Objectives

Ch 24: Electric Potential

24-1 Electric Potential

24-2 Equipotential surfaces and the electric field

24-3 Potential due to a charged particle

24-4 Potential due to a dipole

24-6 Calculating the field from the potential

24-7 Electic potential energy of a system of charged particles

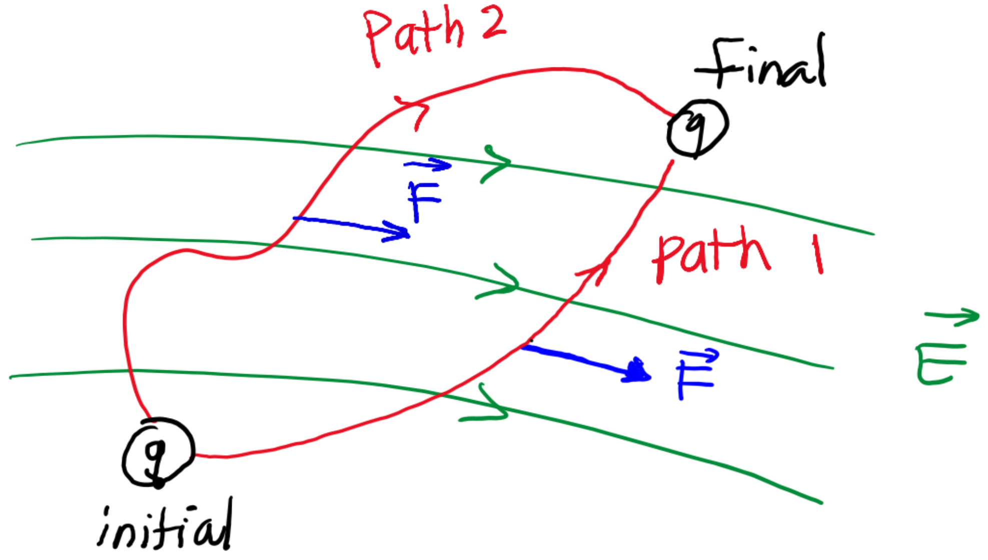
24-8 Potetial of charged isolated conductor

24-1 Electric Potential

The electrostatic force is a conservative force.

⇒ The work done by an electrostatic force does not depend on path.

⇒ We can define an electric potential energy to to a system of charged particles.

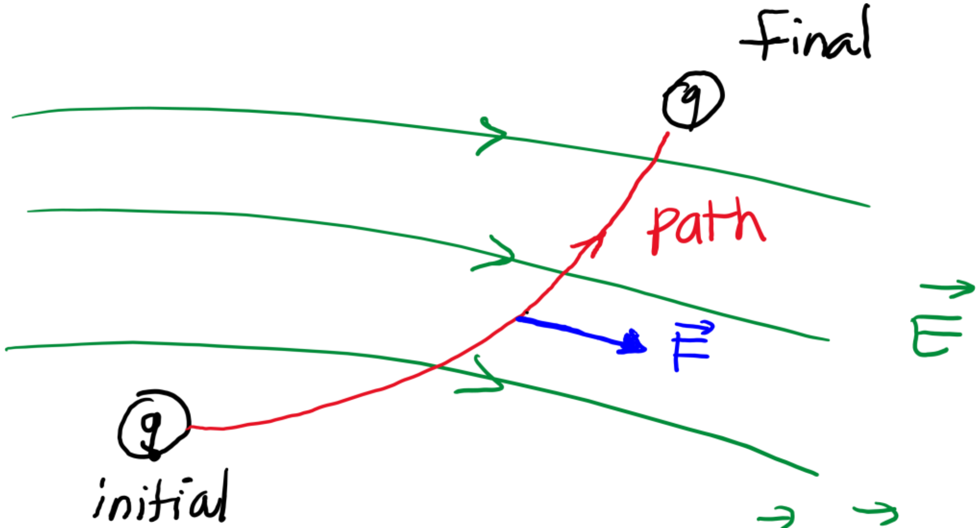


$$W_{path1} = W_{path2}$$

$$W \equiv -\Delta U$$

W: Work done by the electrostatic force on the particle

ΔU: change in electric potential energy of the system.



$$\vec{F} = q\vec{E}$$

$$W_{field} = \int_i^f \vec{F} \cdot d\vec{s} = q \int_i^f \vec{E} \cdot d\vec{s} \equiv -\Delta U \equiv -q\Delta V$$

Electric Potential Energy

Electric Potential

$$V \equiv \frac{U}{q}$$

Electric Potential difference

$$\boxed{\Delta V \equiv -\int_i^f \vec{E} \cdot d\vec{s}}$$

Electric Potential difference

$$\boxed{\Delta V \equiv -\int_i^f \vec{E} \cdot d\vec{s}}$$

The SI unit for the electric potential is volt

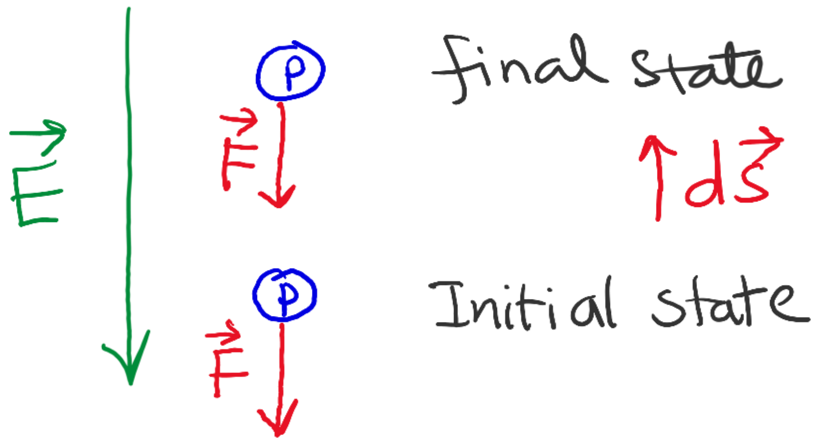
$$V \equiv \frac{U}{q} \Rightarrow 1 \text{ volt} = \frac{1 \text{ joule}}{1 \text{ coulomb}}$$

The SI unit for $\vec{E}$ $=$ $\dfrac{\text{volt}}{m}$ $=$ $\dfrac{\text{newton}}{\text{coulomb}}$

$\Delta V = -\int \vec{E} \cdot d\vec{s}$ $\qquad$ $\vec{F} = q\vec{E}$

Question

Does the electric field do positive or negative work on the proton?



$$W = \int \vec{F} \cdot d\vec{s} \qquad \vec{F}\downarrow \quad \uparrow d\vec{s} \implies \vec{F} \cdot d\vec{s} < 0 \implies \text{Negative work.}$$

Does the electric potential energy of the proton increase or decrease?

$$\Delta U = -W \implies$$ The electric potential energy of the proton increases.

