This is not a replacement for your textbook. It is essential to study from your textbook!

Objetives

Ch 16 : Waves 1

16.1 Transverse waves

16.2 Wave speed on a streched string

16.3 Energy and power of a traveling string wave

~~16.4~~

16.5 Interference of waves

~~16.6~~

16.7 Standing waves and resonance

Mechanical Waves

Obey Newton's Laws

Need medium to propagate

Waves on strings – ch 16

Sound waves – ch 17

Electromagnetic waves

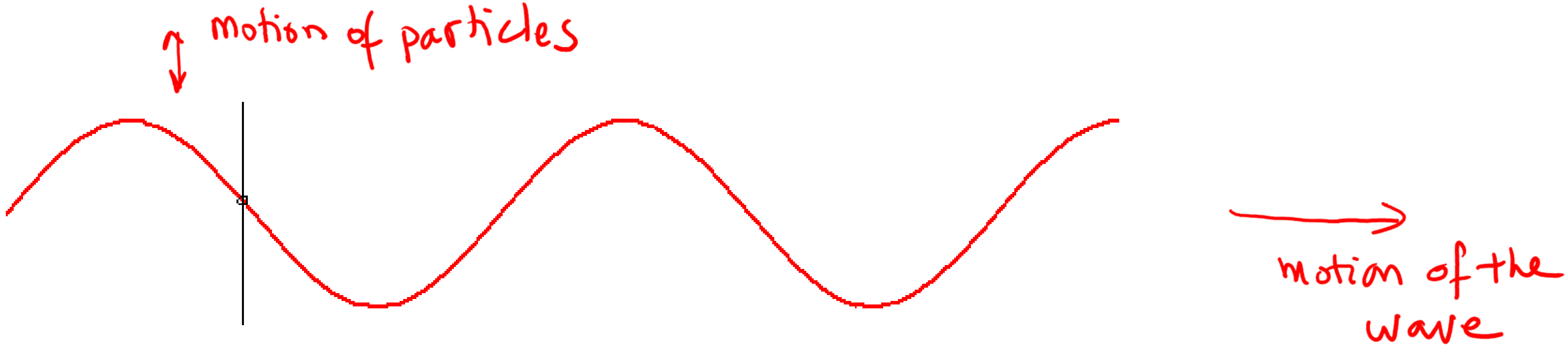
Can propagate in vacuum

Fixed speed in vacuum 3×10^8 m/s

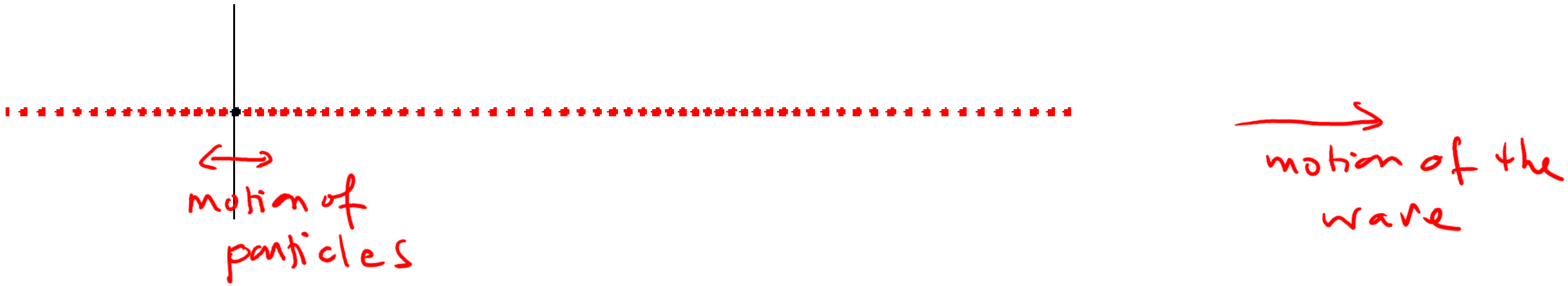
Matter waves

Related to electrons and elementary particles

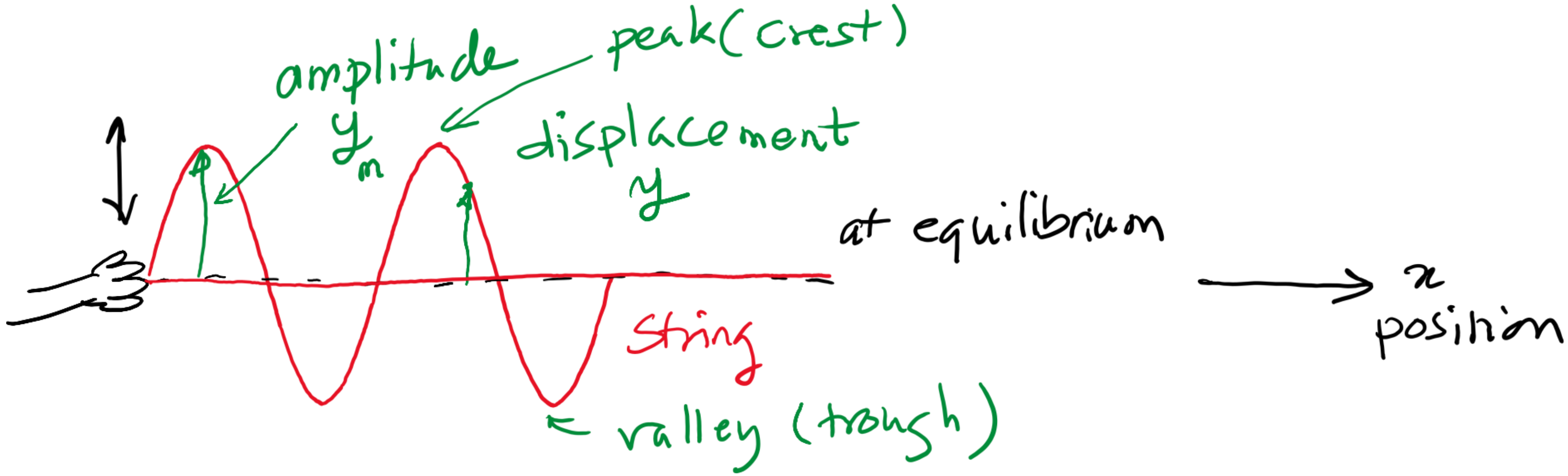
