magnetic fields due to currents

29-1 Magnetic field due to a current

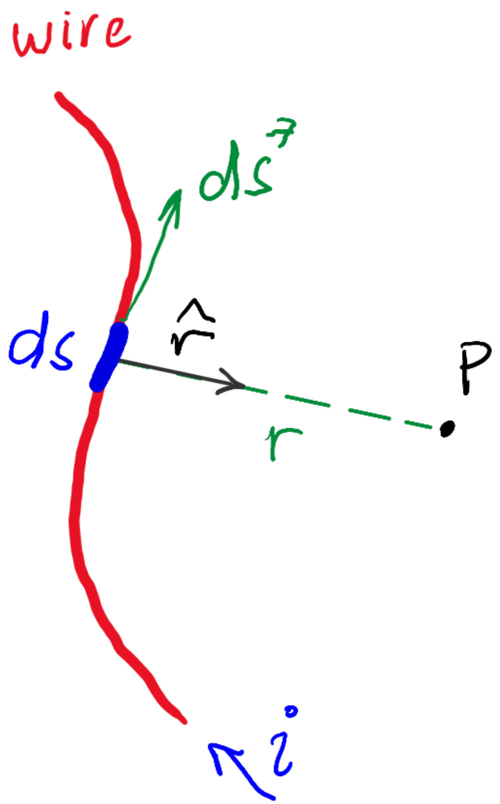
29-2 Force between two parallel currents

29-3 Ampere's law

29-4 Solenoids and toroids

29-5 A current-carrying coil as a magnetic dipole

29.1 Biot-Savart Law



Magnetic field produced at point P by length ds of the wire

$$\boxed{d\vec{B} = \frac{\mu_0}{4\pi} \frac{i\vec{ds} \times \hat{r}}{r^2}}$$

unit vector pointing from ds to point P

$|\hat{r}| = 1$

$4\pi \times 10^{-7} \frac{T.m}{A}$

Permeability Constant

distance between point P and ds

$\vec{ds}$ a length vector

magnitude = length of segment ds

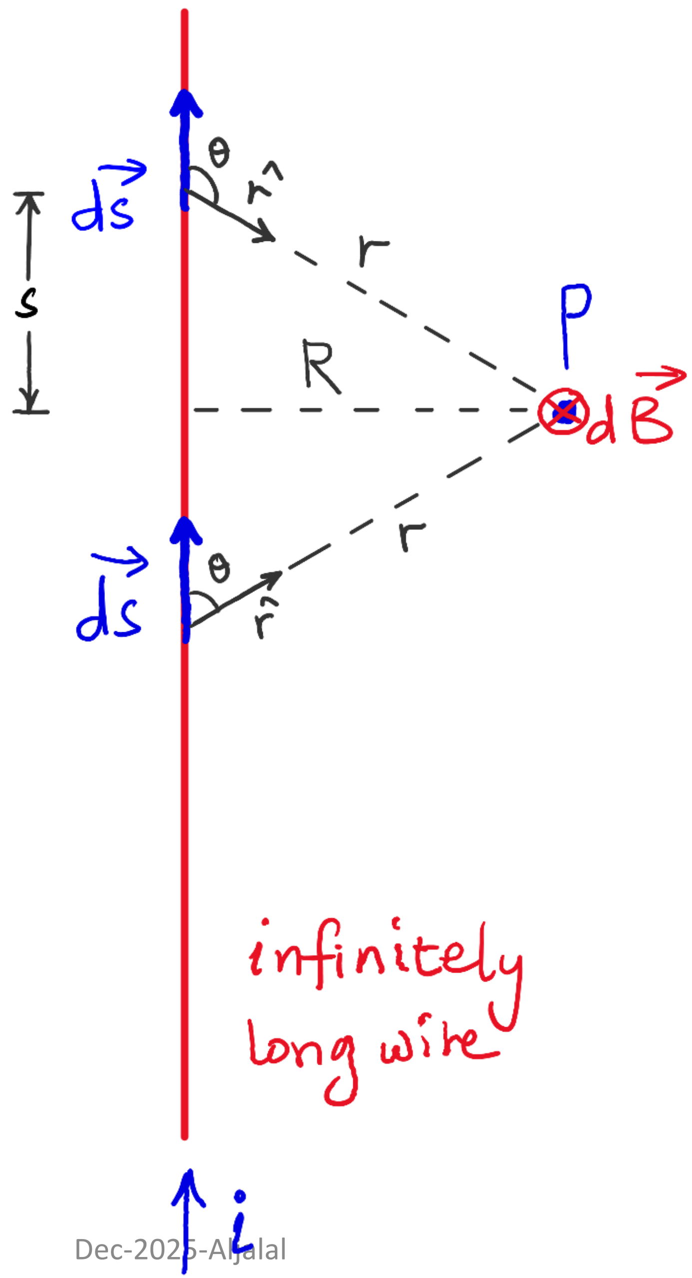
direction = along the current
tangent to the wire

Magnetic field at point P by the whole wire

$$\vec{B} = \int_{\text{wire}} d\vec{B}$$

vector sum

Magnetic Field Due to a Long Straight Wire



$$d\vec{B} = \frac{\mu_0}{4\pi}\,\frac{i\,d\vec{s}\times\hat{r}}{r^2} \qquad \vec{B} = \int_{\text{wire}} d\vec{B}$$

$d\vec{B}$ from all segments points in the same direction.

$\Rightarrow$ sum contributions from all segments algebraically.

$$\sin(\theta) = \sin(180^\circ - \theta) = \frac{R}{r}$$

$$dB = \frac{\mu_0}{4\pi}\,\frac{i\,ds\,\sin\theta}{r^2} \qquad r = \sqrt{s^2 + R^2}$$

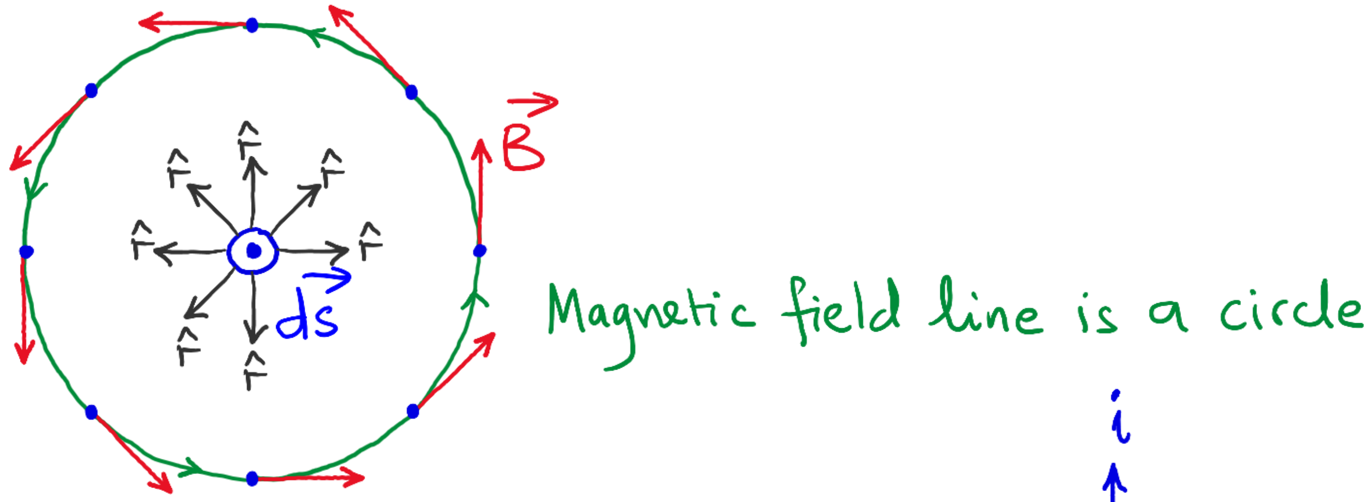
$$dB = \frac{\mu_0}{4\pi}\,\frac{i\,ds\,R}{(s^2+R^2)^{3/2}}$$