Review: Work-kinetic energy theorem

Newton's second law $F = ma$

$$F = m\frac{dv}{dt}$$

$$F\,dx = m\frac{dv}{dt}\,dx$$

$$F\,dx = m\frac{dx}{dt}\,dv$$

$$\int_i^f F\,dx = \int_i^f m v\,dv$$

$$W = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

$$\boxed{W = \Delta K}$$ Work-kinetic energy theorem

$$\Delta K = W$$ Work-kinetic energy theorem

$$\Delta K = W_{field} + W_{applied}$$

$$-\Delta U = -q\Delta V$$

$$\Delta K = -\Delta U + W_{app}$$

If the kinetic energy of the particle does not change

$$K_i = K_f$$

$$0 = -\Delta U + W_{app}$$

$$W_{app} = \Delta U = q\Delta V$$

If there is no external force

$$W_{app} = 0$$

$$\Delta K = -\Delta U + 0$$

$$\Delta K + \Delta U = 0$$

$$\Delta K + q\Delta V = 0$$

Electron-Volt

$$V = \frac{U}{q}$$

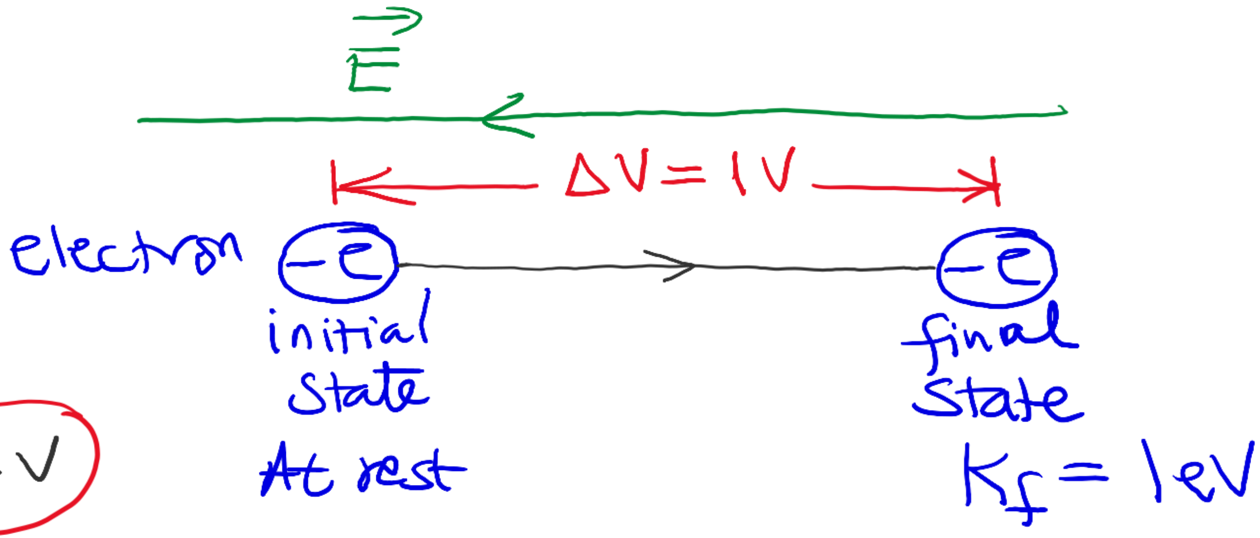
The SI unit for energy is joule $=$ coulomb $\times$ volt

Another unit for energy is electron volt (eV)

$$\Delta K = -\Delta U = -q\,\Delta V$$

$$K_f - \cancel{K_i}^{\,0} = -(-1e)(1V)$$

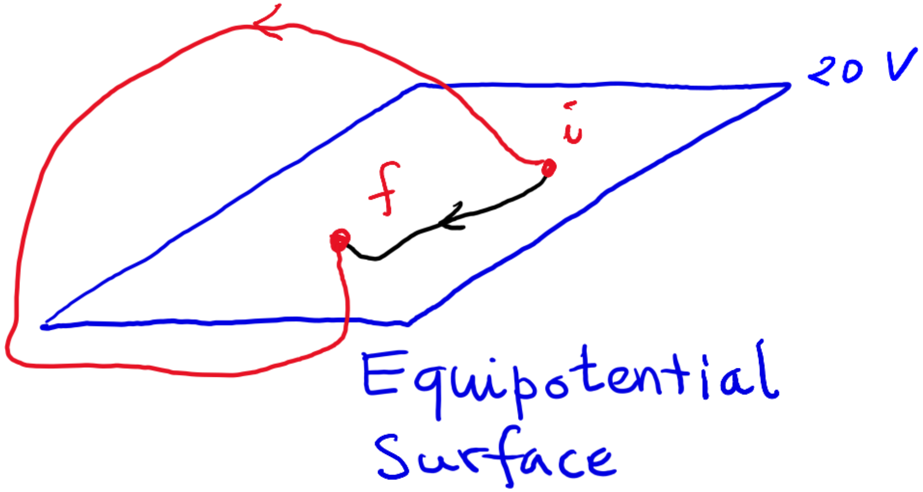
$$K_f = 1\,eV = 1.6\times10^{-19}\,\underbrace{C\cdot V}_{J}$$



$$\boxed{1eV = 1.6\times10^{-19}\,J}$$

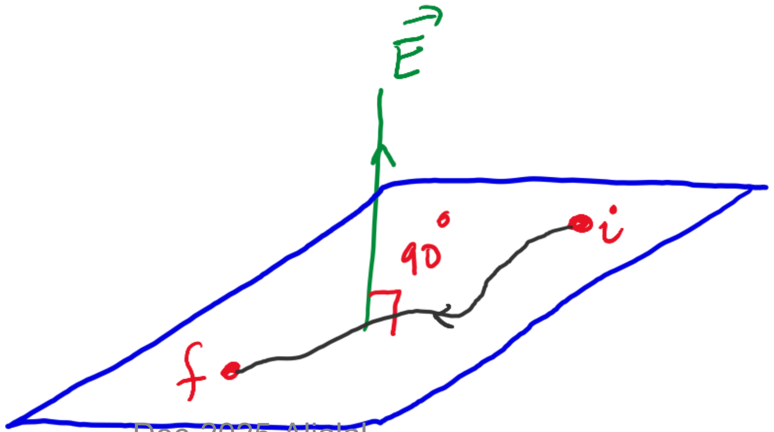
24-2 Equipotential Surfaces

All points on an equipotential surface have the same electric potential.



$$W_{field} = -q\,\Delta V = 0$$

Equipotential surfaces are always perpendicular to the electric field.

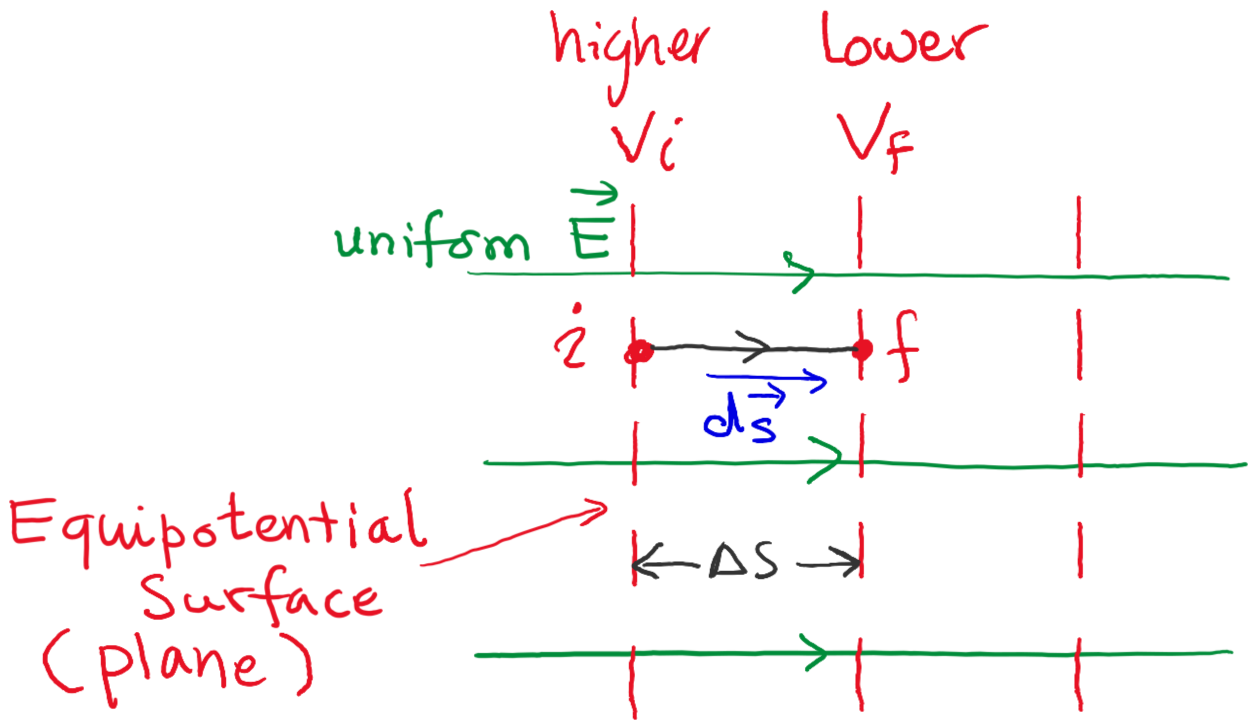


$$W_{field} = 0 = q\int_i^f \vec{E}\cdot d\vec{s} = q\int_i^f E\,ds\,\cos\theta$$

