This is not a replacement for your textbook. It is essential to study from your textbook!

Objectives

Ch27: Circuits

27-1 Single-loop circuits

27-2 Multiloop circuits

27-3 The ammeter and voltmeter

27-4 RC circuits

Emf devices

Devices that maintain potential difference between their terminals by doing work on charge carries.

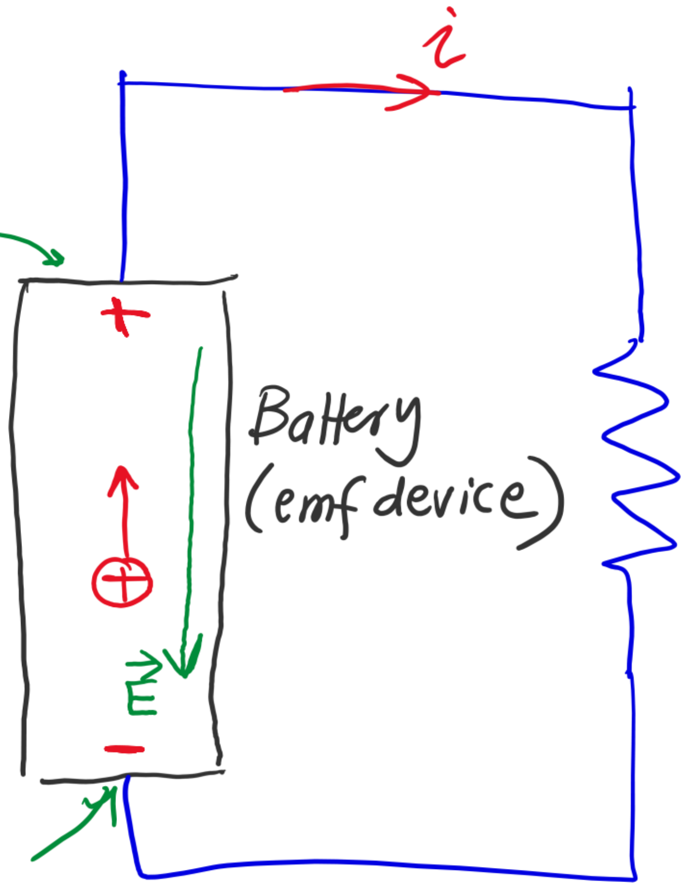
- Battery
- Electric generator
- Solar cell

emf $\equiv$ electromotive force

(outdated phrase)

Positive terminal (higher potential)

<u>Internal chemistry</u> of the battery causes a net flow of positive carriers from the negative terminal to the positive terminal)

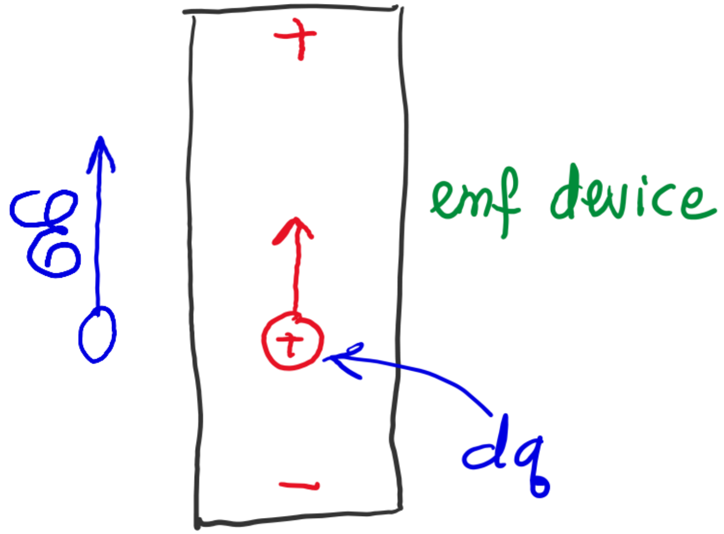


negative terminal (lower potential)

Battery $\longrightarrow$ chemical energy

electric generator $\longrightarrow$ Mechanical energy

Solar cell $\longrightarrow$ light energy

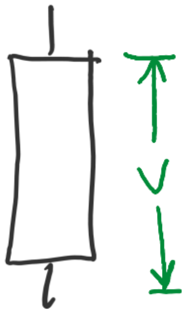


The emf of an emf device is the work per unit charge that an emf device does in moving positive charge from its low-potential terminal to its high-potential terminal

