This is not a replacement for your textbook. It is essential to study from your textbook!

Objectives

Ch 28: Magnetic fields

28-1 Magnetic fields and the definition of $\vec{B}$

28-2 Crossed fields: Discovery of the electron

28-4 A circulating charged particle

28-6 Magnetic force on a current-carrying wire

28-7 Torque on a current loop

28-8 The magnetic dipole moment

28-1 Magnetic Fields and the Definition of $\vec{B}$.

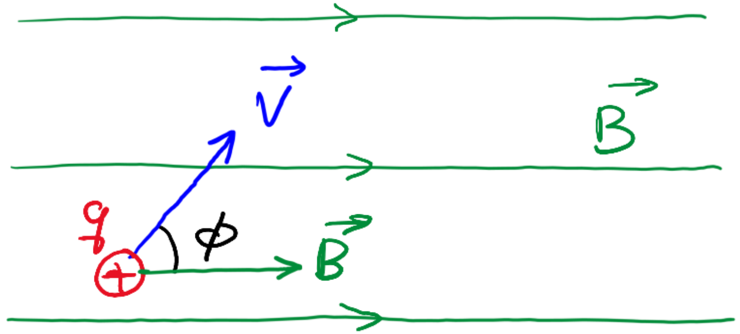
Force on a charged particle due to a magnetic field

$$\vec{F}_B = q\,\vec{v} \times \vec{B}$$

magnetic field

charge of the particle

velocity of the particle

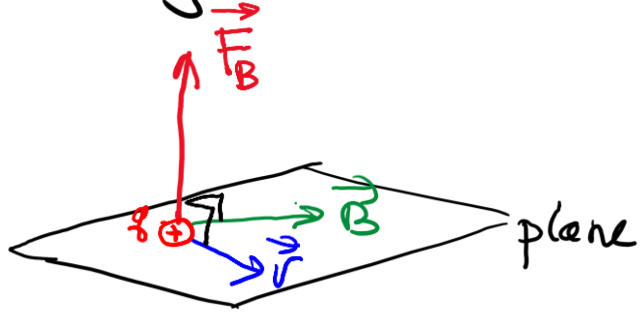


Magnitude: $F_B = |q|\, v B \sin\phi$ the smaller angle between $\vec{v}$ and $\vec{B}$

Direction: normal to the plane containing $\vec{v}$ and $\vec{B}$

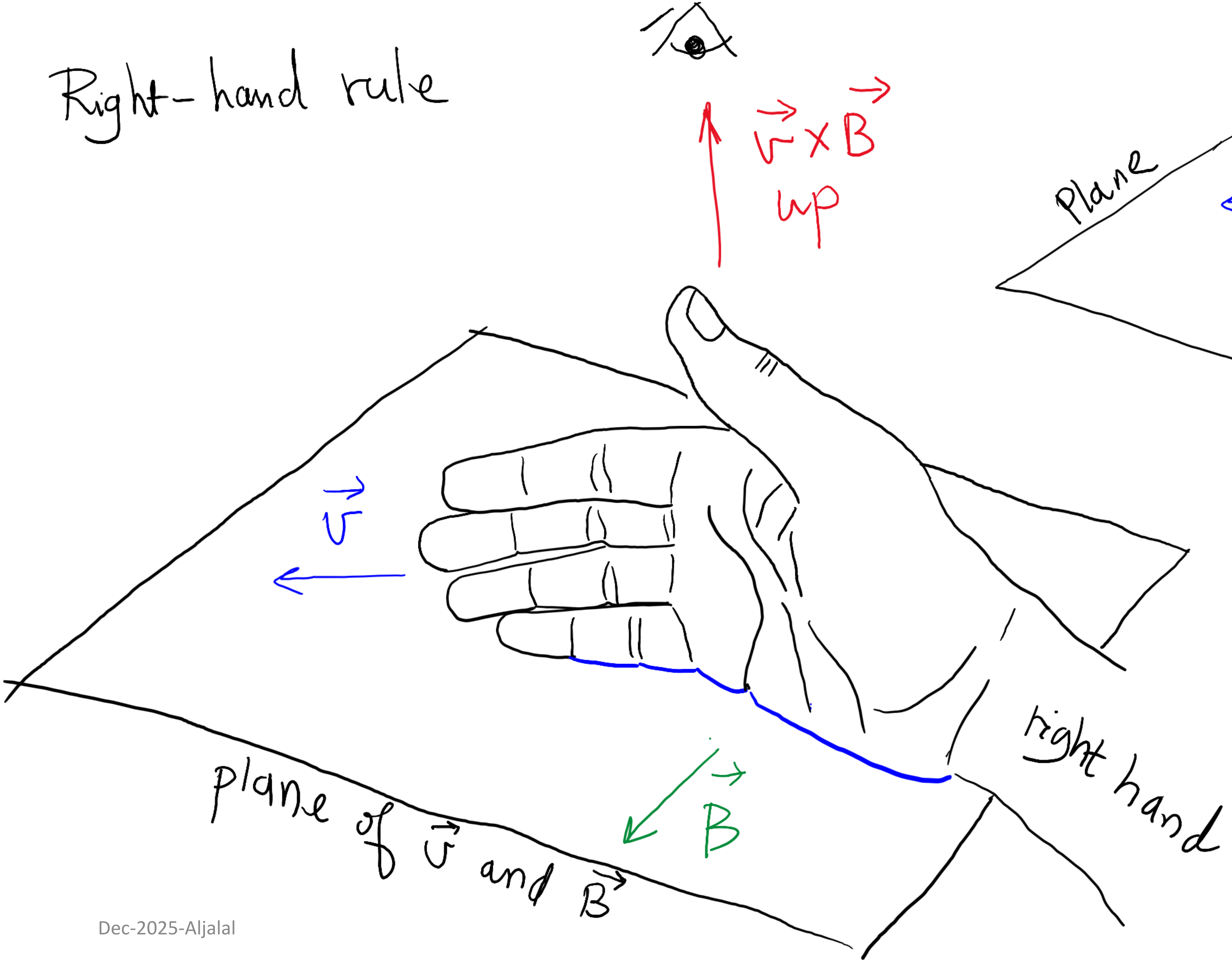
$\vec{F}_B \perp \vec{v}$

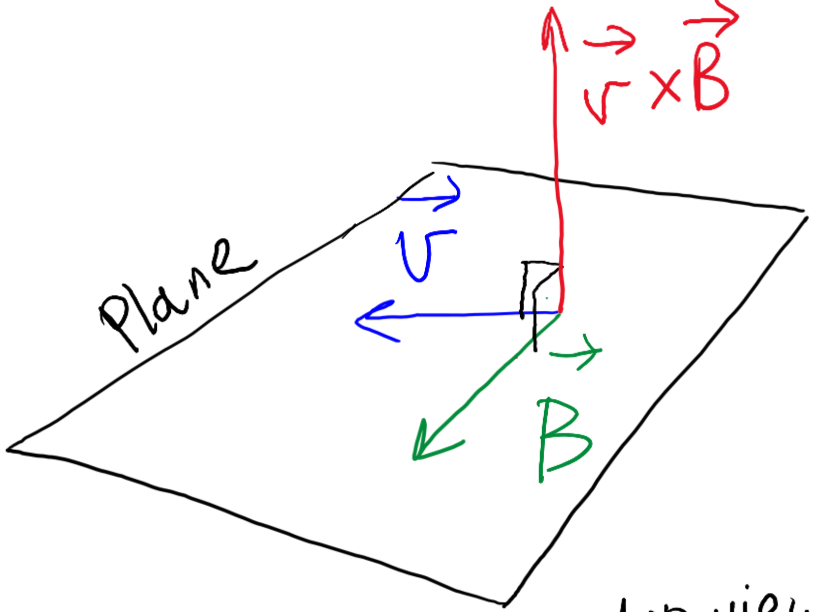
$\vec{F}_B \perp \vec{B}$

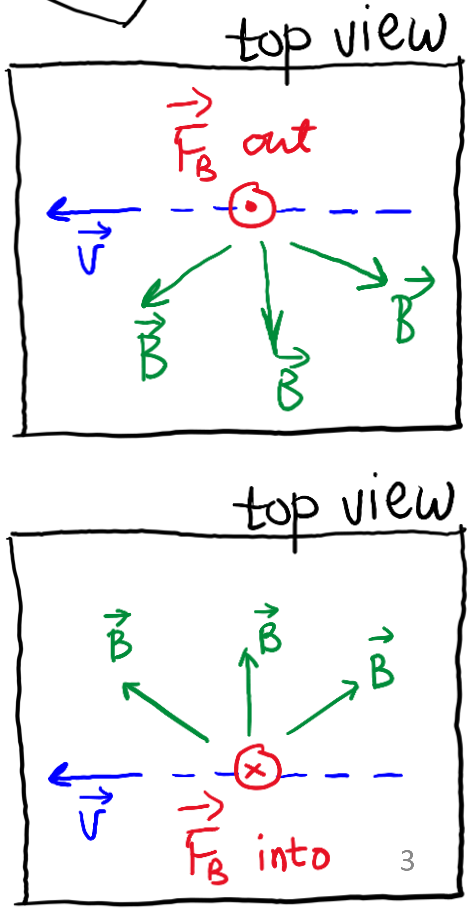


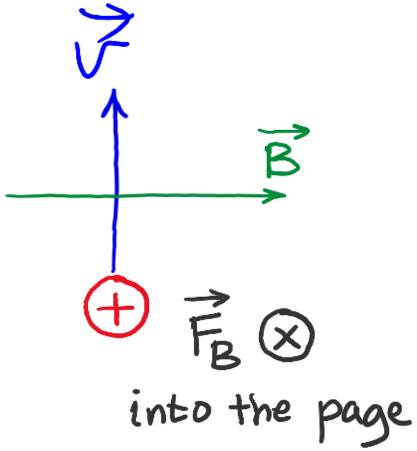
Use the right-hand rule to find if $\vec{F}_B$ points into or out of the plane.