$$B = \frac{\mu_0 i R}{4\pi}\int_{-\infty}^{\infty}\frac{ds}{(s^2+R^2)^{3/2}}$$

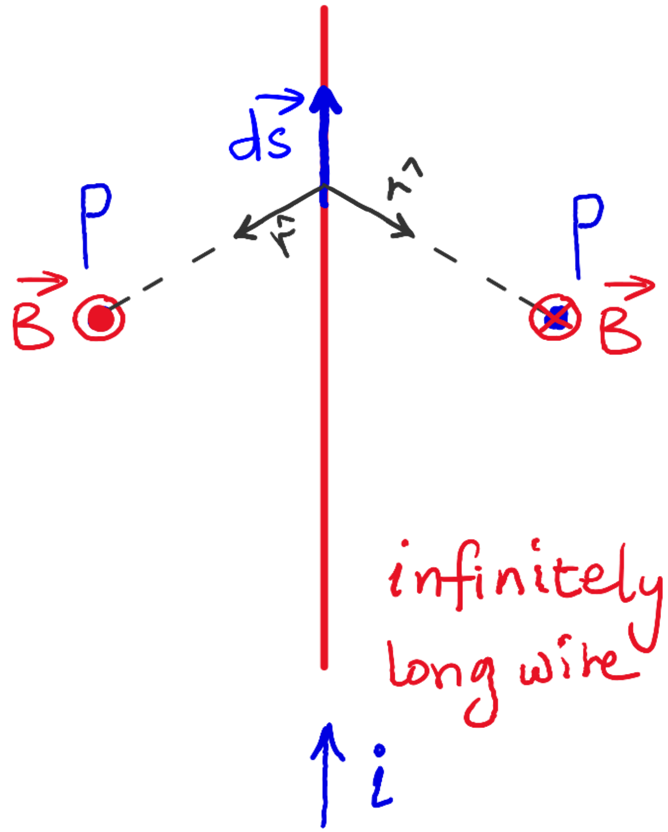
$$= \left[\frac{s}{R^2(s^2+R^2)^{1/2}}\right]_{-\infty}^{\infty} = \frac{2}{R^2}$$

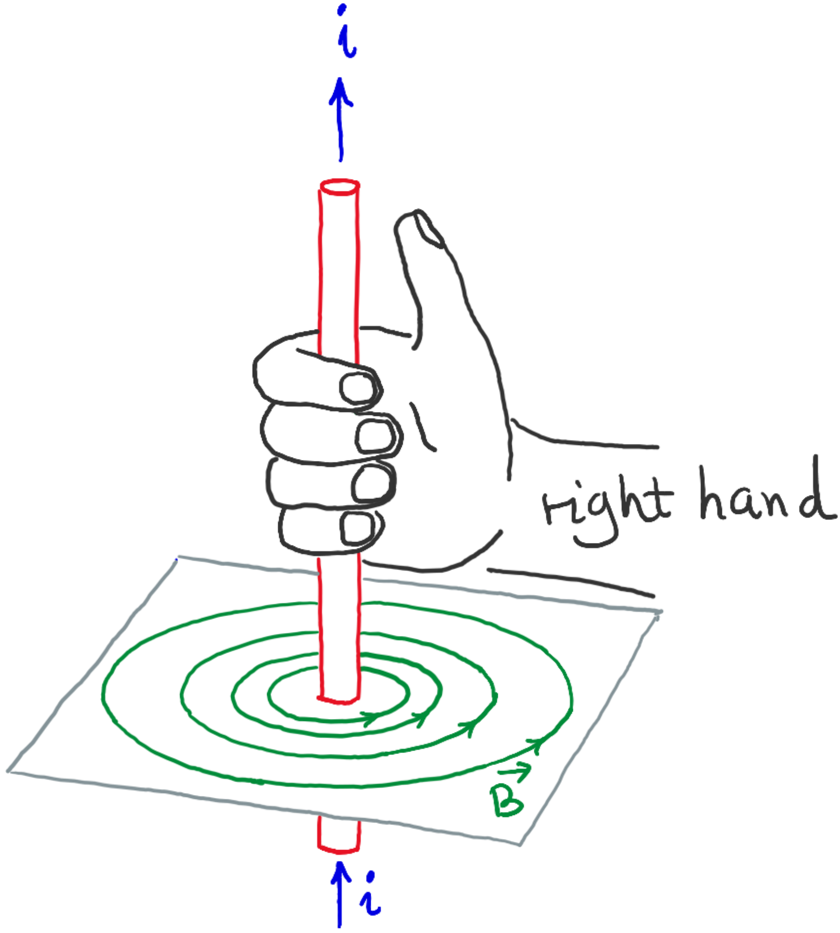
$$\boxed{B = \frac{\mu_0 i}{2\pi R}}$$

Top view



Side view



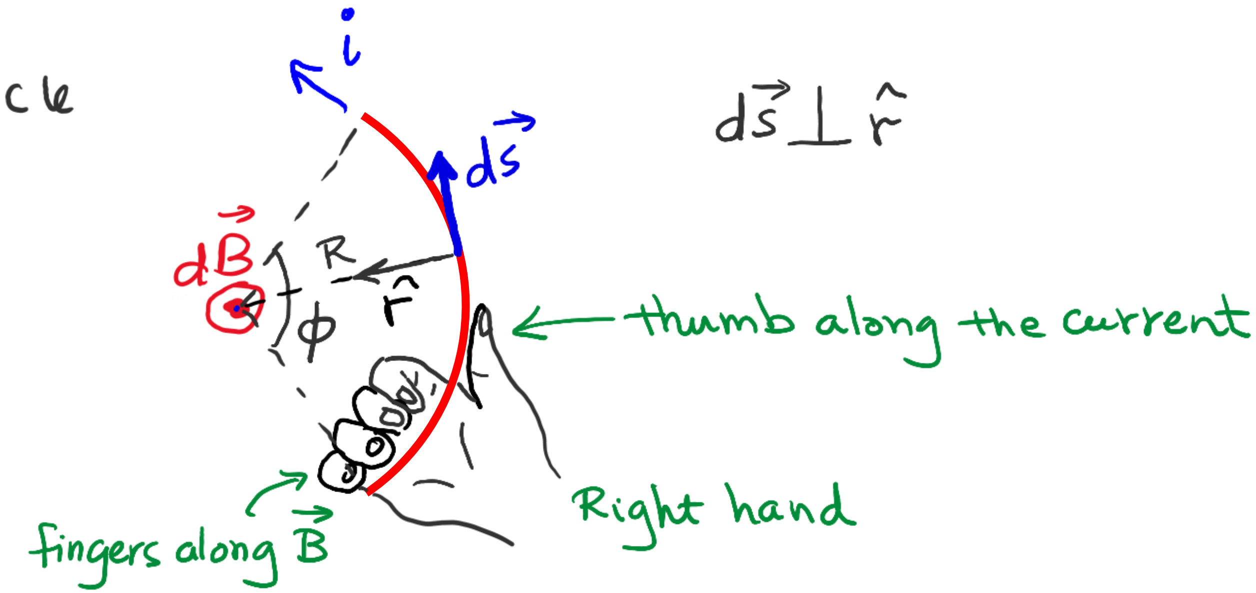


$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{i\, d\vec{s} \times \hat{r}}{r^2}$$

$$B = \frac{\mu_0 i}{2\pi R}$$

Magnetic Field Due to a Circular Arc

A circular arc = part of a circle



$d\vec{s} \perp \hat{r}$

$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{i\, d\vec{s} \times \hat{r}}{r^2}$$

$$\vec{B} = \frac{\mu_0}{4\pi} \int_{arc} \frac{i\, d\vec{s} \times \hat{r}}{r^2}$$