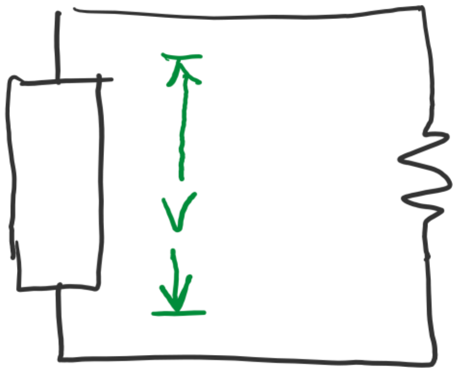
$$\mathcal{E} = \frac{dW}{dq}$$

The SI unit for $\mathcal{E} = \frac{\text{Joul}}{\text{Coulomb}} = \text{Volt}$

Ideal emf device

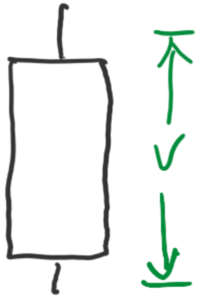


$$\mathcal{E} = V$$

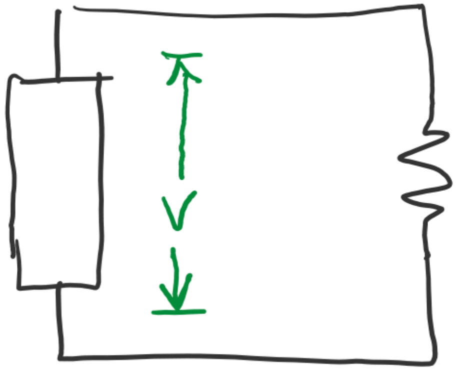


$$\mathcal{E} = V$$

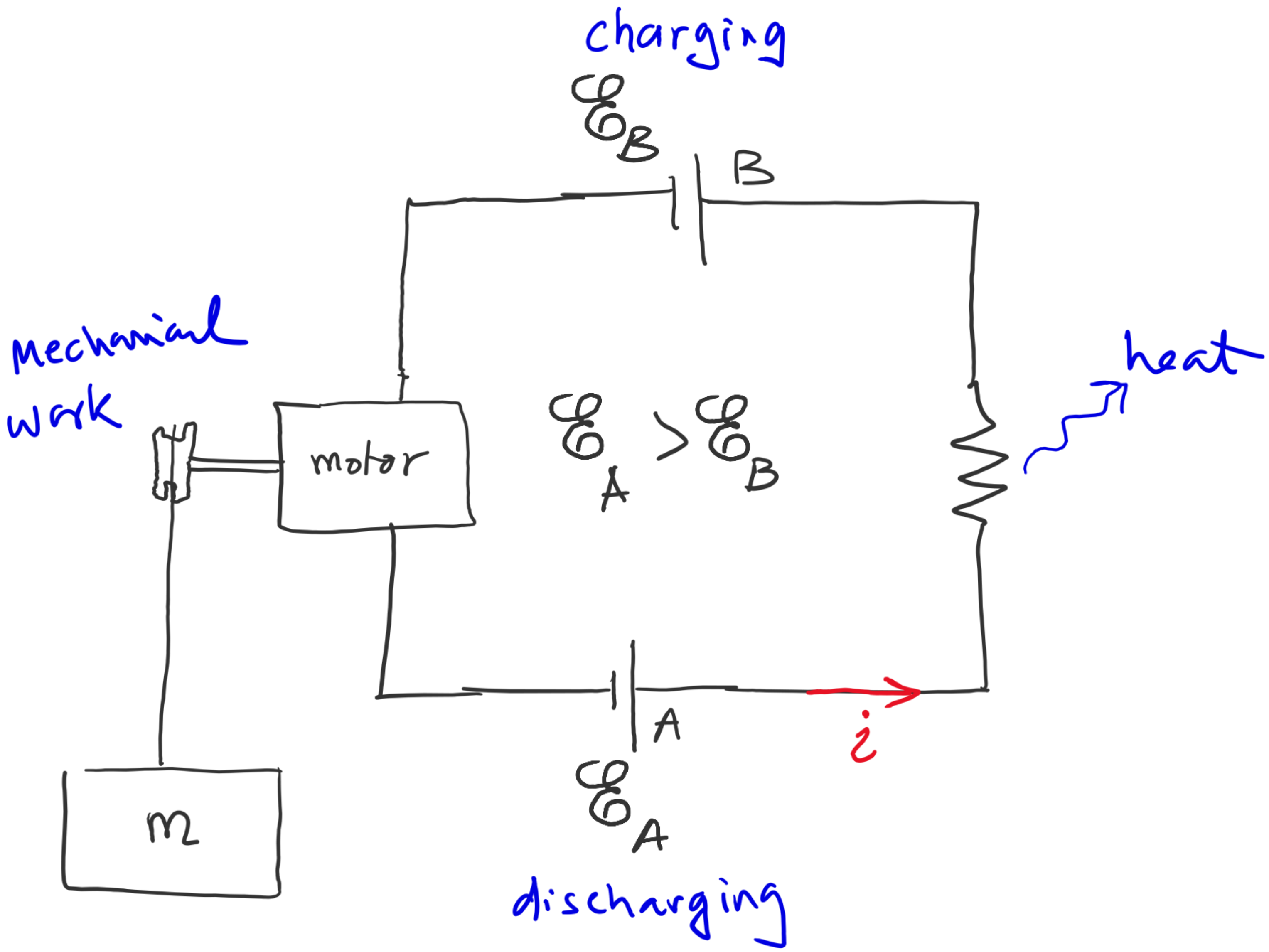
Real emf device



$$\mathcal{E} = V$$



$$\mathcal{E} > V$$

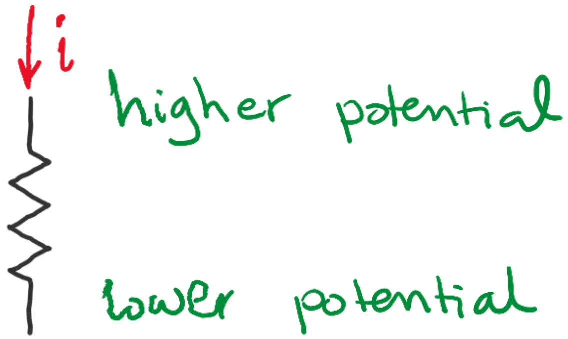

charging
$\mathcal{E}_B$
B
mechanical work
motor
$\mathcal{E}_A > \mathcal{E}_B$
heat
i
A
m
$\mathcal{E}_A$
discharging

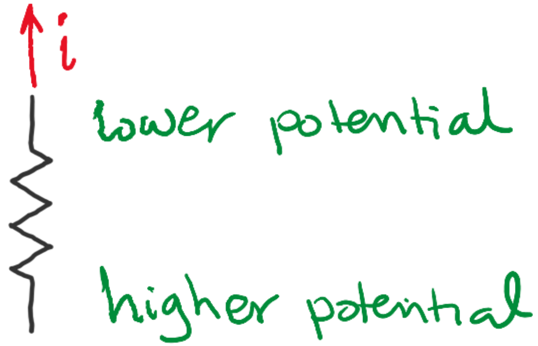
Loop rule (Kirchhoff's loop rule)

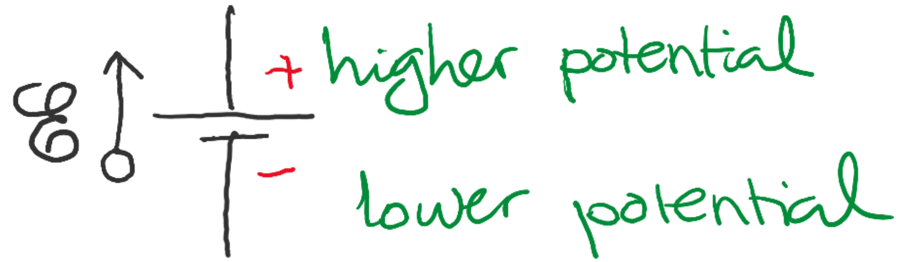
$$\sum_{\text{Loop}} \Delta U = 0$$

conservation of energy

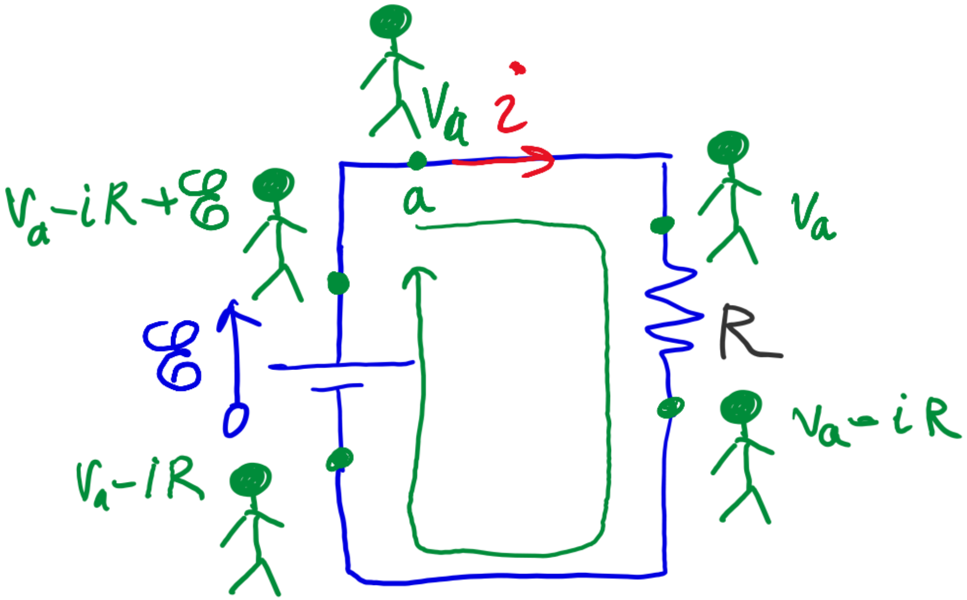
For a resistor, the current flows from higher to lower potential