Transverse waves



Longitudinal waves



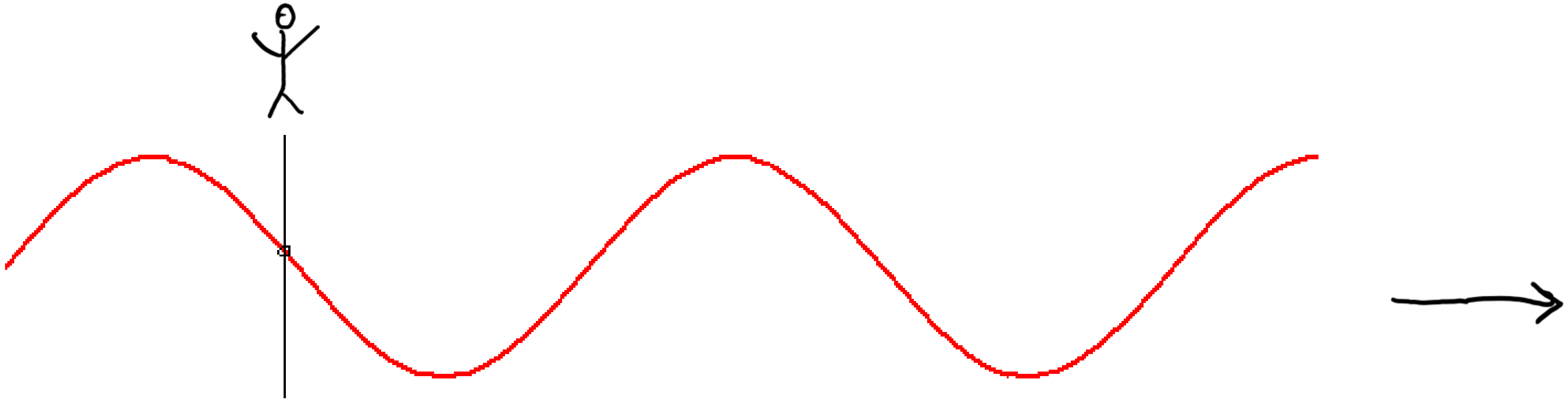
A sinusiodal wave has a shape similar to a sin or cos function



position time phase constant

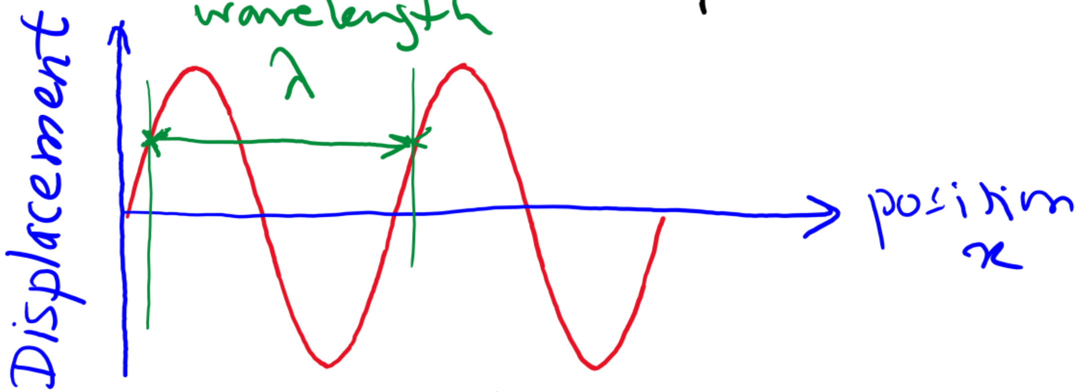
$$y(x,t) = y_m \sin\left(\boxed{2\pi \frac{x}{\lambda} + 2\pi \frac{t}{T} + \phi}\right)$$

displacement amplitude wavelength period phase



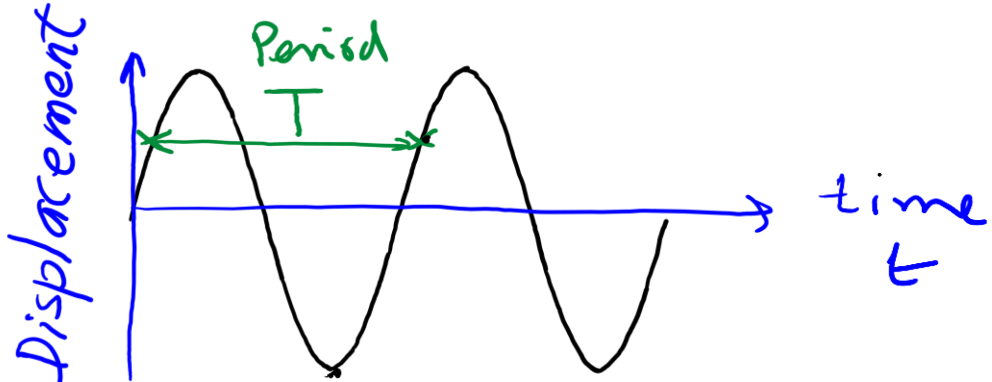
$$y(x,t) = y_m \sin\left(2\pi \frac{x}{\lambda} + 2\pi \frac{t}{T} + \phi\right)$$

At certain time (Snapshot)



wavelength is the distance over which the shape of the wave repeats itself.

At certain location



Period is the time for a string element to make one oscillation.

