This is not a replacement for your textbook. It is essential to study from your textbook!

Objectives

Ch 23: Gauss' Law

23-1 Flux of an electric field

23-2 Gauss' Law

23-3 A charged isolated conductor

23-4 Applying Gauss' Law: cylindrical symmetry

23-5 Applying Gauss' law: planar symmetry

23-6 Applying Gauss' Law: spherical symmetry

Gauss' Law

Useful to find $\vec{E}$ from symmetrical shapes

cylinders
planes
spheres

ε_0 (electric flux through a <u>closed</u> surface)
= total charge inside the surface

$$\varepsilon_0 \Phi = q_{enc}$$

permittivity constant
$8.85 \times 10^{-12} \ \frac{C^2}{N m^2}$

$$k = \frac{1}{4\pi\varepsilon_0}$$

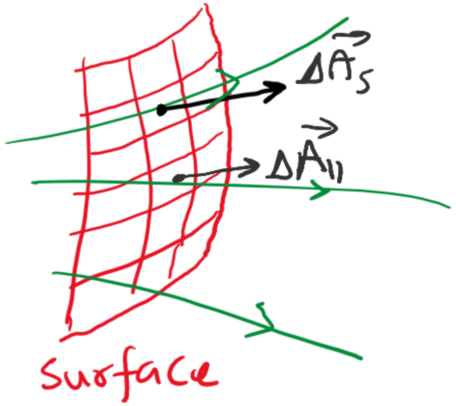
Electric flux through a surface

$$\Phi \equiv \int_{\text{surface}} \vec{E} \cdot d\vec{A}$$

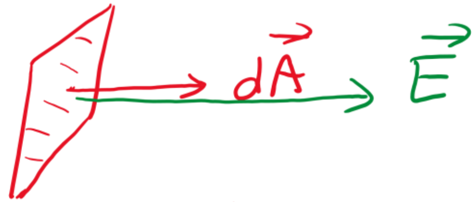
Area vector

magnitude : Area of a small surface

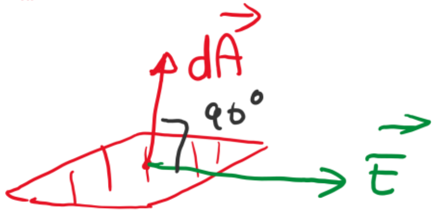
direction : Normal to the surface



$$\Phi \equiv \lim_{\Delta A \to 0} \sum_i \vec{E}_i \cdot \Delta\vec{A}_i = \int_{\text{surface}} \vec{E} \cdot d\vec{A}$$



$$\vec{E} \cdot d\vec{A} = E\,dA \cos 0^\circ = E\,dA$$



$$\vec{E} \cdot d\vec{A} = E\,dA \cos 90^\circ = 0$$

Electric flux through a surface

$$\Phi \equiv \int_{\text{surface}} \vec{E} \cdot \vec{dA}$$

Area vector

magnitude : Area of a small surface

direction : <u>Normal</u> to the surface

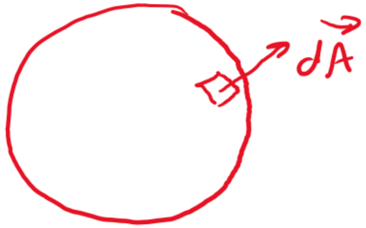
For a <u>closed</u> surface

$$\Phi = \oint_{\text{surface}} \vec{E} \cdot \vec{dA}$$

closed surface

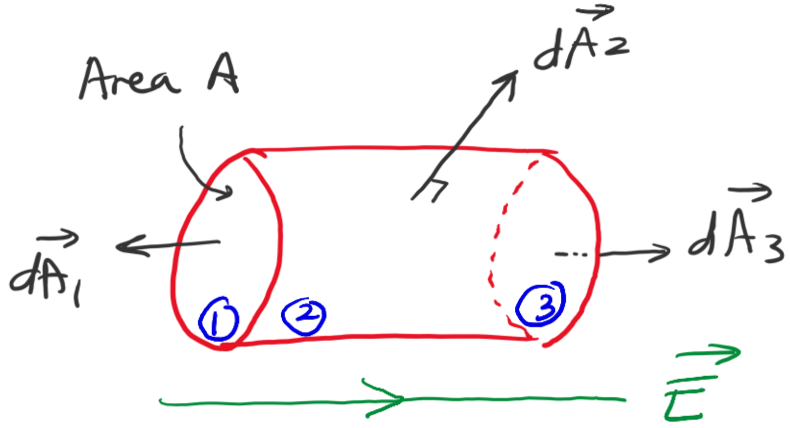
points to the <u>outside</u> of the closed surface

Any closed surface is called Gaussian surface.

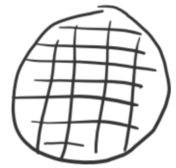


Example

Find the electric flux through the cylinder.



$$\Phi = \Phi_1 + \Phi_2 + \Phi_3$$



$$\Phi_1 = \int_{\text{surface ①}} \vec{E}\cdot d\vec{A}_1 = \int E\,dA\cos 180^\circ = -\int E\,dA = -E\int_{\text{Surface ①}} dA = -EA$$

$$\Phi_2 = \int_{\text{surface ②}} \vec{E}\cdot d\vec{A}_2 = \int E\,dA\cos 90^\circ = 0$$

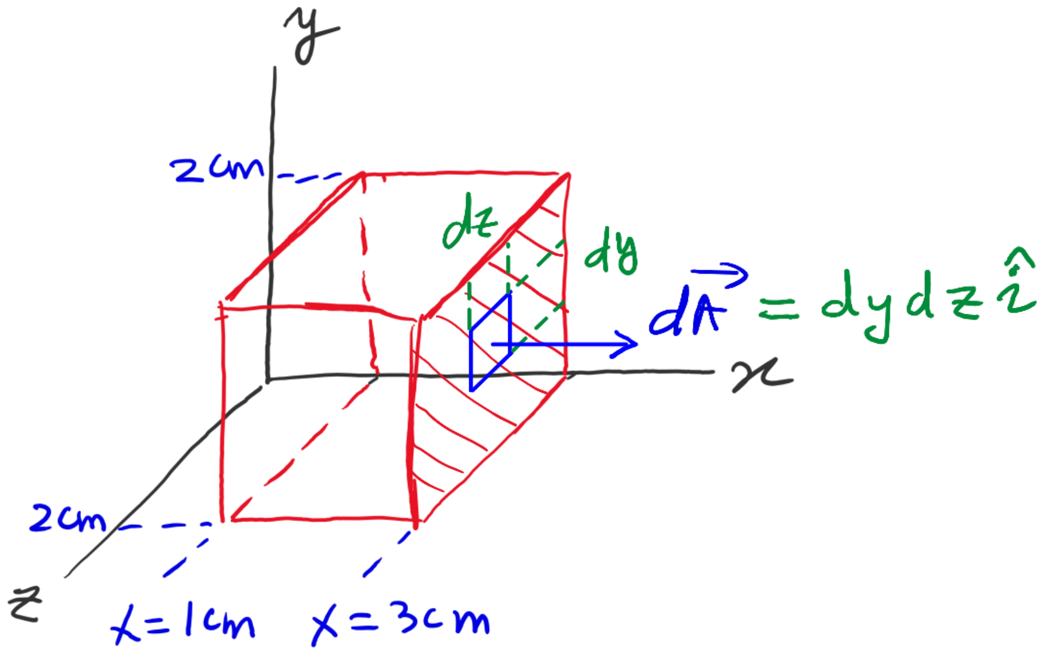
$$\Phi_3 = \int_{\text{surface ③}} \vec{E}\cdot d\vec{A}_3 = \int E\,dA\cos 0^\circ = \int E\,dA = E\int dA = +EA$$