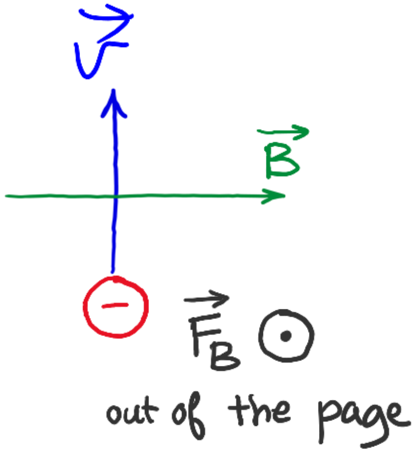
Right-hand rule

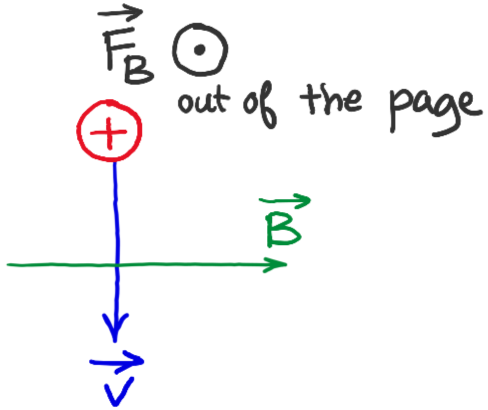


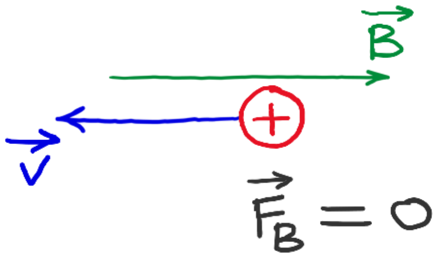




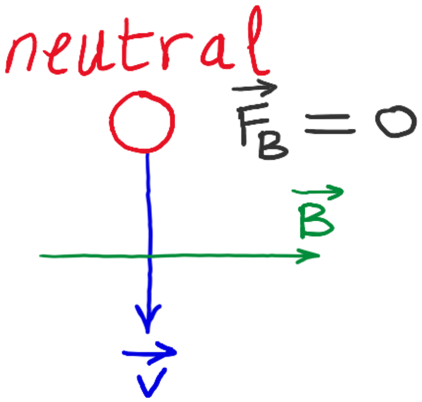


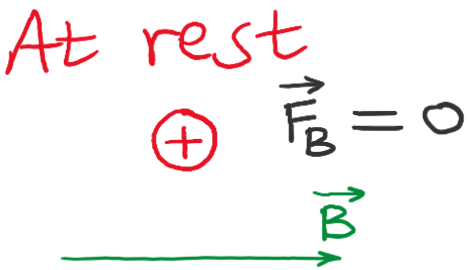






$$\vec{F}_B = q\,\vec{v} \times \vec{B}$$





$\vec{F}_B$ is always perpendicular to $\vec{v}$ $\Longrightarrow$

$\vec{F}_B$ does not have a component along $\vec{v}$.

$\vec{F}_B$ cannot do work on the particle $\quad dW = \vec{F} \cdot d\vec{s} = \vec{F} \cdot \vec{v}\, dt = 0$

$\vec{F}_B$ cannot change the kinetic energy of a particle $\quad \Delta K = W = 0$

$\vec{F}_B$ cannot change the speed of a particle.

$\vec{F}$ can accelerate a particle by changing its direction.

Tesla (T) is the SI unit for magetic field

$$\vec{F} = q\vec{v} \times \vec{B}$$

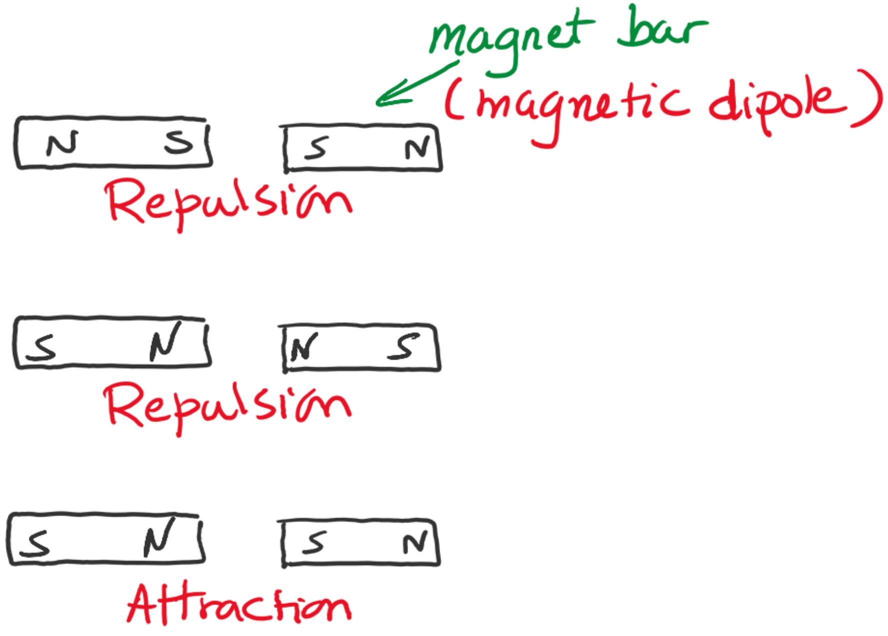
$$1 \text{ Tesla} = \frac{\text{Newton}}{\text{Coulomb (meter/second)}}$$

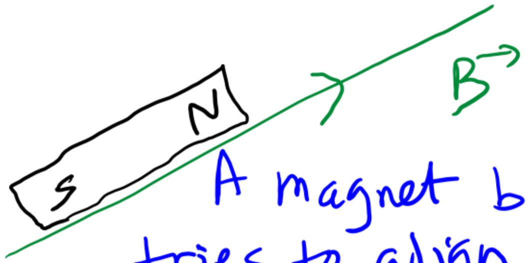
Gauss (G) is another unit for magnetic field

$$1 \text{ Tesla} = 10^4 \text{ Gauss}$$

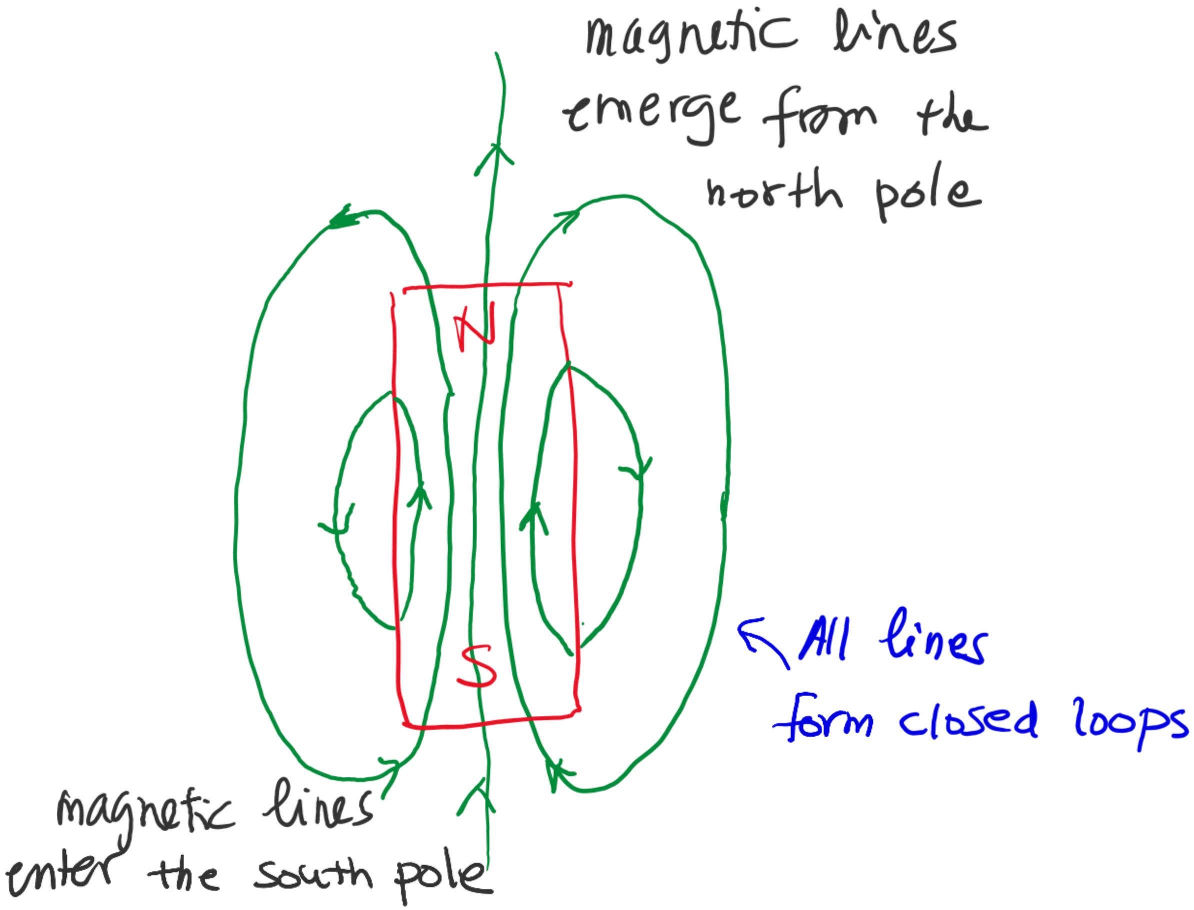
Small magnet bar $B \approx 100\,G = 10^{-2}\,T$