$$\Rightarrow \cos\theta = 0 \Rightarrow \theta = 90^\circ$$

24-2 Equipotential Surfaces

Equipotential surfaces for uniform electric field are planes

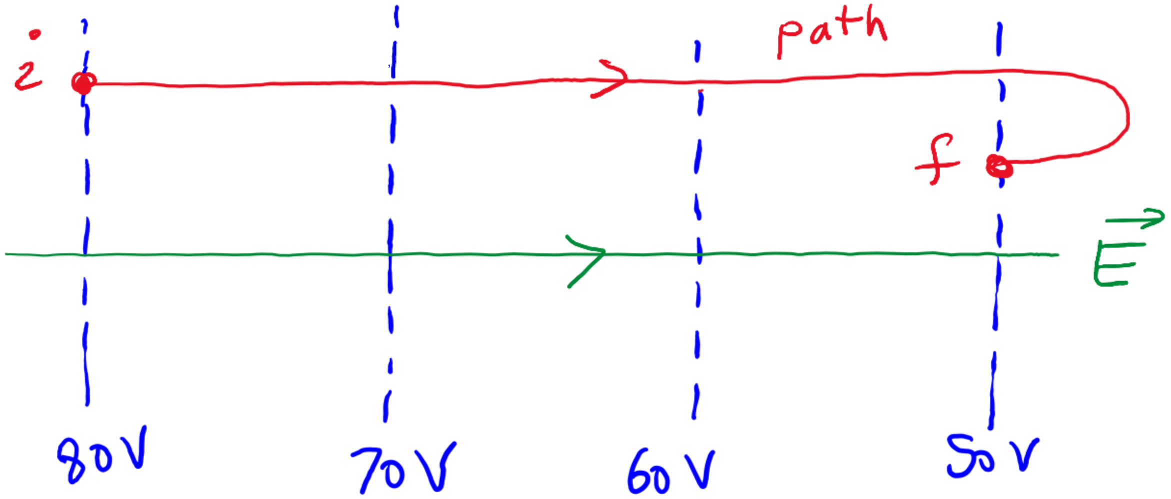


$$\Delta V = -\int_i^f \vec{E}\cdot d\vec{s} = -\int_i^f E\,ds = -E\int_i^f ds = -E\Delta s < 0$$

$$V_f - V_i < 0 \quad \Longrightarrow \quad V_f < V_i$$

Electric fields point toward lower electric potentials.

Question



What is the direction of the electric field ?

Electric fields point toward lower electric potentials $\longrightarrow \vec{E}$

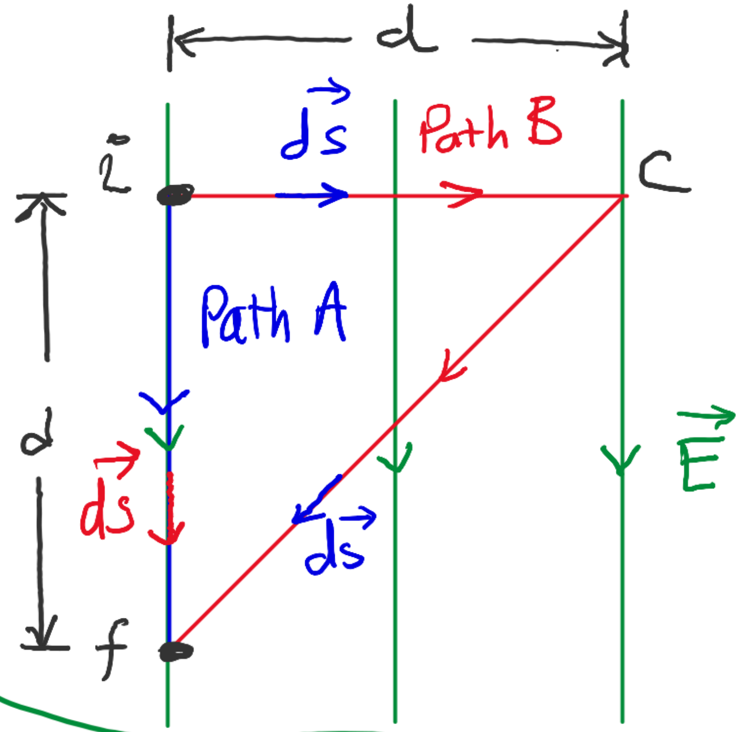
What is the work we do to move an electron along the path ?

Assume the electron is at rest at the initial and final points

$$\Delta K = -\Delta U + W_{app} \Rightarrow W_{app} = \Delta U = (-e)\,\Delta V = -e\,(50-80)\,V = +30\ eV$$

Example: ΔV does not depend on path

Find the electric potential difference between i and f using path A and path B



Path A

$$\Delta V = -\int_i^f \vec{E}\cdot\vec{ds} = -\int_i^f E\,ds = -E\int_i^f ds = -Ed$$

same

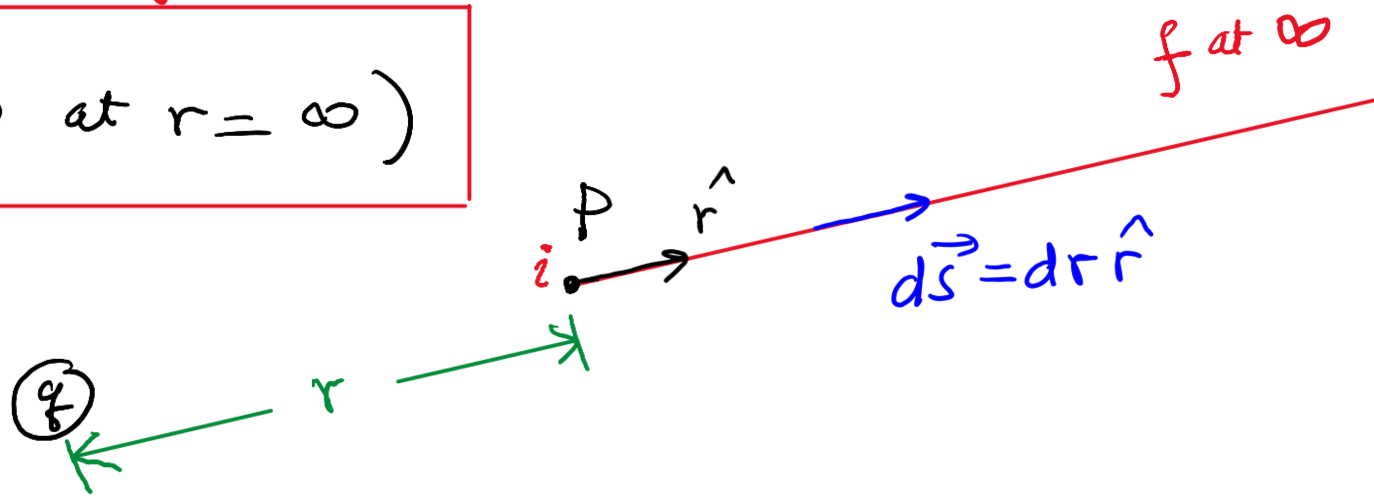
Path B

$$\Delta V = \underbrace{-\int_i^c \vec{E}\cdot\vec{ds}}_{=0} - \int_c^f \vec{E}\cdot\vec{ds} = -\int_c^f E\,ds\cos 45 = -\frac{1}{\sqrt{2}}E\underbrace{\int_c^f ds}_{=\sqrt{2}d} = -Ed$$

