Objectives

Ch 22: Electric fields

22-1 The electric field.

22-2 The electric field due to a point charge.

22-3 The electric field due to an electric dipole.

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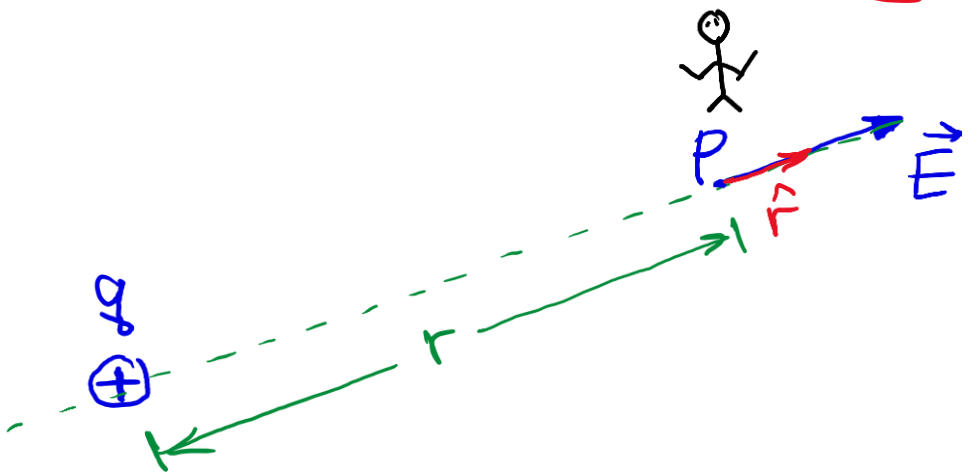
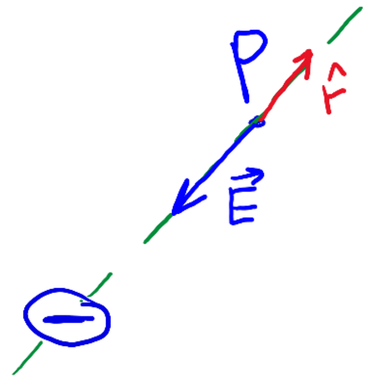
22-6 A point charge in an electric field.

22-7 A dipole in an electric field.

The electric field due to a point charge





The electric field at a point P due to a point charge q.

$$\vec{E} = k \frac{q}{r^2} \hat{r}$$

charge

$8.99 \times 10^9 \frac{Nm^2}{C^2}$

distance between the charge q and the point P

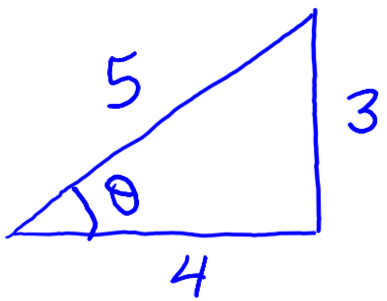
a unit vector pointing away from q toward P.