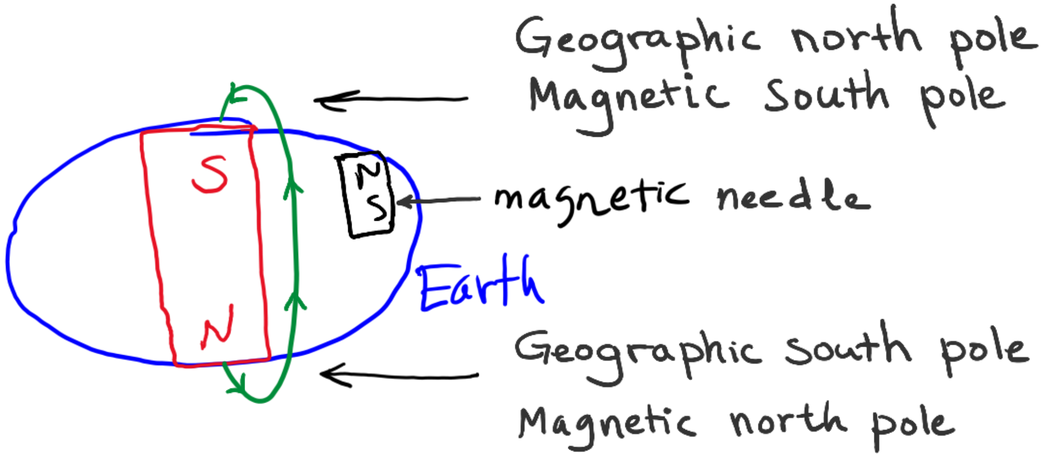
Near Earth's surface $B \approx 0.5\,G = 0.5 \times 10^{-4}\,T$





A magnet bar tries to align itself along external magnetic fields





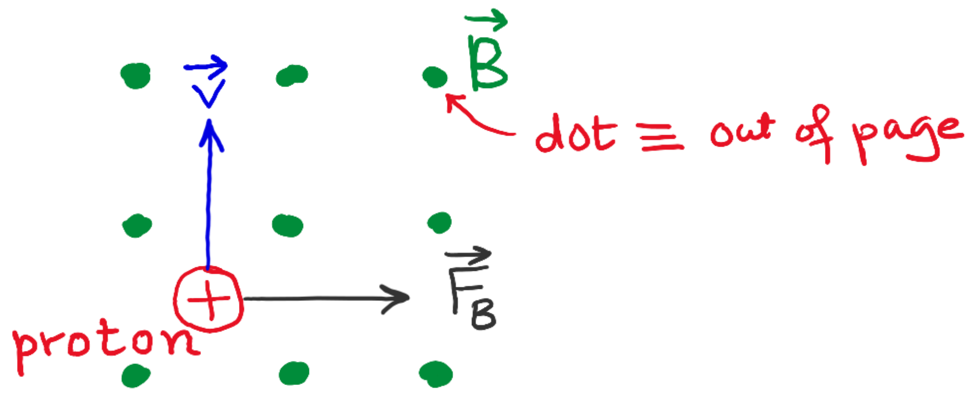
Example

$B = 1.2\times10^{-3}\,T$

Proton: kinetic energy $= 1.0$ keV

$q = 1.6\times10^{-19}\,C$

$m = 1.67\times10^{-27}\,kg$



What is the magnetic force acting on the proton?

$$\vec{F}_B = q\,\vec{v}\times\vec{B}$$

$$F_B = |q|\,v\,B\sin 90^\circ$$

$$K = \tfrac{1}{2}mv^2 \Rightarrow v = \sqrt{\frac{2K}{m}}$$

$$F_B = |q|\sqrt{\frac{2K}{m}}\;B$$

$$K = 1.0\times10^{3}\,\cancel{eV}\left(\frac{1.6\times10^{-19}\,J}{1\,\cancel{eV}}\right) = 1.6\times10^{-16}\,J$$

$$F_B = 8.4\times10^{-17}\,N$$

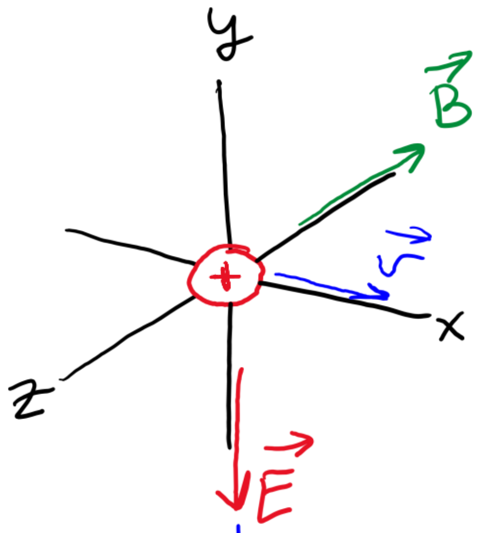
What is the acceleration of the proton?

Newton's second Law

$$F_{net} = ma \Rightarrow F_B = ma \Rightarrow a = \frac{F_B}{m} = 5\times10^{10}\,m/s^2$$

28-2 Crossed Fields: Discovery of the Electron

$$\vec{F}_B = q\vec{v} \times \vec{B}$$



$$\vec{F}_E = q\vec{E}$$

$$\vec{E} \perp \vec{B}$$

$$F_E = |q|E$$

$$F_B = |q|\,vB$$

Suppose $\vec{F}_{net} = 0 = \vec{F}_E + \vec{F}_B$

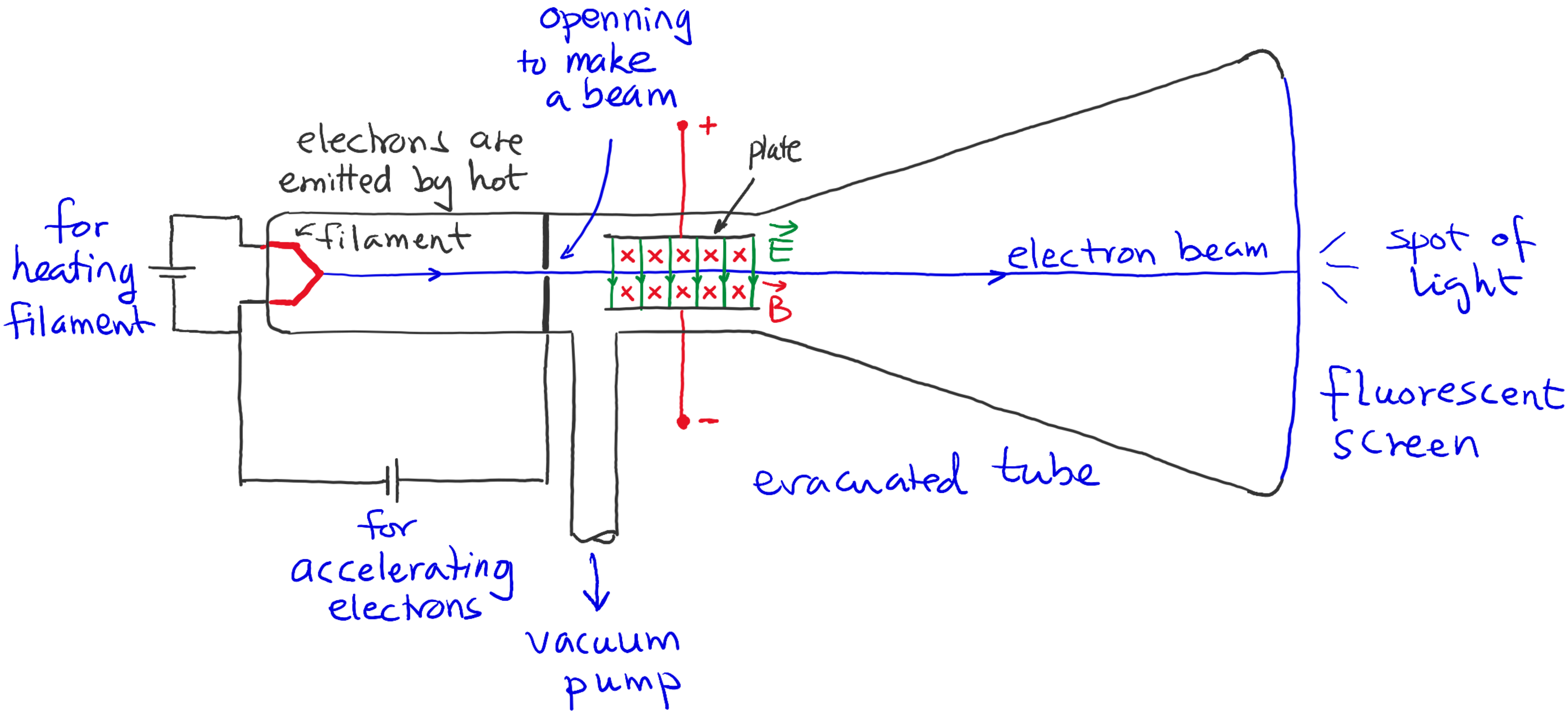
$$\Rightarrow \quad F_E = F_B$$

$$\cancel{|q|}E = \cancel{|q|}\,vB$$

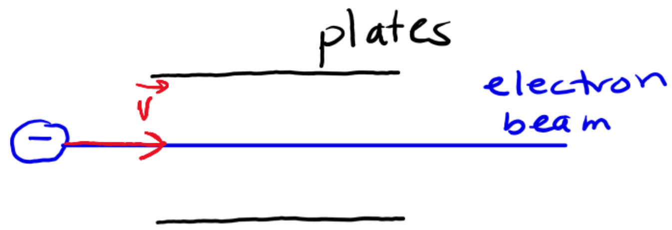
$$\boxed{v = \frac{E}{B}}$$

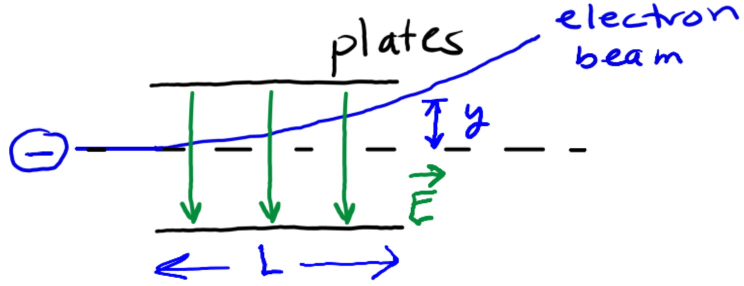
Thomson measured $\frac{m}{q}$ of an electron in 1897