$\vec{E}\perp\vec{ds} \Rightarrow \vec{E}\cdot\vec{ds} = 0$

24-3 Electric Potential Due to a Charged Particle

$$\boxed{V = \frac{kq}{r} \quad (\text{reference } V=0 \text{ at } r=\infty)}$$



$V > 0$ for $q > 0$

$V < 0$ for $q < 0$

$$\Delta V = -\int_i^f \vec{E} \cdot d\vec{s}$$

$$V_f - V_i = -\int_{r_i}^{\infty} \frac{kq}{r^2}\,\hat{r} \cdot dr\,\hat{r} = -kq\int_{r_i}^{\infty} \frac{dr}{r^2} = kq\left[\frac{1}{r}\right]_{r_i}^{\infty}$$

$$0 - V_i = -\frac{kq}{r_i} \implies V = \frac{kq}{r}$$

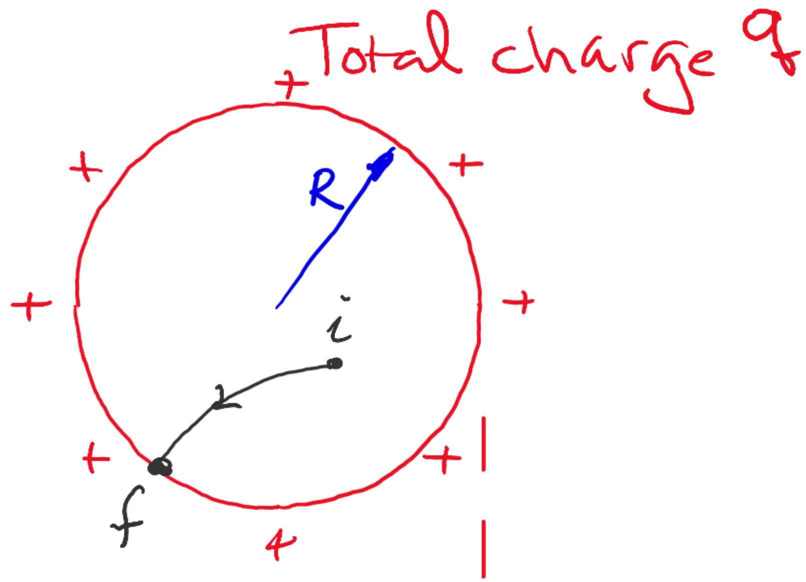
Electric potential due a spherical shell with uniform charge distribution

<u>outside</u>

Similar to a charged particle with charge q located at the center.

$$\vec{E} = k\frac{q}{r^2}\hat{r}$$

$$V = k\frac{q}{r}$$



<u>inside</u>

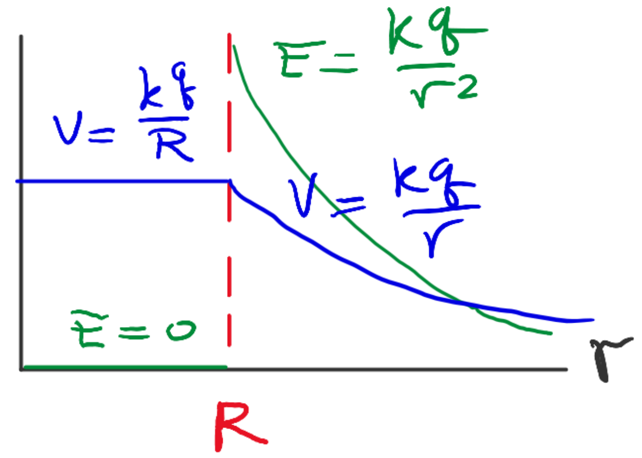
$$E = 0$$

$$V = k\frac{q}{R} = \text{constant}$$

$$= \text{potential on the surface.}$$

$$\Delta V = -\int_i^f \underbrace{\vec{E}}_{=0}\cdot d\vec{s} = 0$$

$$V_f - V_i = 0 \Rightarrow V_i = V_f$$



Example

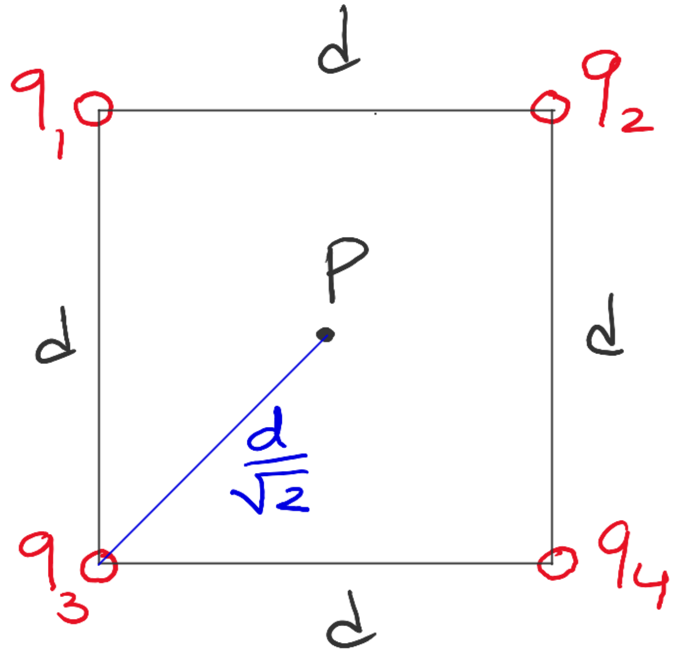
$d = 1.3\ m$

$q_1 = +12\ nC$

$q_2 = -24\ nC$

$q_3 = +31\ nC$

$q_4 = +17\ nC$



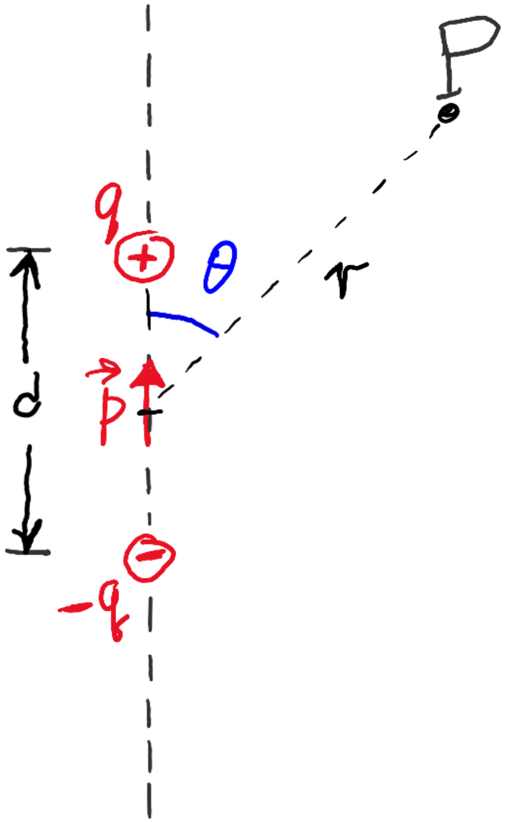
What is the electric potential at the center at P