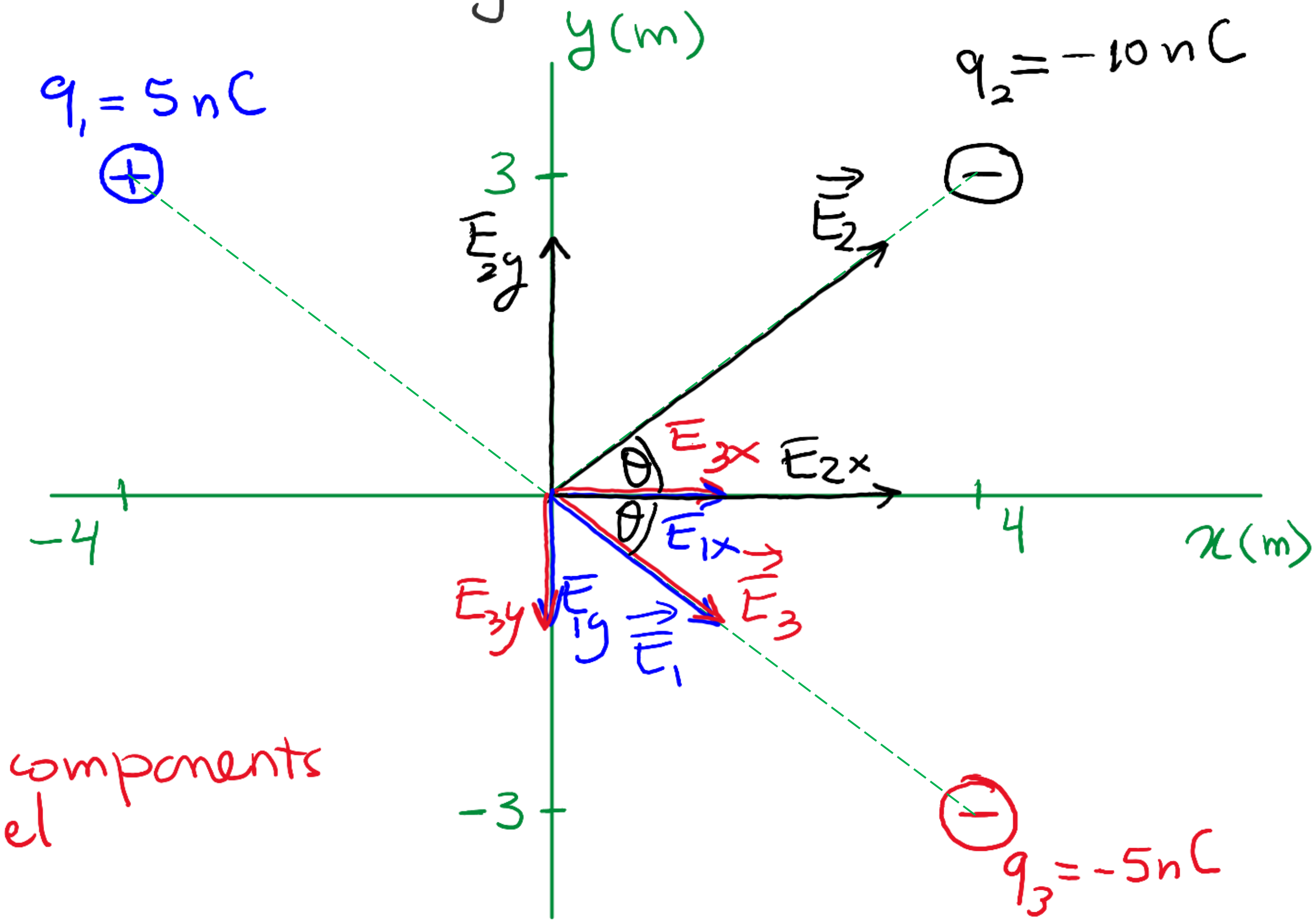
$|\hat{r}| = 1$

Magnitude $\quad E = k \frac{|q|}{r^2}$

Example What is the electric field at the origin?

$$E_1 = \frac{k|q_1|}{r_1^2} = \frac{9\times10^9\times5\times10^{-9}}{5^2} = \frac{9}{5}\ N/C$$





$$E_2 = \frac{k|q_2|}{r_2^2} = \frac{18}{5}\ N/C$$

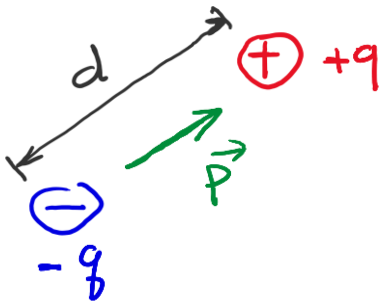
$$E_3 = E_1 = \frac{9}{5}\ N/C$$

the y components cancel

$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 = (E_1\cos\theta + E_2\cos\theta + E_3\cos\theta)\,\hat{i}$$

$$= (E_1 + E_2 + E_3)\cos\theta\,\hat{i} = \left(\left(\frac{9}{5} + \frac{18}{5} + \frac{9}{5}\right)\frac{4}{5}\,\frac{N}{C}\right)\hat{i}$$

Electric Dipole



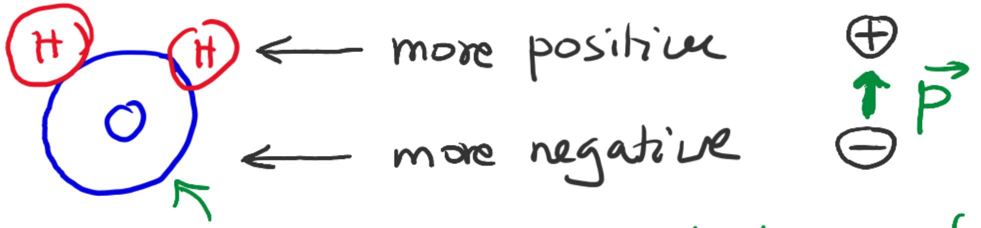
An <u>electric dipole</u> consists of two point charges of equal magnitude q but opposite signs, sperated by a small distance d.

<u>Electric dipole moment</u> $\vec{p}$

Magnitude: $p = qd$

Direction: points towards the positive point charge.

H₂O is an electric dipole.