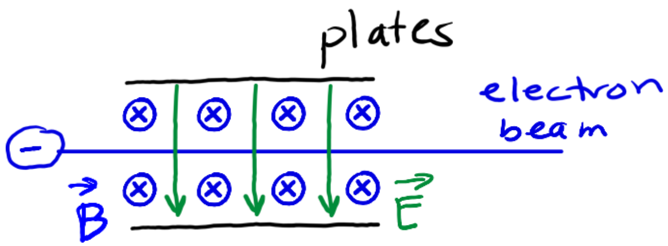
Cathode ray tube



Thomson measured $\frac{m}{q}$ of an electron in 1897







$$F_E = F_B$$

$$\cancel{|q|}E = \cancel{|q|}vB$$

$$v = \frac{E}{B}$$

$$y = \frac{1}{2}at^2 = \frac{1}{2}\left(\frac{|q|E}{m}\right)\left(\frac{L}{v}\right)^2$$

$$F_E = |q|E = ma$$

$$\Rightarrow a = \frac{|q|E}{m}$$

$$t = \frac{L}{v}$$

$$v = \frac{E}{B}$$

$$y = \frac{1}{2}\left(\frac{|q|E}{m}\right)\left(\frac{L}{\frac{E}{B}}\right)^2$$

$$\Rightarrow \frac{m}{|q|} = \frac{1}{2}\frac{B^2L}{Ey}$$

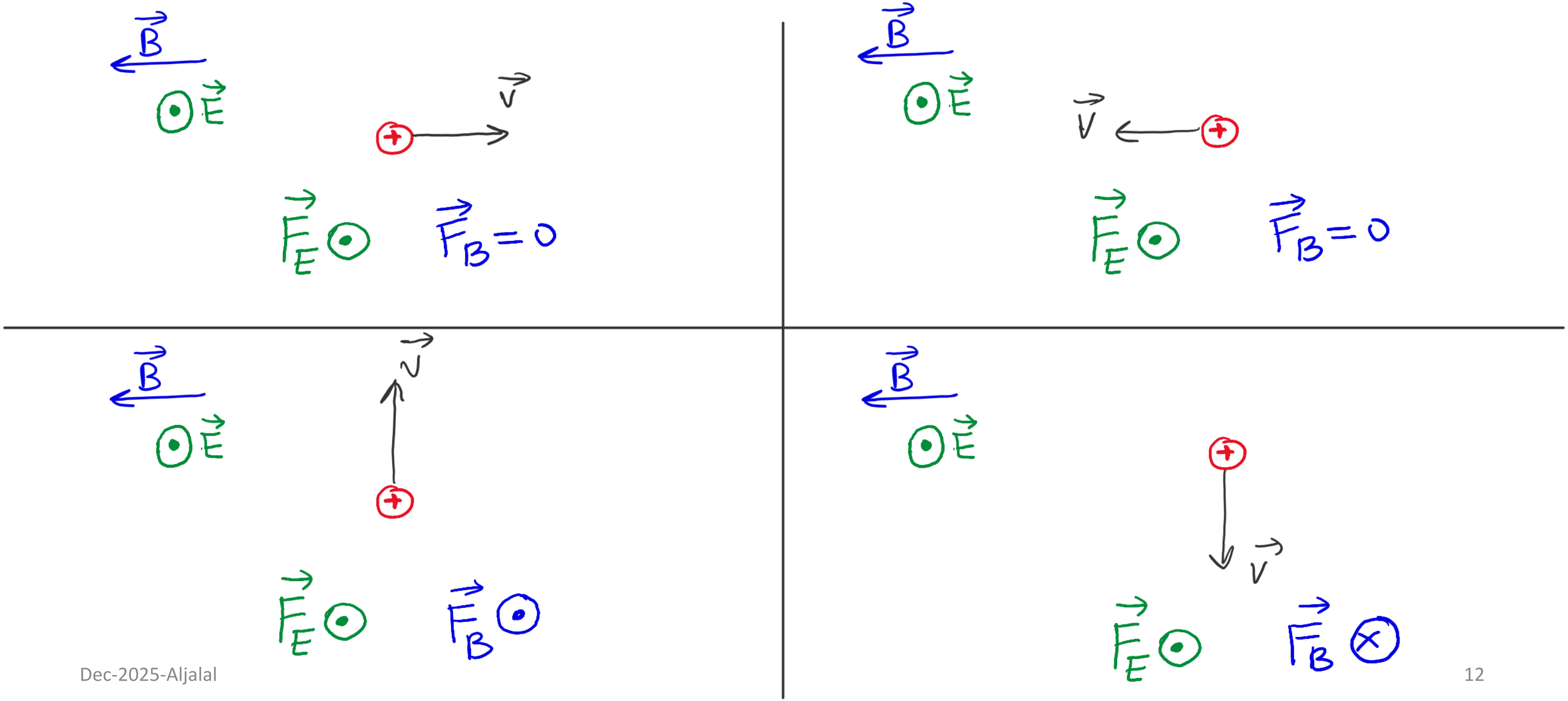
each quantity can be measured

Question

What are the directions of $\vec{F}_E$ and $\vec{F}_B$

$$\vec{F}_E = q\vec{E}$$

$$\vec{F}_B = q\,\vec{v} \times \vec{B}$$



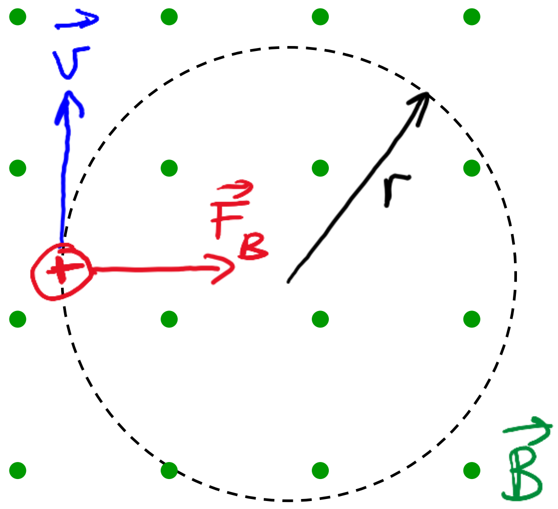
28-4 A Circulating charged Particle

$$\vec{F}_B = m\vec{a}$$

$$|q| v B \sin 90^\circ = m a$$

$$|q| \cancel{v} B = m \frac{v^{\cancel{2}}}{r}$$

Radius $\boxed{r = \frac{mv}{|q|B}}$



$$\vec{F}_B = q\vec{v} \times \vec{B}$$

$\vec{F}_B \perp \vec{v}$

$\vec{F}_B$ does not change the magnitude of $\vec{v}$

$\Rightarrow$ uniform circular motion

$$a = \frac{v^2}{r}$$

Radius $\quad r = \dfrac{mv}{|q|B}$

Period (time to make one revolution)

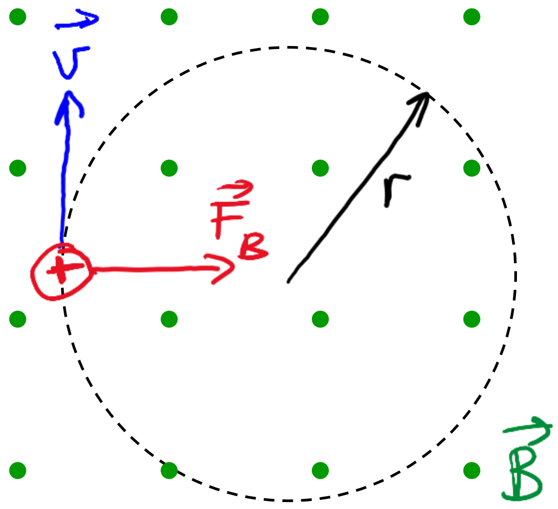
$$T = \frac{2\pi r}{v} = \frac{2\pi m}{|q|B}$$