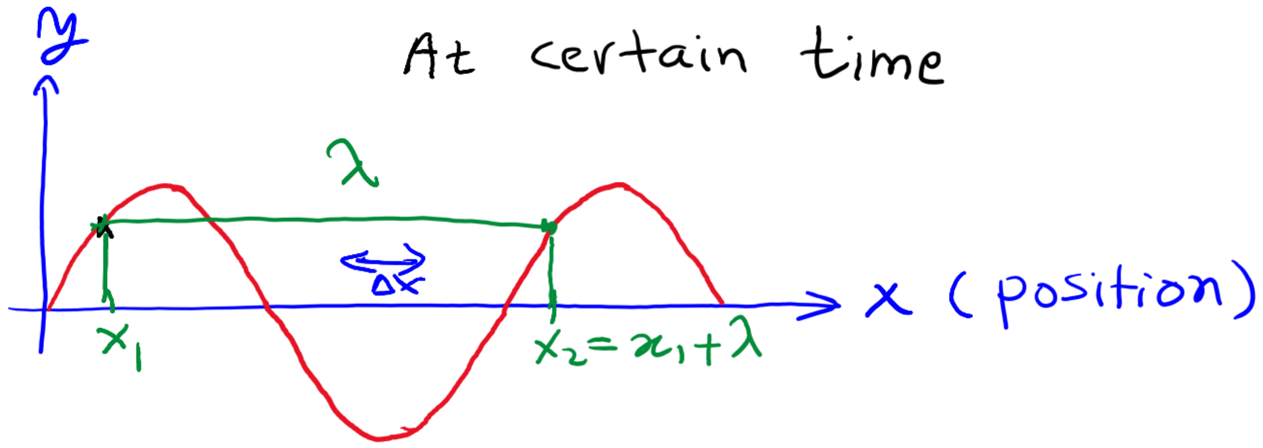
Position and phase

$$y(x_2,t) = y_m \sin\left(2\pi \frac{x_2}{\lambda} + 2\pi \frac{t}{T} + \phi\right)$$

$$= y_m \sin\left(2\pi \frac{x_1+\lambda}{\lambda} + 2\pi \frac{t}{T} + \phi\right)$$

$$= y_m \sin\left(\frac{2\pi x_1}{\lambda} + 2\pi + 2\pi \frac{t}{T} + \phi\right)$$

$$= y(x_1,t)$$



Changing the position by $\lambda \equiv$ changing the phase of the wave by 2π

what is the change in the phase of the wave if the position is changed by Δx?

$$\left.\begin{aligned} \lambda &\equiv 2\pi \\ \Delta x &\equiv \Delta\phi \end{aligned}\right\} \quad \frac{\Delta x}{\lambda} = \frac{\Delta\phi}{2\pi} \Longrightarrow \boxed{\Delta\phi = 2\pi \frac{\Delta x}{\lambda}}$$

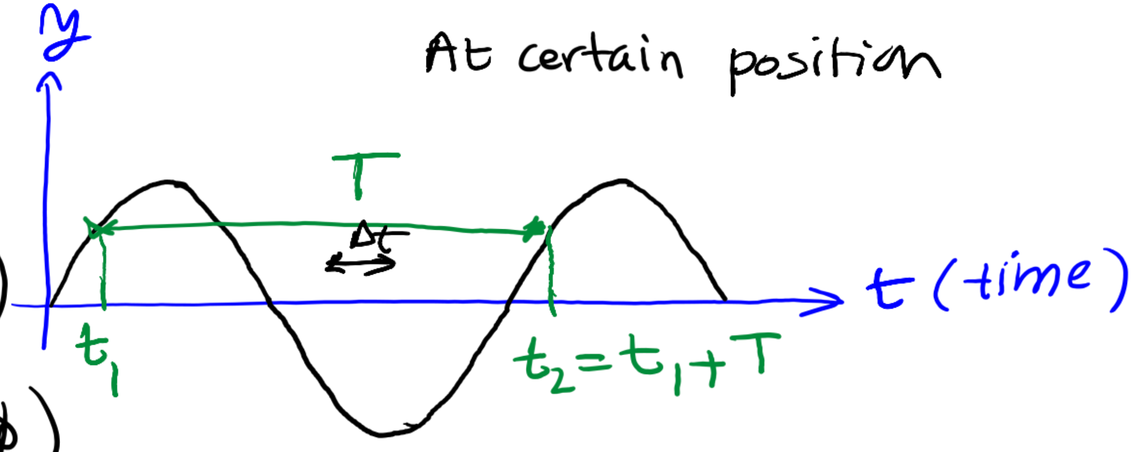
Time and phase

$$y(x,t_2) = y_m \sin\left(2\pi\frac{x}{\lambda} + 2\pi\frac{t_2}{T} + \phi\right)$$

$$= y_m \sin\left(2\pi\frac{x}{\lambda} + 2\pi\frac{t_1+T}{T} + \phi\right)$$

$$= y_m \sin\left(2\pi\frac{x}{\lambda} + \frac{2\pi t_1}{T} + 2\pi + \phi\right)$$

$$= y(x,t_1)$$



changing the time by $T \equiv$ changing the phase by 2π

$$\left.\begin{aligned} T &\equiv 2\pi \\ \Delta t &\equiv \Delta\phi \end{aligned}\right\} \quad \frac{\Delta t}{T} = \frac{\Delta\phi}{2\pi} \Rightarrow \boxed{\Delta\phi = 2\pi\frac{\Delta t}{T}}$$

$$y(x,t) = y_m \sin\left(\underbrace{2\pi\frac{x}{\lambda}}_{k} + \underbrace{2\pi\frac{t}{T}}_{\omega} + \phi\right)$$

$$y(x,t) = y_m \sin(kx + \omega t + \phi)$$

angular wave number
(rad/m)

angular frequency
(rad/s)

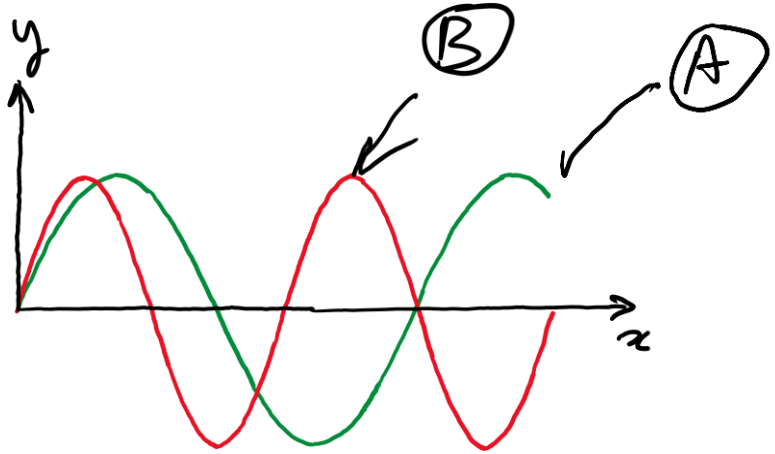
$$f = \frac{1}{T} \qquad (\mathrm{Hz})$$