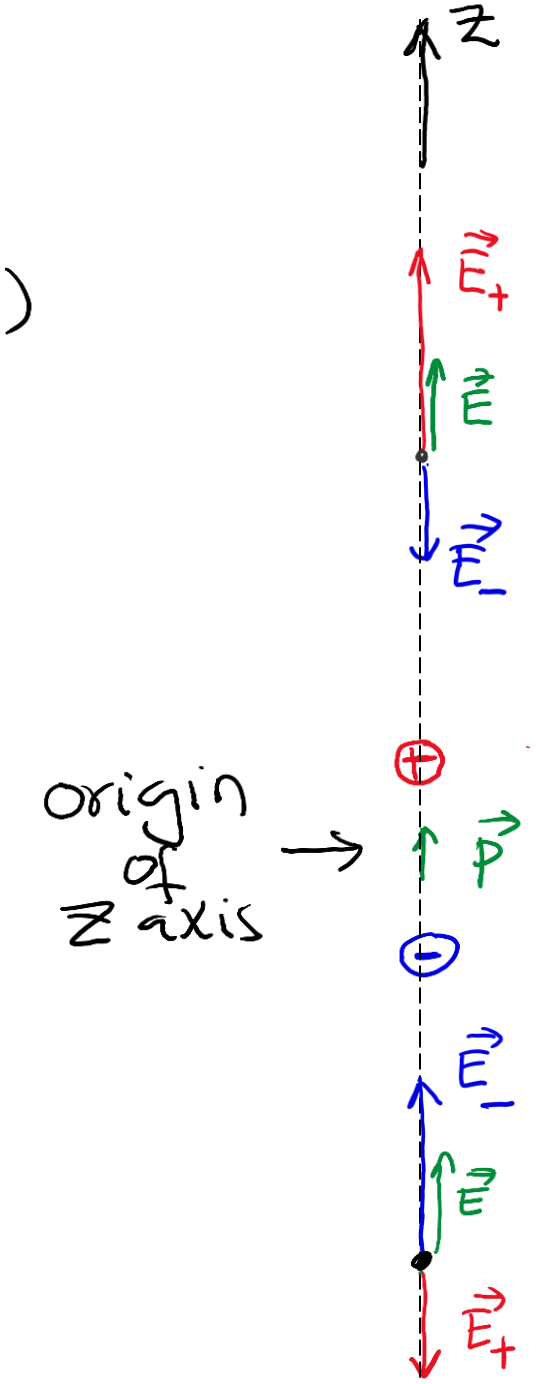
oxygen attracts the electrons of the hydrogen atoms

Electric field from a dipole

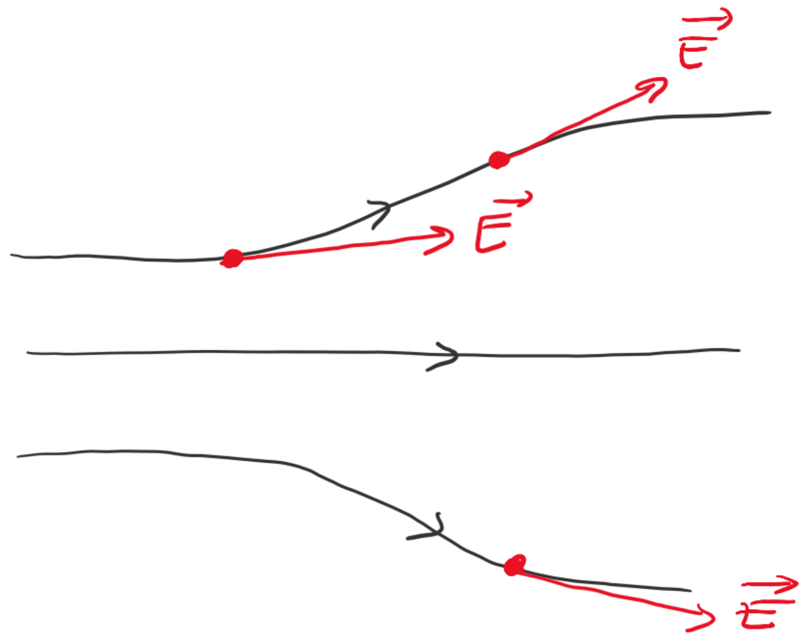
① on the z axis (axis of the dipole)

② $z \gg d$

$$\vec{E} \approx \frac{2k\vec{p}}{z^3}$$



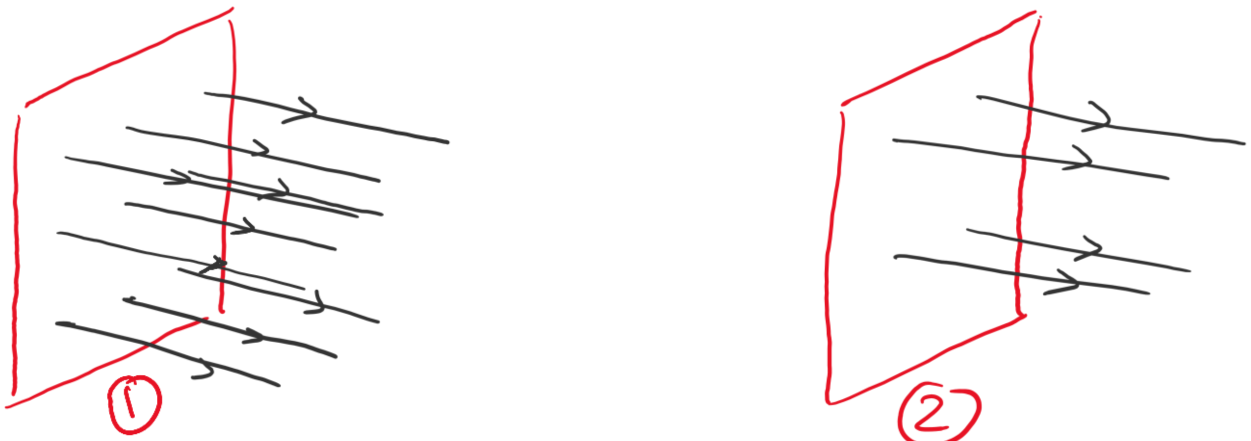
Electric field lines help us representing the electric field