$$\Phi = -EA + 0 + EA = 0$$

Example

$$\vec{E} = 3x\,\hat{i} + 4\,\hat{j}$$

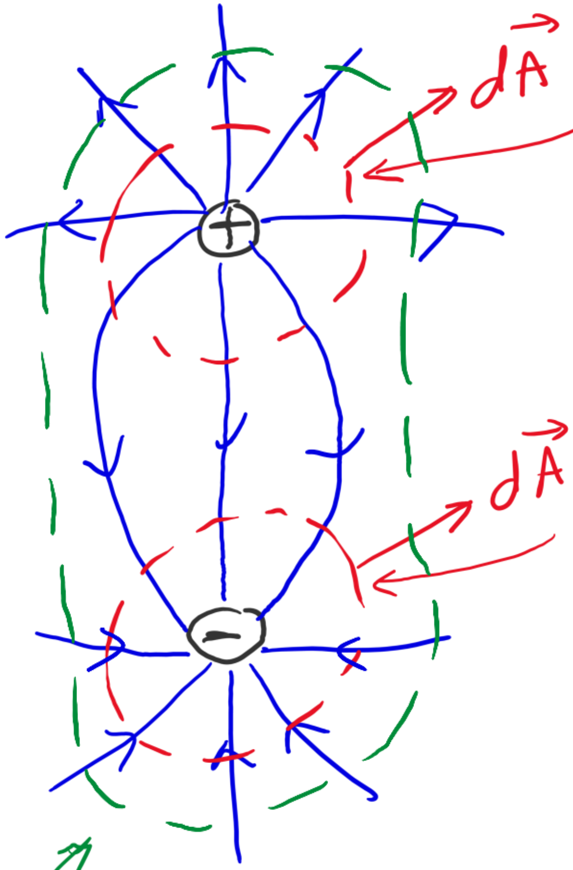
Find the flux through the right face.



$$\Phi = \int_{\substack{\text{right}\\\text{surface}}} \vec{E}\cdot d\vec{A} = \int_{y=0}^{2}\int_{z=0}^{2} \left(\overbrace{3x\,\hat{i} + \underbrace{4\,\hat{j}\,)\cdot dy\,dz\,\hat{i}}_{\hat{j}\cdot\hat{i}=0}}^{\hat{i}\cdot\hat{i}=1}\right.$$

$$= \int_{y=0}^{2}\int_{z=0}^{2} 3\underbrace{x}_{\substack{\text{on the}\\\text{right surface}\\x=3}}\,dy\,dz = 9\int_{y=0}^{2}\int_{z=0}^{2} dy\,dz = 9\times2\times2 = 36\ \frac{N}{C}\,cm^2$$

Dipole



All lines are out

$\Rightarrow$ flux is positive

$\Rightarrow$ net charge inside is positive

All lines are in

$\Rightarrow$ flux is negative

$\Rightarrow$ net charge inside is negative

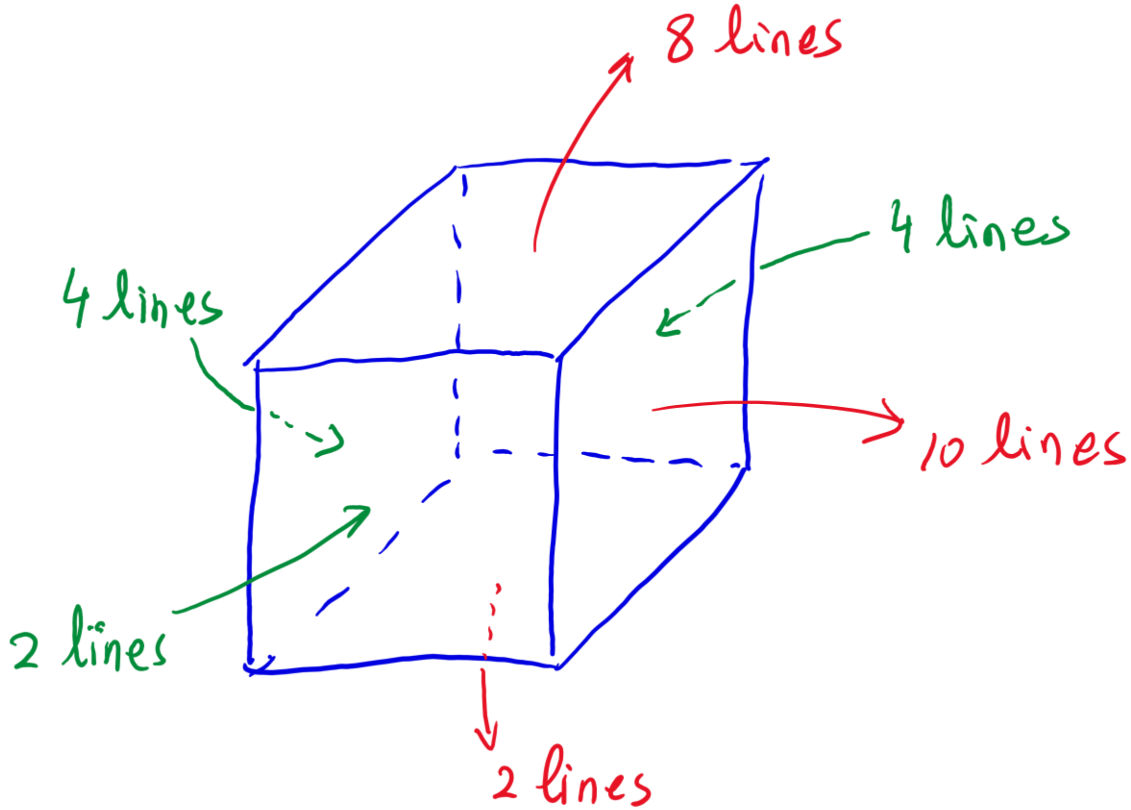
lines in = lines out

$\Rightarrow$ flux = 0

$\Rightarrow$ net charge inside = 0

Question

What is the sign of the charge inside the box?



10 electric lines in
20 electric lines out

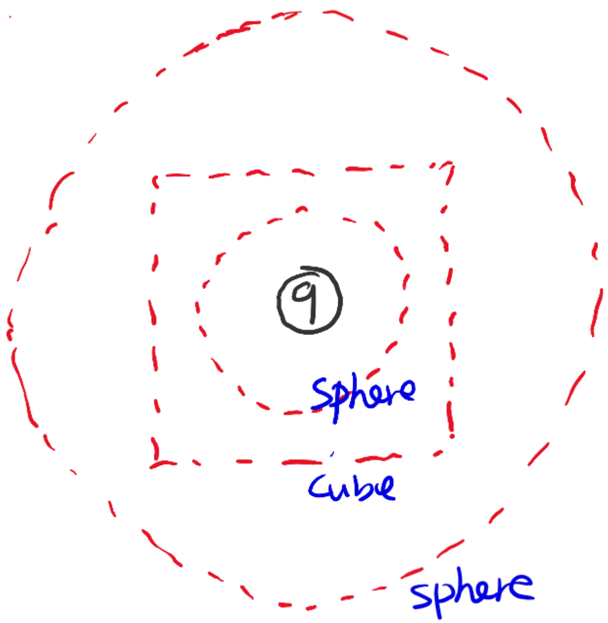
} net 10 electric lines out

$\Rightarrow$ flux is positive

$\Rightarrow$ net charge inside is positive.

Question

What is the relation between the electric flux through the three Gaussian surfaces?



ε_0(flux through a closed surface) = charge inside the surface.

All have the same flux.