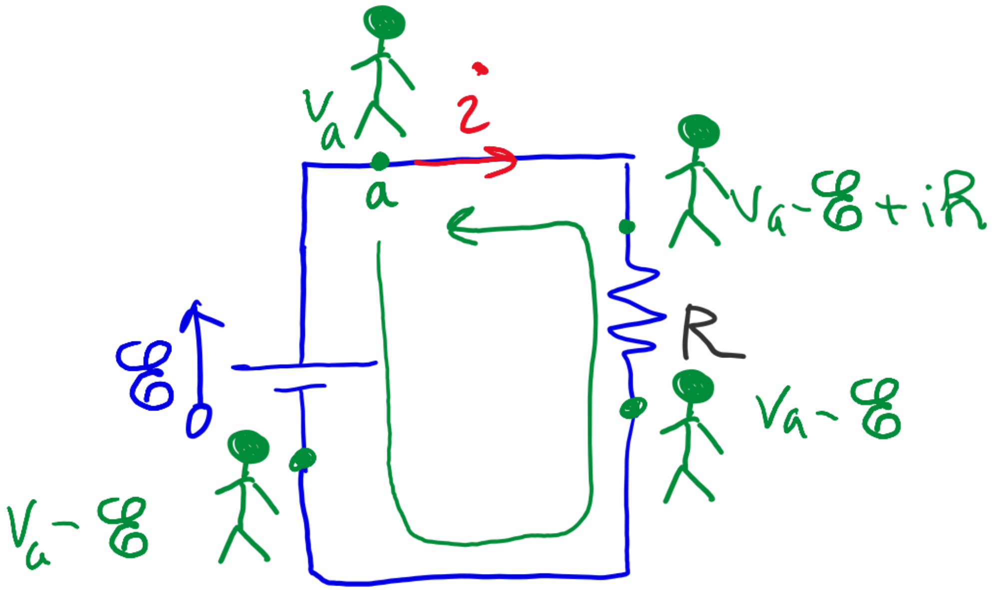
$$V = iR$$



$$\cancel{V_a} - iR + \mathcal{E} = \cancel{V_a}$$

$$-iR + \mathcal{E} = 0$$

$$i = \frac{\mathcal{E}}{R}$$

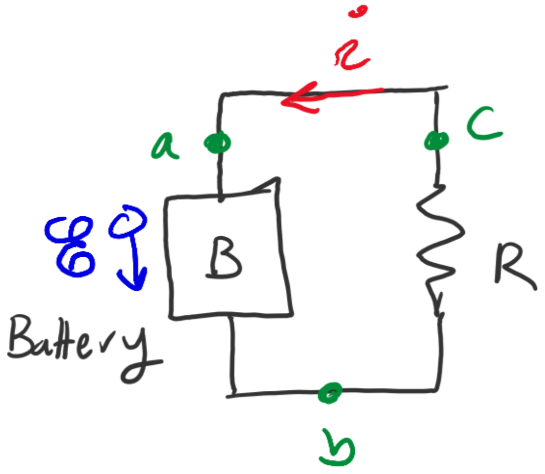


$$\cancel{V_a} - \mathcal{E} + iR = \cancel{V_a}$$

$$-\mathcal{E} + iR = 0$$

$$i = \frac{\mathcal{E}}{R}$$

Question



Draw the emf arrow

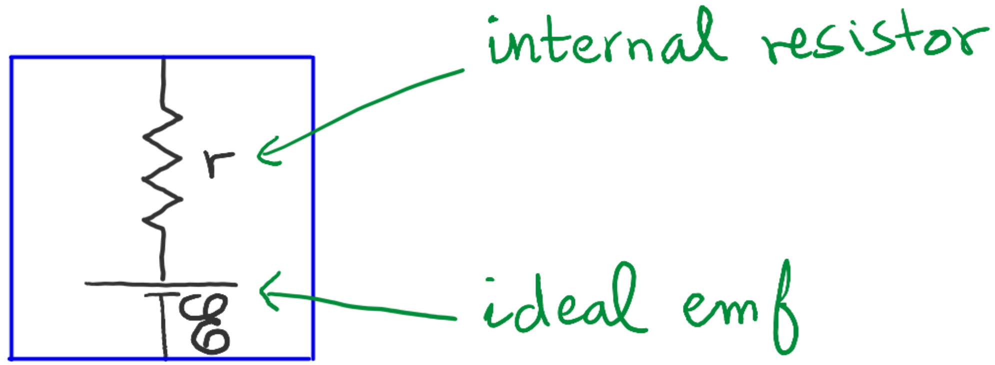
At points a, b, and c, rank

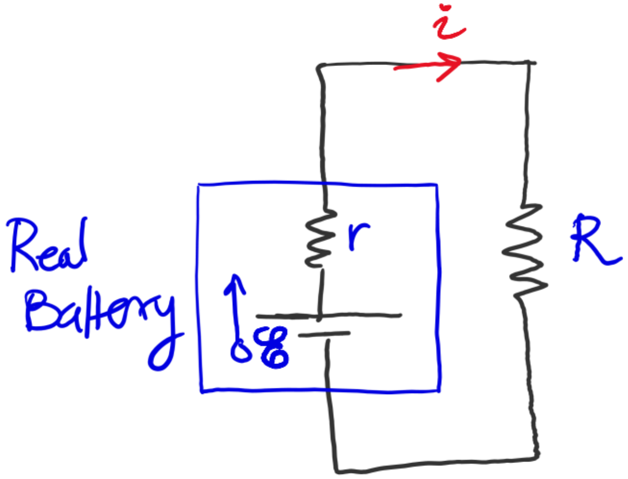
current — all tie

electric potential — $V_b > V_c = V_a$

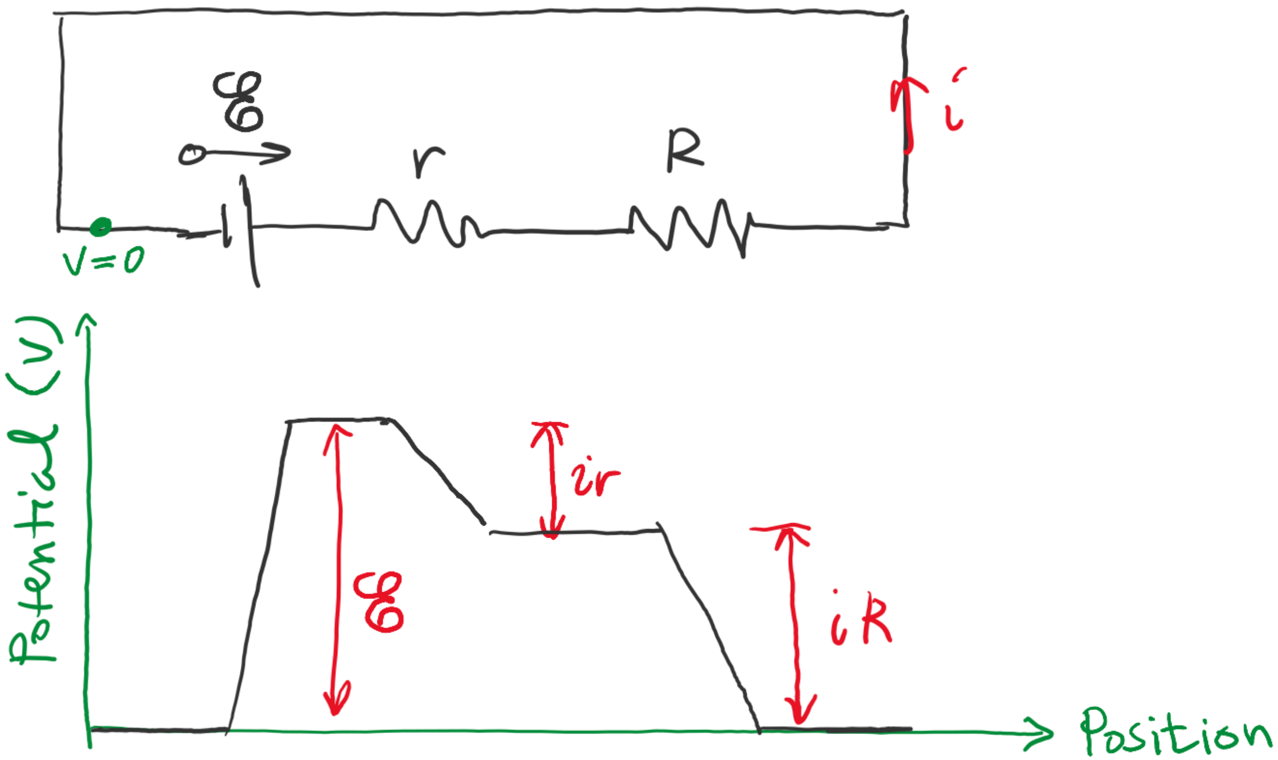
electric potential energy of the positive charge carriers — $U_b > U_c = U_a$