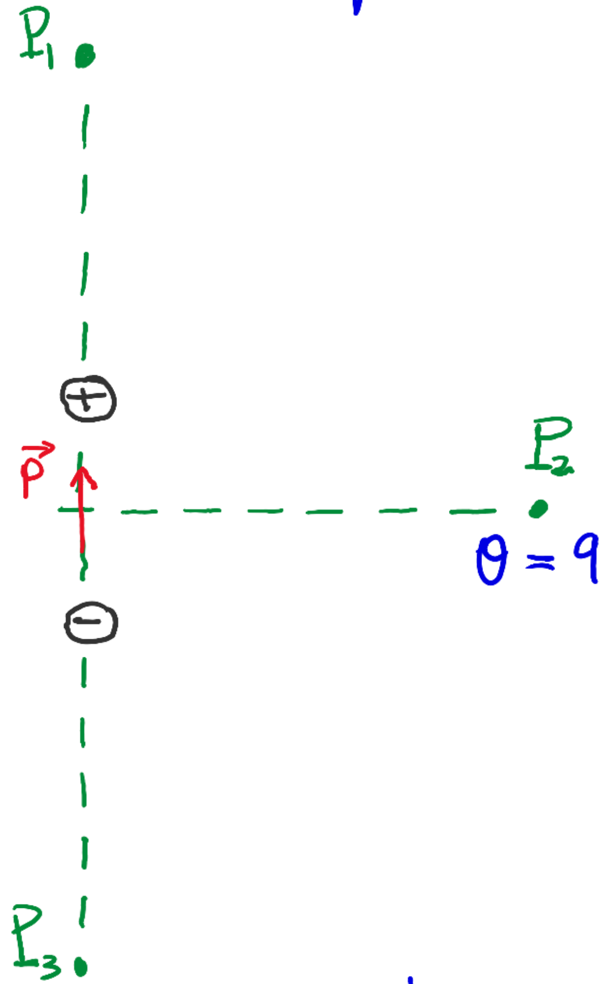
$$V = V_1 + V_2 + V_3 + V_4$$

$$V = \frac{kq_1}{r_1} + \frac{kq_2}{r_2} + \frac{kq_3}{r_3} + \frac{kq_4}{r_4} = \frac{k}{\frac{d}{\sqrt{2}}}(q_1 + q_2 + q_3 + q_4) = 350\ V$$

24-4 Potential Due to an Electric Dipole

For $r \gg d$ $\quad V = k\dfrac{p\cos\theta}{r^2}$ $\quad p = qd$



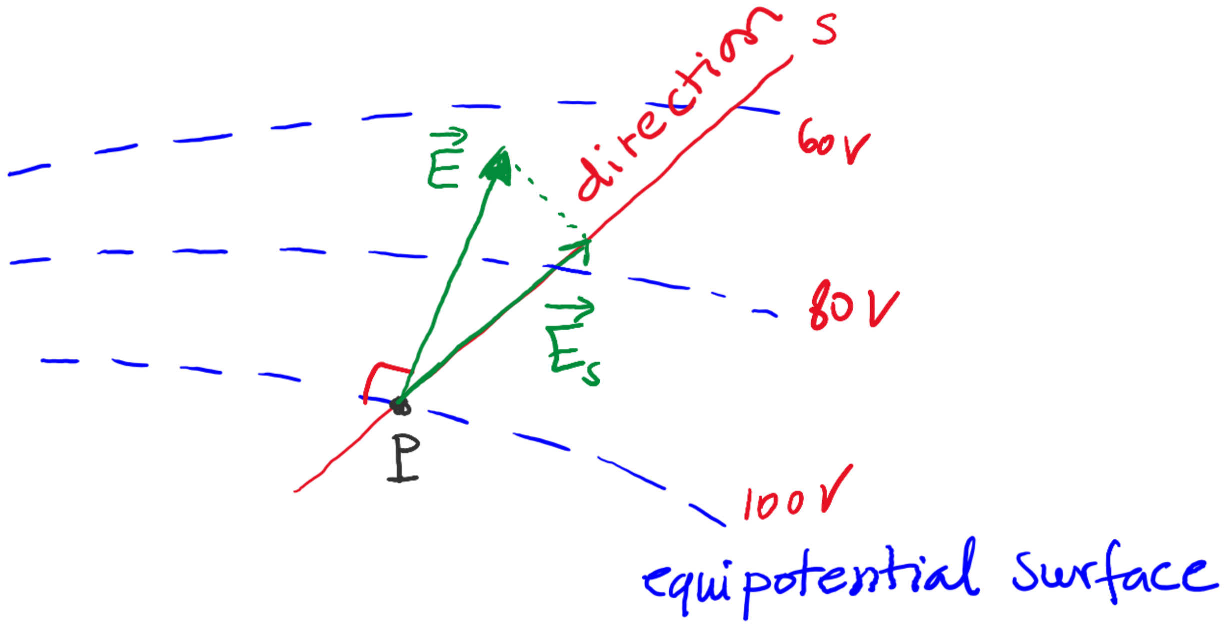


$\theta = 0 \Rightarrow V = \dfrac{kp}{r^2}$

$\theta = 90^\circ \Rightarrow V = 0$

$\theta = 180^\circ \rightarrow V = -\dfrac{kp}{r^2}$

24-6 Calculating the field from the potential

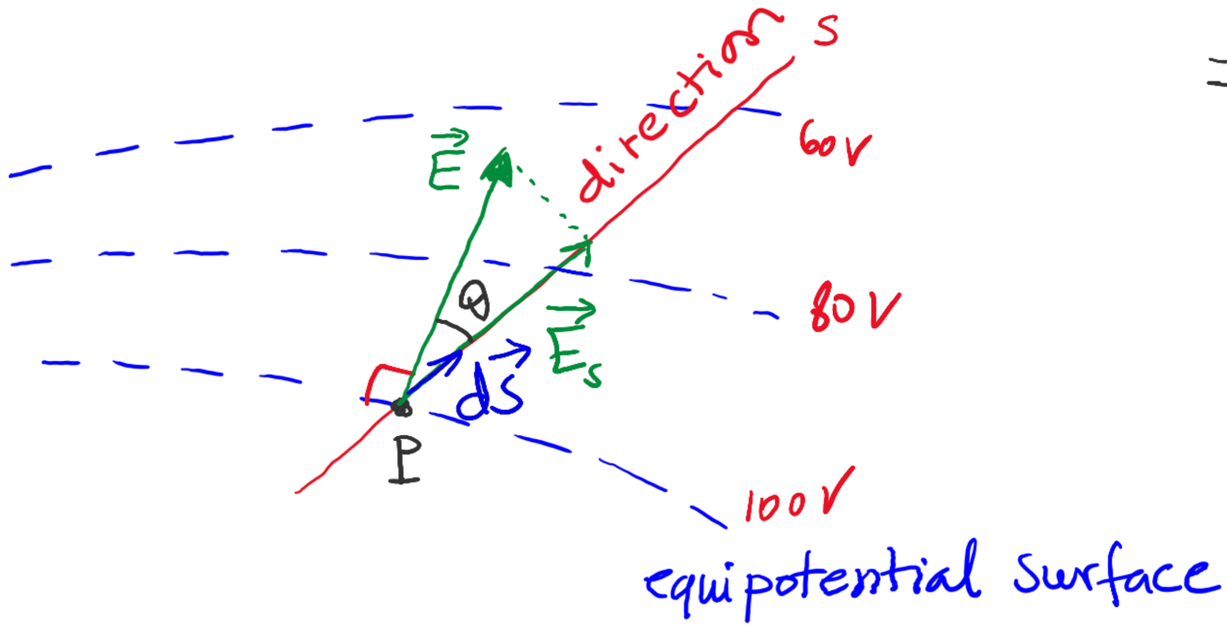


The component of electric field along direction s

$$E_s = -\frac{\partial V}{\partial s}$$

Partial derivative
treat all variables
except s as constants.