Question

Match the snapshot to the correct phase

A. phase $= 2x - 8t$

B. phase $= 4x + 10t$



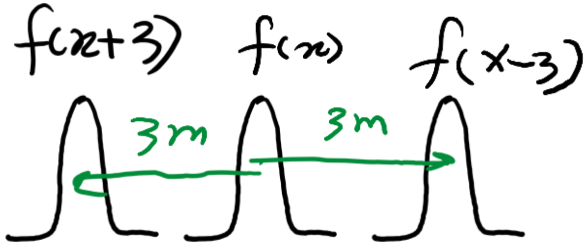
$$kx + \cancel{\omega t}$$

small

$$k = \frac{2\pi}{\lambda}$$

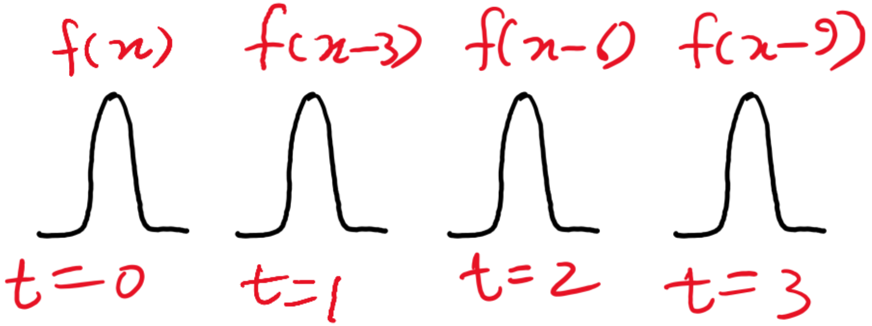
big

Shifting a function



$f(x-3t)$

Speed



$f(x-vt)$ traveling wave moving to the right with speed v

$f(x+vt)$ traveling wave moving to the left with speed v

Example

$\sqrt{5(x-2t)}$ ✓

$\sqrt{5(x^2-2t)}$ ✗

Direction of sinusoidal waves

$f(x-vt) \longrightarrow$

$f(x+vt) \longleftarrow$

$$y = y_m \sin(kx+\omega t+\phi) = y_m \sin\left(k\left(x+\overset{v}{\boxed{\frac{\omega}{k}}}\,t\right)+\phi\right) \qquad \longleftarrow$$

$$y = y_m \sin(kx-\omega t+\phi) = y_m \sin\left(k\left(x-\boxed{\frac{\omega}{k}}\,t\right)+\phi\right) \qquad \longrightarrow$$

$$y = y_m \sin(-kx+\omega t+\phi) = y_m \sin\left(-k\left(x-\boxed{\frac{\omega}{k}}\,t\right)+\phi\right) \qquad \longrightarrow$$

$$y = y_m \sin(-kx-\omega t+\phi) = y_m \sin\left(-k\left(x+\boxed{\frac{\omega}{k}}\,t\right)+\phi\right) \qquad \longleftarrow$$