Real emf device

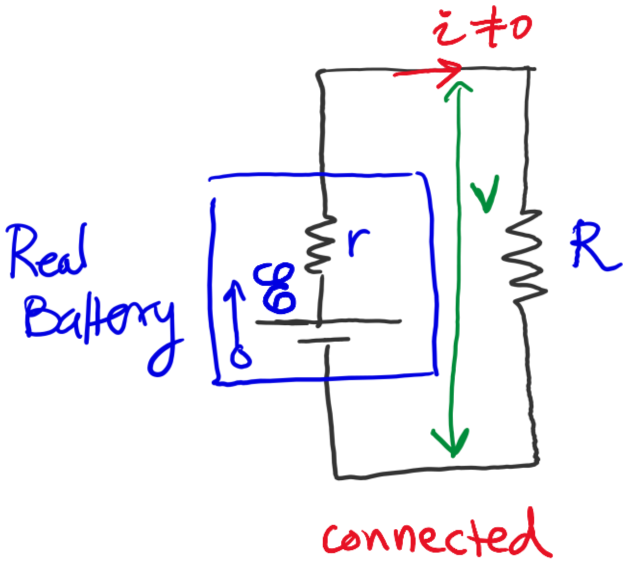




i
Real Battery
r
R
ℰ



ℰ
r
R
i
V=0
Potential (V)
Position
ir
iR
ℰ

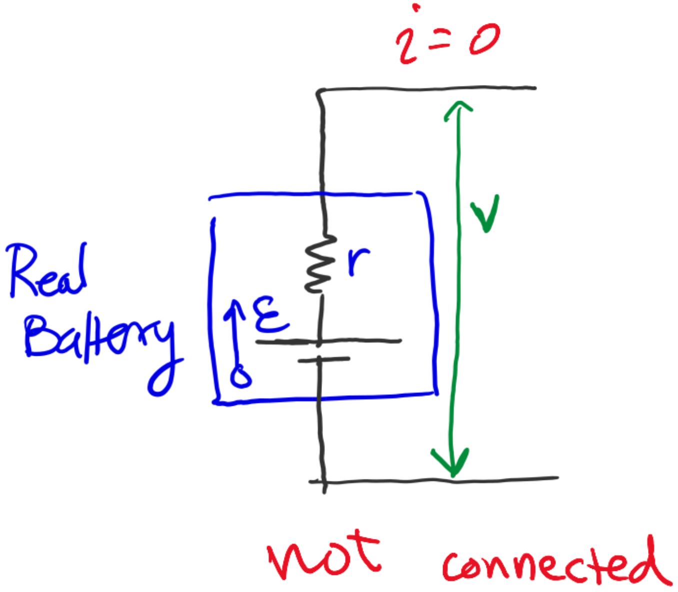


$$\mathcal{E} - ir - iR = 0$$

$$i = \frac{\mathcal{E}}{r + R}$$

$$V = \mathcal{E} - ir = \mathcal{E}\left(1 - \frac{r}{r+R}\right)$$

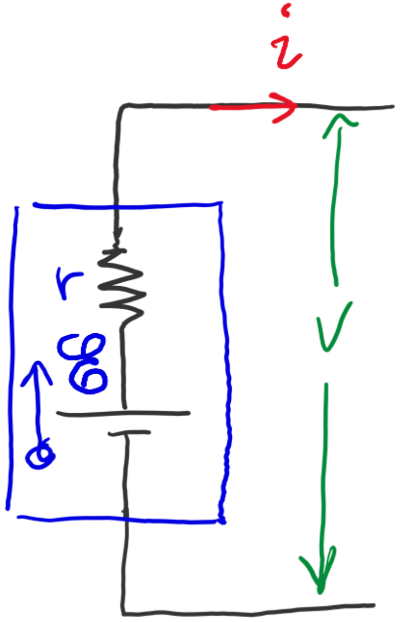
$$V < \mathcal{E}$$



$$V = \mathcal{E}$$

Question

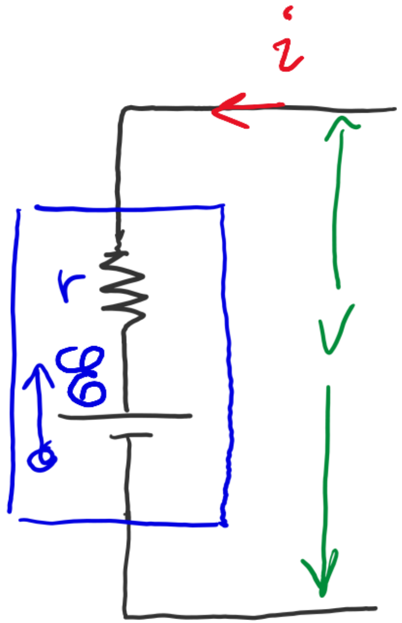
compare $\mathcal{E}$ and V



connected

$V = \mathcal{E} - ir$

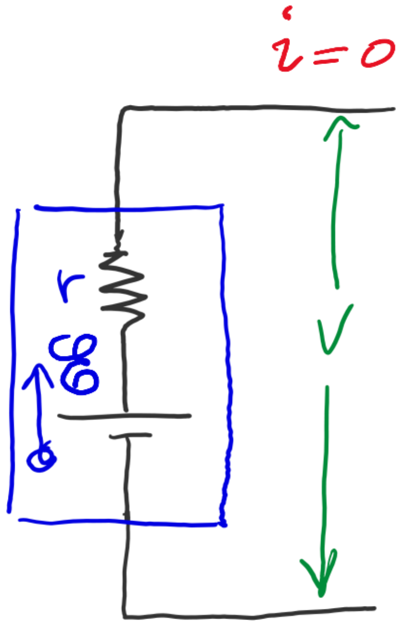
$V < \mathcal{E}$



connected

$V = \mathcal{E} + ir$

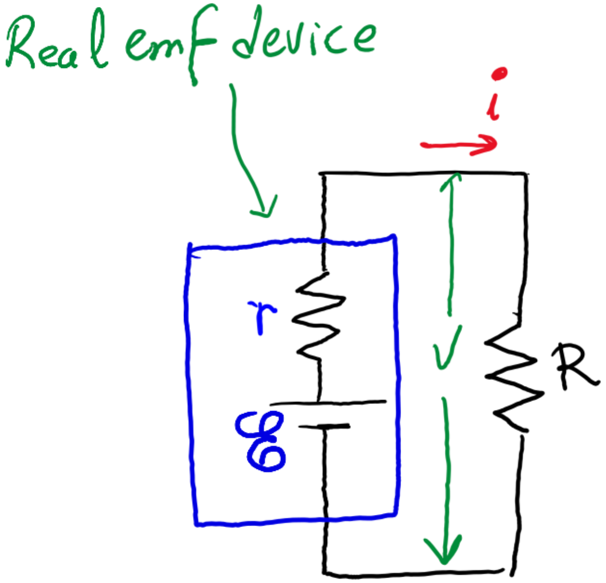
$V > \mathcal{E}$



not connected

$V = \mathcal{E}$

Power from Real emf Devices



$$V = \mathscr{E} - ir$$

Power provided by the real emf device

$$P = iV$$

$$P = i(\mathscr{E} - ir)$$

$$P = i\mathscr{E} - i^2 r$$

$$P = P_{emf} - i^2 r$$

Power provided by the ideal emf

Power dissipated in the internal resistance (thermal)

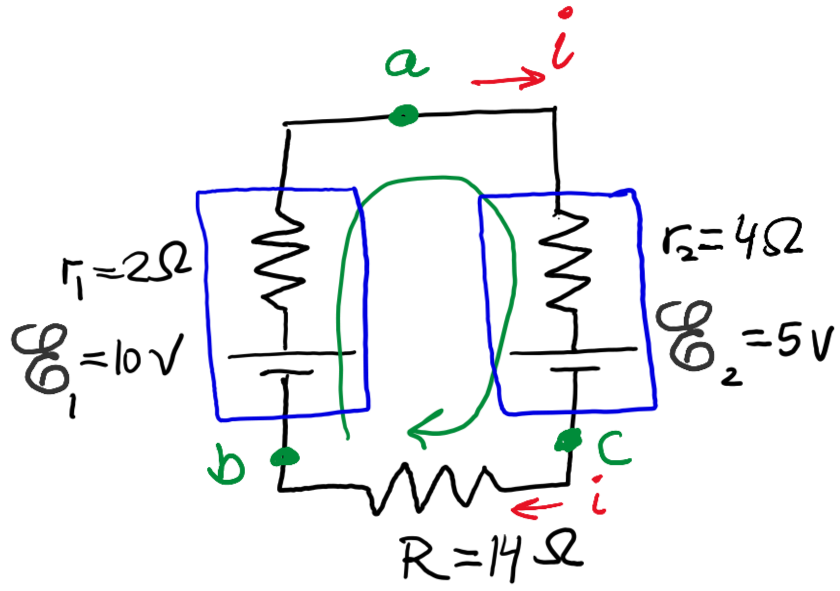
Example

What is the current?

Pick any direction for current.

If your guess is wrong, your current will be negative.

Pick any direction for the loop rule.



$$\mathcal{E}_1 - ir_1 - ir_2 - \mathcal{E}_2 - iR = 0$$

$$i = \frac{\mathcal{E}_1 - \mathcal{E}_2}{r_1 + r_2 + R} = \frac{10-5}{20} = 0.25\,A$$

$i > 0$

$\Rightarrow$ our guess is correct

What is $V_a - V_b$?