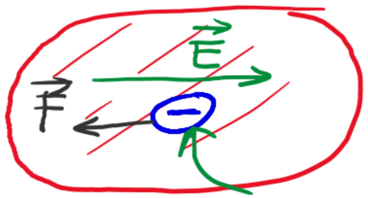
23-3 A charged isolated conductor

Electrostatic equilibrium $\equiv$ charges do not move

<u>In electrostatic equilibrium:</u>

① Electric field inside a conductor is zero.

Conductor



Free electron

$\vec{E} \neq 0 \Rightarrow \vec{F} \neq 0$

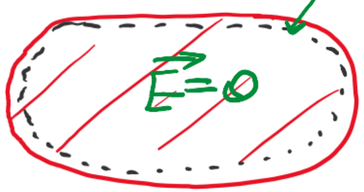
$\Rightarrow$ free electrons move

$\Rightarrow$ not an electrostatic equilibrium

② All excess charge on a conductor is on the conductor's <u>outer</u> surface.

Gaussian surface inside the conductor

conductor



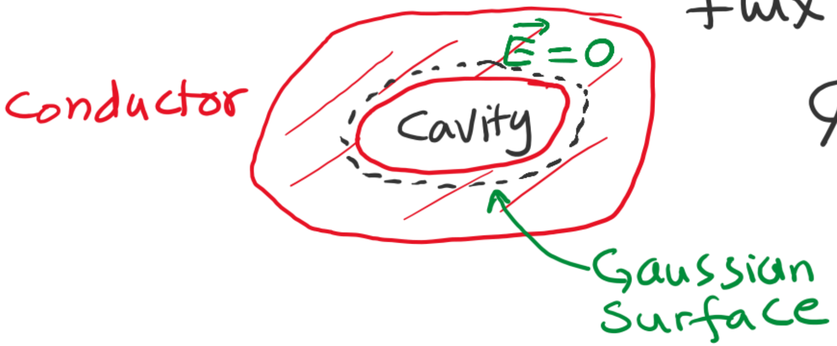
Flux through the Gaussian surface $=0$

$$\Phi = 0 \xrightarrow{\varepsilon_0 \Phi = q_{enc}} q_{enc} = 0$$

$\Rightarrow$ No net charge inside the conductor

In electrostatic equilibrium:

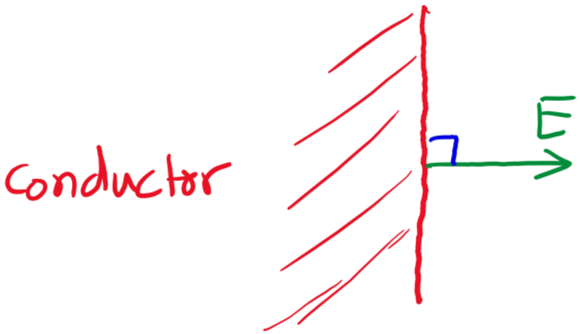
③ No net charge on the wall of a cavity which is totally within a conductor.

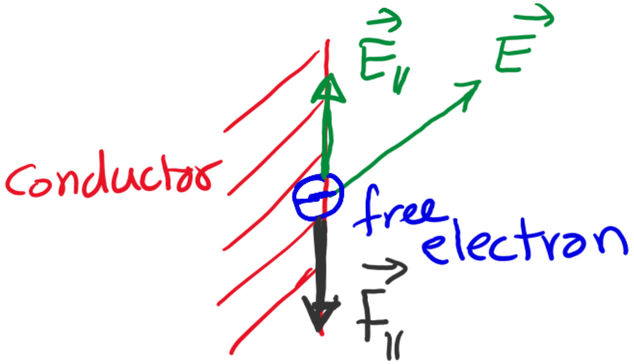


Flux through the Gaussian surface $= 0$

$$\phi = 0 \xrightarrow{\varepsilon_0 \phi = q_{enc}} q_{enc} = 0$$

④ Electric field is perpendicular to the surface of a conductor.



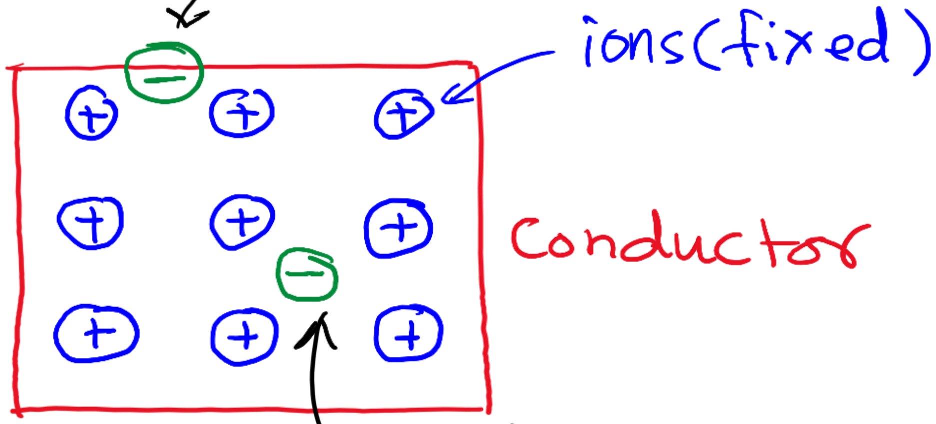


$\vec{E}_{\parallel} \neq 0 \Rightarrow \vec{F}_{\parallel} \neq 0$

$\Rightarrow$ electrons move

$\Rightarrow$ Not electrostatic equilibrium.

edge: the ions pull the free electron in.



inside: average force from the ions on the free electron is zero.

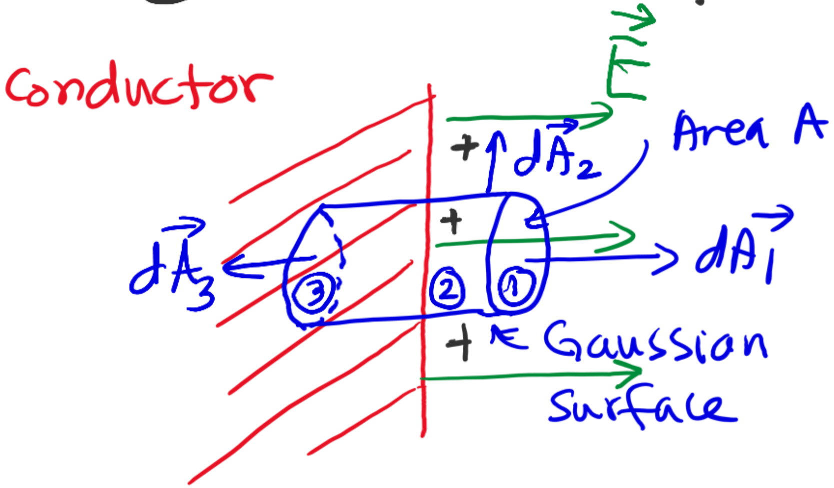
In electrostatic equilibrium

(5) Near the surface of a conductor

$$\boxed{E = \frac{\sigma}{\varepsilon_0}}$$

σ = surface charge density

$$\sigma = \frac{\text{charge}}{\text{area}}$$

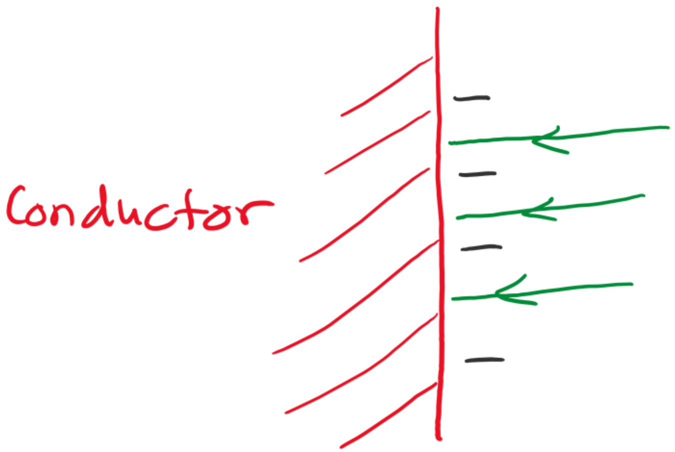


$$\phi = \phi_1 + \phi_2 + \phi_3$$

$$\phi = EA + 0 + 0 = EA$$

$$\varepsilon_0 \phi = q_{enc}$$

$$\varepsilon_0 EA = q_{enc} \Rightarrow E = \frac{q_{enc}}{\varepsilon_0 A} = \frac{\sigma}{\varepsilon_0}$$



Example

Net charge on the shell = 20 μC

Put a point charge − 5 μC inside

What is the charge on the inner surface of the shell?