Speed of a sinusoidal wave

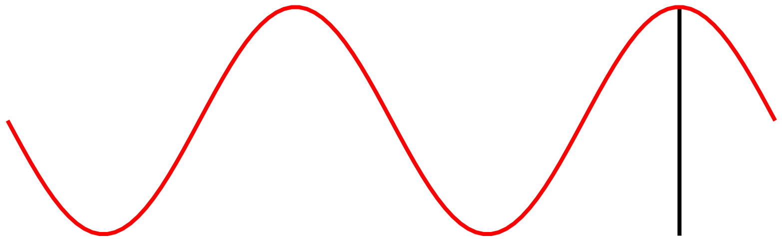
$$\frac{2\pi}{T}$$

$$v = \frac{\omega}{k}$$

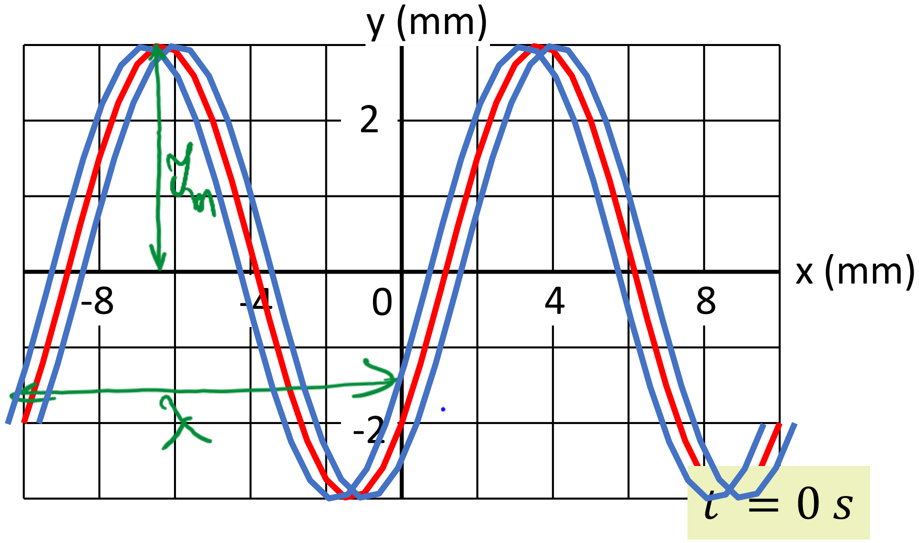
$$\frac{2\pi}{\lambda}$$

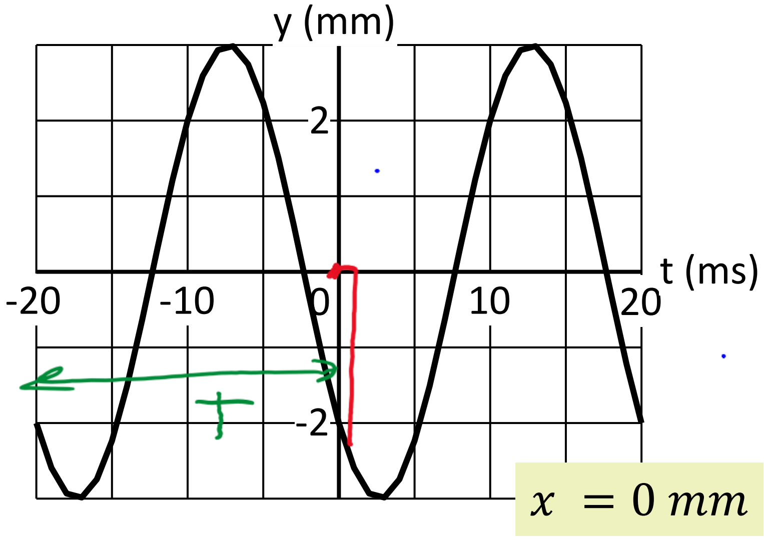
$$\boxed{v = \frac{\lambda}{T}} = \lambda f \qquad\qquad f = \frac{1}{T}$$

A sinusiodal wave moves a distance of one wavelength in a time of one perid.



Example





$$y(x,t) = y_m \sin(kx \pm \omega t + \phi)$$

$$y_m = 3\ mm$$

$$k = \frac{2\pi}{\lambda} = \frac{2\pi}{10\times10^{-3}} = 200\pi\ rad/m$$

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{20\times10^{-3}} = 100\pi\ rad/s$$

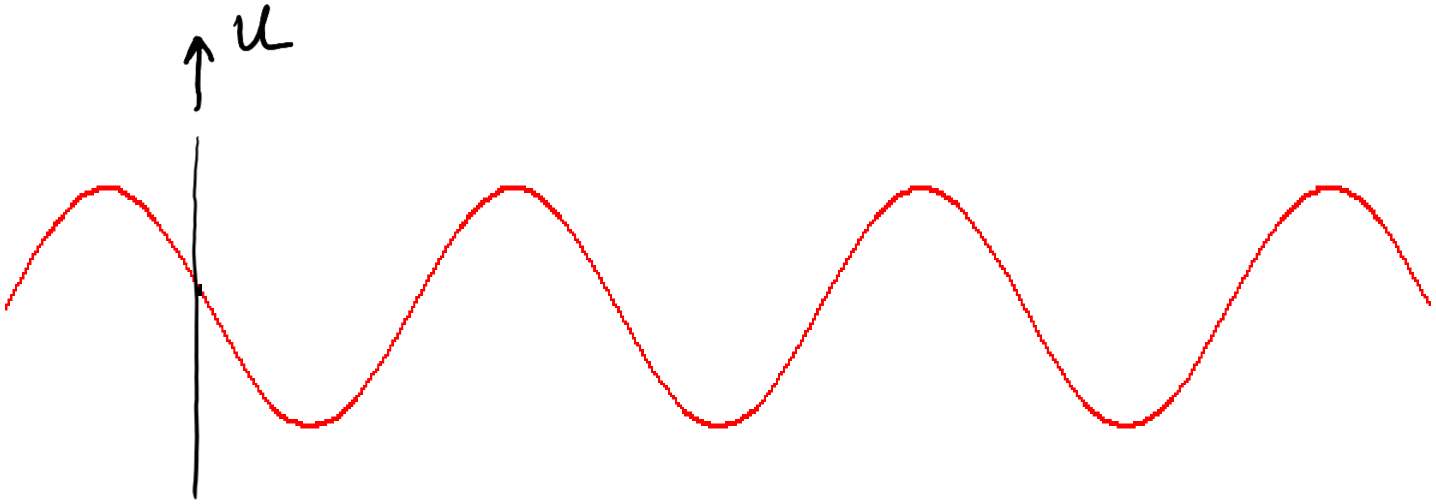
$$y(x=0, t=0) = -2 = 3\sin\phi \Rightarrow \phi = \sin^{-1}\left(\frac{-2}{3}\right) = -0.73\ rad$$

$$y(x,t) = (3\,mm)\sin(200\pi x - 100\pi t - 0.73)$$

Transverse velocity and acceleration

$$y = y_m \sin(kx - \omega t + \phi)$$

A string element moves transversely in a harmonic manner



u = transverse velocity of a string element

$$u = \frac{\partial y}{\partial t} = -\omega y_m \cos(kx - \omega t + \phi)$$

$$a = \frac{\partial u}{\partial t} = -\omega^2 \boxed{y_m \sin(kx - \omega t + \phi)} = -\omega^2 y(x,t)$$