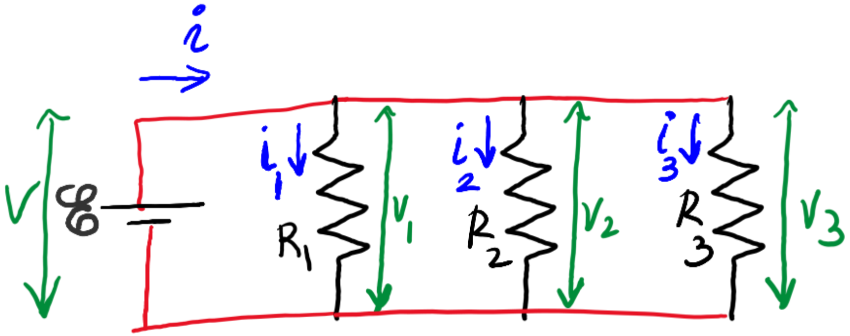
$$V_b + \mathcal{E}_1 - ir_1 = V_a \implies V_a - V_b = \mathcal{E}_1 - ir_1 = 9.5\,V$$

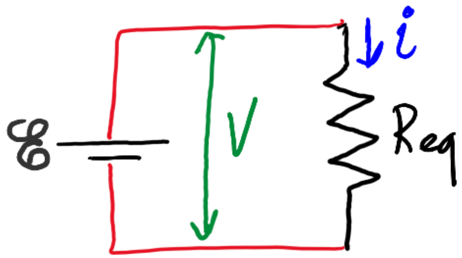
What is $V_a - V_c$?

$$V_c + \mathcal{E}_2 + ir_2 = V_a \implies V_a - V_c = \mathcal{E}_2 + ir_2 = 6\,V$$

$$V = iR$$

In Parallel





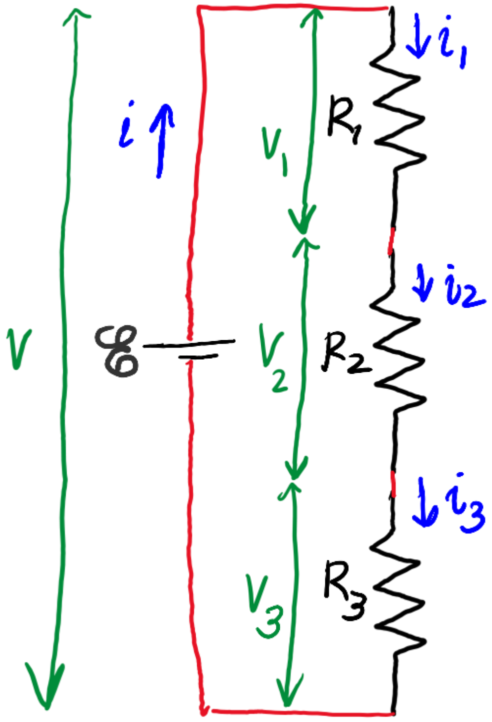
$$V = V_1 = V_2 = V_3$$

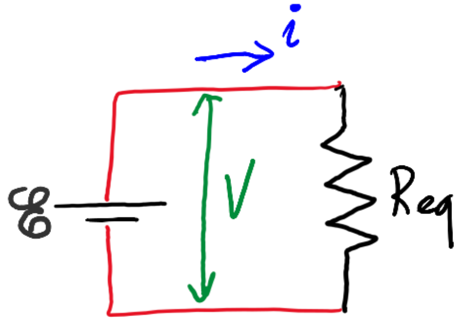
$$i = i_1 + i_2 + i_3$$

$$\frac{\cancel{V}}{R_{eq}} = \frac{\cancel{V_1}}{R_1} + \frac{\cancel{V_2}}{R_2} + \frac{\cancel{V_3}}{R_3}$$

$$\boxed{\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}$$

In Series





$$i = i_1 = i_2 = i_3$$

$$V = V_1 + V_2 + V_3$$

$$\cancel{i} R_{eq} = \cancel{i_1} R_1 + \cancel{i_2} R_2 + \cancel{i_3} R_3$$

$$\boxed{R_{eq} = R_1 + R_2 + R_3}$$

Example

What is the current?

$$i = \frac{\mathcal{E}}{R} = \frac{12}{40} = 0.3 \text{ A}$$

What is the current through R_2?

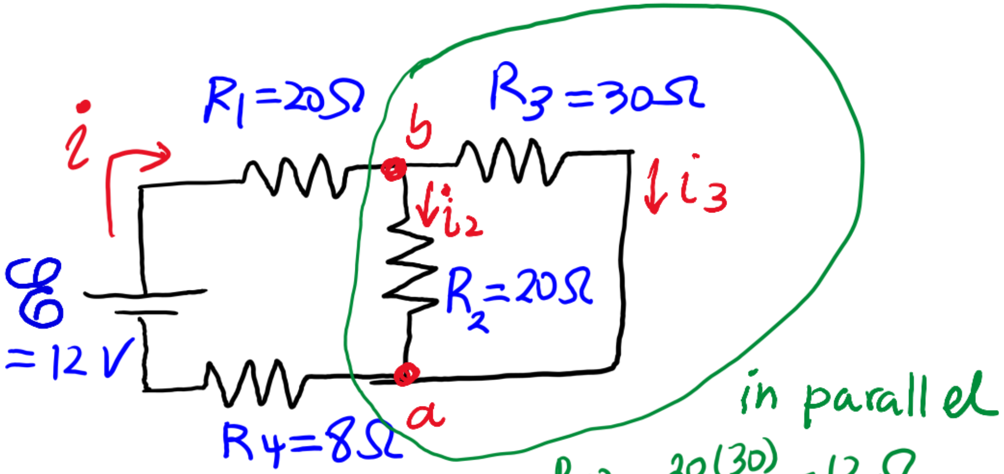
$$V_{ba} = i R_{23} = (0.3)(12) = 3.6 \text{ V}$$

$$V_{ba} = i_2 R_2$$

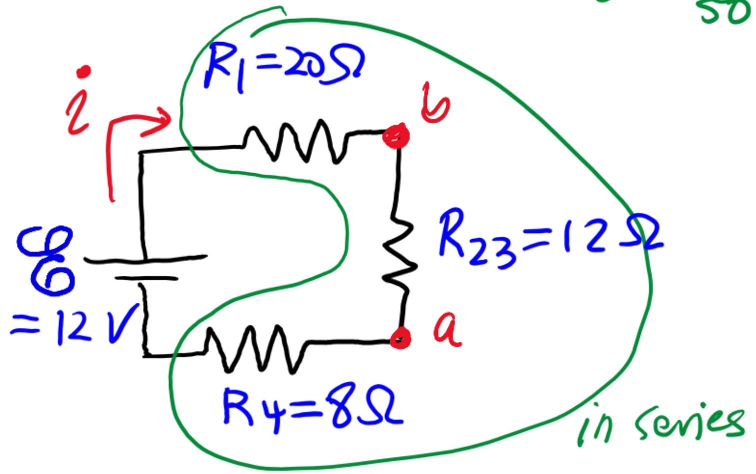
$$\Longrightarrow i_2 = \frac{V_{ba}}{R_2} = \frac{3.6}{20} = 0.18 \text{ A}$$

What is the current through R_3?