At the center of a circular arc

$|d\vec{s} \times \hat{r}| = ds\,|\hat{r}| \sin 90^\circ = ds$

$d\hat{s} \times \hat{r}$ has the same direction for all segments

$r = R$ constant

$$B = \frac{\mu_0 i}{4\pi} \int_{arc} \frac{ds}{R^2} = \frac{\mu_0 i}{4\pi R^2} \int_{arc} ds = \frac{\mu_0 i}{4\pi} \frac{S}{R^2}$$

length of the arc

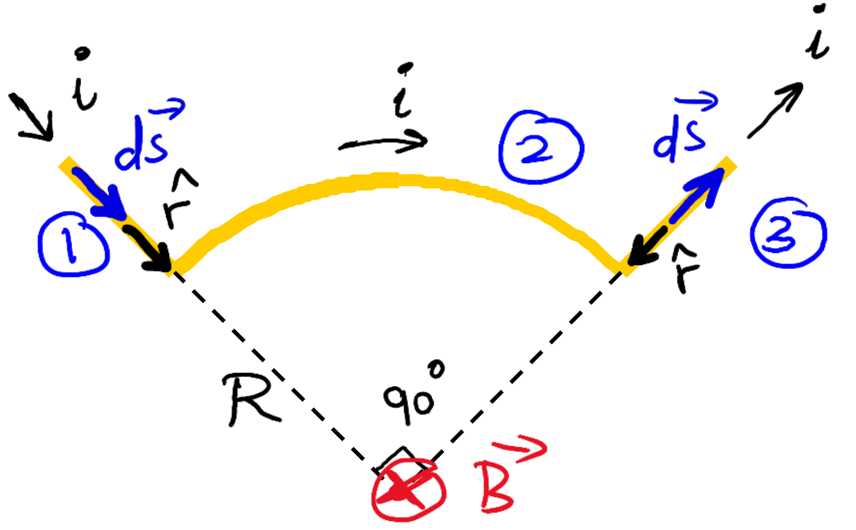
$S = R\phi$

$\phi =$ angle subtended by the arc

At the center $\boxed{B = \dfrac{\mu_0 i}{4\pi R} \phi}$ for a complete circle $\phi = 2\pi$

Example

What magnetic field does the current produce at the center?



$$\vec{B} = \vec{B}_1 + \vec{B}_2 + \vec{B}_3$$

$$\left.\begin{array}{ll} \text{for } 1 & |d\vec{s} \times \hat{r}| = ds \sin 0^\circ = 0 \\ \text{for } 3 & |d\vec{s} \times \hat{r}| = ds \sin 180^\circ = 0 \end{array}\right\}$$

$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{i\, d\vec{s} \times \hat{r}}{r^2}$$

$$B_1 = B_3 = 0$$

$$B = B_2 = \frac{\mu_0 i}{4\pi} \frac{\phi}{R} = \frac{\mu_0 i}{4\pi} \frac{\pi/2}{R} = \frac{\mu_0 i}{8R}$$ into the page.

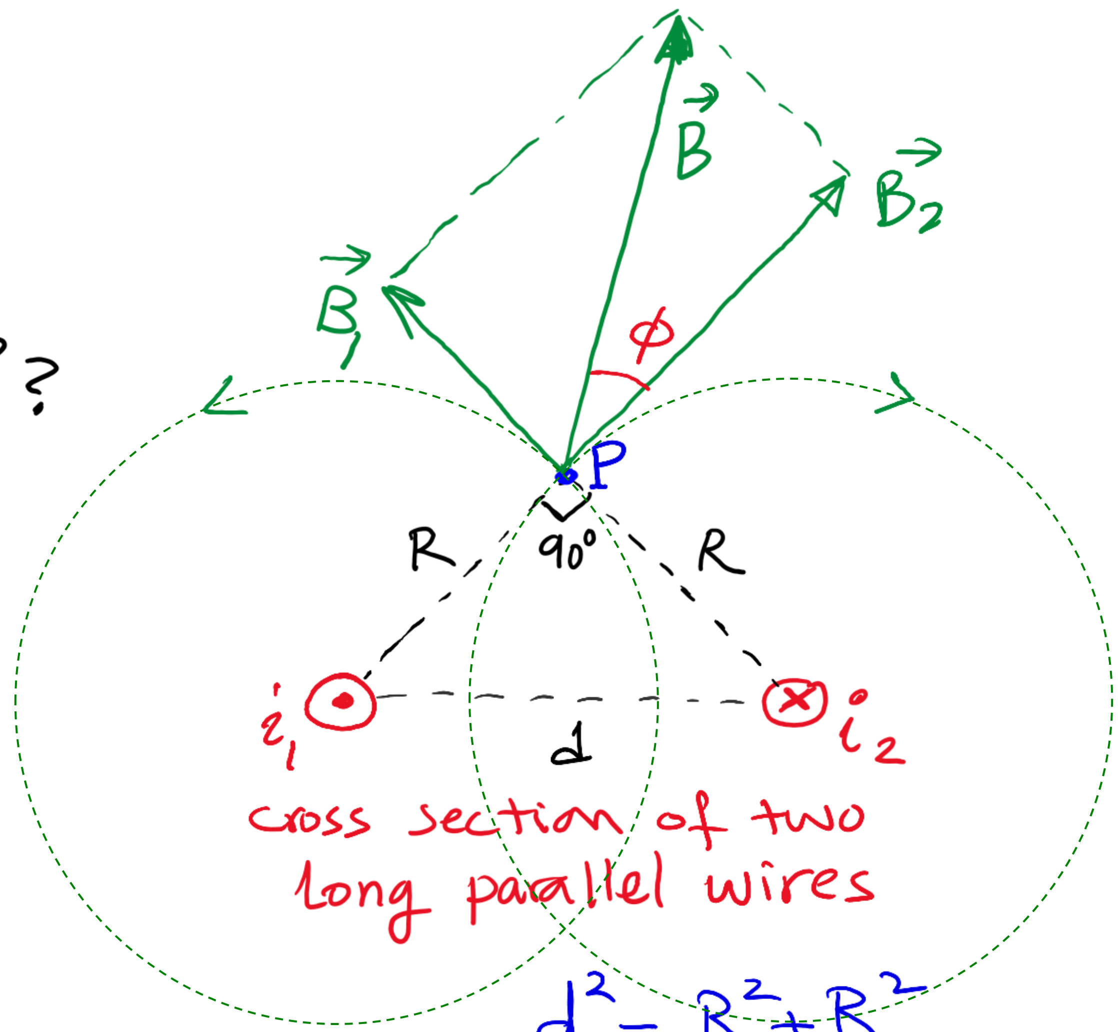
Example

$i_1 = 15 A$

$i_2 = 30 A$

$d = 5$ cm

What is the magnetic field at point P ?



$$B_1 = \frac{\mu_0 i_1}{2\pi R} = \frac{\mu_0 \sqrt{2}}{2\pi d} i_1$$

$$B_2 = \frac{\mu_0 i_2}{2\pi R} = \frac{\mu_0 \sqrt{2}}{2\pi d} i_2$$

$$B = \sqrt{B_1^2 + B_2^2} = \frac{\mu_0 \sqrt{2}}{2\pi d} \sqrt{i_1^2 + i_2^2}$$

$$\phi = \tan^{-1}\left(\frac{B_1}{B_2}\right) = \tan^{-1}\left(\frac{i_1}{i_2}\right)$$