Frequency (number of revolutions per second)

$$f = \frac{1}{T} = \frac{|q|B}{2\pi m}$$

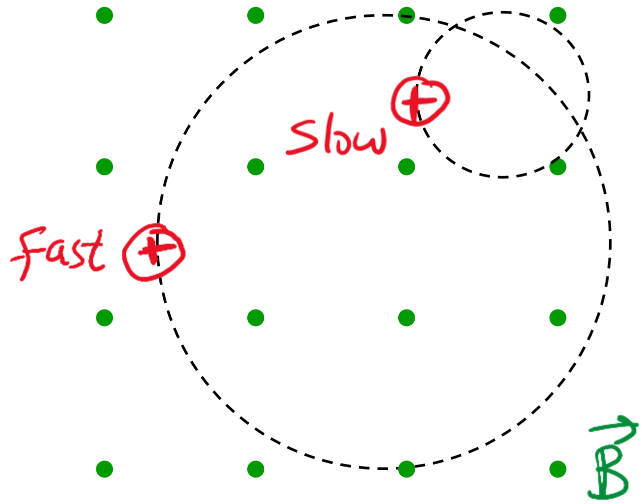
Angular frequency

$$\omega = 2\pi f = \frac{|q|B}{m}$$



Period (time to make one revolution)

$$T = \frac{2\pi r}{v} = \frac{2\pi m}{|q| B}$$



Periods do not depend on the speed of the particles

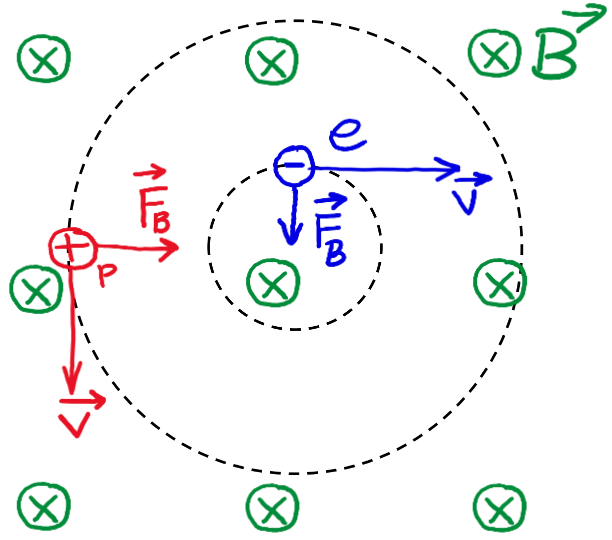
$$\frac{m_1}{|q_1|} = \frac{m_2}{|q_2|} \Longrightarrow T_1 = T_2$$

Fast particles move in big circles

slow particles move in small circles

Question

$$\vec{F}_B = q\,\vec{v} \times \vec{B}$$



An electron and a proton travel at the same speed in a uniform magnetic field

Which particle does follow the smaller circle?

$$r = \frac{mv}{|q|B}$$

$$r_e = \frac{m_e v}{eB} \qquad r_p = \frac{m_p v}{eB}$$

$$\frac{r_e}{r_p} = \frac{m_e}{m_p} \ll 1 \Rightarrow r_e \ll r_p$$

$m_e = 9.1 \times 10^{-31}$ kg

$m_p = 1.7 \times 10^{-27}$ kg

$m_p = 1800\, m_e$

$|q_p| = |q_e| = e$

What is the travel direction of the particles?

electron : clockwise

proton : counterclockwise

Example

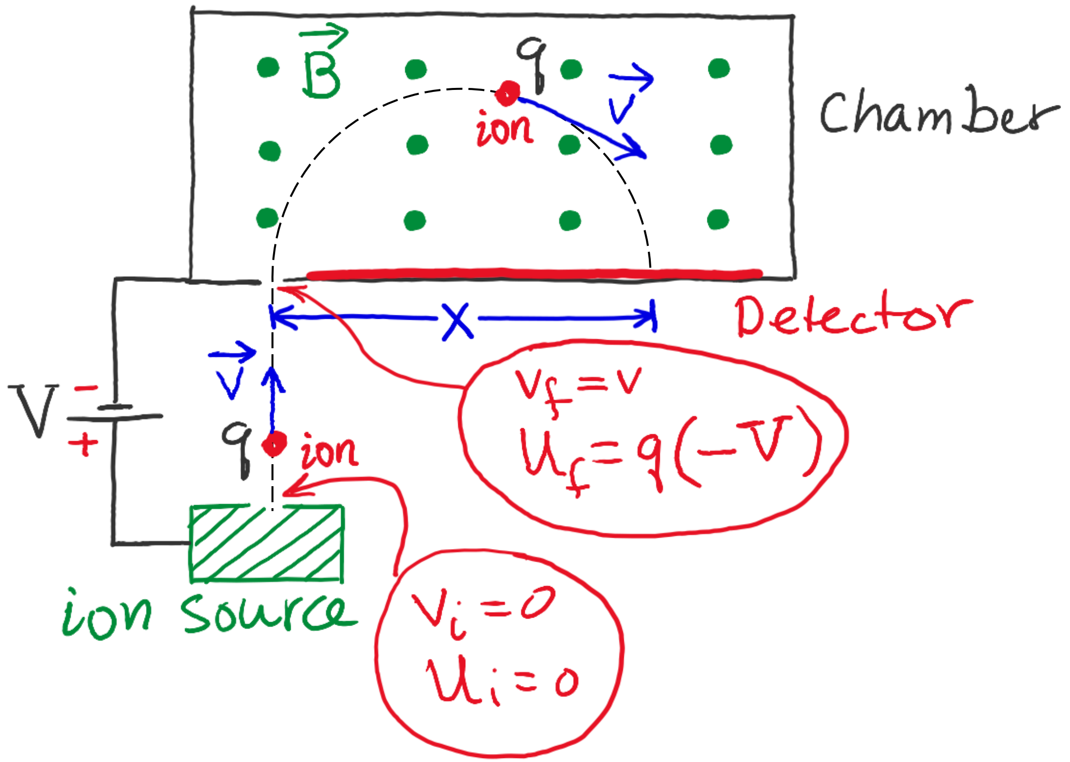
Mass spectrometer (device to measure mass of ions)

charge $q = 1.6 \times 10^{-19}$ C

$x = 1.64$ m

$B = 80$ mT

$V = 1.0$ kV



What is the mass of the ion?

Before chamber

$$\Delta K = -\Delta U$$

$$K_f - K_i = U_i - U_f$$

$$\frac{1}{2}mv^2 - 0 = 0 - (q(-V))$$

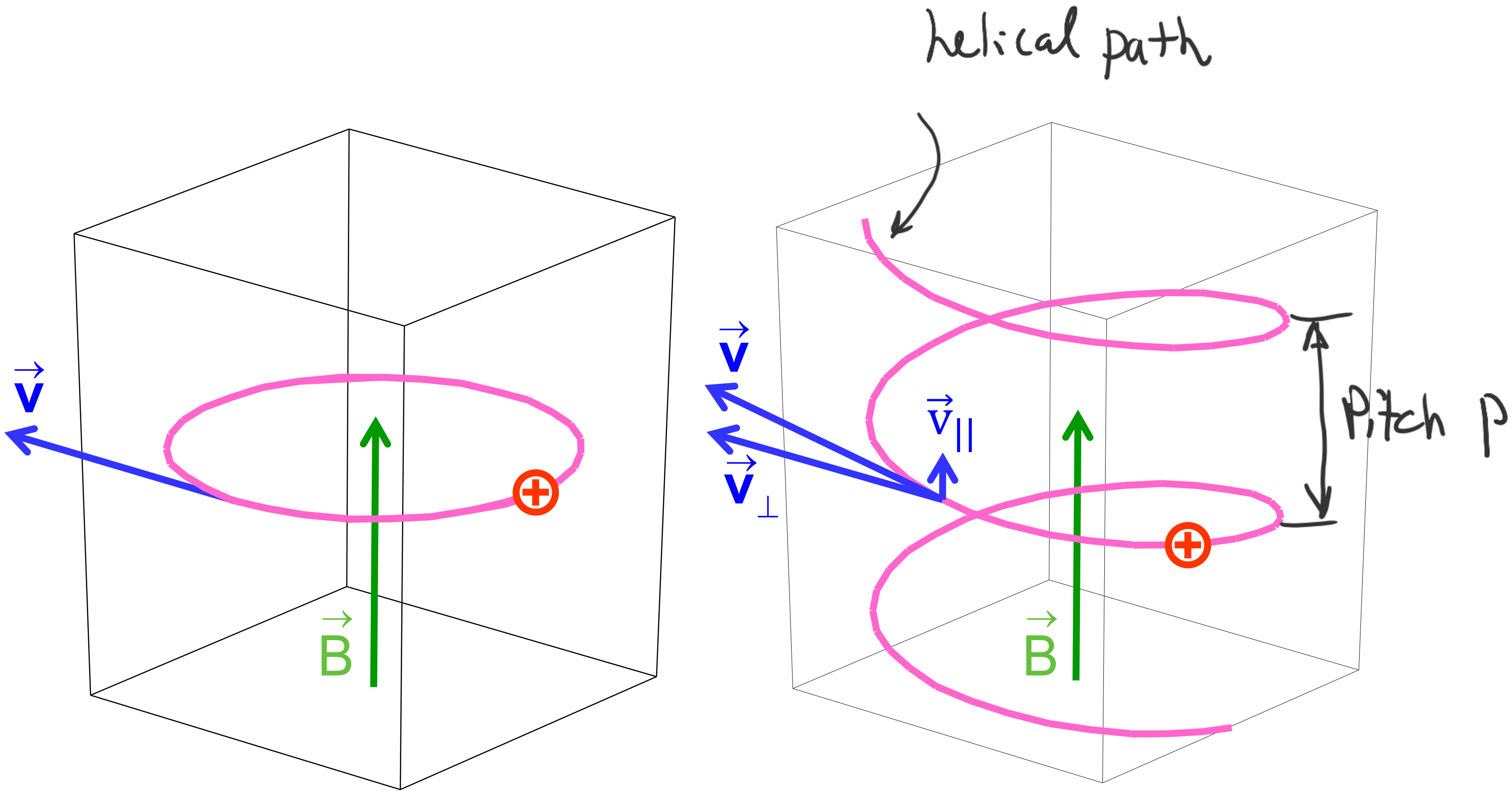
$$v = \sqrt{\frac{2qV}{m}}$$

Inside chamber $\quad r = \dfrac{mv}{qB} \Rightarrow \dfrac{x}{2} = \dfrac{m\sqrt{\frac{2qV}{m}}}{qB} = \dfrac{1}{B}\sqrt{\dfrac{2mV}{q}}$

$$\Rightarrow m = \frac{x^2}{4}\,\frac{B^2 q}{2V} = 3.39 \times 10^{-25}\,\text{kg}\left(\frac{1\ u}{1.66 \times 10^{-27}\,\text{kg}}\right) = 207\ u$$