At any point, $\vec{E}$ is tangent to the electric field lines

Electric field lines help us representing the electric field

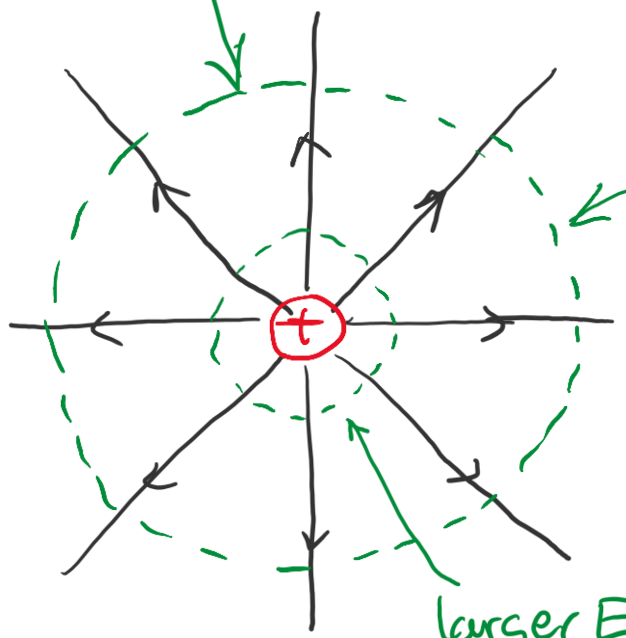
The magnitude of the electric field is proportional to the number of lines per unit area in a plane perpendicular to the electric field lines.



$E_1 > E_2$

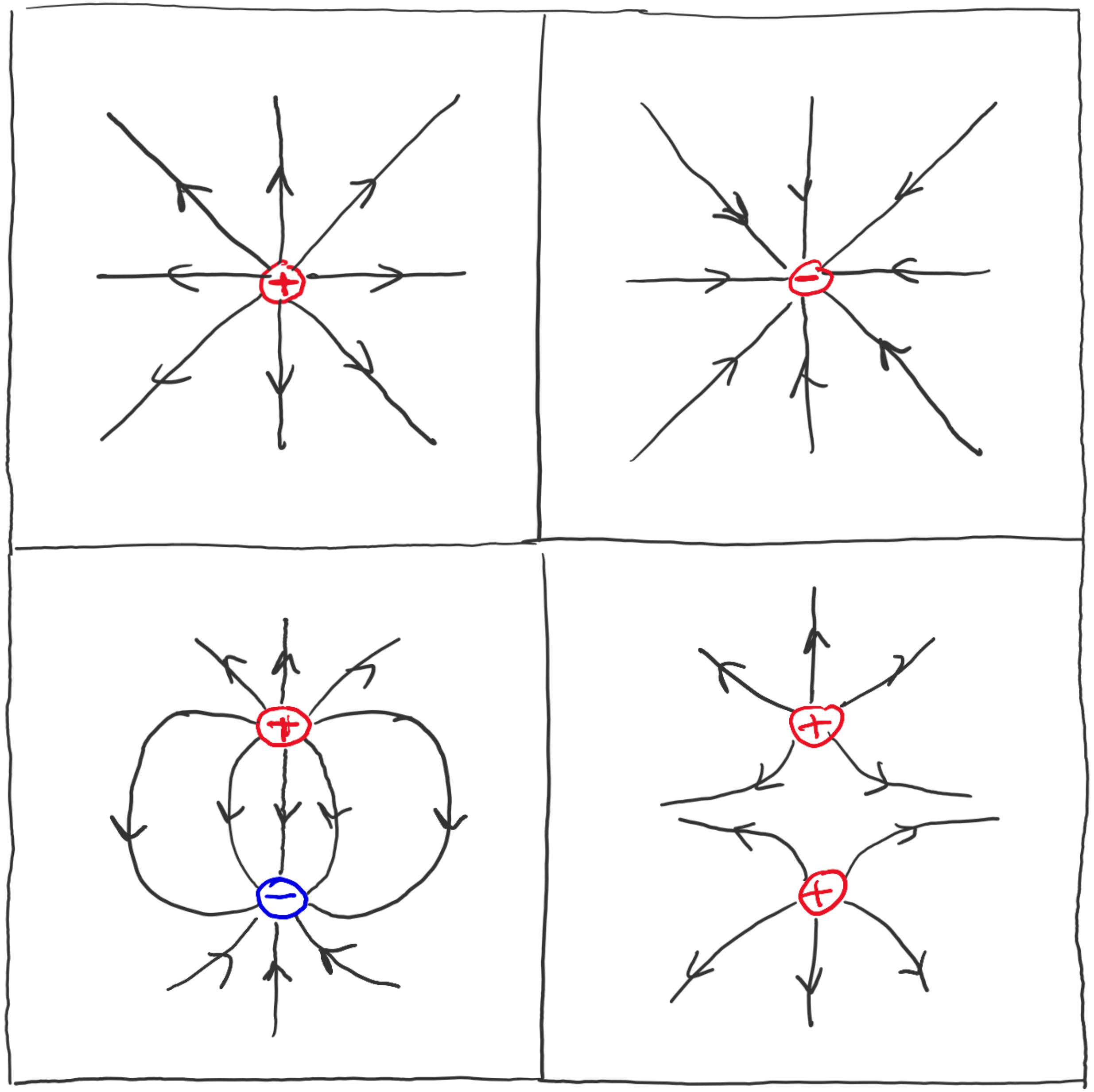
Electric field lines from a point chagre.