$$d^2 = R^2 + R^2$$

$$\Rightarrow R = \frac{d}{\sqrt{2}}$$

29-2 Force between two parallel currents

$$B_a = \frac{\mu_0 i_a}{2\pi d} \qquad B_b = \frac{\mu_0 i_b}{2\pi d}$$

$$\vec{F}_{ab} = i_a \vec{L}_a \times \vec{B}_b$$

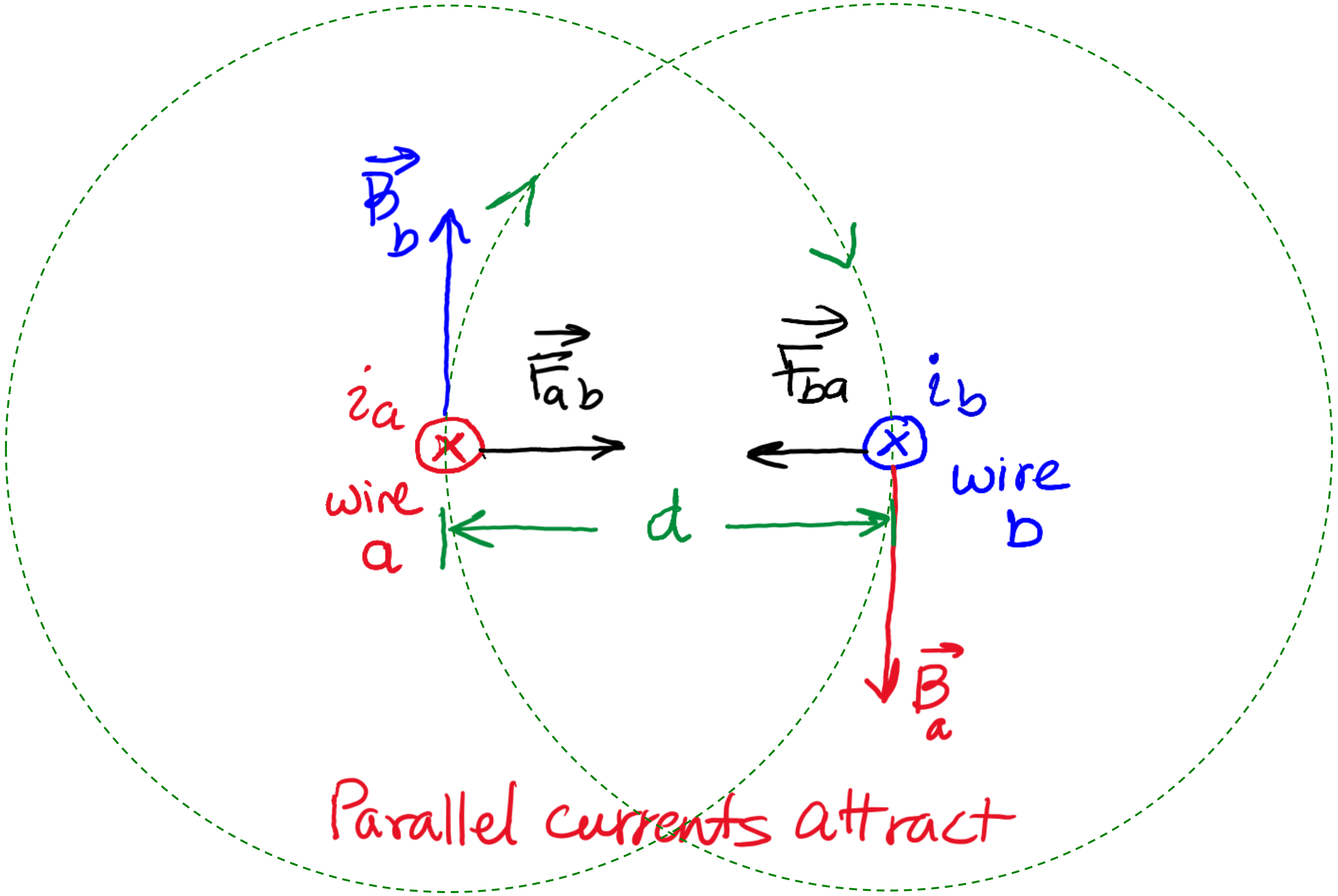
on (a) due (b)

$$F_{ab} = i_a L_a \frac{\mu_0 i_b}{2\pi d} = \frac{\mu_0 L_a i_a i_b}{2\pi d}$$

$$\vec{F}_{ba} = i_b \vec{L}_b \times \vec{B}_a$$

on (b) due (a)

$$F_{ba} = i_b L_b \frac{\mu_0 i_a}{2\pi d} = \frac{\mu_0 L_b i_a i_b}{2\pi d}$$



$$\frac{F_{ab}}{L_a} = \frac{F_{ba}}{L_b} = \frac{\mu_0 i_a i_b}{2\pi d}$$

29-2 Force between two parallel currents

$$B_a = \frac{\mu_0 i_a}{2\pi d} \qquad B_b = \frac{\mu_0 i_b}{2\pi d}$$

$$\vec{F}_{ab} = i_a \vec{L}_a \times \vec{B}_b$$

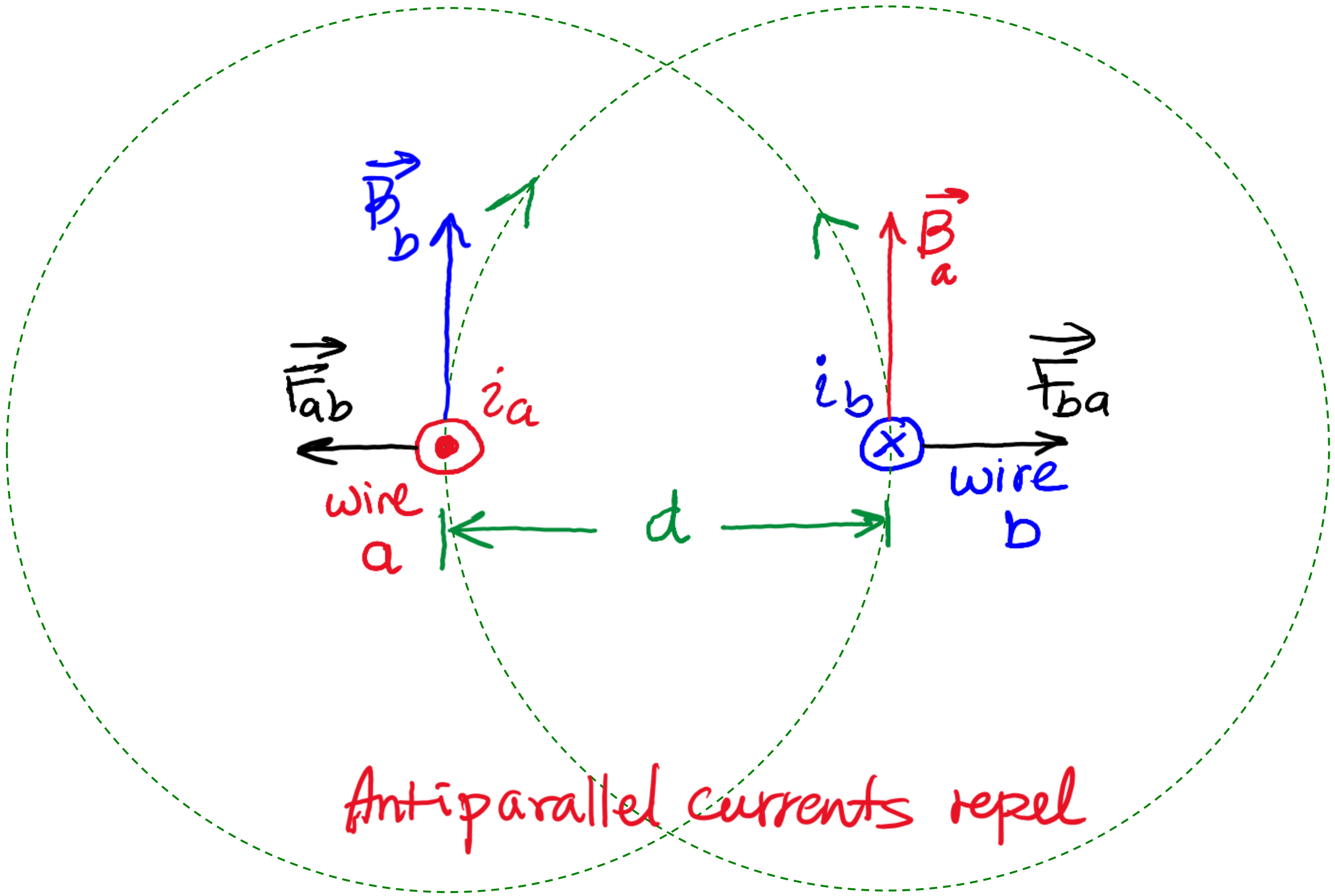
on due

$$F_{ab} = i_a L_a \frac{\mu_0 i_b}{2\pi d} = \frac{\mu_0 L_a i_a i_b}{2\pi d}$$

$$\vec{F}_{ba} = i_b \vec{L}_b \times \vec{B}_a$$

on due

$$F_{ba} = i_b L_b \frac{\mu_0 i_a}{2\pi d} = \frac{\mu_0 L_b i_a i_b}{2\pi d}$$



$$\frac{F_{ab}}{L_a} = \frac{F_{ba}}{L_b} = \frac{\mu_0 i_a i_b}{2\pi d}$$

Gauss' Law

Closed Surface

charge enclosed within the surface

$$\varepsilon_0 \oint \vec{E} \cdot d\vec{A} = q_{enc}$$

Surface

Area vector

$\varepsilon_0 = 8.85 \times 10^{-12} \ \frac{C}{Vm}$

Closed Surface = Gaussian Surface

Ampere's Law

current enclosed within the closed line (loop)

Closed line (loop)