Derivation



$$\Delta V = -\int \vec{E} \cdot d\vec{s}$$

$$\Rightarrow dV = -\vec{E} \cdot d\vec{s}$$

$$dV = -E\, ds \cos\theta$$

$$dV = -E \cos\theta\, ds$$

$$dV = -E_s\, ds$$

$$\Rightarrow E_s = -\left(\frac{dV}{ds}\right)$$ only with respect to s

$$\Rightarrow E_s = -\frac{\partial V}{\partial s}$$

The component of electric field along direction s

$$E_s = -\frac{\partial V}{\partial s}$$

Partial derivative treat all variables except s as constants.

Example

Find the electric field associated with

$$V(x,y,z) = \frac{\sigma}{2\varepsilon_0}\left(\sqrt{z^2+R^2} - z\right)$$

Solution

$$E_x = -\frac{\partial V}{\partial x} = 0$$

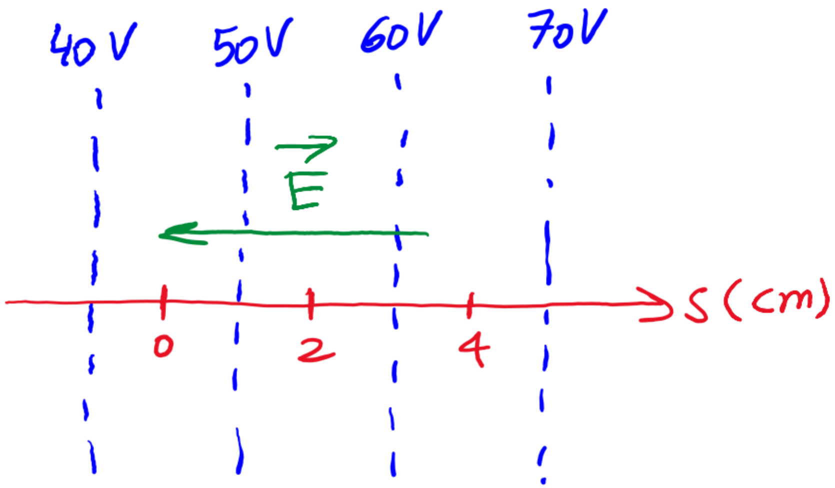
$$E_y = -\frac{\partial V}{\partial y} = 0$$

$$E_z = -\frac{\partial V}{\partial z} = -\frac{\sigma}{2\varepsilon_0}\left(\frac{1}{2}\frac{2z}{\sqrt{z^2+R^2}} - 1\right) = \frac{\sigma}{2\varepsilon_0}\left(1 - \frac{z}{\sqrt{z^2+R^2}}\right)$$

$$\vec{E} = \frac{\sigma}{2\varepsilon_0}\left(1 - \frac{z}{\sqrt{z^2+R^2}}\right)\hat{k}$$

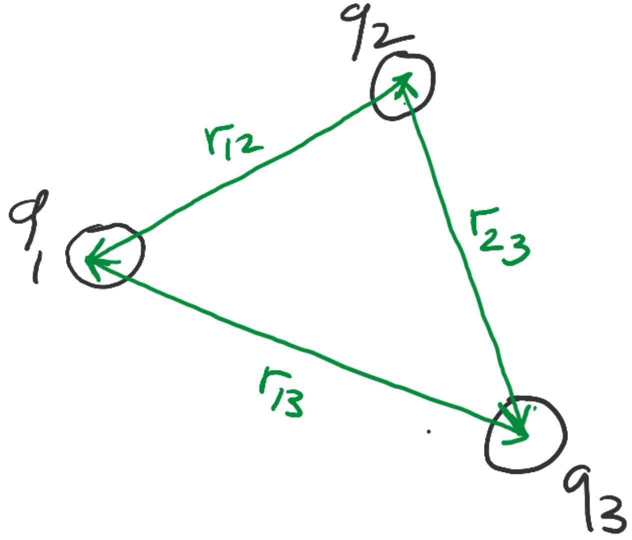
For uniform electric field, the equipotential surfaces are equally spaced.

$$E_s = -\frac{\partial V}{\partial s} \Longrightarrow E_s = -\frac{\Delta V}{\Delta s}$$



$$E_s = -\frac{60-50}{0.03-0.01} = -\frac{10}{0.02} = -500\ V/m$$

24-7 Electric Potential Energy of a System of charged Particles



The electric potential for a pair of particles

$$U = U_{12} + U_{13} + U_{23}$$

The electric potential energy of a system of three fixed point charges

U = the sum of the potential energies for every pair of particles.

For a pair of particles $\quad U_{ij} = k\dfrac{q_i q_j}{r_{ij}}$

k: $9\times10^{9}\ \dfrac{N\,m^2}{C^2}$

r_{ij}: distance between particle i and j

For a system of three particles $\quad U = k\dfrac{q_1 q_2}{r_{12}} + k\dfrac{q_1 q_3}{r_{13}} + k\dfrac{q_2 q_3}{r_{23}}$

Why is U = the sum of the potential energies for every pair of particles?

Initial state: all point charges are stationary at infinity

Assume $U_\infty = 0$

Final state: fixed point charges

q_1

q_2 q_3

U = Electric potential energy of the fixed point charges.

$$\Delta K = -\Delta U + W_{app,\infty}$$ ← work applied to bring from infinity

$$0 = -(U - U_\infty) + W_{app,\infty}$$

$$U = W_{app,\infty}$$

The electric potential energy of a system of fixed point charges

Work done to assemble the system from inifinite position

Why is U = the sum of the potential energies for every pair of particles?

$$U = W_{app,\infty}$$

First, bring q_1 from infinity

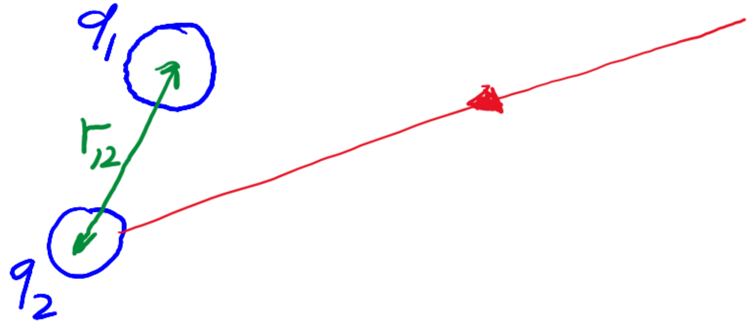
q_1 moves in zero electric field

$$\Rightarrow W_{1\text{-}app,\infty} = 0$$

Then, bring q_2 from infinity

q_2 moves in the electric field of q_1

$$\Rightarrow W_{2\text{-}app,\infty} = q_2 V = q_2 \frac{k q_1}{r_{12}} \equiv U_{12}$$