Transverse velocity and acceleration

Displacement

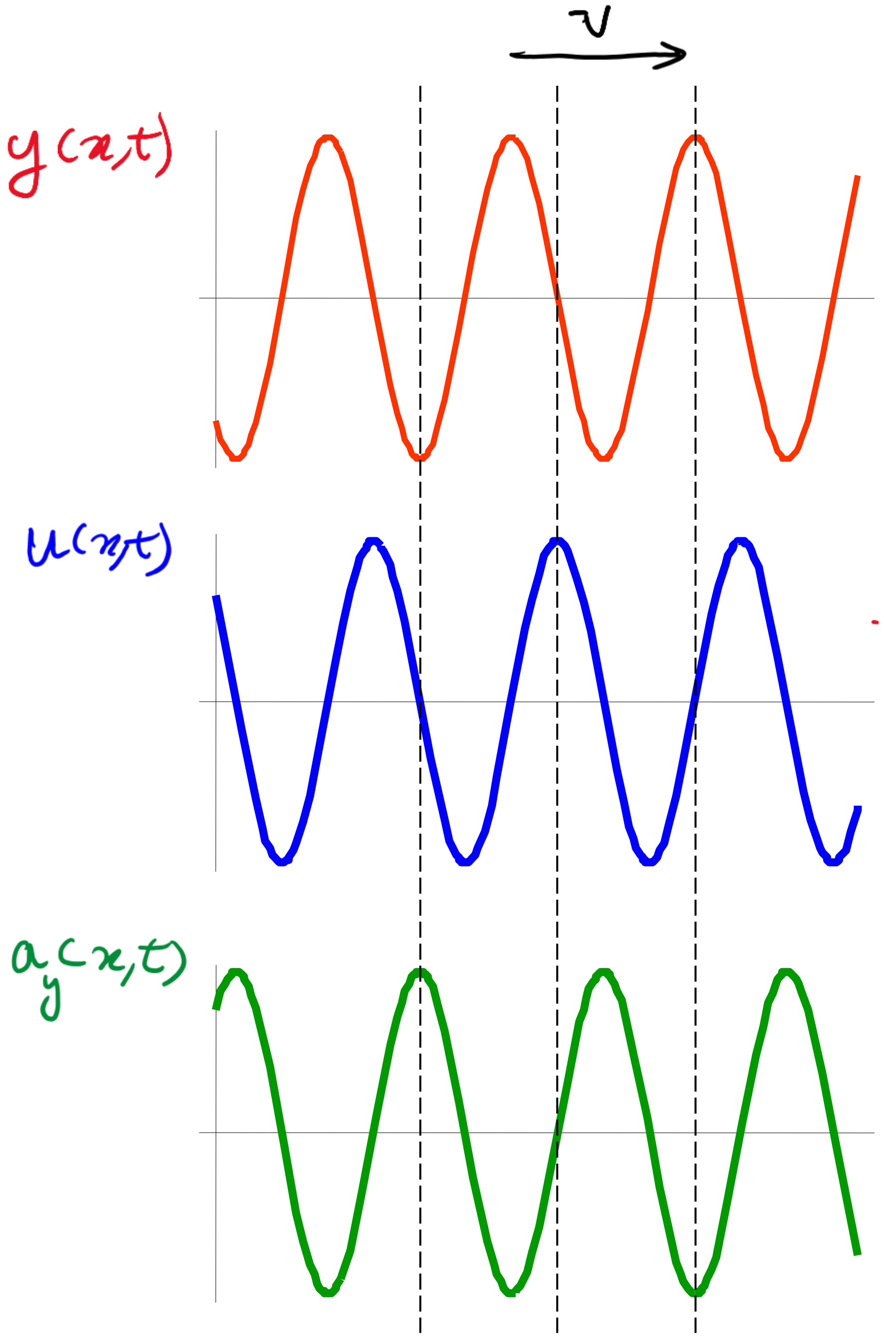
$$y(x,t) = y_m \sin(kx - \omega t + \phi)$$

Transverse velocity

$$u(x,t) = -\omega y_m \cos(kx - \omega t + \phi)$$

Transverse acceleration

$$a_y(x,t) = -\omega^2 y(x,t)$$



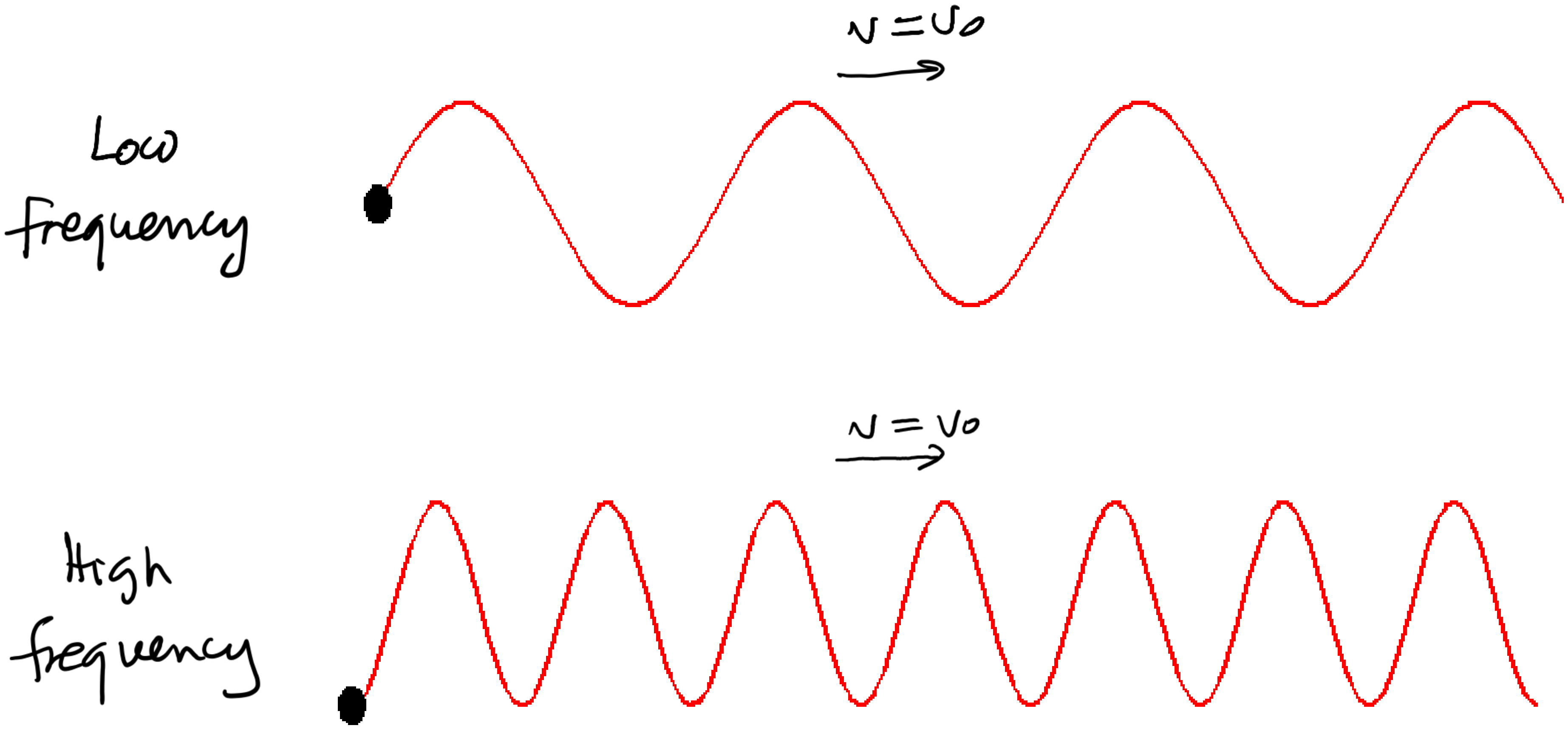
Question

$y(x,t)$	wave speed v	maximum transverse velocity u_{max}
$y(x,t) = 2\sin(\underset{k}{4}x - \underset{\omega}{2}t)$	$\frac{2}{4}$	2×2
$y(x,t) = \sin(3x - 4t)$	$\frac{4}{3}$	4×1
$y(x,t) = 2\sin(3x - 3t)$	$\frac{3}{3}$	3×2