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 \, i_{enc}$$

line

length vector

$\mu_0 = 4\pi \times 10^{-7} \ \frac{T.m}{A}$

closed loop = Amperian loop

Ampere's Law

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 \, i_{enc}$$

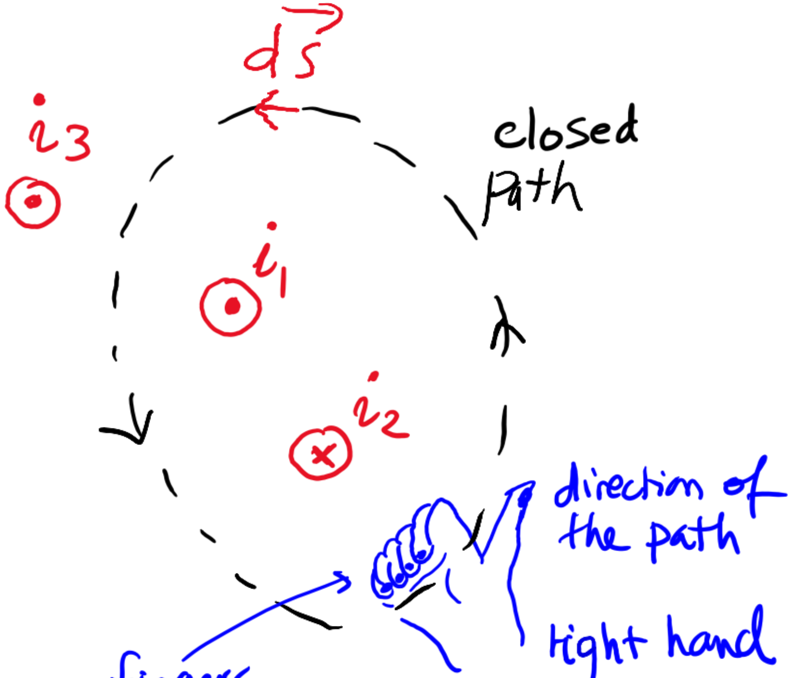
Line integral along a closed path

length vector

magnitude : length of a small segment

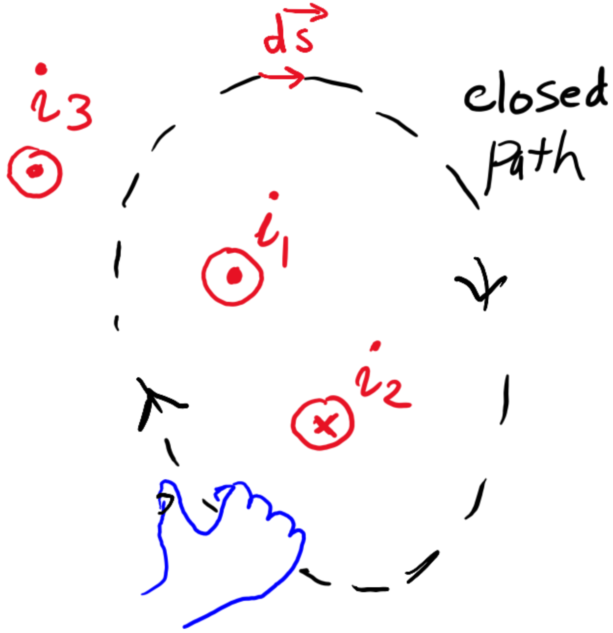
direction : tangent to the path

current enclosed by the closed path



direction of positive currents

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 (i_1 - i_2)$$



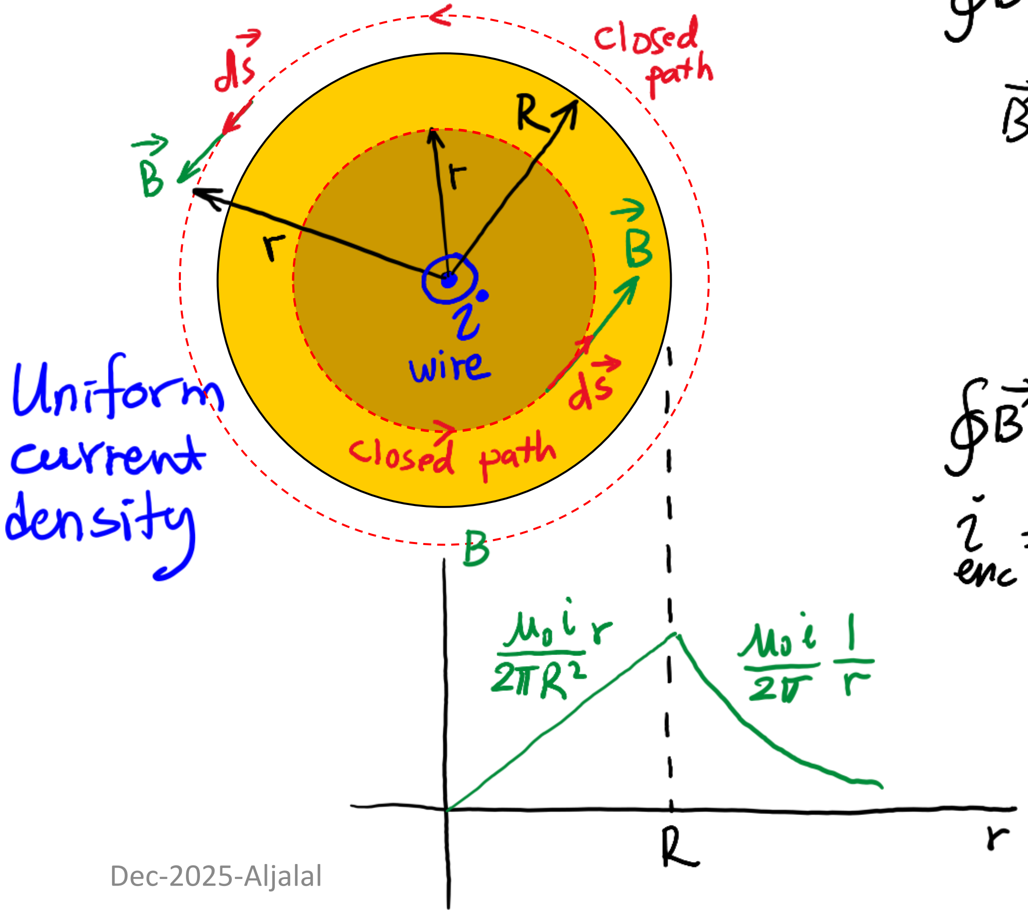
$$\oint \vec{B} \cdot d\vec{s} = \mu_0 (-i_1 + i_2)$$

Magnetic Field Inside a Long Straight Wire

From symmetry, the magnetic field has the same magnitude at all points on a circle of radius r.

Ampere's law

$$\oint \vec{B}\cdot d\vec{s} = \mu_0 i_{enc}$$



Outside $r > R$

$$\oint \vec{B}\cdot d\vec{s} = \oint B\,ds = B\oint ds = B(2\pi r)$$

$$\vec{B}\cdot d\vec{s} = B\,ds\cos 0°$$

$$i_{enc} = i$$

$$\left.\begin{matrix} \oint \vec{B}\cdot d\vec{s} = B(2\pi r) \\ i_{enc} = i \end{matrix}\right\} \begin{matrix} B(2\pi r) = \mu_0 i \\ \Rightarrow B = \frac{\mu_0 i}{2\pi} \frac{1}{r} \end{matrix}$$

