This is not a replacement for your textbook. It is essential to study from your textbook!

Objectives

Ch26: Currents and resistance

26-1 Electric currents

26-2 Current density

26-3 Resistance and resistivity

26-4 Ohm's law

26-5 Power in electric circuits

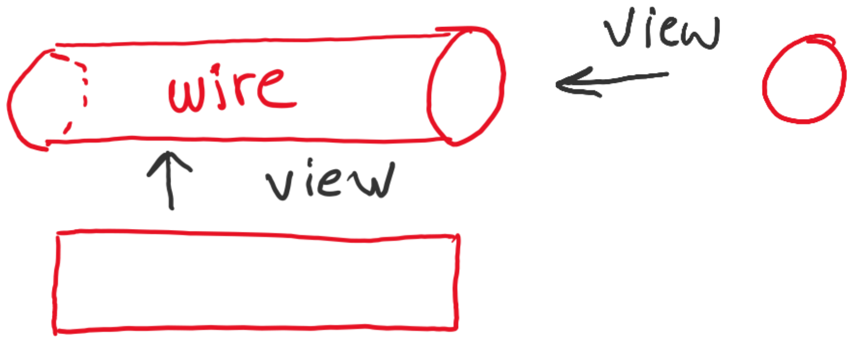
26-1 Electric current

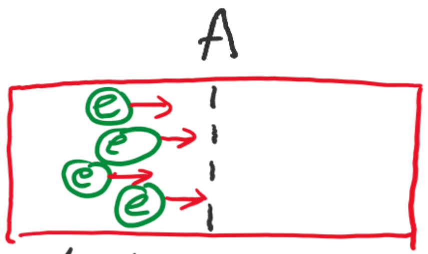
Electric current through a surface = the rate of **net** charge through the surface

current through a surface

$$i = \frac{dq}{dt}$$

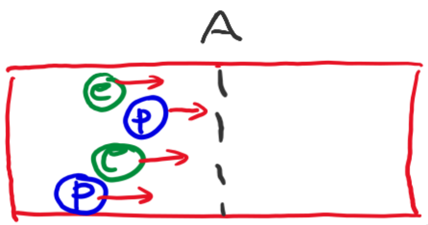
charge dq passes through the surface in time dt





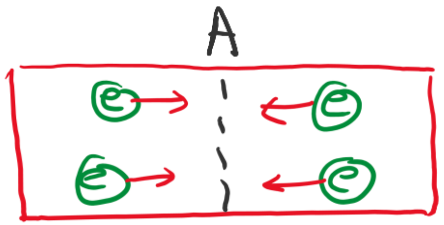
4 electrons moving to the right

current $\neq 0$



2 electrons and 2 protons moving to the right

current $= 0$



2 electrons moving right
2 electrons moving left

current $= 0$

current through a surface $i = \dfrac{dq}{dt}$ charge dq passes through the surface in time dt