Since $\vec{E}$ inside the conductor = 0

$\Rightarrow \phi = 0 \Rightarrow q_{enc} = 0$

$q_{enc} = -5\mu C + q_{in} = 0$

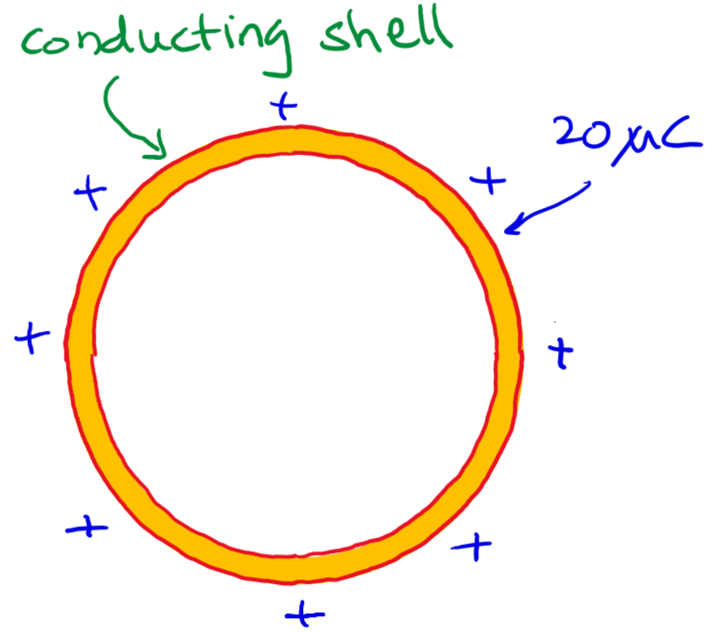
$\Rightarrow q_{in} = +5\mu C$

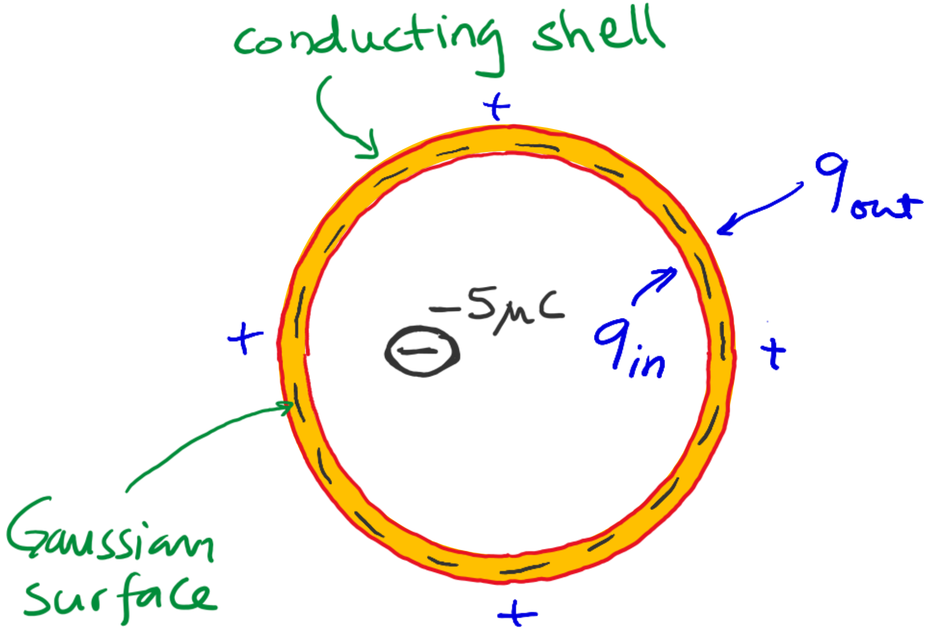
What is the charge on the outer surface of the shell?