Smaller $E \propto \dfrac{8 \text{ lines}}{\text{area of a large sphere}}$

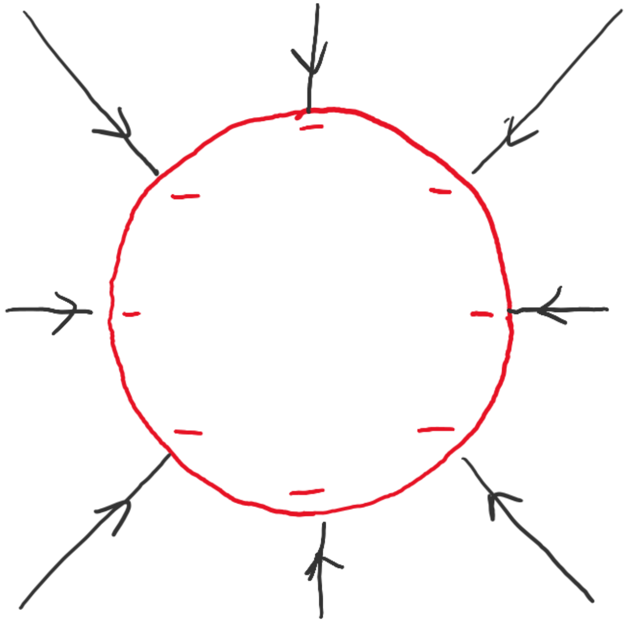


surfare perpendicular to the electric field lines.

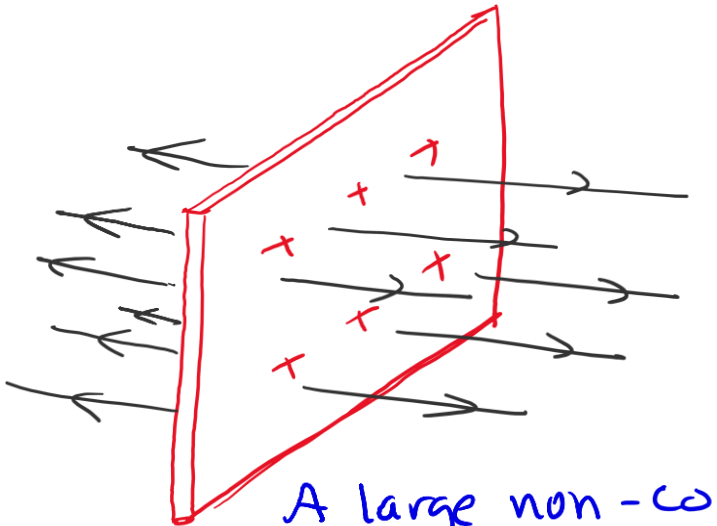
larger $E \propto \dfrac{8 \text{ lines}}{\text{area of a small sphere}}$



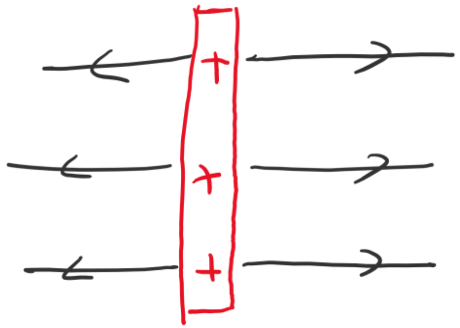
Electric field lines extend away from positive charge and toward negative charge.