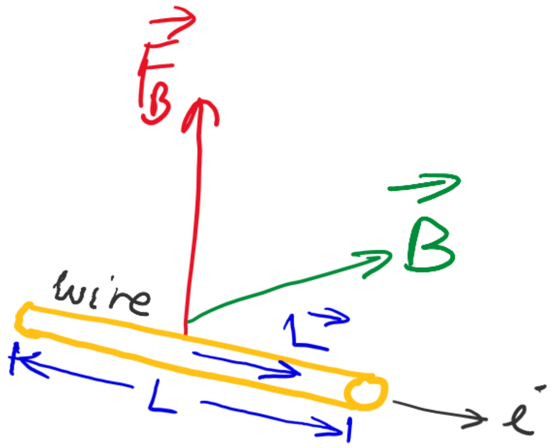
atomic mass unit

$\Rightarrow$ Lead Pb ion



$$p = v_{||} T = v_{||} \frac{2\pi m}{qB}$$

28-6 Magnetic force on a current-carrying wire



magnetic force on a wire of length L.

magnetic field

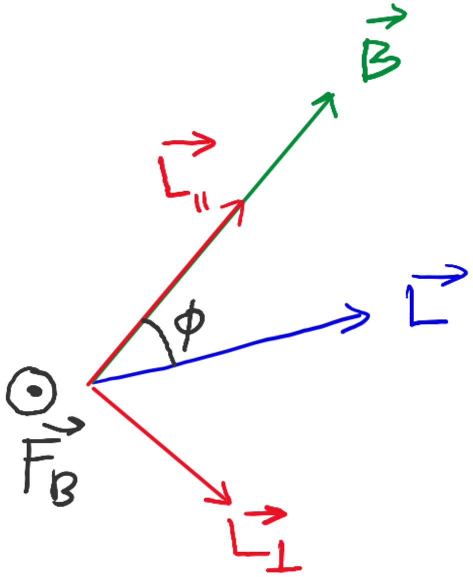
$$\vec{F}_B = i\vec{L} \times \vec{B}$$

current through the wire

length vector

magnitude: length of the wire

Direction: along the wire segment in the direction of current.



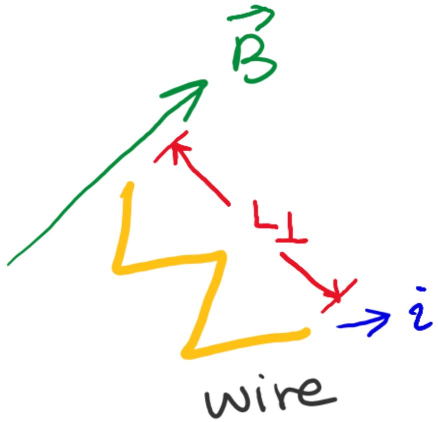
$$\vec{F}_B = i\,\vec{L} \times \vec{B}$$

$$F_B = i\,L\,B\,\sin\phi$$

$$F_B = i\,L\sin\phi\;B$$

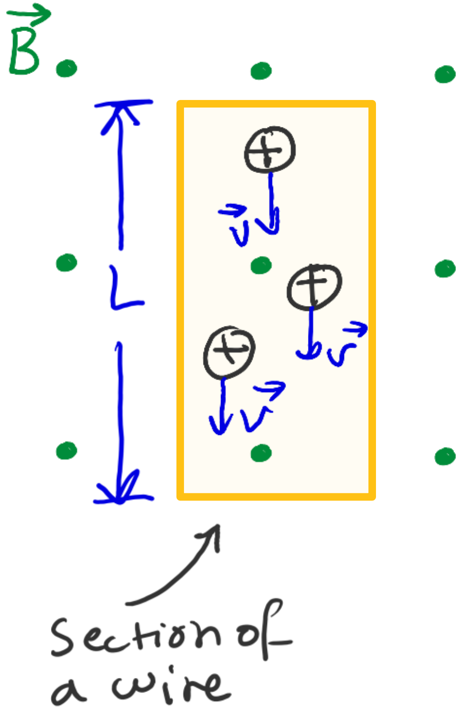
$$F_B = i\,L_{\perp}\,B$$

Only the length perpendicular to the magnetic field $L_{\perp}$ contributes to the force.



$$F_B = i\,L_{\perp}\,B$$

Derivation of $\vec{F}_B = i\vec{L}\times\vec{B}$ from $\vec{F}_B = q\vec{v}\times\vec{B}$



All the "moving" ions in a section of length L enter the section during time

$$dt = \frac{L}{v}$$

The charge in Length L is

$$|q| = i\,dt = i\,\frac{L}{v}$$

Magnetic force on the section

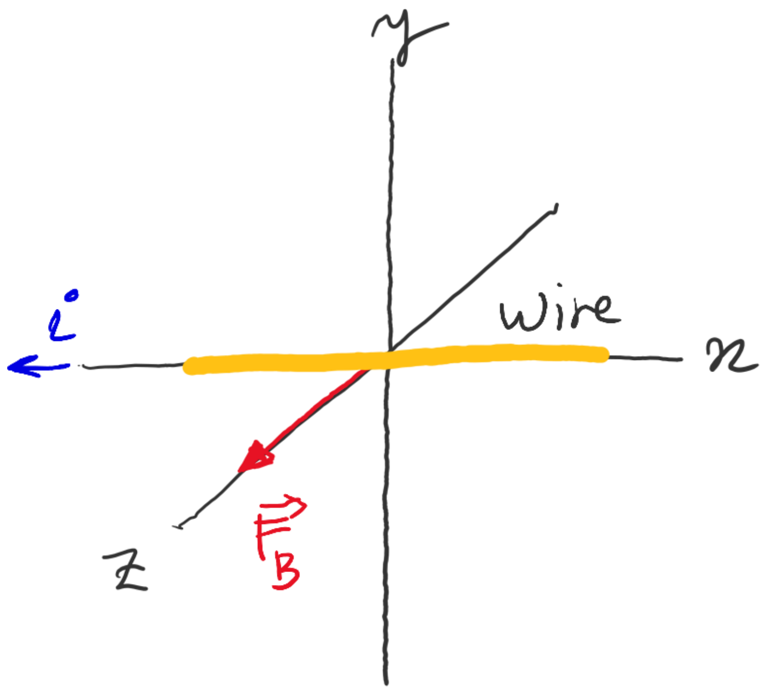
$$F_B = |q|\,v B \sin\phi$$

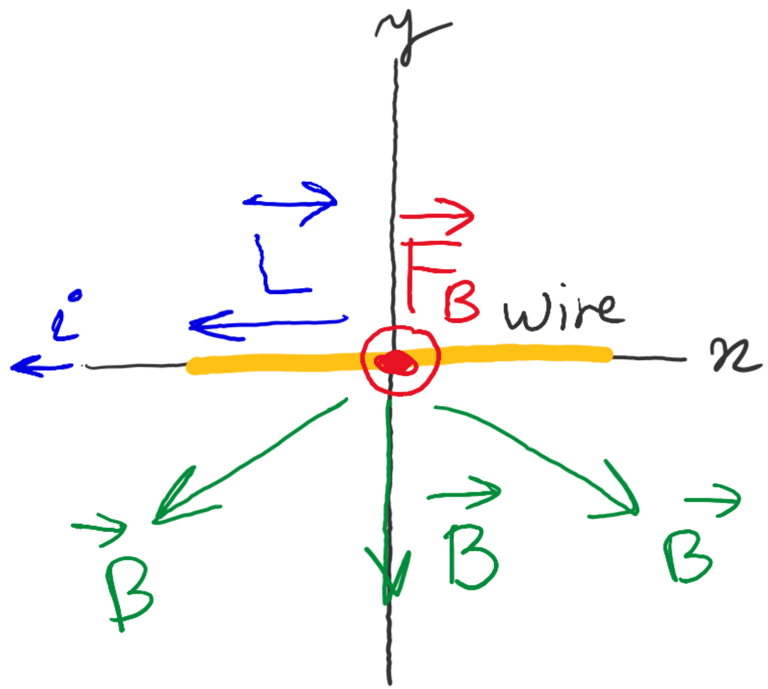
$$F_B = i\frac{L}{\cancel{v}}\,\cancel{v}\,B \sin\phi$$

$$F_B = i L B \sin\phi$$

Question

What is the direction of the field?