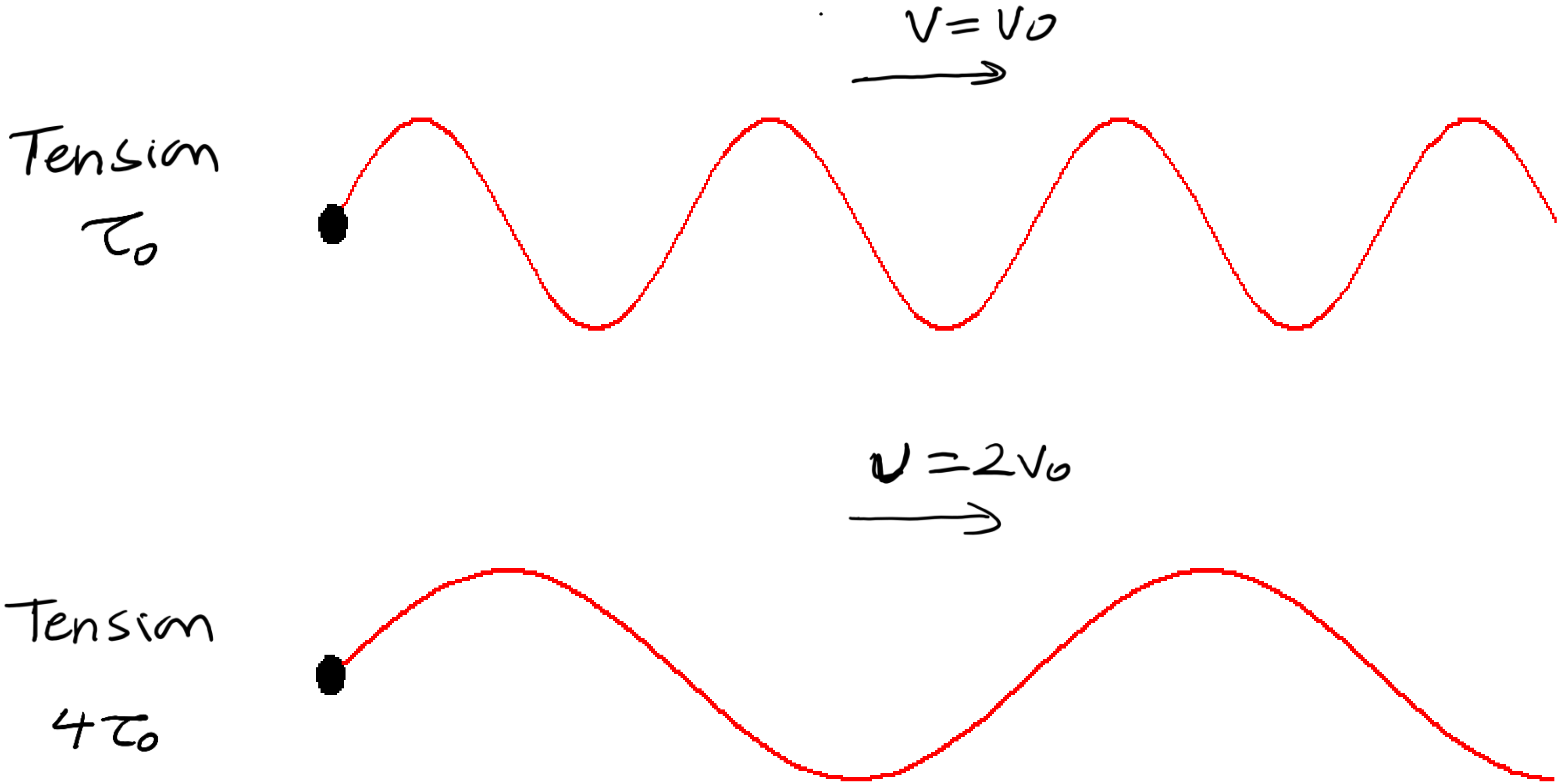
$$v = \frac{\omega}{k}$$

$$u_{max} = \omega\, y_m$$

16.2 Wave speed on a stretched string



The string adjusts the wavelength such that the speed $v = \lambda f$ is kept constant



Increasing the tension in the string by a factor of four increases the speed by a factor of two.