$q_{net} = 20\mu C = q_{in} + q_{out}$

$\Rightarrow q_{out} = 20\mu C - 5\mu C$

$q_{out} = 15\mu C$





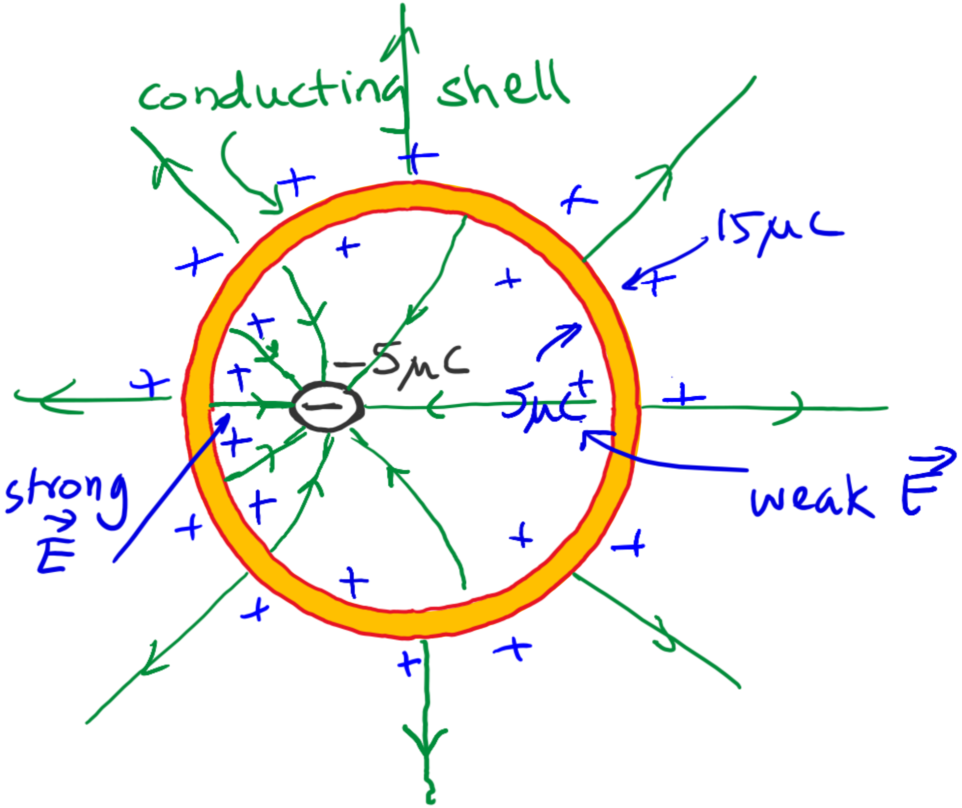
Example

Net charge on the shell = $20\mu C$

Put a point charge $-5\mu C$ inside

Sketch the electric field lines inside

$$E = \frac{\sigma}{\varepsilon_0}$$

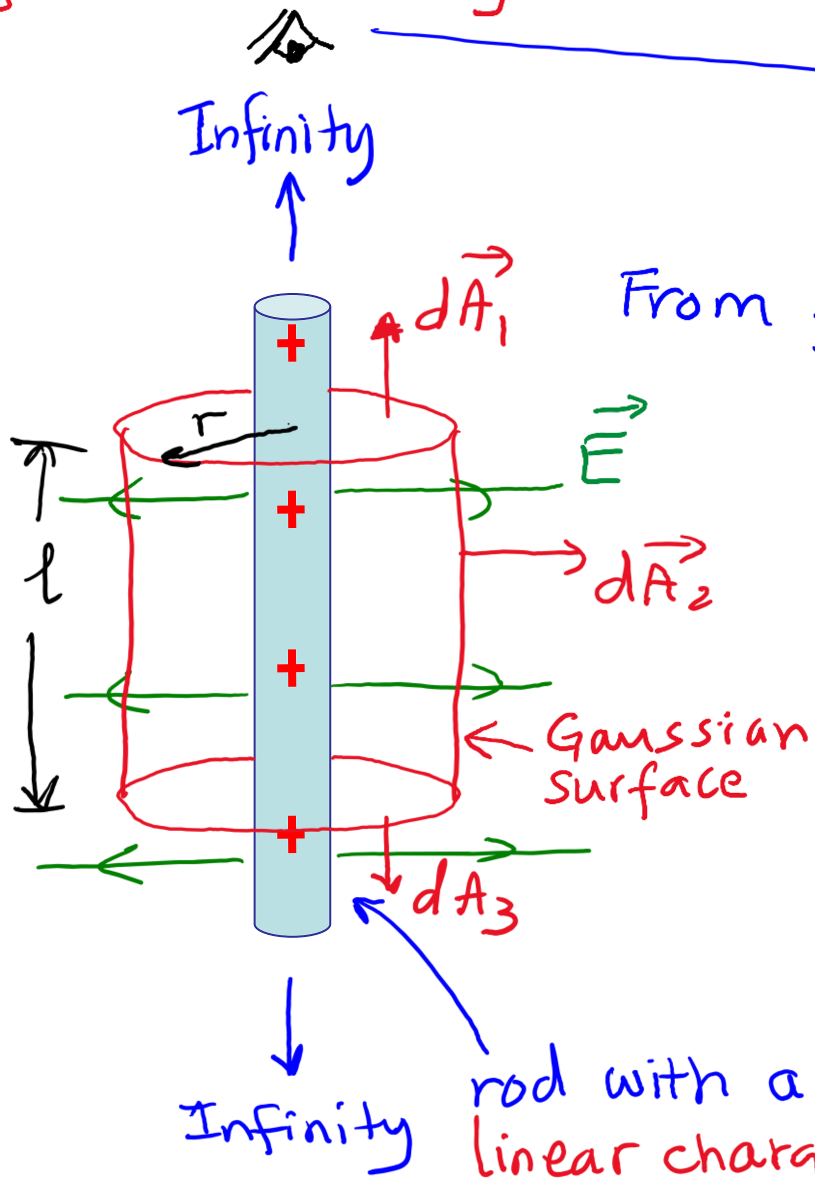


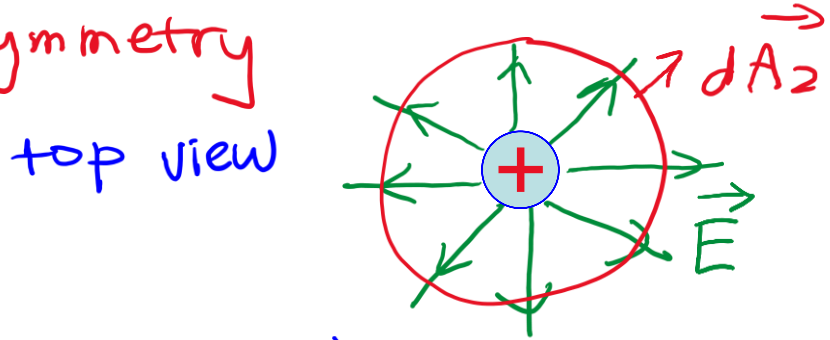
Sketch the electric field lines outside

Since $\vec{E} = 0$ inside the conductor,
the charge outside is not influenced by the charge inside.

$\Rightarrow$ Outside charge is distributed uniformly.

23-4 Applying Gauss' Law: Cylindrical Symmetry



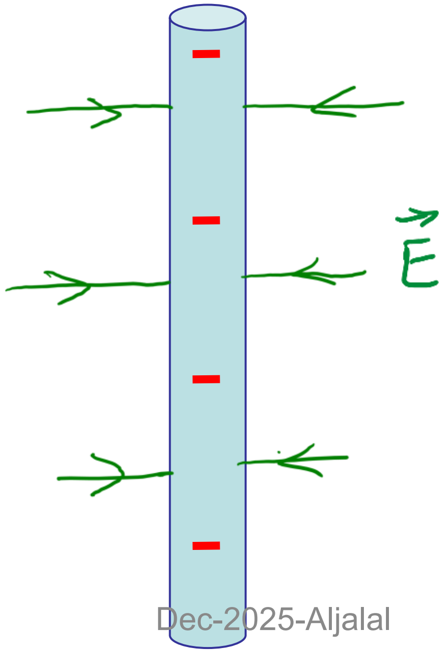


From symmetry, $\vec{E}$ points radially out

$$\phi = \phi_1 + \phi_2 + \phi_3$$
$$\phi = 0 + EA_2 + 0$$
$$\phi = E(2\pi r \ell)$$

$$\varepsilon_0 \phi = q_{enc}$$
$$\varepsilon_0 E(2\pi r \ell) = q_{enc}$$
$$E = \frac{q_{enc}/\ell}{2\pi \varepsilon_0 r}$$

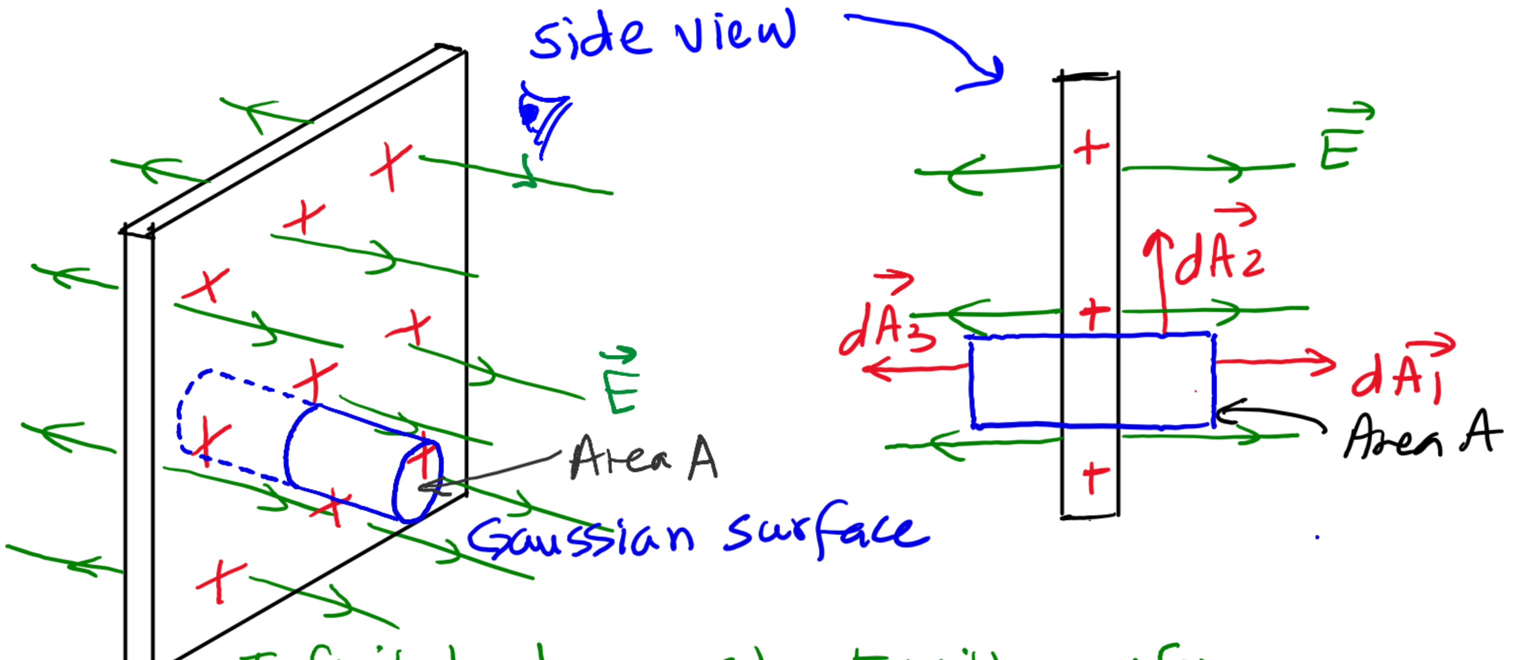
$$\boxed{E = \frac{\lambda}{2\pi \varepsilon_0 r}}$$