Ampere (A) is the SI unit for current

$$\text{Ampere} = \frac{\text{Coulomb}}{\text{second}} \qquad A = \frac{C}{s}$$

$dq = i\,dt \Rightarrow$

Total charge q passes through the surface in time t

$$q = \int_0^t i\,dt$$

current through a surface $\searrow$

$$i = \frac{dq}{dt}$$

charge dq passes through the surface in time dt

steady-state condition $\equiv$ The charge density does not vary with time

Under a steady-state condition,

$$i = \frac{q}{t}$$

charge q passes through the surface in time t

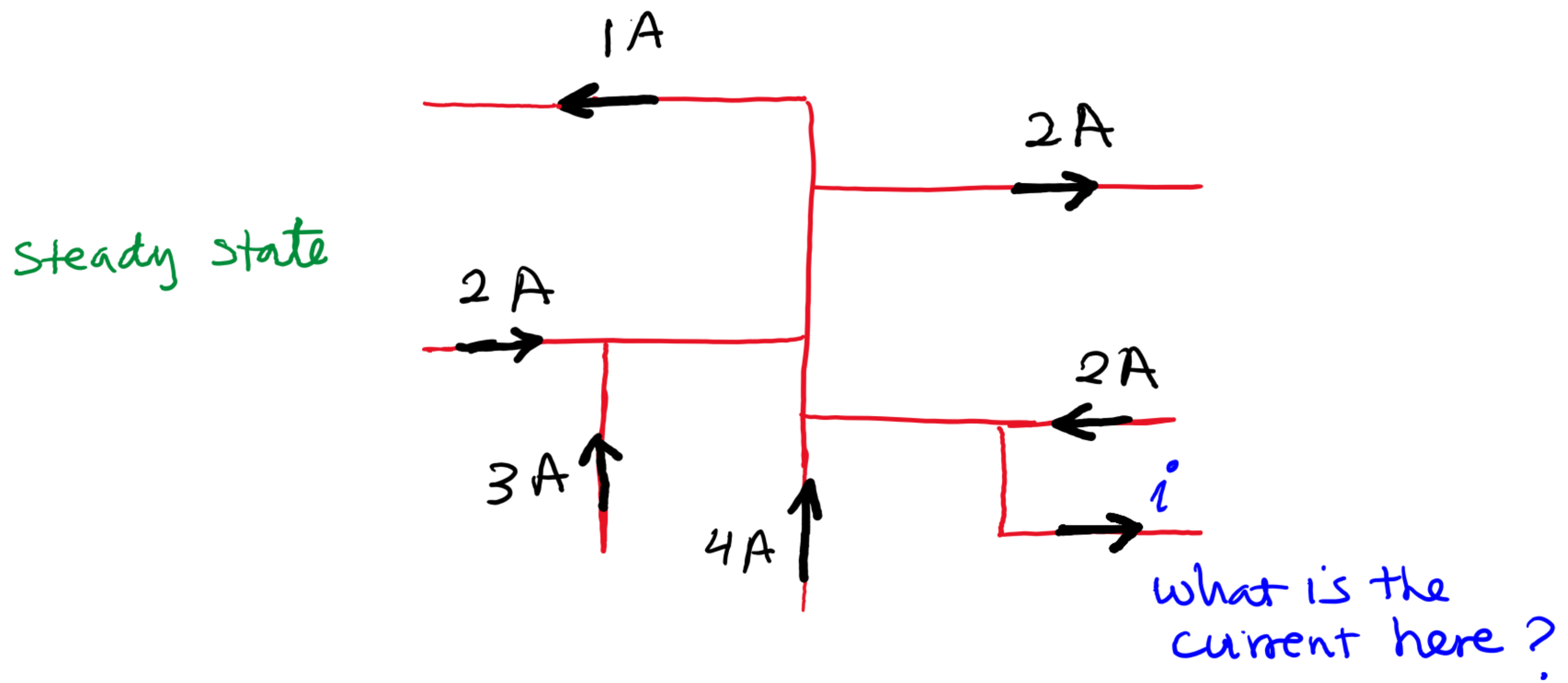
Under steady-state conditions,

the current is the same in any cross section.

wire (side view)

Since charge is conserved, any electron passes through A_1 must pass through A_2 and A_3.

Question



Answer

Charge is conserved.

current in = current out

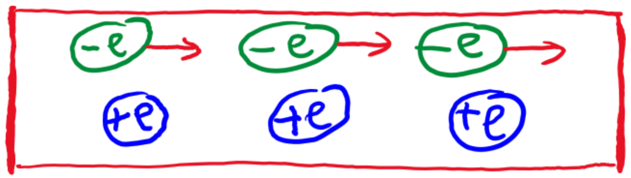
given current in $= 2+2+3+4 = 11A$

given current out $= 2+1 = 3A$

$i = 8A$ out

current direction

Real word



conductor

Electrons can move

Electrons ≡ charge carriers

Positive ion are fixed

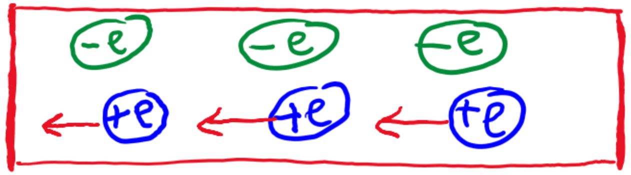
Current arrow is in the <u>opposite</u> direction to the motion of eletrons

current

$\longleftarrow$

i

Convention



conductor

Positive ions can move

Positive ions ≡ charge carriers

Electrons are fixed

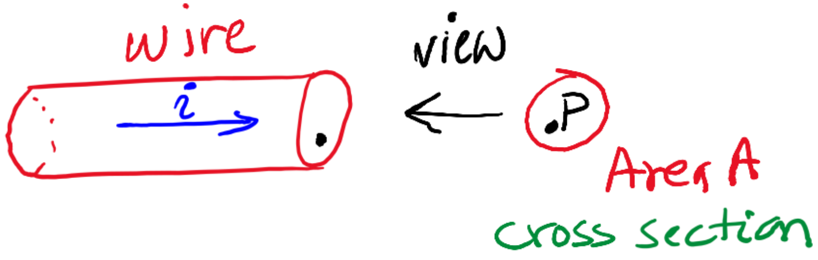
Current arrow is in the <u>same</u> direction as the motion of positive ions