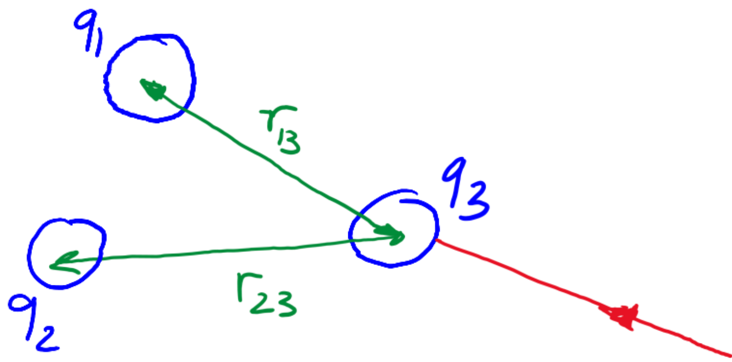


Then, bring q_3 from infinity

q_3 moves in the electric field of q_1 and q_2

$$\Rightarrow W_{3\text{-}app,\infty} = q_3 V = q_3 \left(\frac{k q_1}{r_{13}} + \frac{k q_2}{r_{23}} \right)$$

$$\equiv U_{13} + U_{23}$$

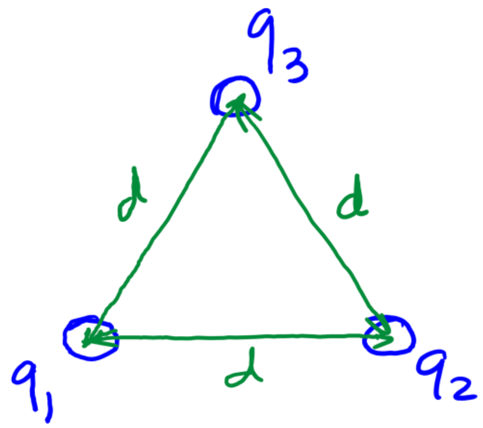


$$U = W_{app,\infty} = U_{12} + U_{13} + U_{23}$$

Example:

Find the electric potential energy of the system?

$q_1 = 100\ nC$
$q_2 = -50\ nC$
$q_3 = 200\ nC$
$d = 1\ cm$



Solution

U = the sum of the potential energies for every pair of particles.

$$U = U_{12} + U_{13} + U_{23}$$

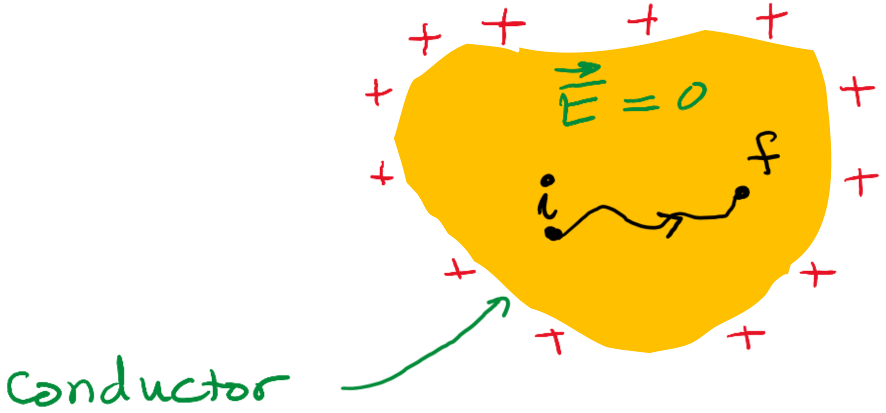
$$U = k\frac{q_1 q_2}{d} + k\frac{q_1 q_3}{d} + k\frac{q_2 q_3}{d} = +4.5\ mJ$$

Work needed to assemble $= +4.5\ mJ$

Work needed to dissemble $= -4.5\ mJ$

24-9 Potential of a Charged Isolated Conductor

In electrostatic equilibrium,
all points within a conductor have the same potential.



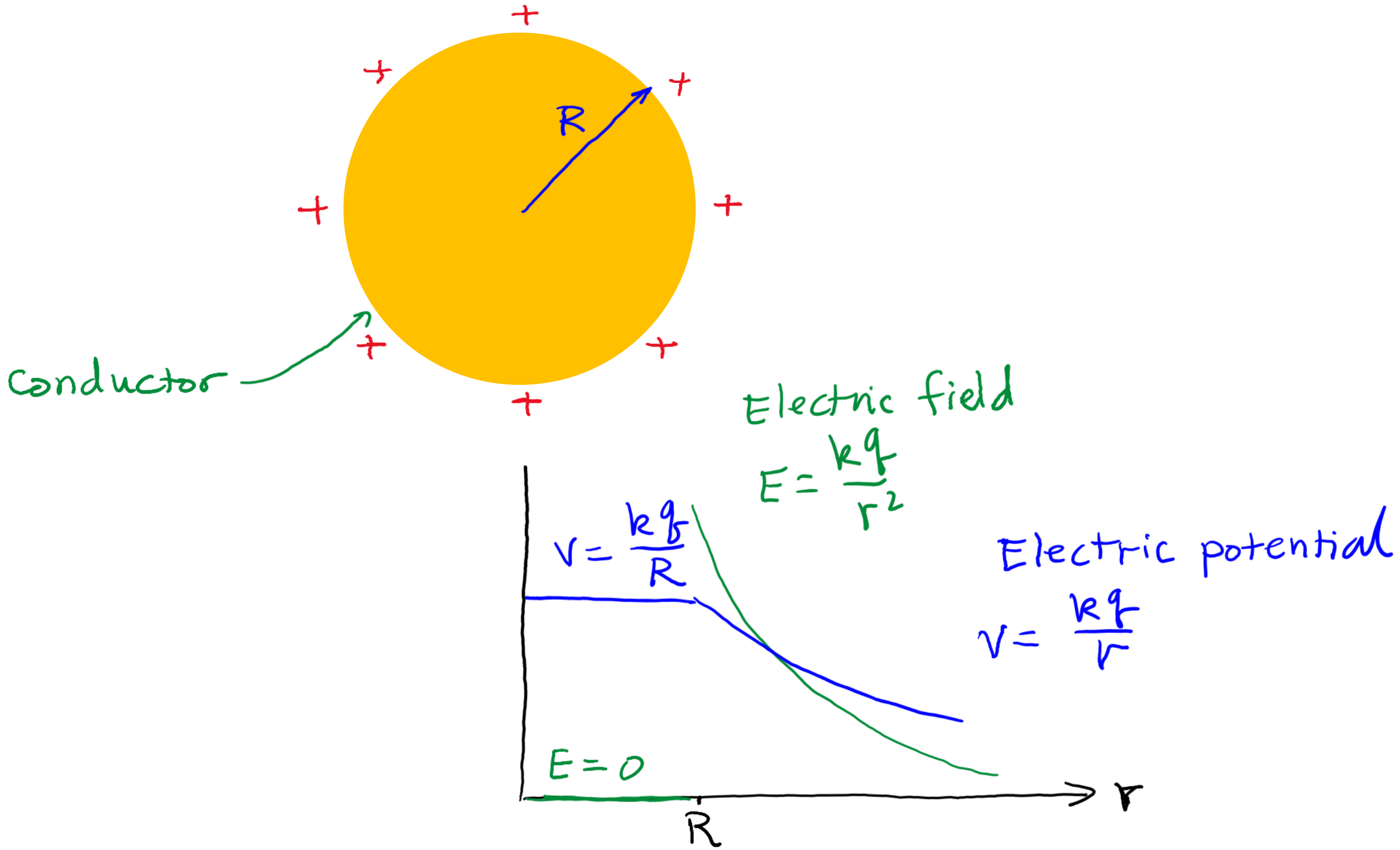
$$\Delta V = -\int_i^f \vec{E} \cdot d\vec{s} = 0$$