a sphere with uniform negative charge



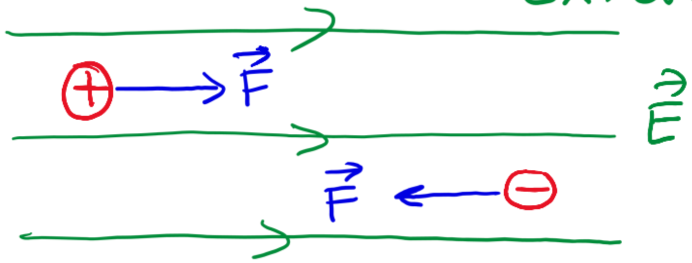
A large non-conducting sheet



Side view

22-6 A point charge in an electric field

External electric field



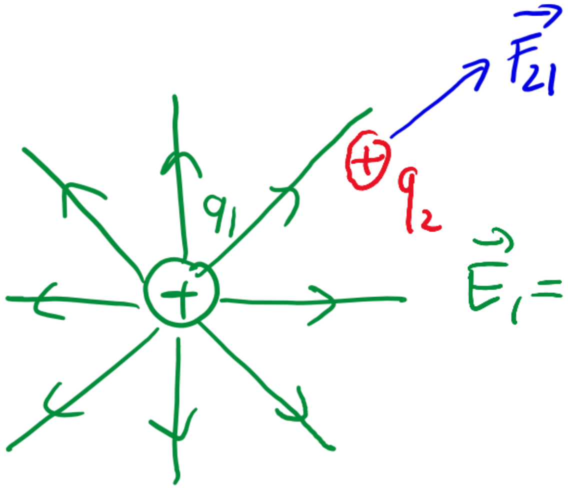
$$\underbrace{\vec{F}}_{\text{electrostatic force}} = q \underbrace{\vec{E}}_{\text{external field}}$$

positive point charge

$\Rightarrow \vec{F}$ same direction as $\vec{E}$

negative point charge

$\Rightarrow \vec{F}$ opposite to $\vec{E}$



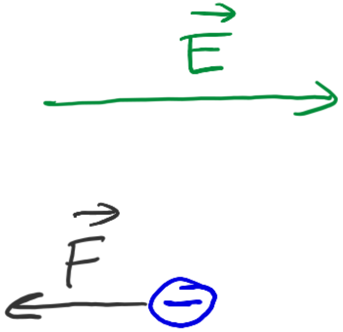
$$\vec{E}_1 = k\frac{q_1}{r^2}\hat{r}$$

$$\vec{F}_{21} = q_2\vec{E}_1 = k\frac{q_1 q_2}{r_{12}^2}\hat{r}$$

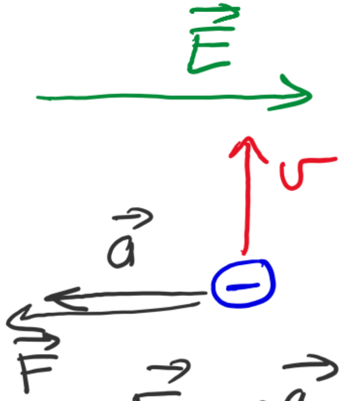
Coulomb's law

Question

Direction of $\vec{F}$?

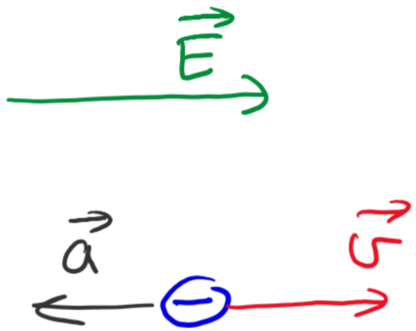


Direction of acceleration?