$$i_3 = \frac{V_{ba}}{R_3} = \frac{3.6}{30} = 0.12 \text{ A}$$

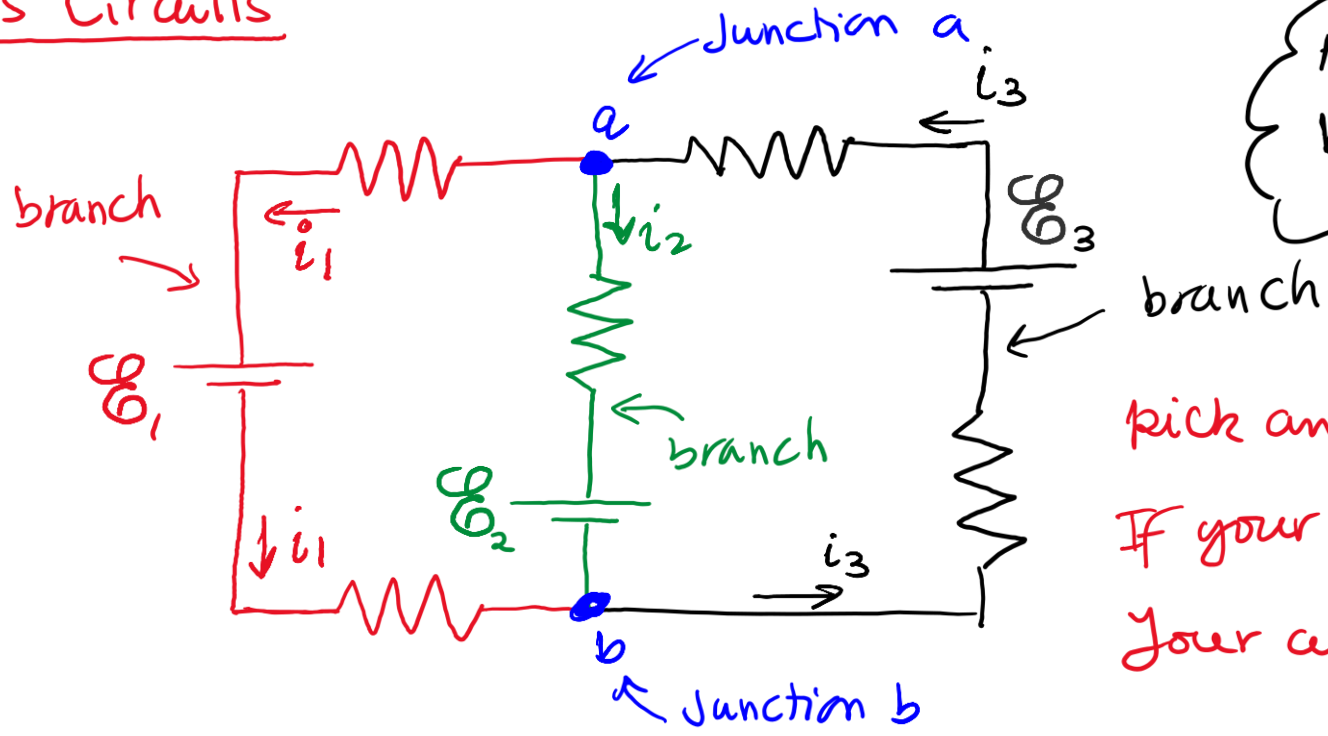


$$R_{23} = \frac{20(30)}{50} = 12\,\Omega$$



$$R = R_1 + R_{23} + R_4 = 40\,\Omega$$

Multiloops Circuits



Any point in a branch has the same current

Pick any direction for currents.

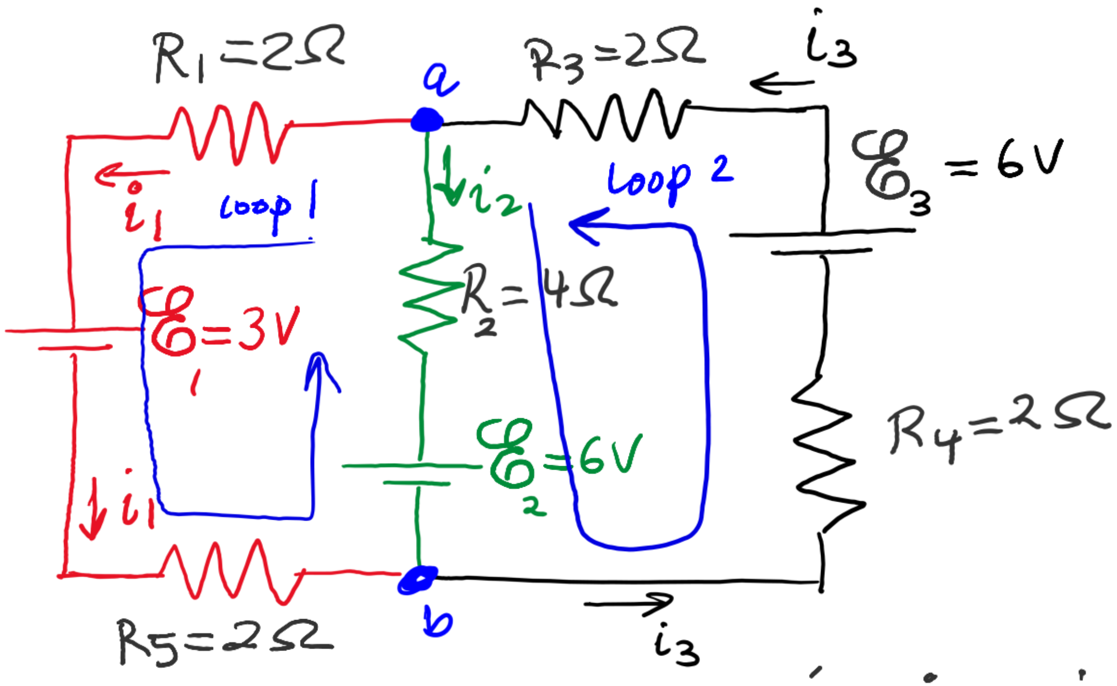
If your guess is wrong,

Your current will be negative.

(Kirchhoff's) Junction Rule:

From conservation of charge

<u>For a junction</u> Sum of currents in = Sum of currents out



number of unknowns = 3

We need as many independent equations as the number of unknowns

Find the current in each branch. i_1, i_2, i_3

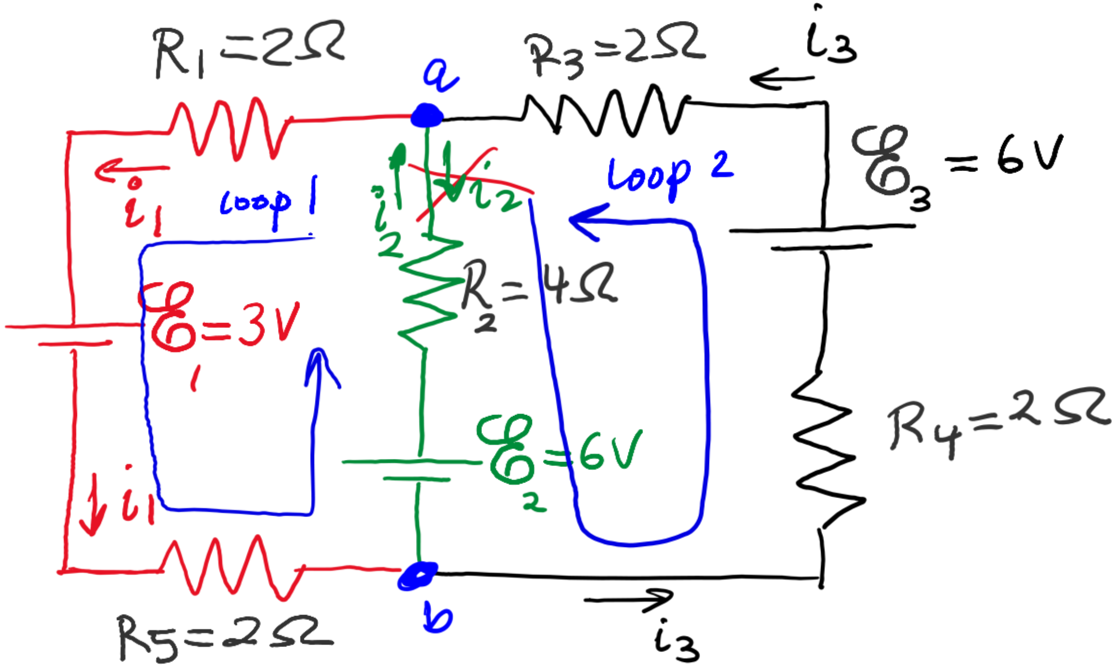
Junction rule at a : $\quad i_3 = i_1 + i_2 \quad$ —— ①

Loop rule on loop 1 : $\quad -i_1R_1 - \mathcal{E}_1 - i_1R_5 + \mathcal{E}_2 + i_2R_2 = 0$

$$-4i_1 + 3 + 4i_2 = 0 \quad \text{—— ②}$$

Loop rule on Loop 2 : $\quad -i_2R_2 - \mathcal{E}_2 - i_3R_4 + \mathcal{E}_3 - i_3R_3 = 0$

$$-4i_2 - 4i_3 = 0 \quad \text{—— ③}$$



$$i_3 = i_1 + i_2$$

$$-4i_1 + 3 + 4i_2 = 0$$

$$-4i_2 - 4i_3 = 0 \Longrightarrow -8i_2 - 4i_1 = 0 \Rightarrow i_2 = -\frac{i_1}{2}$$