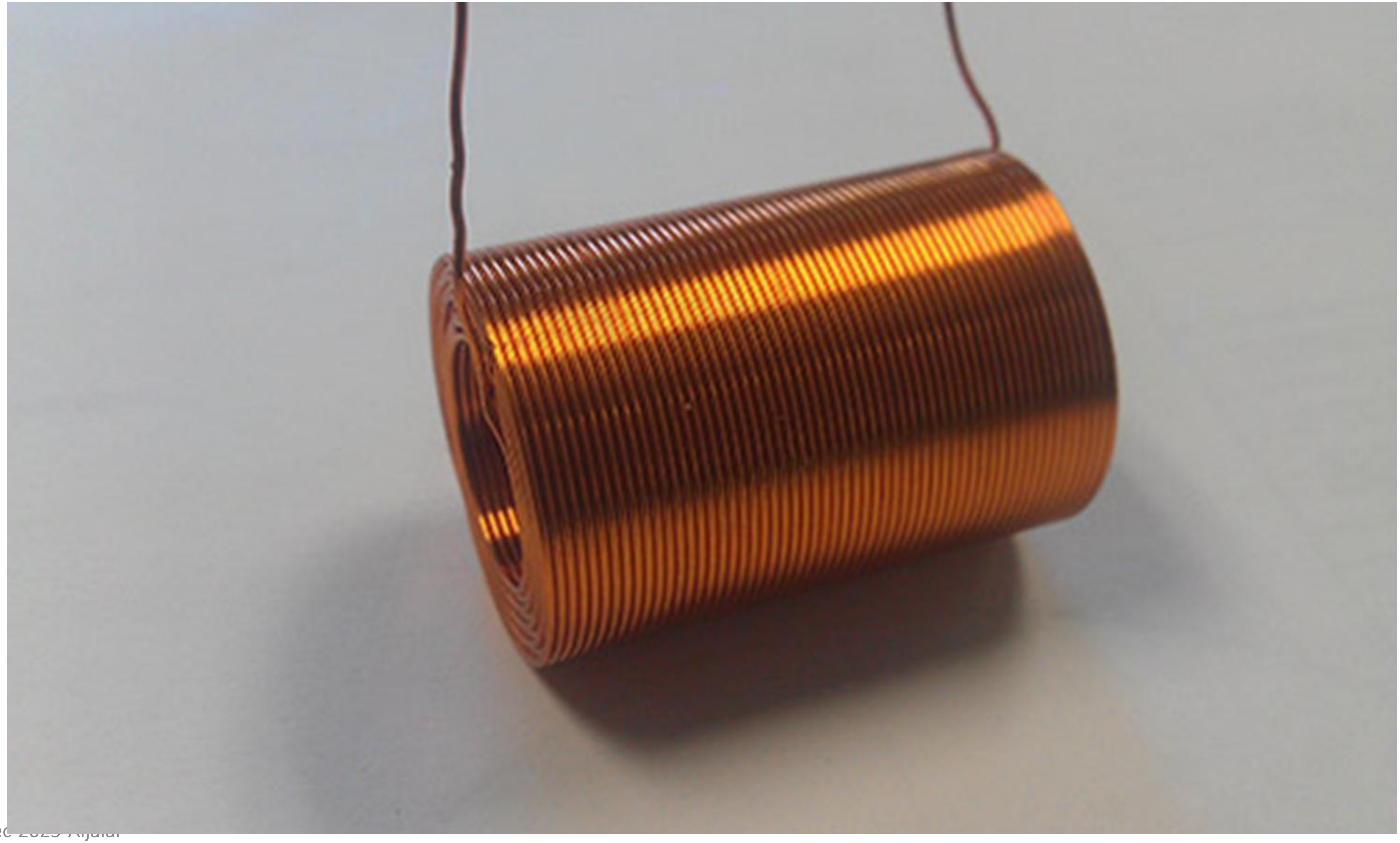
Inside $r < R$

$$\oint \vec{B}\cdot d\vec{s} = \oint B\,ds = B\oint ds = B(2\pi r)$$

$$i_{enc} = i \frac{\text{enclosed area}}{\text{total area}} = i \frac{\pi r^2}{\pi R^2} = i \frac{r^2}{R^2}$$

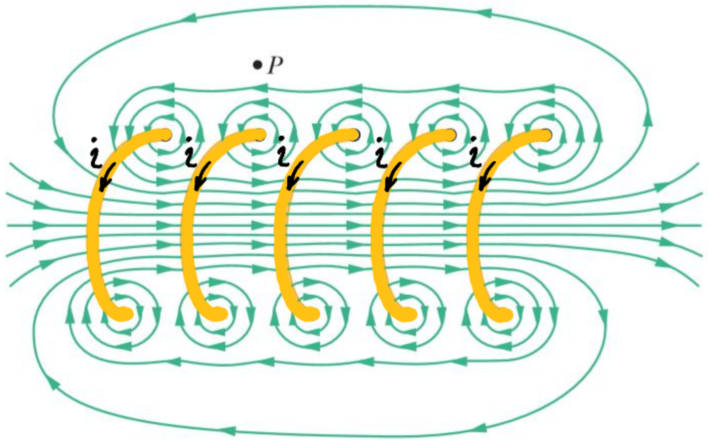
$$\left.\begin{matrix} \oint \vec{B}\cdot d\vec{s} = B(2\pi r) \\ i_{enc} = i\frac{r^2}{R^2} \end{matrix}\right\} \begin{matrix} B(2\pi r) = \mu_0 i \frac{r^2}{R^2} \\ \Rightarrow B = \frac{\mu_0 i}{2\pi R^2} r \end{matrix}$$