current

$\longleftarrow$

i

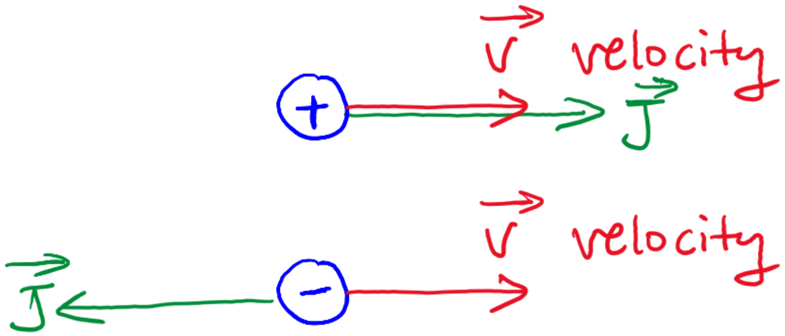
Current Density



$$J \equiv \text{current density at point P} = \frac{\text{current}}{\text{Area}} = \frac{i}{A}$$

Current density is a <u>vector quantity</u>

<u>direction</u>



<u>magnitude</u>

$$J = \frac{\text{current}}{\text{area normal to velocity}}$$

$$J = \frac{\text{current}}{\text{area normal to velocity}}$$

$$i = \int_{\text{surface}} \vec{J} \cdot d\vec{A}$$

area vector

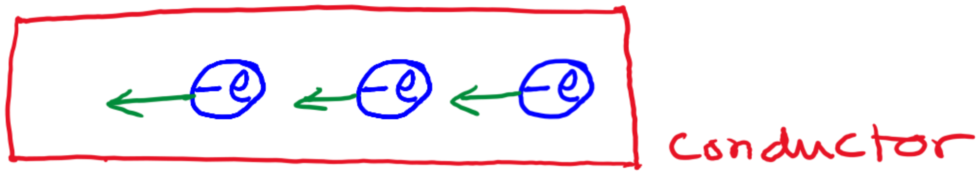
magnitude : Area of the surface

direction : Normal to the surface

If $\vec{J}$ is uniform and parallel to $d\vec{A}$

$$\Longrightarrow J = \frac{i}{A}$$

Question:



what is the direction of

Current ? Rightward →

Current density ? Right ward →

Electric field ? Right ward →

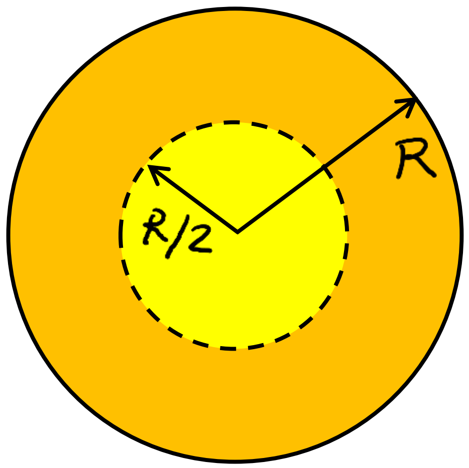
Example

uniform current density

$J = 2000 \frac{A}{m^2}$

$R = 1$ cm

What is the current in area between $R/2$ and R?