$$-4i_1 + 3 + 4\left(-\frac{i_1}{2}\right) = 0 \Longrightarrow -6i_1 + 3 = 0$$

$$\Longrightarrow i_1 = 0.5\ A$$

$$i_2 = -0.25\ A$$

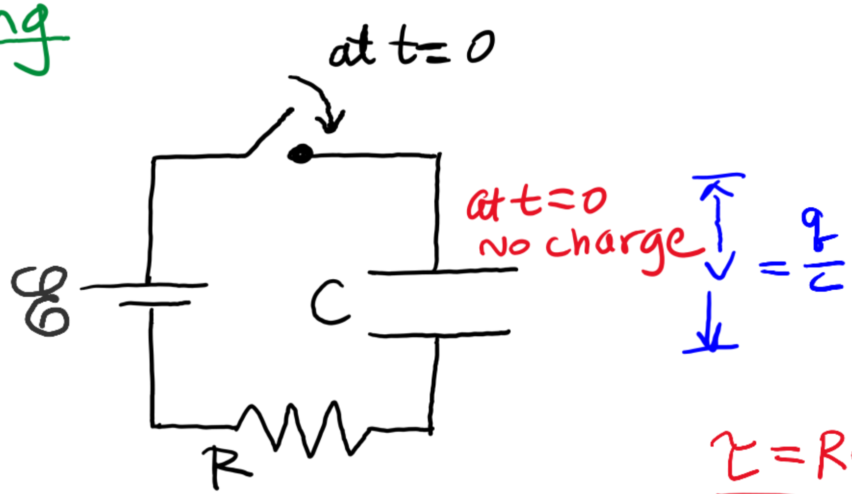
$$i_3 = +0.25\ A$$

Do not change direction untill you find all the currents

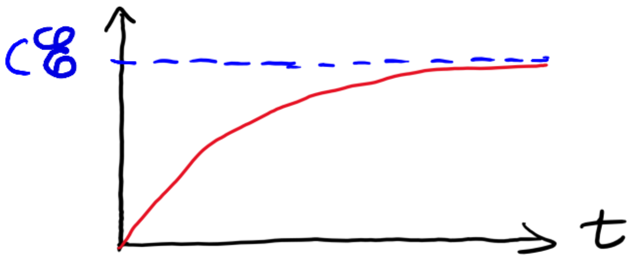
our guess for i_2 is wrong.

27-4 RC Circuits

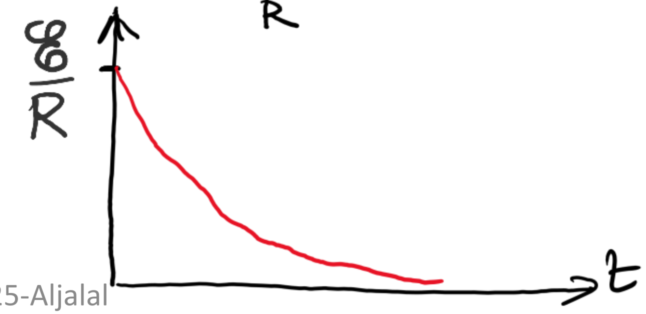
Charging



$$q(t) = C\mathcal{E}\left(1 - e^{-t/\tau}\right)$$



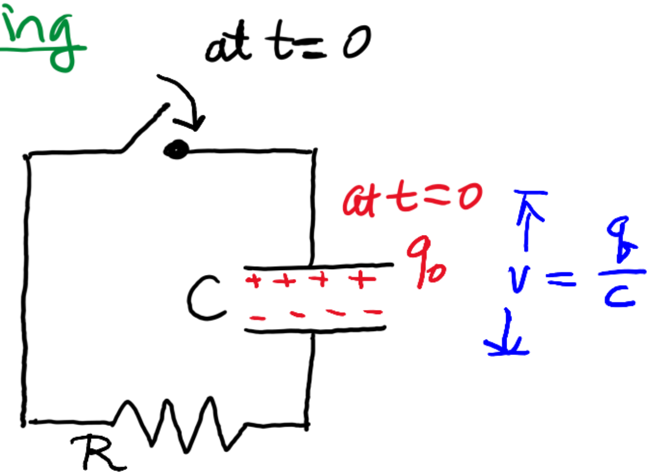
$$i(t) = \frac{\mathcal{E}}{R} e^{-t/\tau}$$



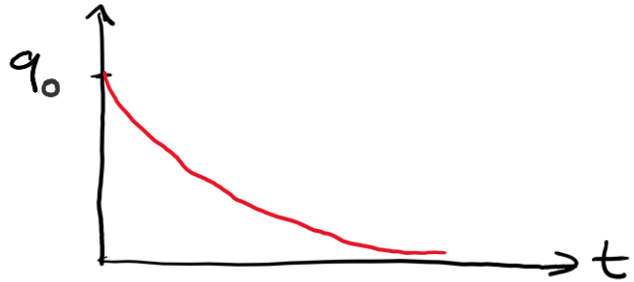
$\tau = RC$ time constant

$$i = \frac{dq}{dt}$$

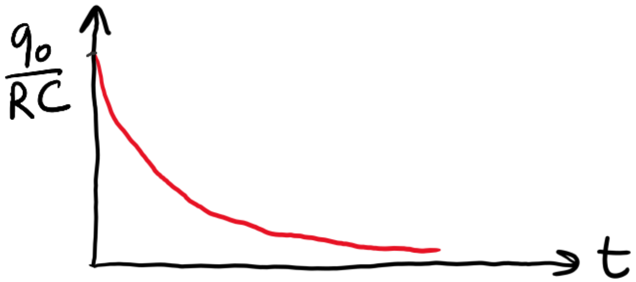
Discharging



$$q(t) = q_0 e^{-t/\tau}$$

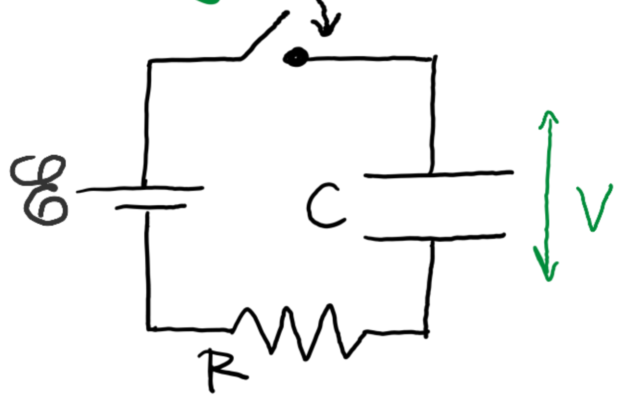


$$i(t) = \frac{q_0}{RC} e^{-t/\tau}$$

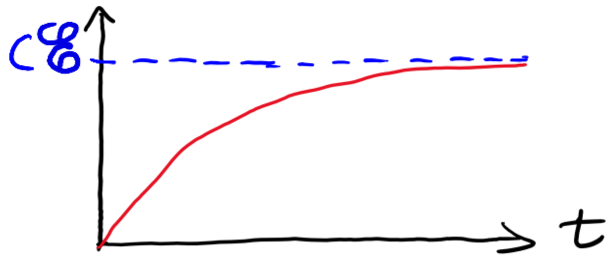


Charging

at $t=0$

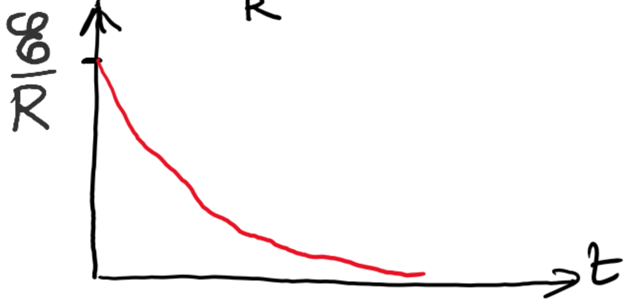


$$q(t) = C\mathcal{E}\left(1 - e^{-t/\tau}\right)$$



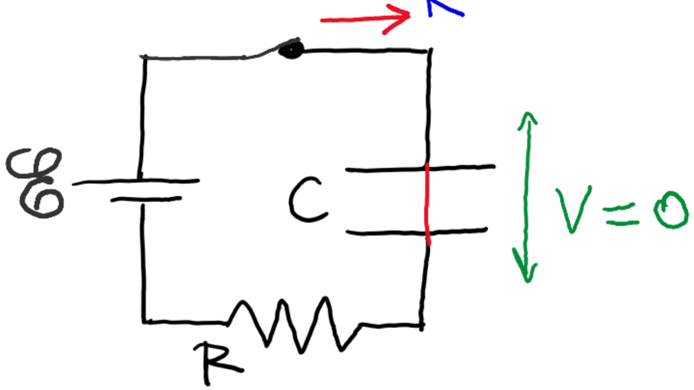
$$V(t) = \frac{q(t)}{C}$$

$$i(t) = \frac{\mathcal{E}}{R} e^{-t/\tau}$$



At $t=0$

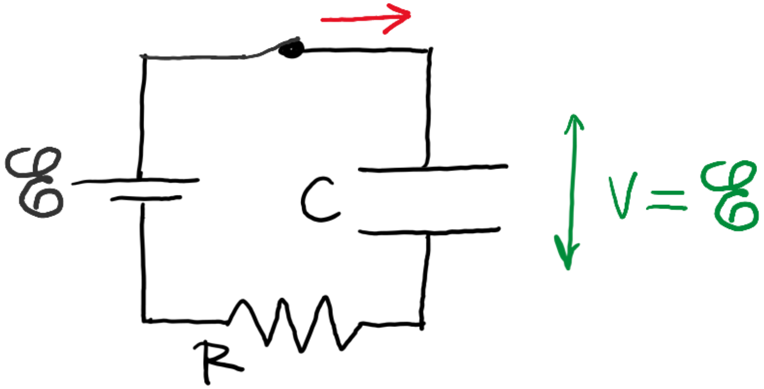
$i = \frac{\mathcal{E}}{R}$



The capacitor acts as a connecting wire

At $t=\infty$

$i = 0$



The capacitor acts as a broken wire

Dimension of $\tau \equiv RC$

$$[\tau] = [R][C] = \frac{\cancel{[V]}}{[i]}\frac{[Q]}{\cancel{[V]}} = \frac{[Q]}{[i]} = \frac{\cancel{[Q]}}{\frac{\cancel{[Q]}}{[t]}} = [t]$$