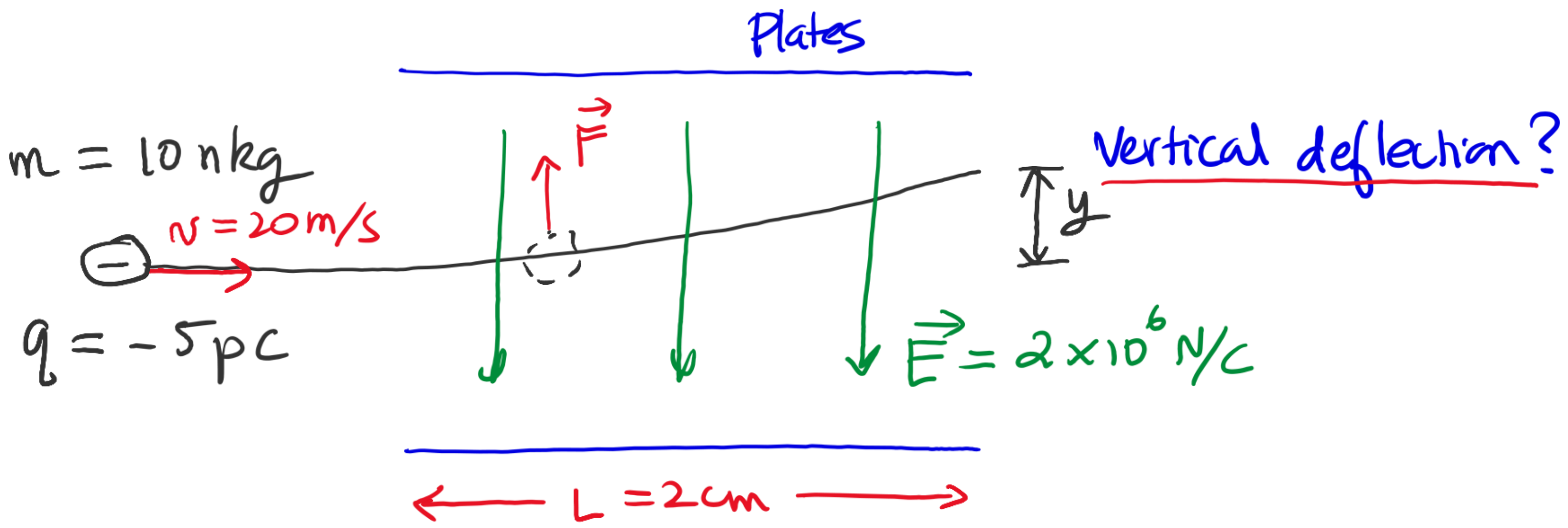
$\vec{F} = m\vec{a}$

How does velocity change?



$\vec{v}$ decreases.

Example



$$y = \frac{1}{2} a_y t^2 = \frac{1}{2}\left(\frac{qE}{m}\right)\left(\frac{L}{v_x}\right)^2 = 0.5\ mm$$

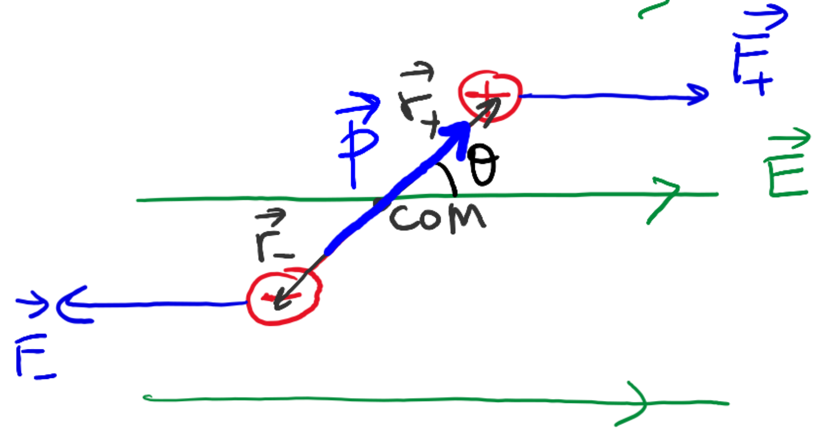
$$F_y = m a_y$$
$$qE = m a_y$$
$$a_y = \frac{qE}{m}$$

$L = v_x t$ (no acceleration along x-direction)

$$t = \frac{L}{v_x}$$

22-7 A dipole in an electric field

$\vec{F} = q\vec{E}$



Net force on the dipole $= 0$

Net torque $\vec{\tau} = \vec{r}_+ \times \vec{F}_+ + \vec{r}_- \times \vec{F}_-$

$$\boxed{\vec{\tau} = \vec{p} \times \vec{E}}$$

electric field

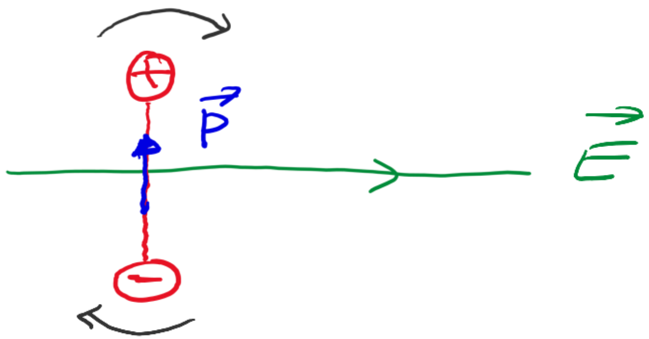
torque   electric dipole moment

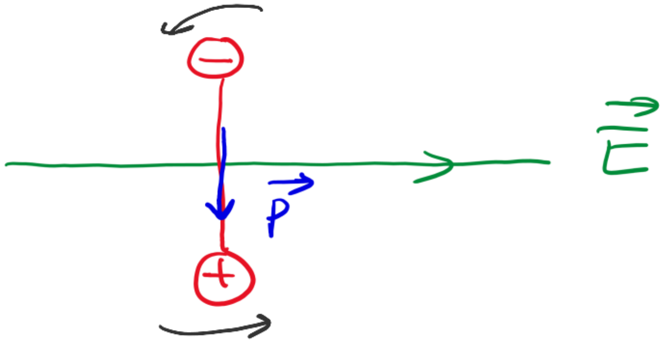
$$\vec{\tau} = \vec{r}_+ \times q\vec{E} + \vec{r}_- \times (-q)\vec{E}$$