cross section of a wire

Answer

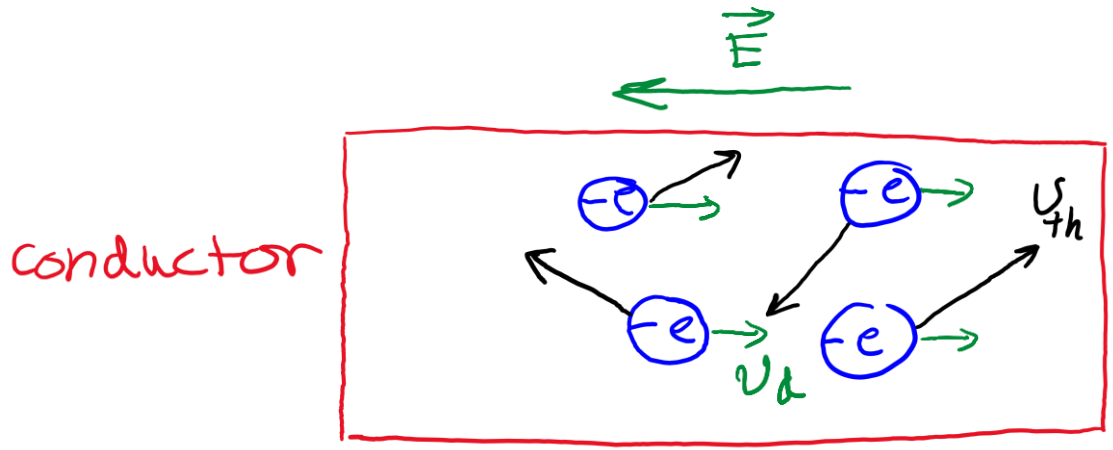
$$i = JA = J\left(\pi R^2 - \pi\left(\frac{R}{2}\right)^2\right)$$

$$= J\left(\frac{3}{4}\pi R^2\right) = 0.47\ A$$

Drift Speed

Conduction electrons: free to move

collide with atoms of the metal



motion due thermal energies:

→ Random motion

→ $v_{th} \approx 10^6$ m/s

→ has no specific direction

⟹ current = 0

motion due to $\vec{E}$:

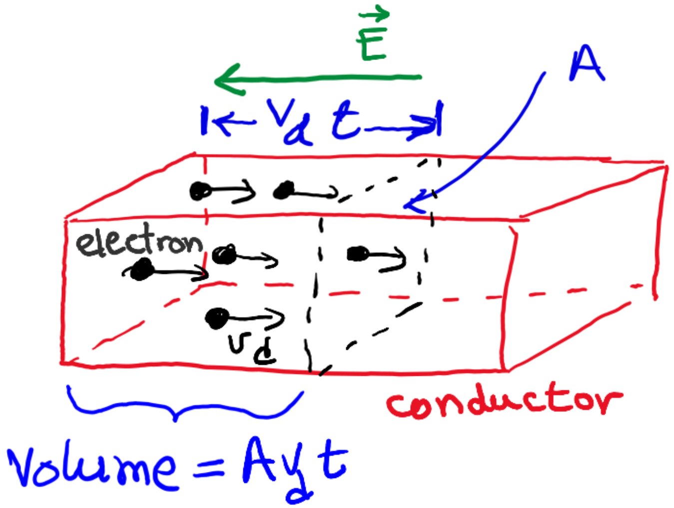
→ drift motion

→ drift speed $v_d \approx 10^{-5}$ m/s

→ opposite to the direction of $\vec{E}$

⟹ current ≠ 0

Drift Speed



Assume all electrons have the same drift speed v_d

charge crossing area A in time t

$$q = e\,(\text{number of electrons in volume } A v_d t)$$

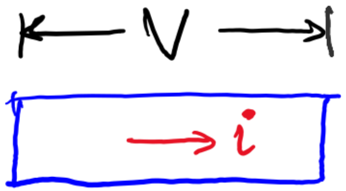
$$q = e\,(\text{number of electons per unit volume})(A v_d t)$$

$$q = e\, n A v_d t$$

current $i = \dfrac{q}{t} = e n A v_d$

current density $J = \dfrac{i}{A} \Rightarrow \boxed{J = e n v_d}$

26-3 Resistance and Resistivity



Resistor

Symbol

Resistor

Potential difference

current

$$V = R\, i$$

Resistance

Property of an objet

Electric field

current density

$$\vec{E} = \rho\, \vec{J}$$

Resistivity

Property of a material

Ohm (Ω) is the SI unit for resistance

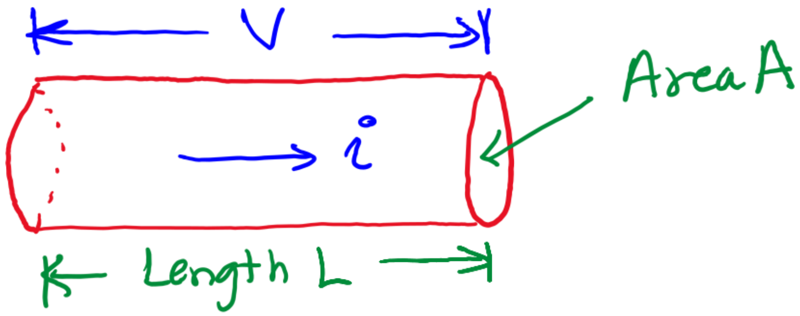
$$\text{ohm} = \frac{\text{Volt}}{\text{Ampere}}$$