A soleniod



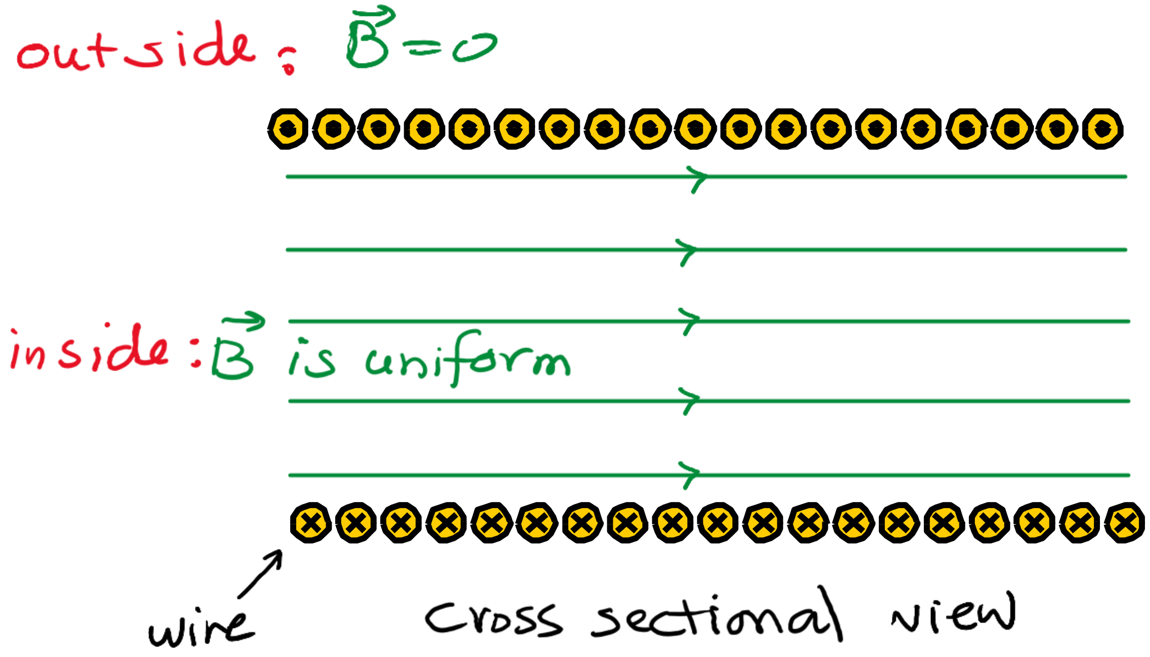
Solenoid

short



Infinitely long

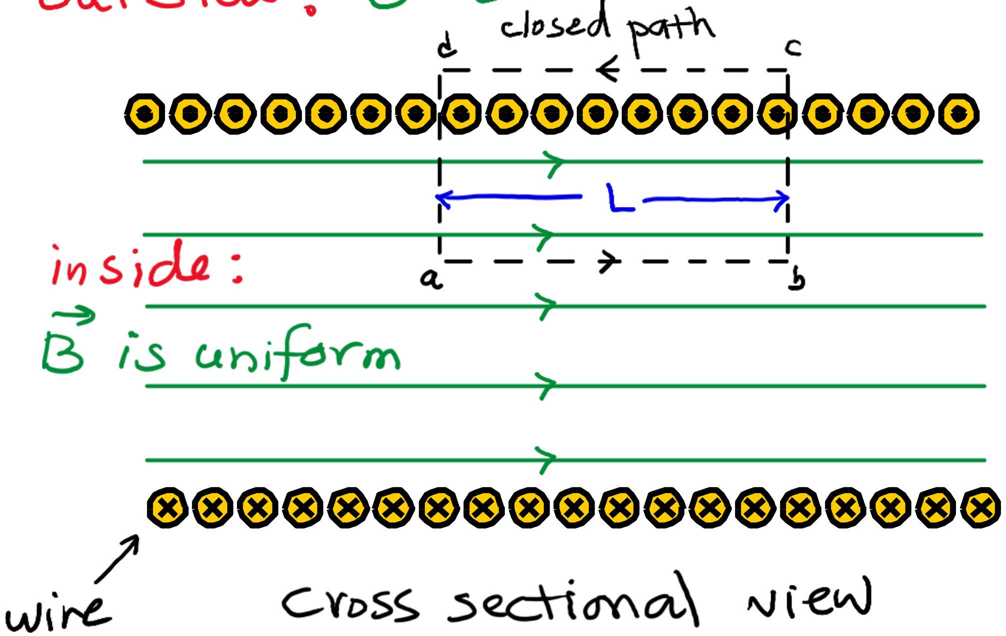
Turns are tightly packed



Infinitely long solenoid

Solenoid

outside: $\vec{B} = 0$



inside:
$\vec{B}$ is uniform

cross sectional view

$$\oint \vec{B}\cdot d\vec{s} = \mu_0 i_{enc}$$

$$\int_a^b \vec{B}\cdot d\vec{s} + \int_b^c \vec{B}\cdot d\vec{s} + \int_c^d \vec{B}\cdot d\vec{s} + \int_d^a \vec{B}\cdot d\vec{s} = \mu_0 i_{enc}$$