$$\vec{\tau} = \underbrace{q(\vec{r}_+ - \vec{r}_-)}_{\vec{p}} \times \vec{E}$$

Magnitude $\tau = pE\sin\theta$

Torque on the dipole tries to align $\vec{p}$ along $\vec{E}$





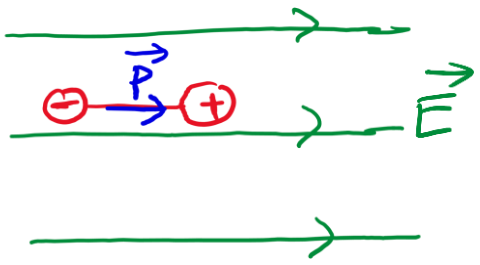
Potential energy of an electric dipole

$$U = -\vec{p}\cdot\vec{E} = -pE\cos\theta$$

$\uparrow$ angle between $\vec{p}$ and $\vec{E}$

$$\vec{\tau} = \vec{p}\times\vec{E}$$

$$\tau = pE\sin\theta$$

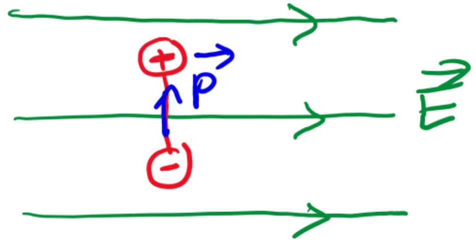


$\theta = 0^\circ$

$U = -pE$

$\tau = 0$

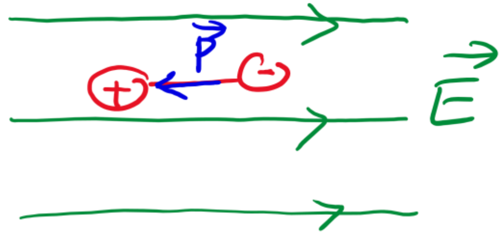
stable equilibrium



$\theta = 90^\circ$

$U = 0$

$\tau = pE$



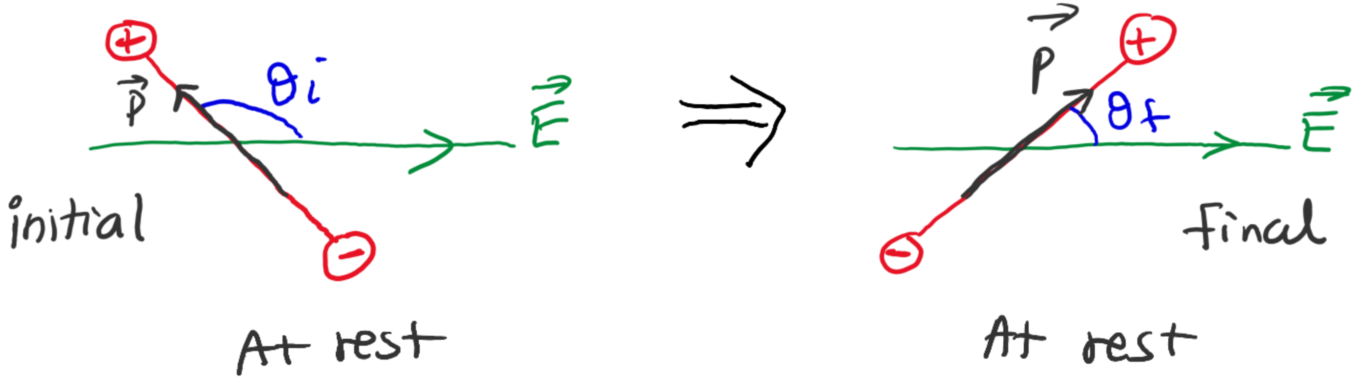
$\theta = 180^\circ$

$U = +pE$

$\tau = 0$

unstable equilibrium

You want to rotate an electric dipole from θ_i to θ_f



Work done by you = − work done by the electric field

Work done by the field $W \equiv -\Delta U$

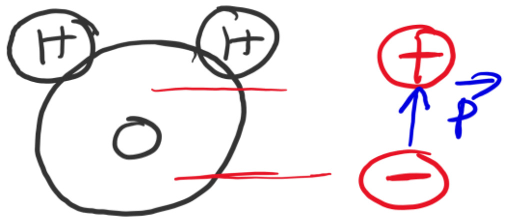
$\Rightarrow$ Work done by $\underbrace{\text{you}}_{\text{applied force}}$ $W_a = \Delta U$

Example

The electric dipole moment for H_2O is 6×10^{-30} C·m

How far apart are the molecule's centers for positive and negative charges?

Hydrogen 1 proton
oxygen 8 protons



$$q = 2\times1e + 1\times8e = 10e$$

$$p = qd \Rightarrow d = \frac{p}{q} = \frac{6\times10^{-30}}{10\times1.6\times10^{-19}} = 4\,\text{pm}$$

H_2O is put in an external electric field $= 2\times10^{-4}$ N/m

How much work you need to do to rotate it from $\theta = 0^\circ$ to $\theta = 180^\circ$

$$W_a = \Delta U = U_f - U_i = -pE\cos\theta_f - (-pE\cos\theta_i)$$

$$= pE + pE = 2pE = +24\times10^{-26}\ \text{J}$$