$$\Delta V = V_f - V_i = 0$$

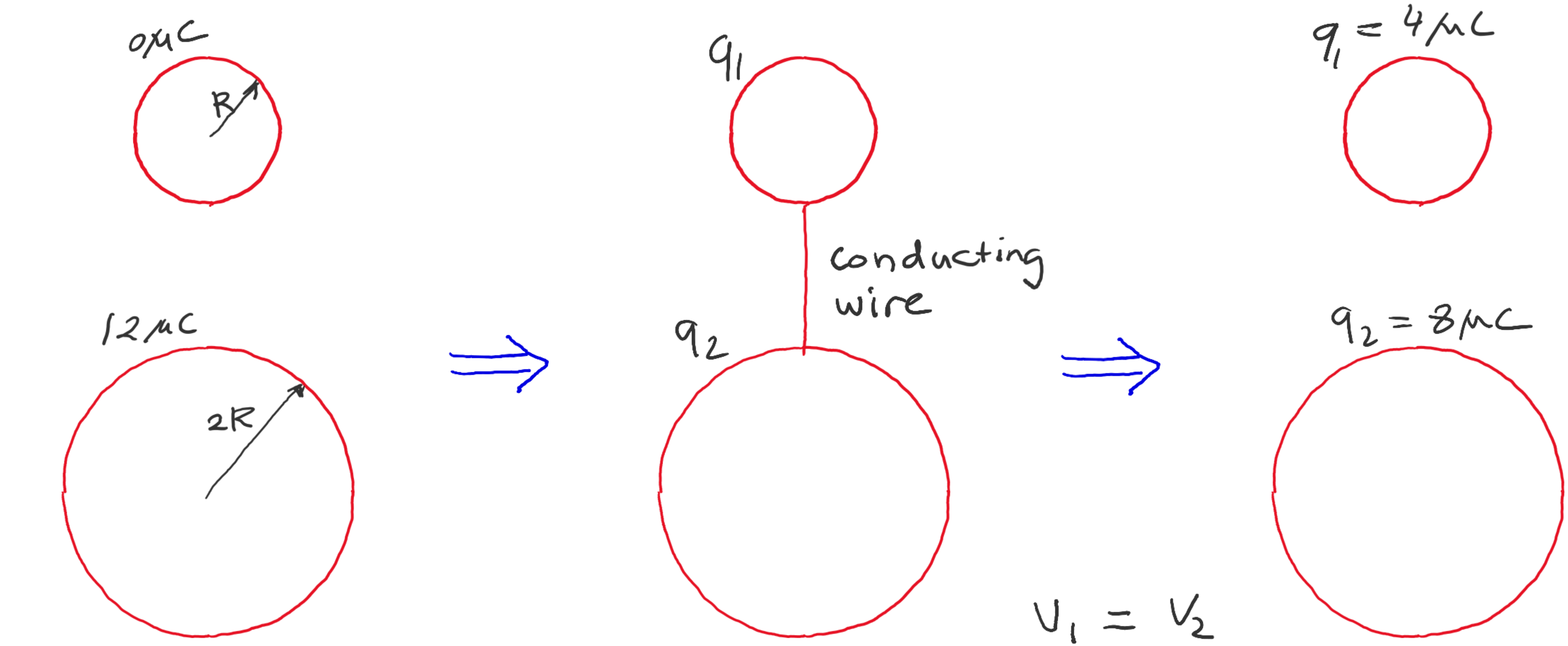
$$V_f = V_i$$

The surface of a conductor is an equipotential surface.

Electric Potential of an Isolated Conductor



Question :



conducting spherical shells

Conservation of charge

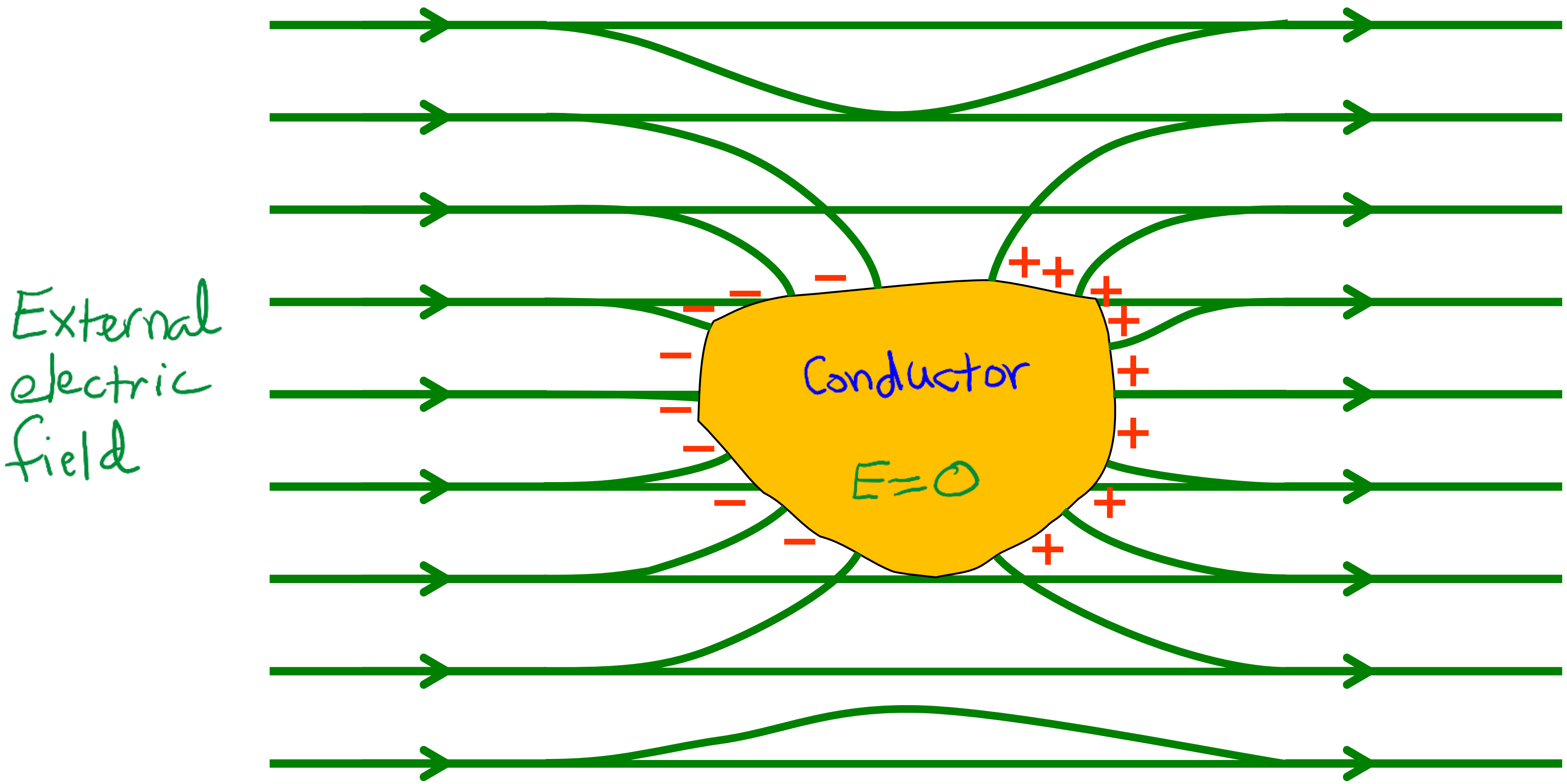
$$q_1 + q_2 = 12\,\mu C$$

$$V_1 = V_2$$

$$\frac{\cancel{k} q_1}{\cancel{R}} = \frac{\cancel{k} q_2}{2\cancel{R}}$$

$$q_1 = \frac{q_2}{2}$$

$$\Rightarrow q_1 = 4\,\mu C \quad \text{and} \quad q_2 = 8\,\mu C$$



Electrons distribute themselves on the surface such that

① E=0 inside the conductor

② $\vec{E}$ is perpendicular to the surface of the conductor.