rod with a uniform linear charge density

$$\lambda \equiv \frac{\text{charge}}{\text{length}} = \frac{q_{enc}}{\ell}$$

23-5 Applying Gauss' Law: Planar Symmetry



Infinitely large sheet with uniform surface charge density $\sigma = \dfrac{\text{charge}}{\text{Area}} = \dfrac{q_{enc}}{A}$

From symmetry, $\vec{E}$ is perpendicular to the sheet.

$$\Phi = \Phi_1 + \Phi_2 + \Phi_3$$

$$\Phi = EA + 0 + EA$$

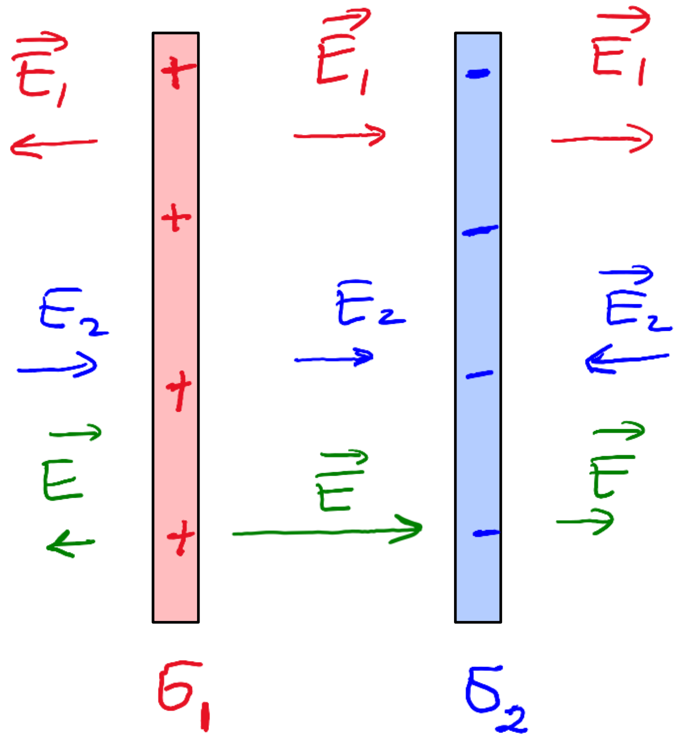
$$\varepsilon_0 \Phi = q_{enc} \Rightarrow \varepsilon_0 (2EA) = q_{enc} \Rightarrow E = \frac{q_{enc}/A}{2\varepsilon_0}$$

$$\boxed{E = \frac{\sigma}{2\varepsilon_0}}$$

Two infinite nonconducting sheets.

Find $\vec{E}$ between and outside the two parallel sheets.

Cross section
(side view)



Assume $\sigma_1 > \sigma_2$

superposition $\vec{E} = \vec{E}_1 + \vec{E}_2$

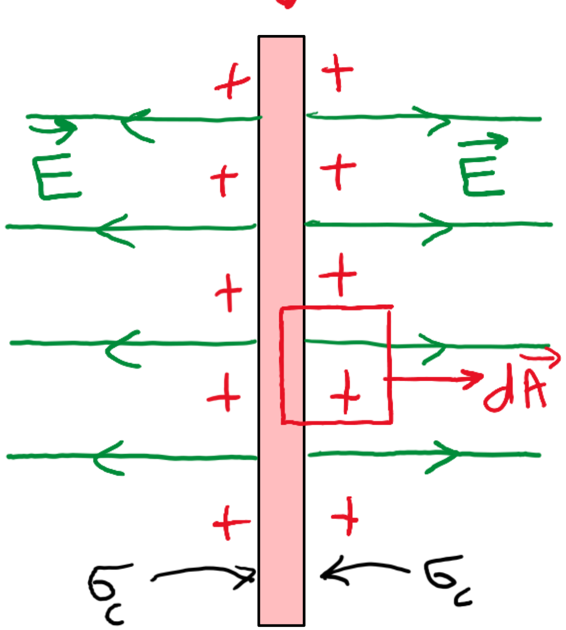
A conducting sheet

No excess charge inside conductors

The excess charge lies on the surface

Define charge density for each side of the conducting sheet

$2\sigma_c$ (total charge density)