$$\Omega = \frac{V}{A}$$

Conductivity $\sigma = \dfrac{1}{\rho}$

$$\vec{J} = \sigma \vec{E}$$

Resistance and Resistivity



$$V = R\,i$$

$$V = EL$$

$$J = \frac{i}{A}$$

$$EL = RAJ$$

$$E = \frac{RA}{L}J$$

$$E = \rho J$$

$$\rho = \frac{A}{L}R$$

$$\boxed{R = \frac{L}{A}\rho}$$

Resistivity and Temperature

Temperature

$$\rho - \rho_0 = \rho_0 \alpha (T - T_0)$$

resistivity at T

resistivity at T_0

Temperature coefficient of resistivity

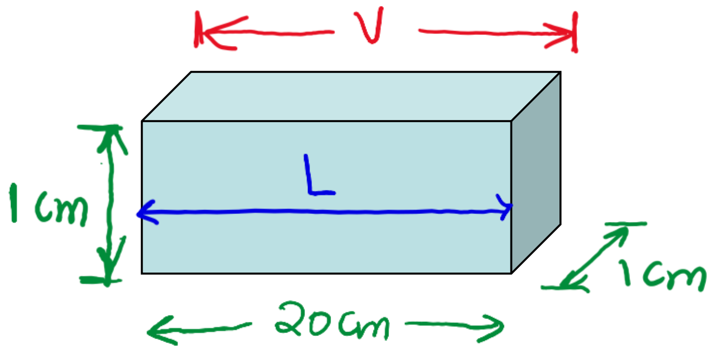
reference temperature.

$$R - R_0 = R_0 \alpha (T - T_0)$$

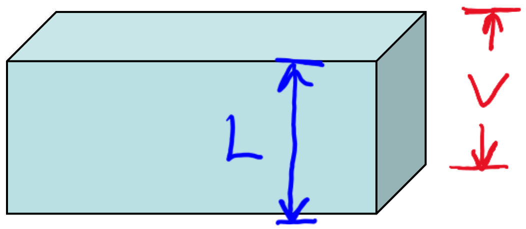
Example

What is the resistance of the iron block?