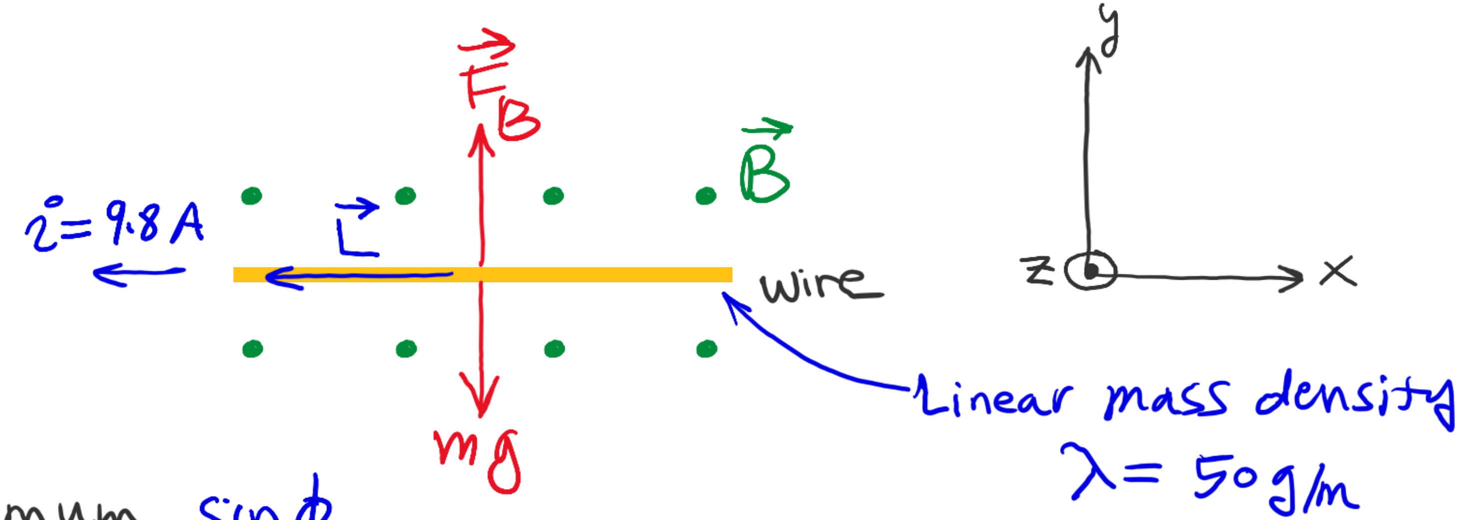
$\vec{B}$ is in the x-y plane

Example

What is the magnitude and direction of the minimum magnetic field to balance the gravitational force on the wire ?

$$\vec{F}_B = i\,\vec{L} \times \vec{B}$$

$$\left.\begin{array}{l} \vec{F}_B \perp \vec{L} \\ \vec{F}_B \perp \vec{B} \end{array}\right\} \Rightarrow \vec{B} \text{ in } xz \text{ plane}$$



$$F_B = iLB\sin\phi$$

minimum $B \Rightarrow$ maximum $\sin\phi$

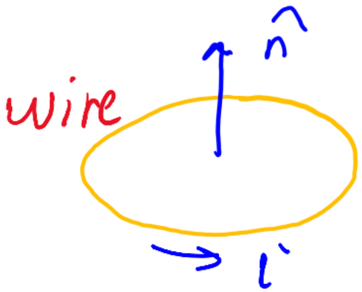
$$\Rightarrow \phi = 90^\circ$$

$$F_B = iLB = mg = \lambda L g$$

$$B = \frac{\lambda g}{i} = \frac{50\times10^{-3} \times 9.8}{9.8} = 50\times10^{-3}\,T$$

Video

Magnetic Dipole



Magnetic dipole moment

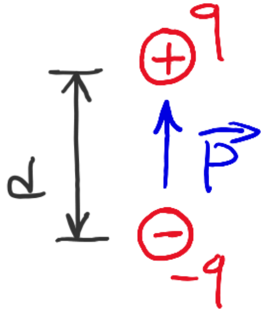
$$\vec{\mu} = i A \hat{n}$$

i: current

A: area of the loop

$\hat{n}$: normal unit vector

Electric Dipole



$$p = q d$$

Torque due to an External field

$$\vec{\tau} = \vec{\mu} \times \vec{B} \qquad \vec{\tau} = \vec{p} \times \vec{E}$$

Potential (orientation) energy

$$U(\theta) = -\vec{\mu} \cdot \vec{B} \qquad U(\theta) = -\vec{p} \cdot \vec{E}$$

Work done by an external force ($\Delta K = 0$)

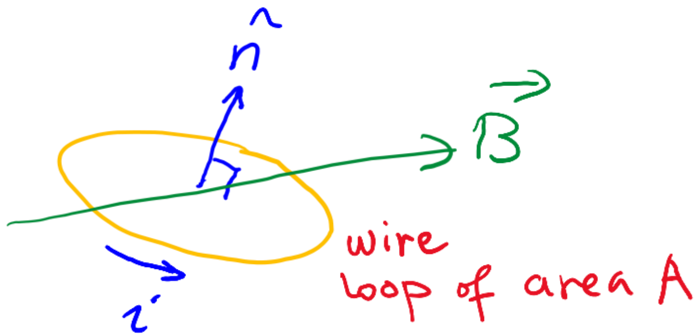
$$W_{app} = \Delta U \qquad W_{app} = \Delta U$$

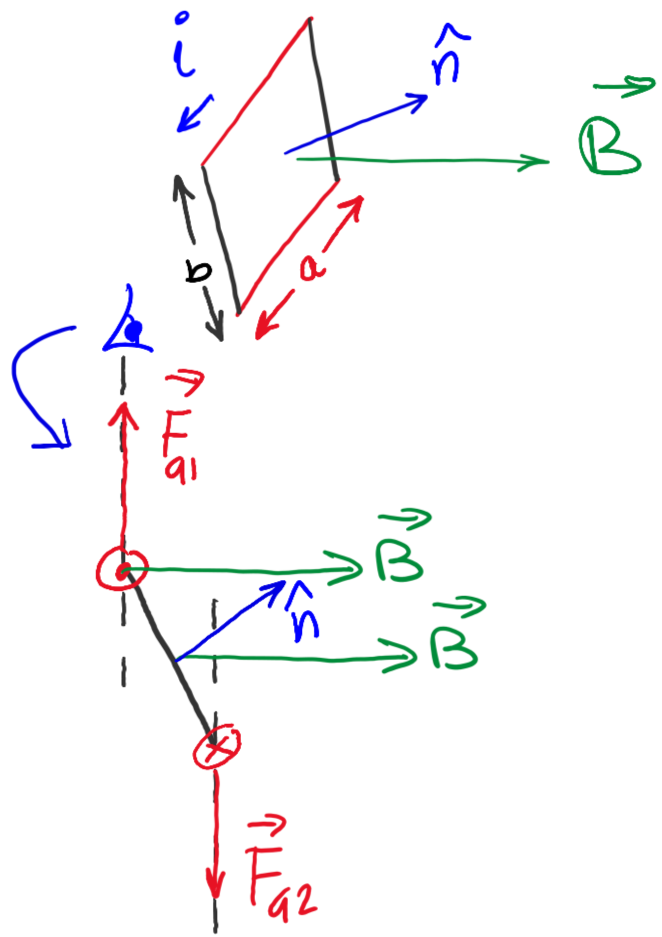
Torque on a Current Loop

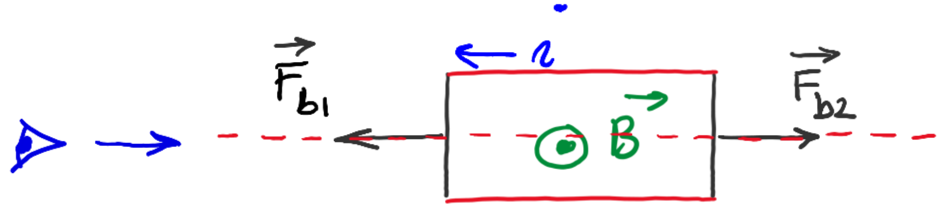
$$\vec{F}_B = i\vec{L}\times\vec{B}$$

$$\vec{\tau} = \underbrace{iA\,\hat{n}}_{\vec{\mu}}\times\vec{B}$$

$\vec{\mu}$ magnetic dipole moment







$\vec{F}_{b1}$ and $\vec{F}_{b2}$ share the same line of action

$\Longrightarrow$ They produce <u>no</u> torque

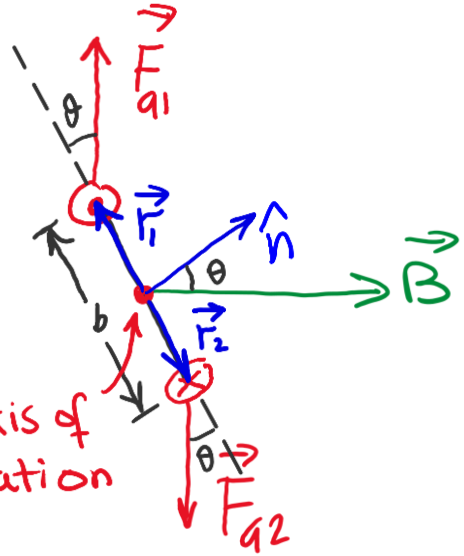
$\vec{F}_{a1}$ and $\vec{F}_{a2}$ have different lines of action

$\Longrightarrow$ They produce torque

$$F_{a1} = F_{a2} = iaB$$

Torque on a Current Loop

$F_{a1} = F_{a2} = iaB$



$$\vec{\tau} = \vec{\tau}_1 + \vec{\tau}_2 \qquad \vec{\tau} = \vec{r} \times \vec{F}$$

$$\vec{\tau} = \underbrace{\vec{r}_1 \times \vec{F}_{a1}}_{} + \underbrace{\vec{r}_2 \times \vec{F}_{a2}}_{}$$

into the page (clockwise rotation)

$$\tau = \underbrace{r_1}_{\frac{b}{2}} F_{a1} \sin\theta + \underbrace{r_2}_{\frac{b}{2}} F_{a2} \sin\theta$$

$$\tau = b(iaB)\sin\theta$$

$$\tau = i\,\underbrace{ab}_{\text{area of the loop } A}\,B\sin\theta$$

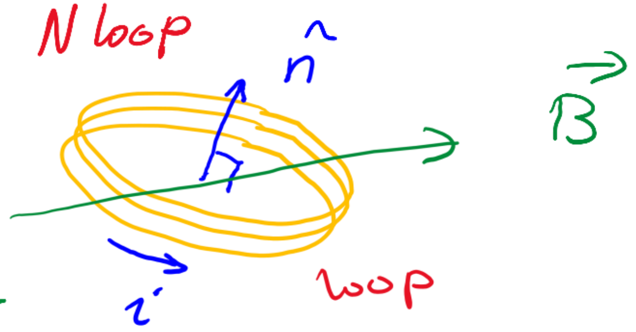
$$\boxed{\vec{\tau} = iA\,\hat{n} \times \vec{B}}$$