$V = iR$

$R = \frac{V}{i}$

$Q = CV$

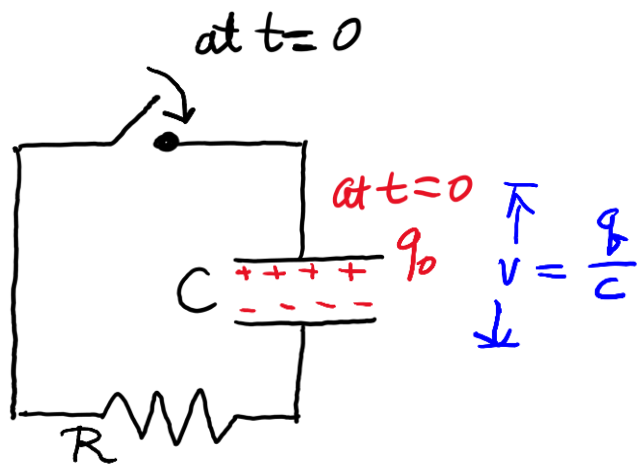
$C = \frac{Q}{V}$

$i = \frac{Q}{t}$

Example

In terms of RC, when will the charge on the capacitor be half of its initial value?

Discharging



$\frac{q_0}{2}$

$$q(t) = q_0\, e^{-t/\tau}$$

$$\frac{\cancel{q_0}}{2} = \cancel{q_0}\, e^{-t/\tau}$$

$$\frac{1}{2} = e^{-t/\tau}$$

$$\ln \frac{1}{2} = \ln e^{-t/\tau}$$

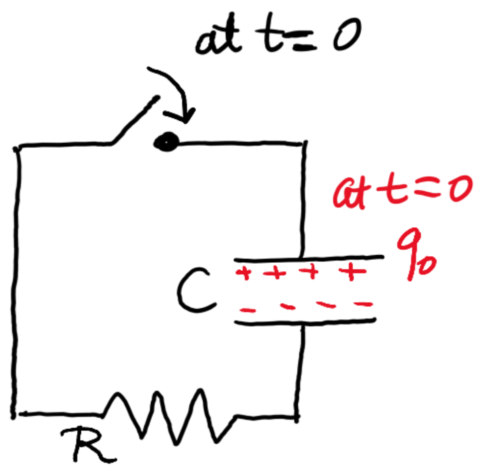
$$-\ln 2 = -t/\tau$$

$$t = \tau \ln 2 = 0.69\, \tau = 0.69\, RC$$

Example

In terms of RC, when will the energy stored in the capacitor be half of its initial value?

Discharging



$$U(t) = \frac{q^2(t)}{2C} = \frac{\left(q_0 e^{-t/\tau}\right)^2}{2C}$$

$$U(t) = \boxed{\frac{q_0^2}{2C}}\, e^{-2t/\tau}$$

$U(0)$ = initial energy stored in the capacitor

$\frac{U(0)}{2}$

$$U(t) = U(0)\, e^{-2t/\tau}$$

$$\frac{\cancel{U(0)}}{2} = \cancel{U(0)}\, e^{-2t/\tau}$$

$$\frac{1}{2} = e^{-2t/\tau} \Longrightarrow \ln\frac{1}{2} = \ln e^{-2t/\tau}$$

$$\Longrightarrow -\ln 2 = -2\frac{t}{\tau} \Longrightarrow t = \frac{\ln 2}{2}\tau$$

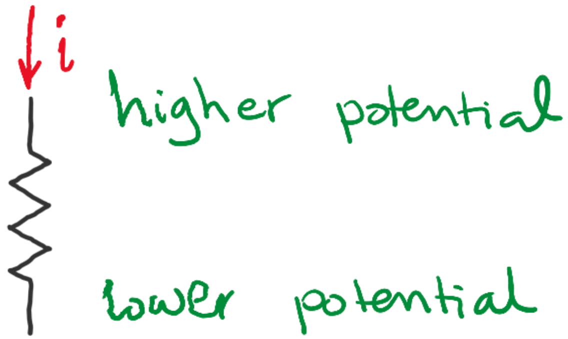
$$\Longrightarrow t = 0.35\ RC$$

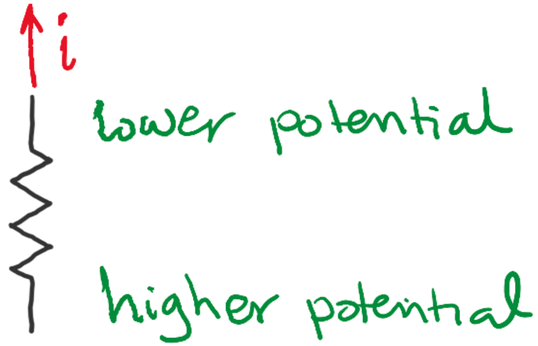
Review

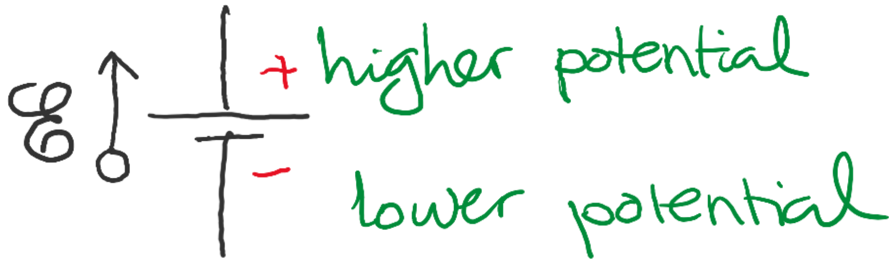
Loop rule: $\sum_{\text{Loop}} \Delta V = 0$

Junction Rule: Sum of currents in = Sum of currents out

For a resistor, the current flows from higher to lower potential







Example

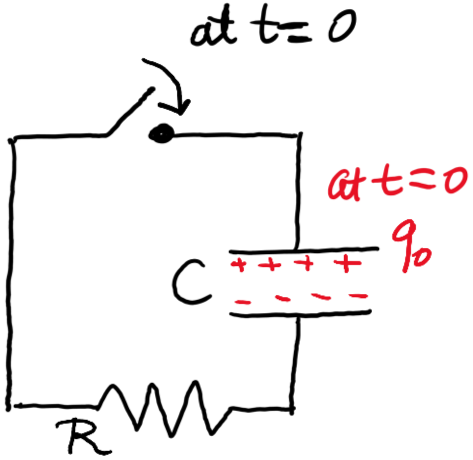
In terms of RC, when will the energy stored in the capacitor be half of its initial value?

$$U(t) = \frac{q^2(t)}{2C} = \frac{\left(q_0 e^{-t/\tau}\right)^2}{2C}$$

$$U(t) = \boxed{\frac{q_0^2}{2}} e^{-2t/\tau}$$

$U(0)$ = initial energy stored

Discharging



$$U = \frac{q^2}{2C} = \frac{q_0^2}{2C} e^{-2t/\tau}$$

$U_0 \equiv U$

$$U(t) = U_0 e^{-2t/\tau}$$

$\frac{U_0}{2}$

$$\frac{\cancel{U_0}}{2} = \cancel{U_0} e^{-2t/\tau}$$

$$\ln 2 = \frac{2t}{\tau}$$

$$t = \frac{RC}{2} \ln 2 = 0.35 R$$