Resistivity of iron $= 10^{-7}\ \Omega\cdot m$

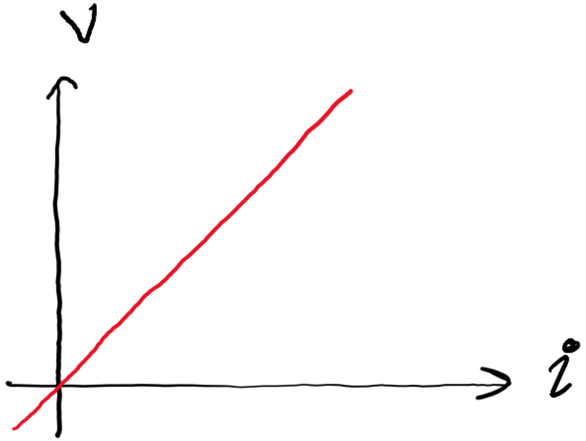


$$R = \rho \frac{L}{A} = 10^{-7} \frac{0.2}{(0.01)(0.01)} = 200\,\mu\Omega$$



$$R = \rho \frac{L}{A} = 10^{-7} \frac{0.01}{(0.2)(0.01)} = 0.5\,\mu\Omega$$

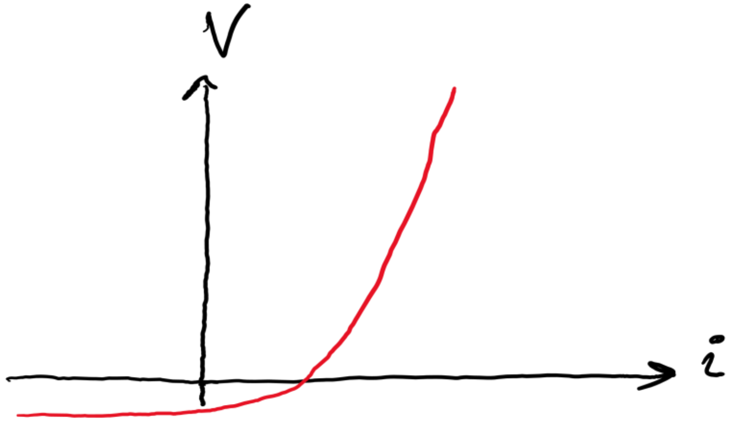
26-4 Ohm's Law



$V = R\,i$

constant

The device obeys omh's Law



$V = R\,i$

function of current

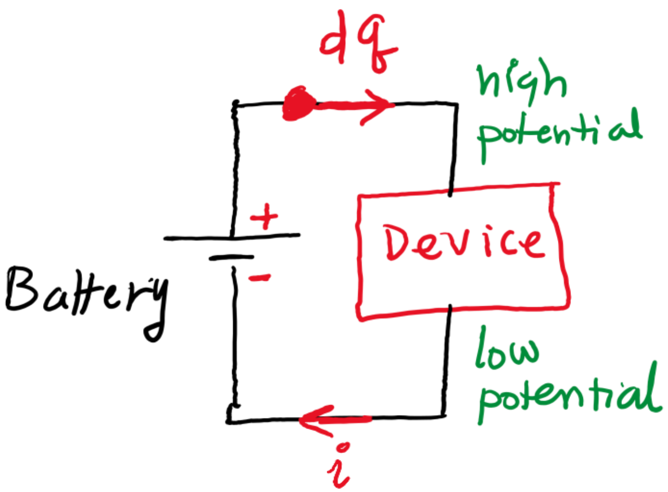
The device does <u>not</u> obey ohm's Law

26-5 Power in Electric Circuit

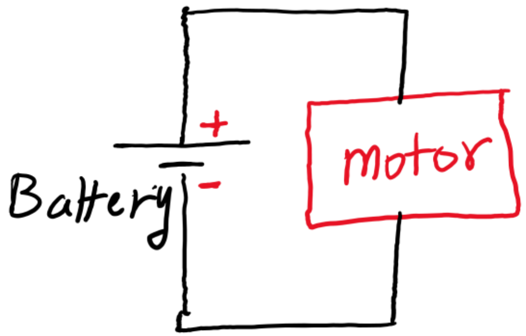
Conservation of energy

$$\begin{bmatrix}\text{The rate at which the} \\ \text{device consumes energy}\end{bmatrix} = -\begin{bmatrix}\text{The rate at which the potential} \\ \text{energy of charge } dq \text{ changes} \\ \text{as it moves through the device}\end{bmatrix}$$

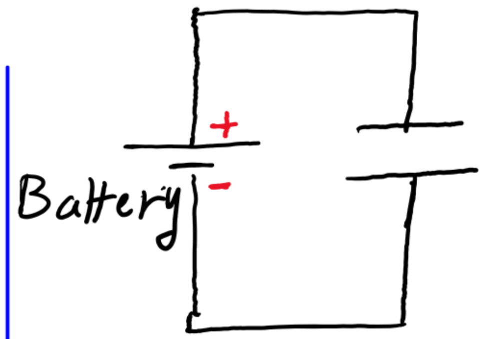
Power Cosumed by the device $= -\dfrac{dU}{dt} = -\dfrac{dq(-V)}{dt} = iV$



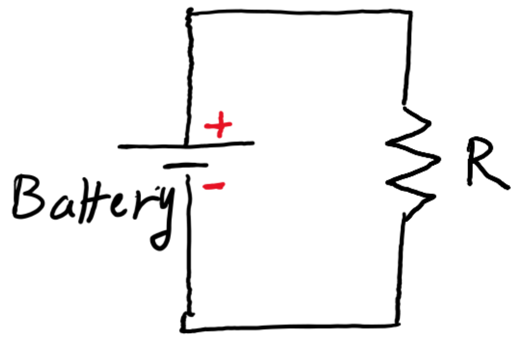
$$\boxed{P = iV}$$



Energy from the battery is transferred at a rate of $P = iV$ to rotate the motor



Energy from the battery is transferred at a rate of $P = iV$ to charge the capacitor

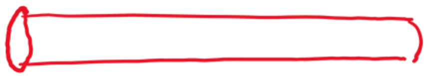


Energy from the battery is transferred at a rate of $P = iV$ to heat up the resistor (collisions between electrons and atoms)

For a resistor

$$P = iV \xrightarrow{V = Ri} \begin{cases} P = i^2 R \\ P = \dfrac{V^2}{R} \end{cases}$$

Example



wire

$R = 50\,\Omega$

$V = 100\,V$

At which rate is the energy dissipated?

Answer

$$P = \frac{V^2}{R} = 200\ W$$