conducting sheet

$$E = \frac{\sigma_c}{\varepsilon_0}$$

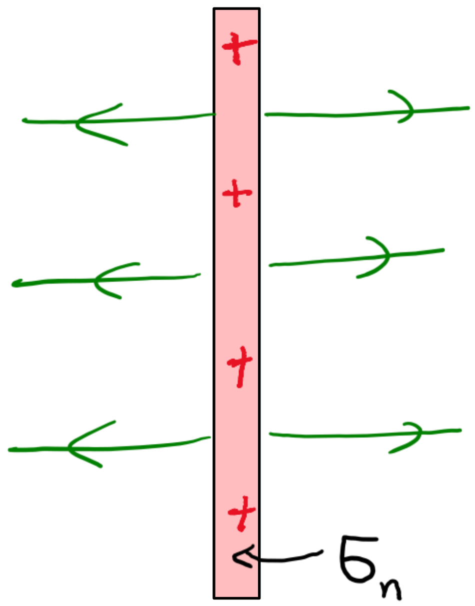
$$\phi = EA$$

Gauss' Law

$$\varepsilon_0 \phi = q_{enc}$$

$$\varepsilon_0 E A = q_{enc}$$

$$E = \frac{q_{enc}/A}{\varepsilon_0} = \frac{\sigma_c}{\varepsilon_0}$$



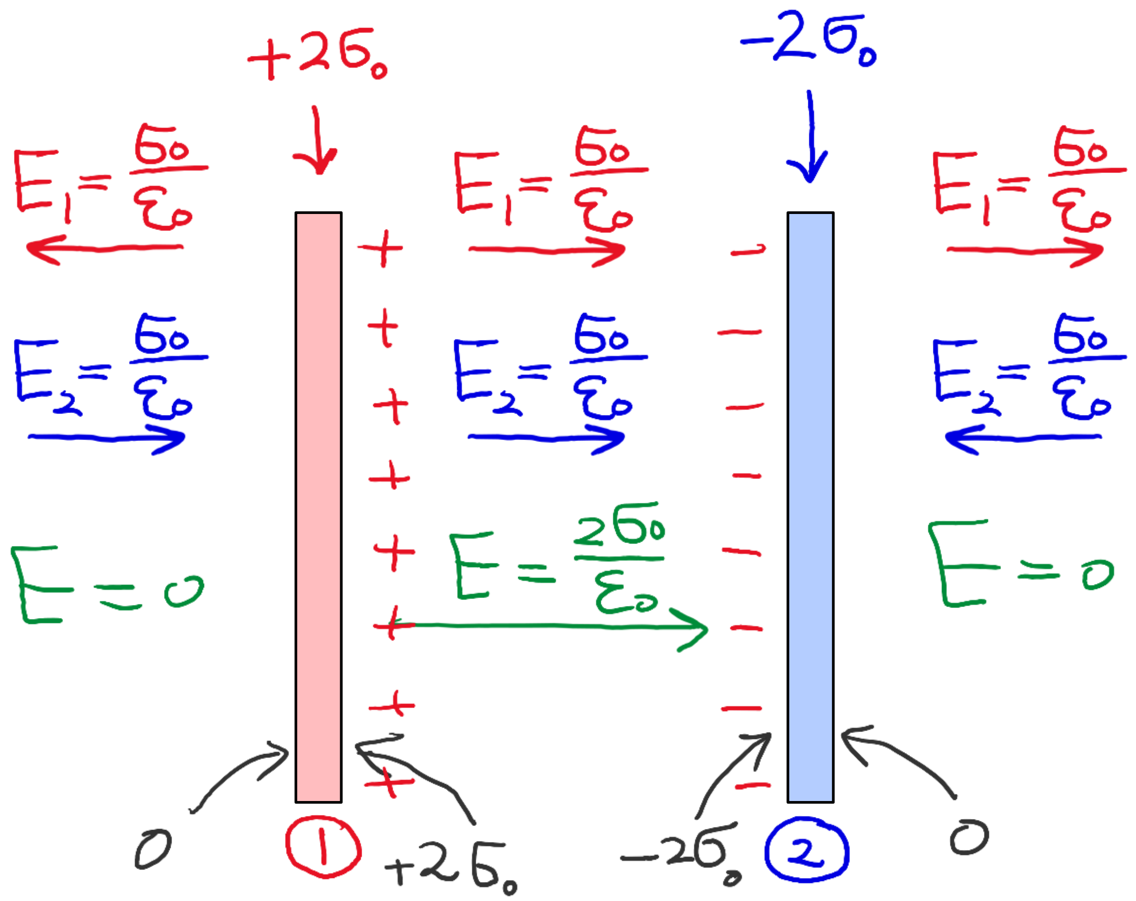
nonconducting sheet

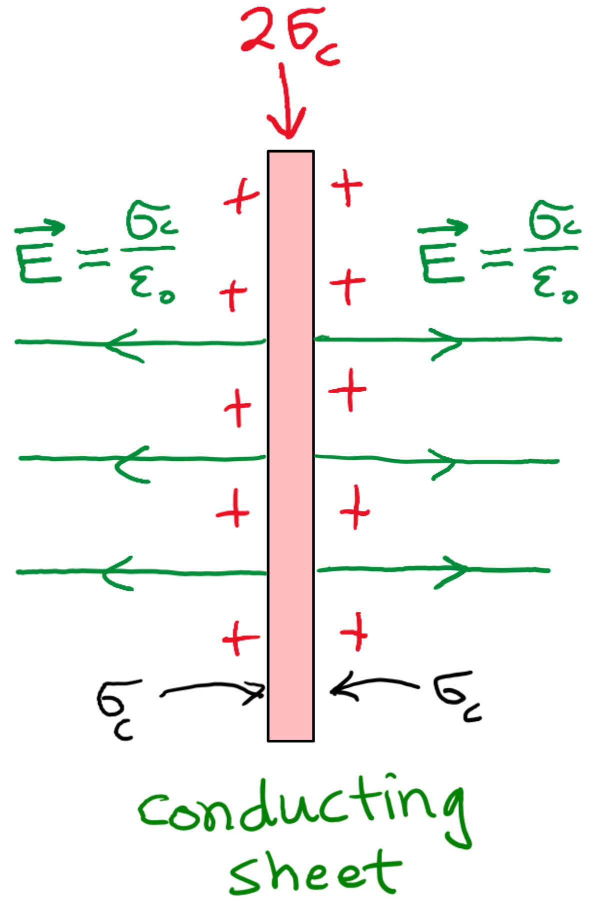
$$E = \frac{\sigma_n}{2\varepsilon_0}$$

If $2\sigma_c = \sigma_n$, you get the same $\vec{E}$

Two infinite conducting sheets

Assume both sheets have the same charge density





Near the surface of a conductor $E = \frac{\sigma}{\varepsilon_0}$

Uniformly charged spherical shell

From symmetry, $\vec{E}$ points along the radial direction.

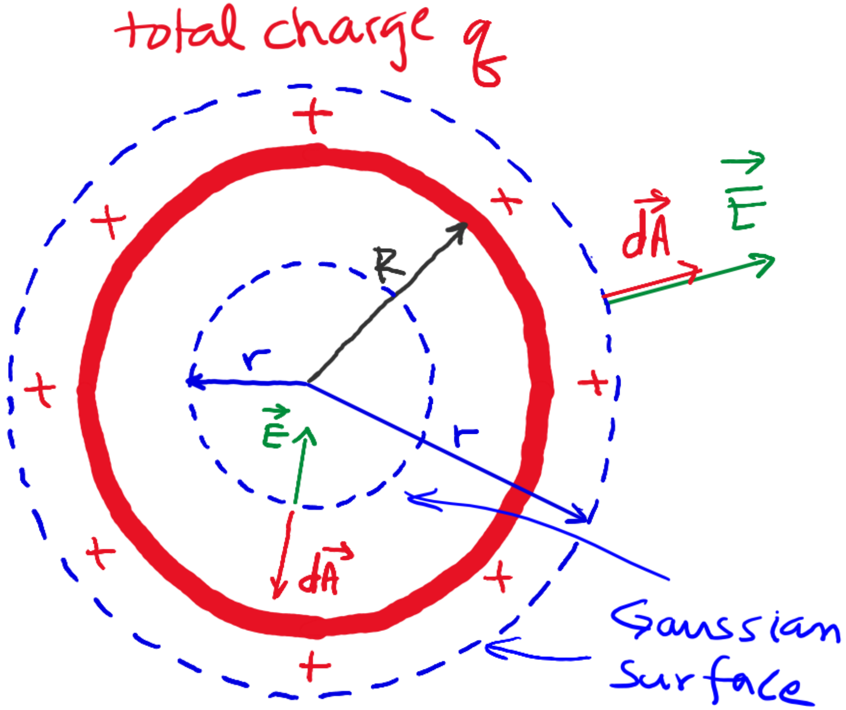
Inside the shell $r < R$

$\Phi = -EA$

area of a sphere of radius r

$\varepsilon_0 \Phi = q_{enc} = 0 \Rightarrow \Phi = 0$

$\Rightarrow E = 0$



Outside the shell $r > R$

$\Phi = EA = E(4\pi r^2)$

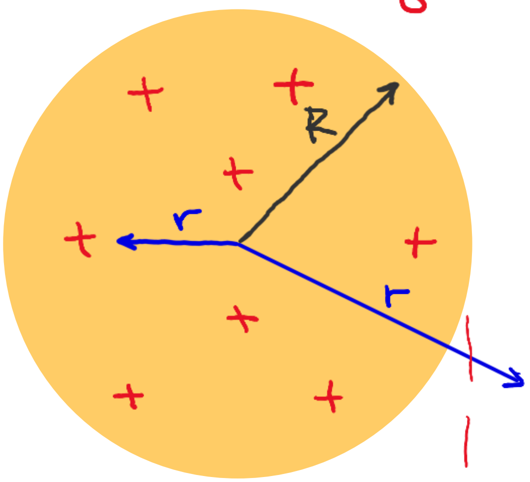
$\varepsilon_0 \Phi = q_{enc} \Rightarrow \varepsilon_0 E(4\pi r^2) = q \Rightarrow E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2} = k\frac{q}{r^2}$

Same as $\vec{E}$ from a charged particle with charge q at the center.