For any loop

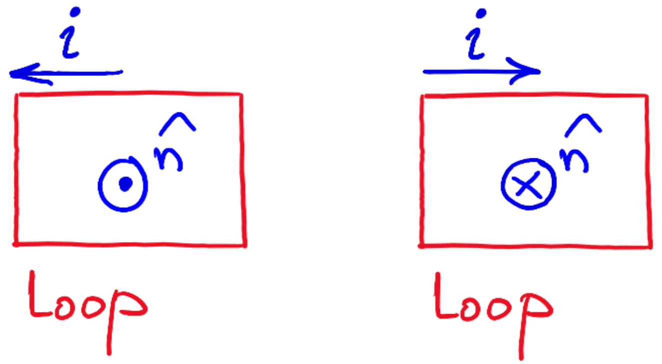
$$\vec{\tau} = N i A \, \hat{n} \times \vec{B}$$

N: Number of loops

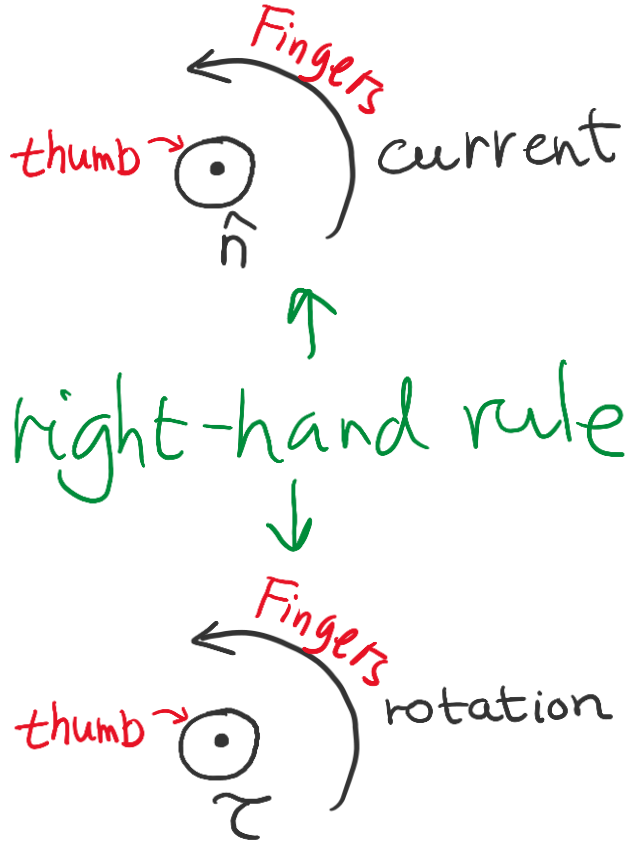
A: Area of a loop



$$|\hat{n}| = 1$$



$\hat{n}$ is a unit vector perpendicular to the surface of the loop and it points along your right hand's thumb when you algin your fingers along the current



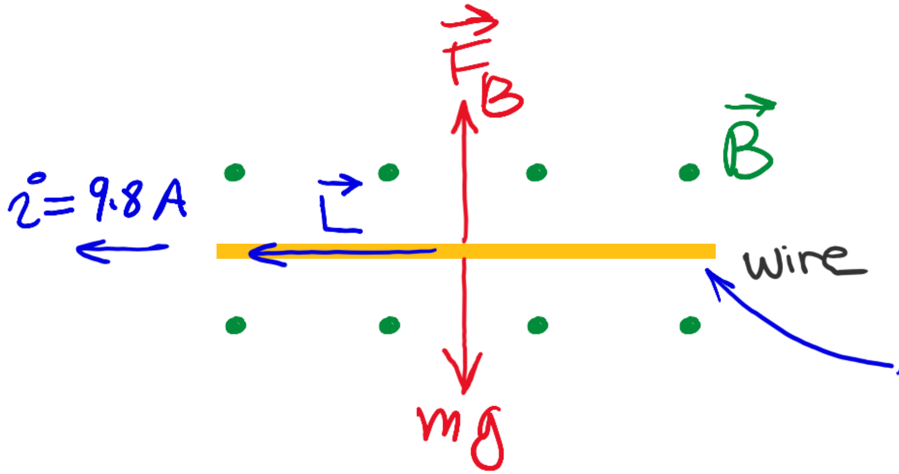
Any current-carrying loop is a magnetic dipole

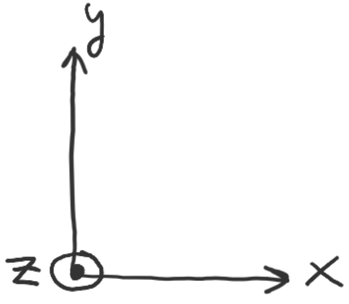
The magnetic dipole moment $\vec{\mu} \equiv N i A \hat{n}$

$$\vec{\tau} = \vec{\mu} \times \vec{B}$$

Example

What is the magnitude and direction of the minimum magnetic field to balance the gravitational force on the wire?





Linear mass density $\lambda = 50\,g/m$

$$\vec{F}_B = i\vec{L} \times \vec{B}$$

$$F_B\hat{j} = iL(-\hat{i}) \times (B_x\hat{i} + B_y\hat{j} + B_z\hat{k})$$

$\hat{i} \times \hat{i} = 0$, $\hat{i} \times \hat{j} = \hat{k}$, $\hat{i} \times \hat{k} = -\hat{j}$

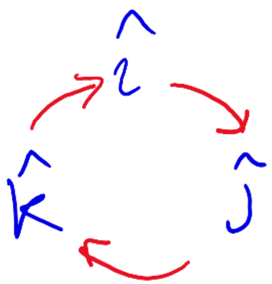
$$F_B\hat{j} = iL(-\underbrace{B_y}_{=0}\hat{k} + B_z\hat{j}) = iLB_z\hat{j} \Rightarrow B_z = \frac{F_B}{iL}$$

$\vec{F}_B$ has no component along $\hat{k}$

$$B = \sqrt{B_x^2 + B_y^2 + B_z^2} = B_z = \frac{F_B}{iL}$$

B_x: can be any value; $B_y = 0$

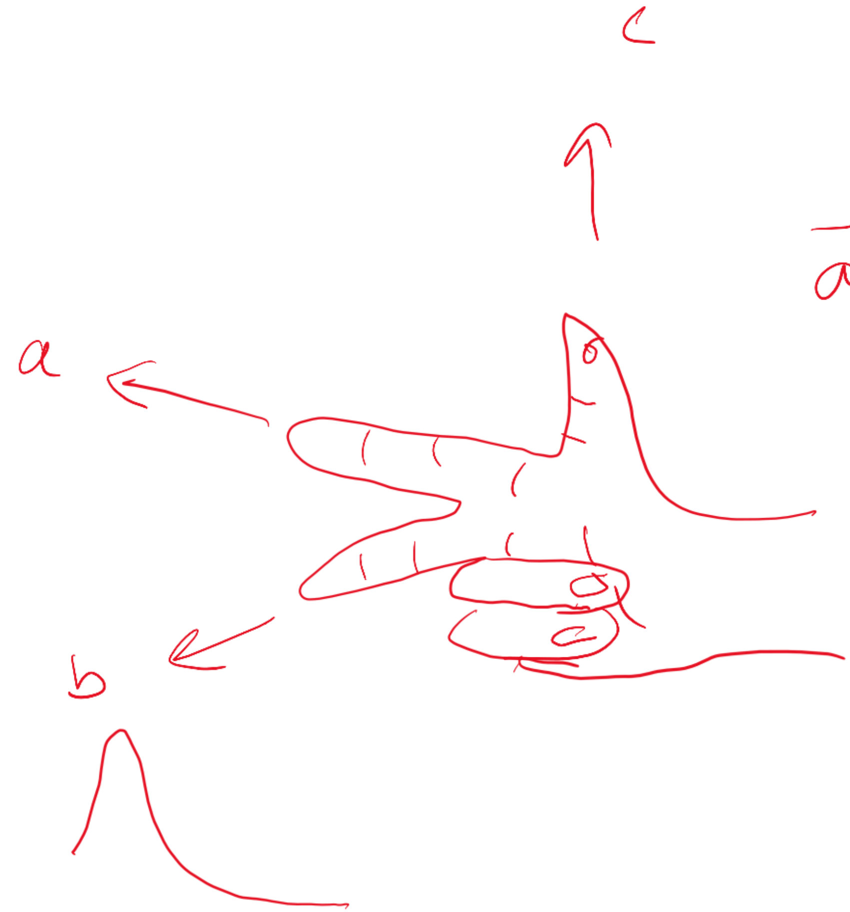
Minimum $B \Rightarrow B_x = 0$



$\hat{i} \times \hat{j} = \hat{k}$

$\hat{j} \times \hat{k} = \hat{i}$

$\hat{k} \times \hat{i} = \hat{j}$



$$\vec{a} \times \vec{b} = \vec{c}$$