$$BL + 0 + 0 + 0 = \mu_0 i_{en}$$

number of turns within L

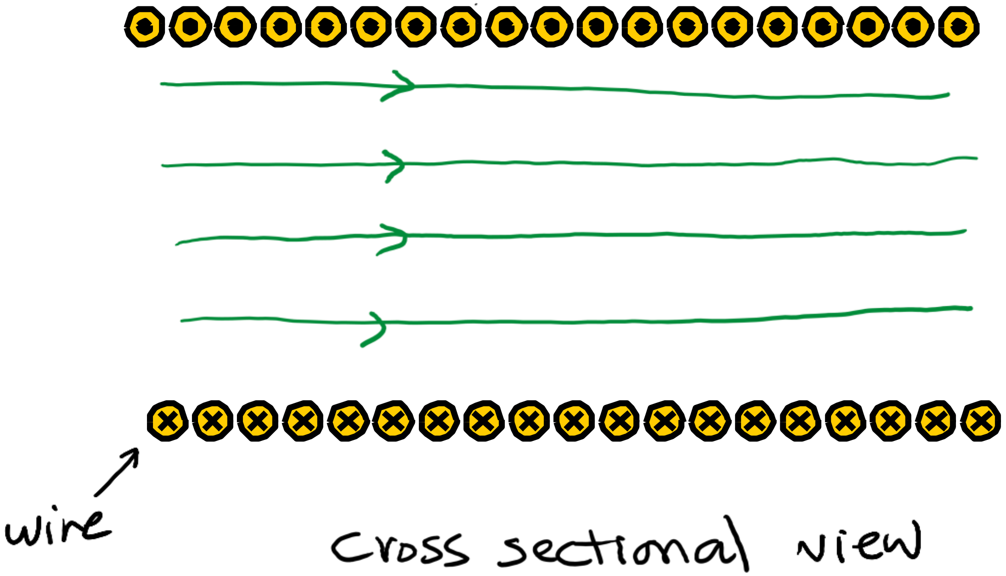
$$BL = \mu_0 N i$$

number of turns per unit Length

$$B = \mu_0 \frac{N}{L} i = \mu_0 n i$$

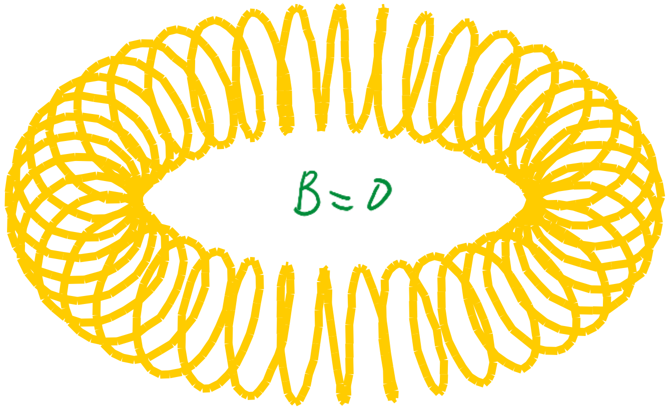
$$\boxed{B = \mu_0 n i}$$

Soleniods



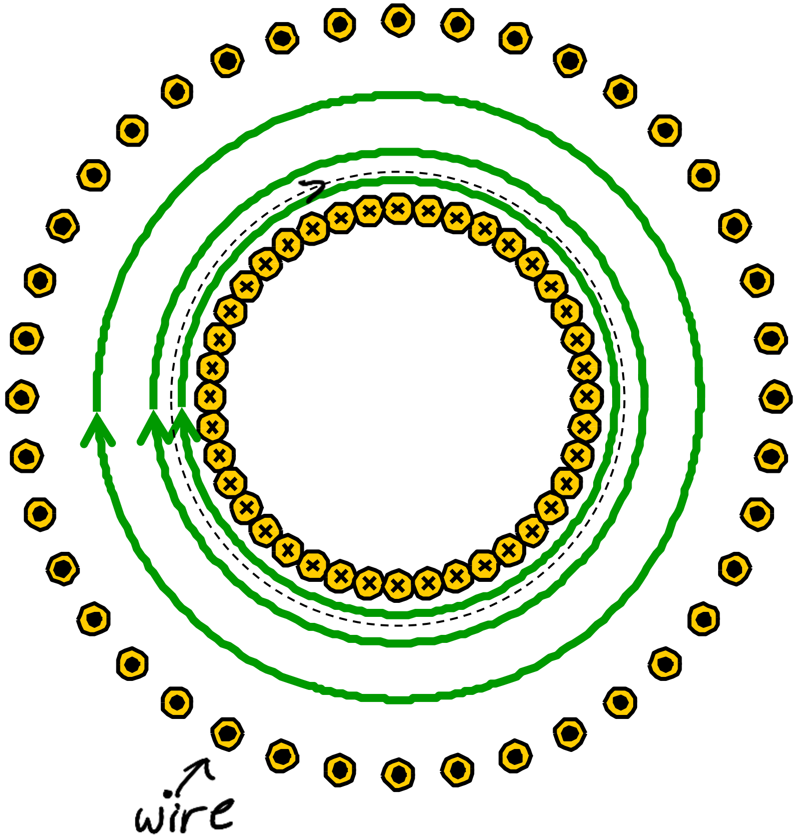
A toroid

outside:
$B = 0$



Toriod

Cross sectional view

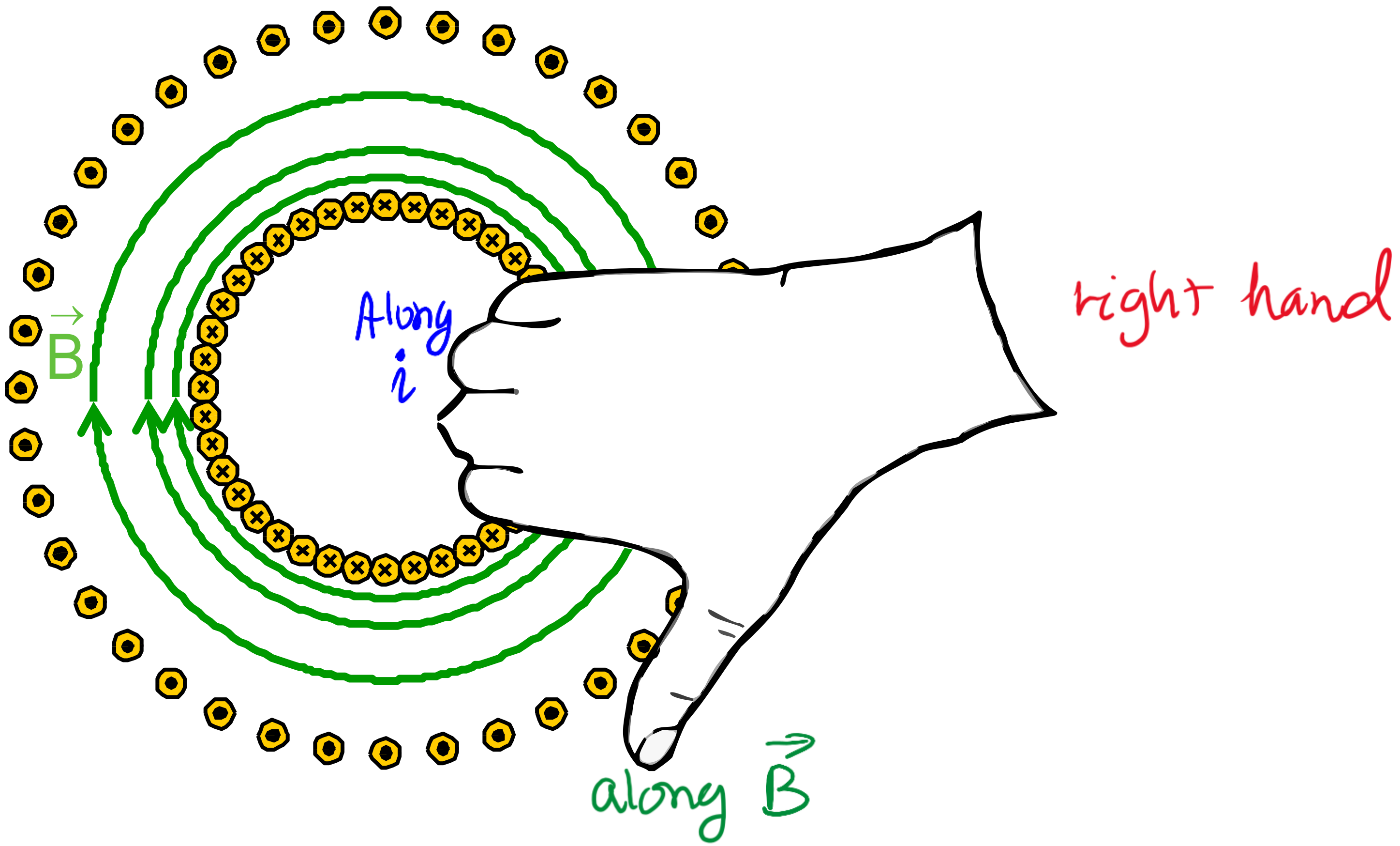


$$\oint \vec{B} \cdot d\vec{s} = \mu_0 i_{enc}$$

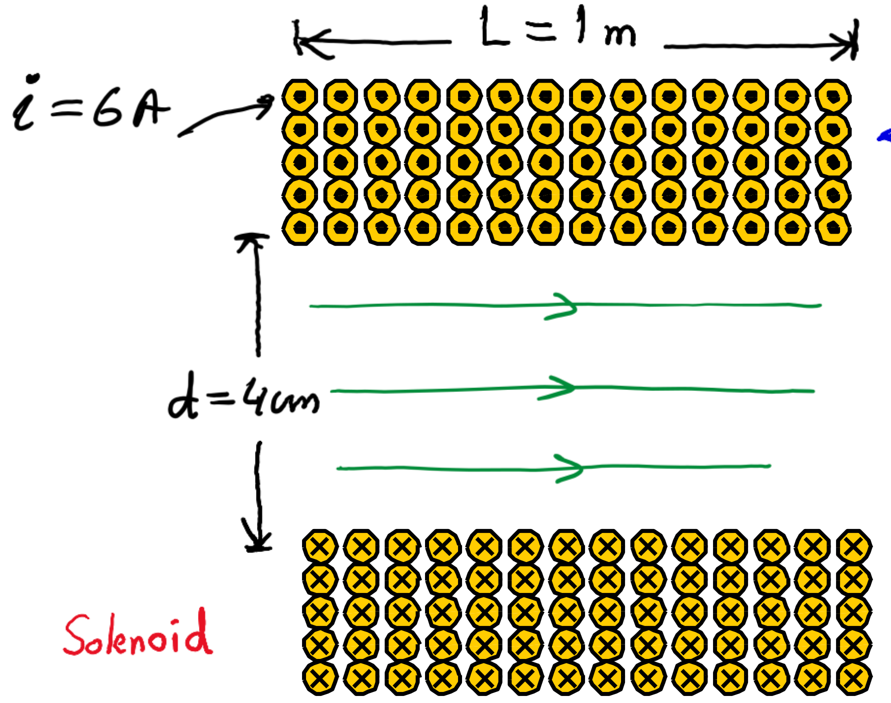
total number of turns

$$B(2\pi r) = \mu_0 N i$$

$$\boxed{B = \frac{\mu_0 N i}{2\pi} \frac{1}{r}}$$


$\vec{B}$
Along i
right hand
along $\vec{B}$

Example



5 layers

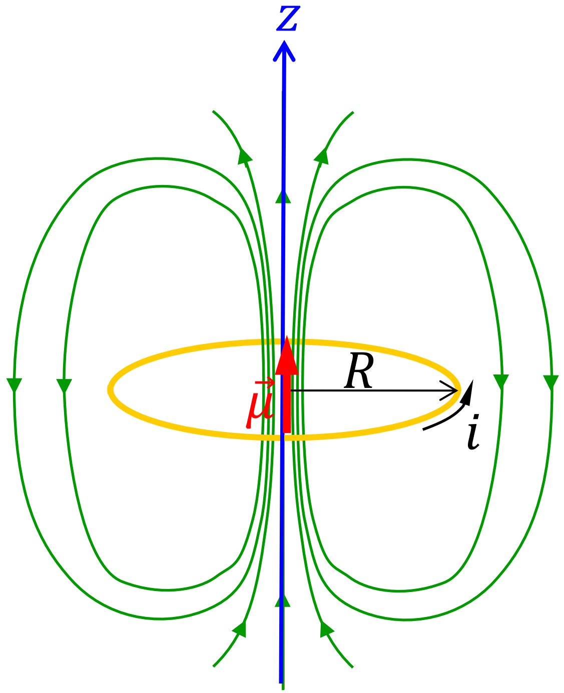
each layer has 600 turns

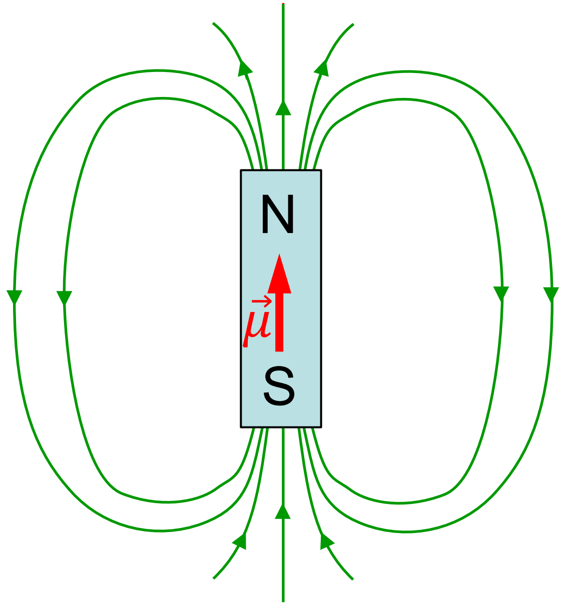
What is the magnetic field at the center?

$$B = \mu_0 n i = 4\pi \times 10^{-7} \left(\frac{600 \times 5}{1} \right) 6 \ \text{T}$$

29-5 A Current-Carrying as a Magnetic Dipole

A current loop produces a magnetic field like that of a bar magnet.





<u>Along the z-axis</u>

$$B(z) = \frac{\mu_0 i R^2}{2(R^2+z^2)^{3/2}}$$

<u>For $z \gg R$</u>

Magnetic dipole moment for a coil

$$B(z) \approx \frac{\mu_0 i R^2}{2z^3} = \frac{\mu_0 i \pi R^2}{2\pi z^3} = \frac{\mu_0 \underbrace{iA}}{2\pi z^3}$$

$$\vec{B}(z) \approx \frac{\mu_0 \vec{\mu}}{2\pi z^3}$$