Uniformly charged sphere

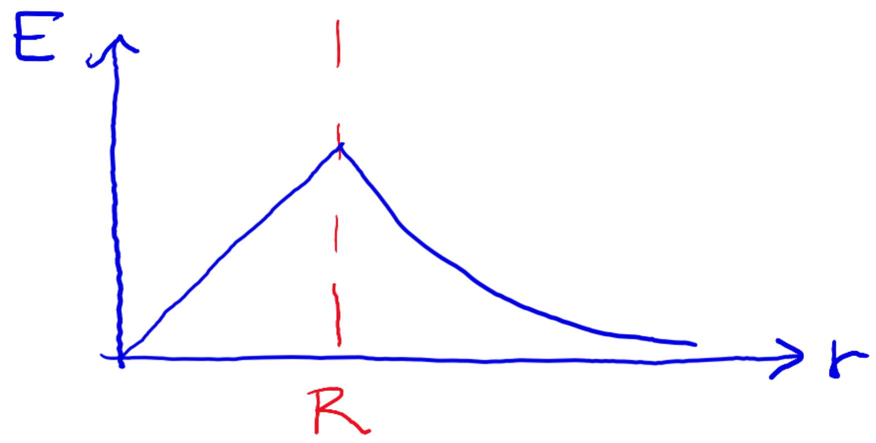
Inside the sphere $r < R$

$$E = \frac{kq}{R^3} r$$



Outside the sphere $r > R$

$$E = \frac{kq}{r^2}$$

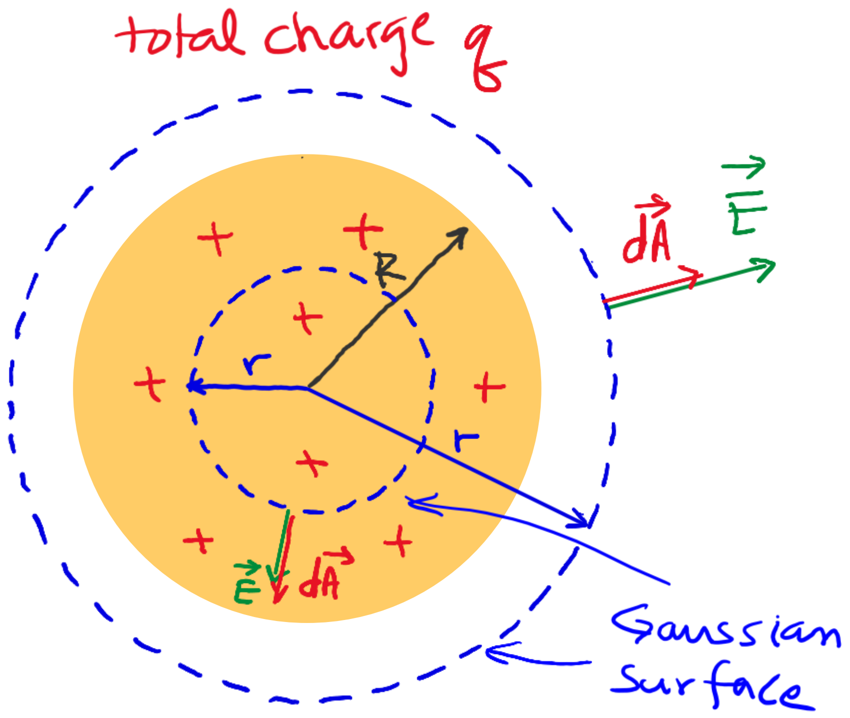


similar to $\vec{E}$ from a charged particle with charge q at the center.

Uniformly charged sphere

From symmetry, $\vec{E}$ points along the radial direction.

· Inside the sphere $r < R$



$$\varepsilon_0 \phi = q_{enc} \Rightarrow \varepsilon_0 (E(4\pi r^2)) = q\,\frac{r^3}{R^3}$$

$$\Rightarrow \quad E = \frac{q}{4\pi\varepsilon_0}\frac{r}{R^3} = \frac{kq}{R^3}\,r$$

$$\frac{\text{Charge enclosed by } r}{\text{Volume enclosed by } r} = \frac{\text{Total charge}}{\text{Total volume}}$$

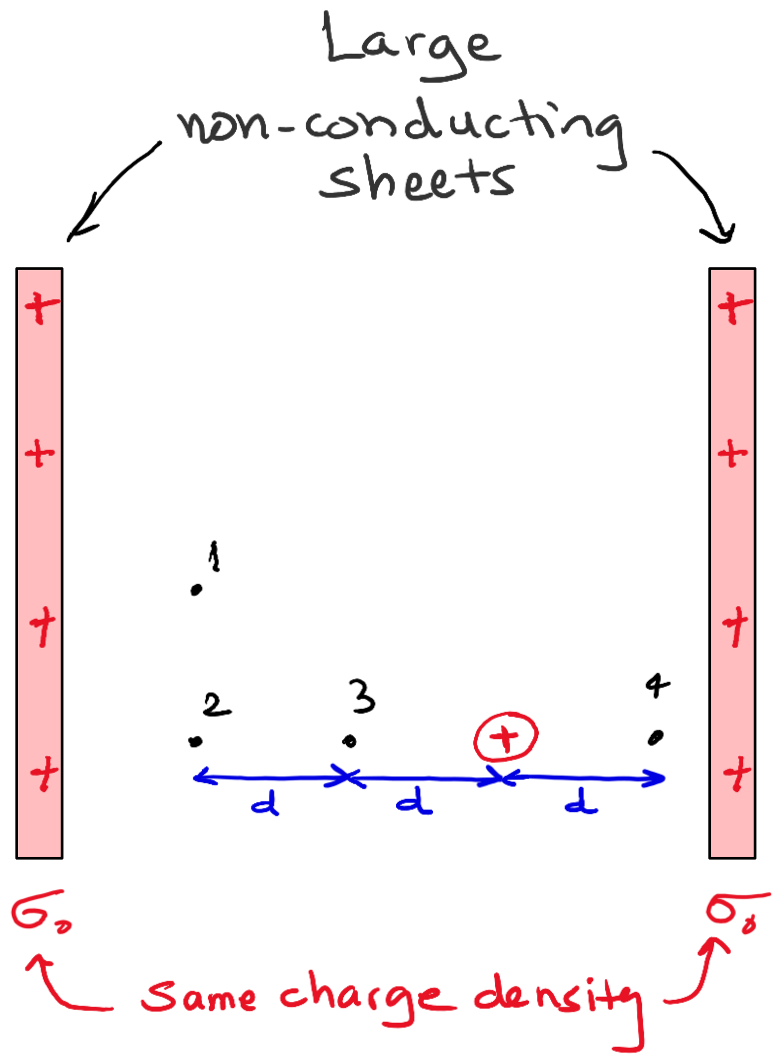
$$\frac{q_{enc}}{\frac{4\pi r^3}{3}} = \frac{q}{\frac{4\pi R^3}{3}} \Rightarrow q_{enc} = q\,\frac{r^3}{R^3}$$

Outside the sphere $r > R$

$$\varepsilon_0 \phi = q_{enc} \Rightarrow \varepsilon_0 (E(4\pi r^2)) = q$$

$$\Rightarrow E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2} = \frac{kq}{r^2}$$

Question Rank electric field at the four points, greatest first.



3 and 4 tie
then 2
then 1