Linear mass density

The linear mass density of a string μ is the mass per unit length of the string

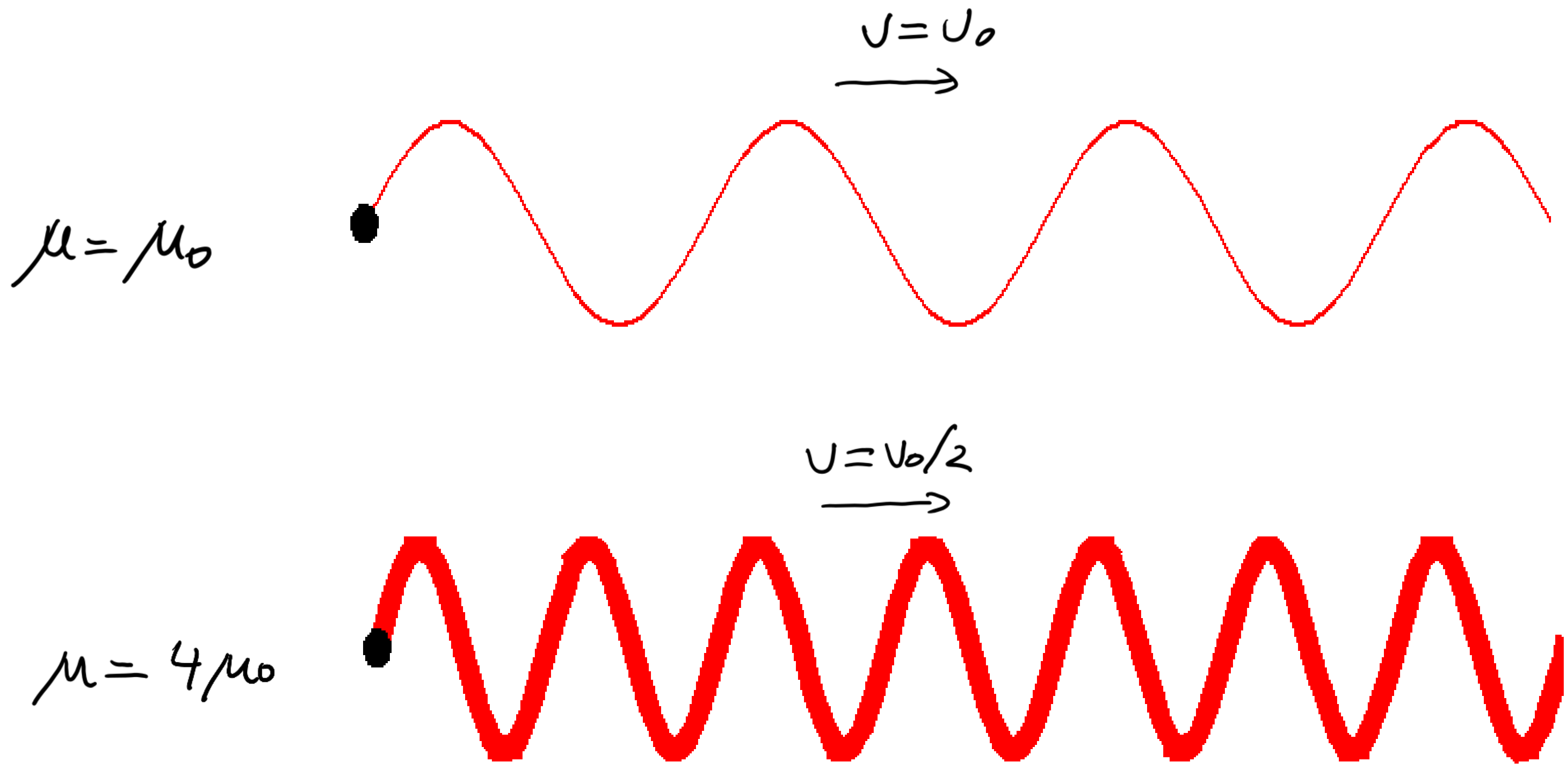
linear mass density

$$\mu = \frac{m}{L}$$

mass of the string

length of the string

The linear mass density of a string depends on the type of the string not on its length.



Increasing the linear mass density of a string by a factor of four, decreases the speed of the wave by a factor of two.

speed of the wave

tension in the string

wavelength

$$v = \sqrt{\frac{\tau}{\mu}} = \lambda f$$

frequency

linear mass density of the string

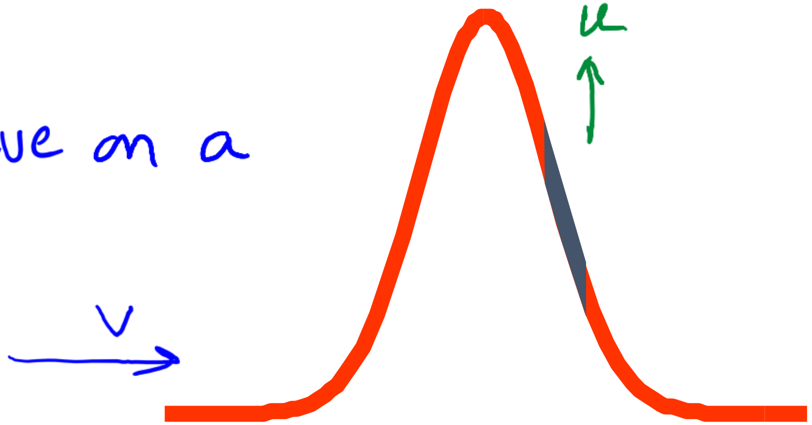
The speed of the wave on a stretched string does <u>not</u> depend on the frequency of the wave.

16.3 Energy and power of a traveling string wave

No wave on the string

A string element

A wave on a string

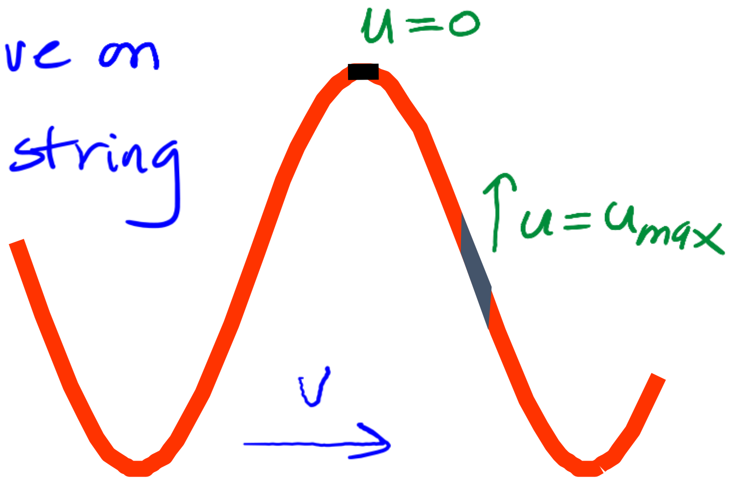


The string element has <u>kinetic energy</u> because it is moving transversely with speed u.

The <u>elastic potential energy</u> of the string element increases because it is stretched more by the wave.

No wave on the string

A wave on the string



At maximum displacement

Zero transverse velocity

$\Rightarrow$ Zero kinetic energy

No extra stretch

$\Rightarrow$ Zero elastic potential energy

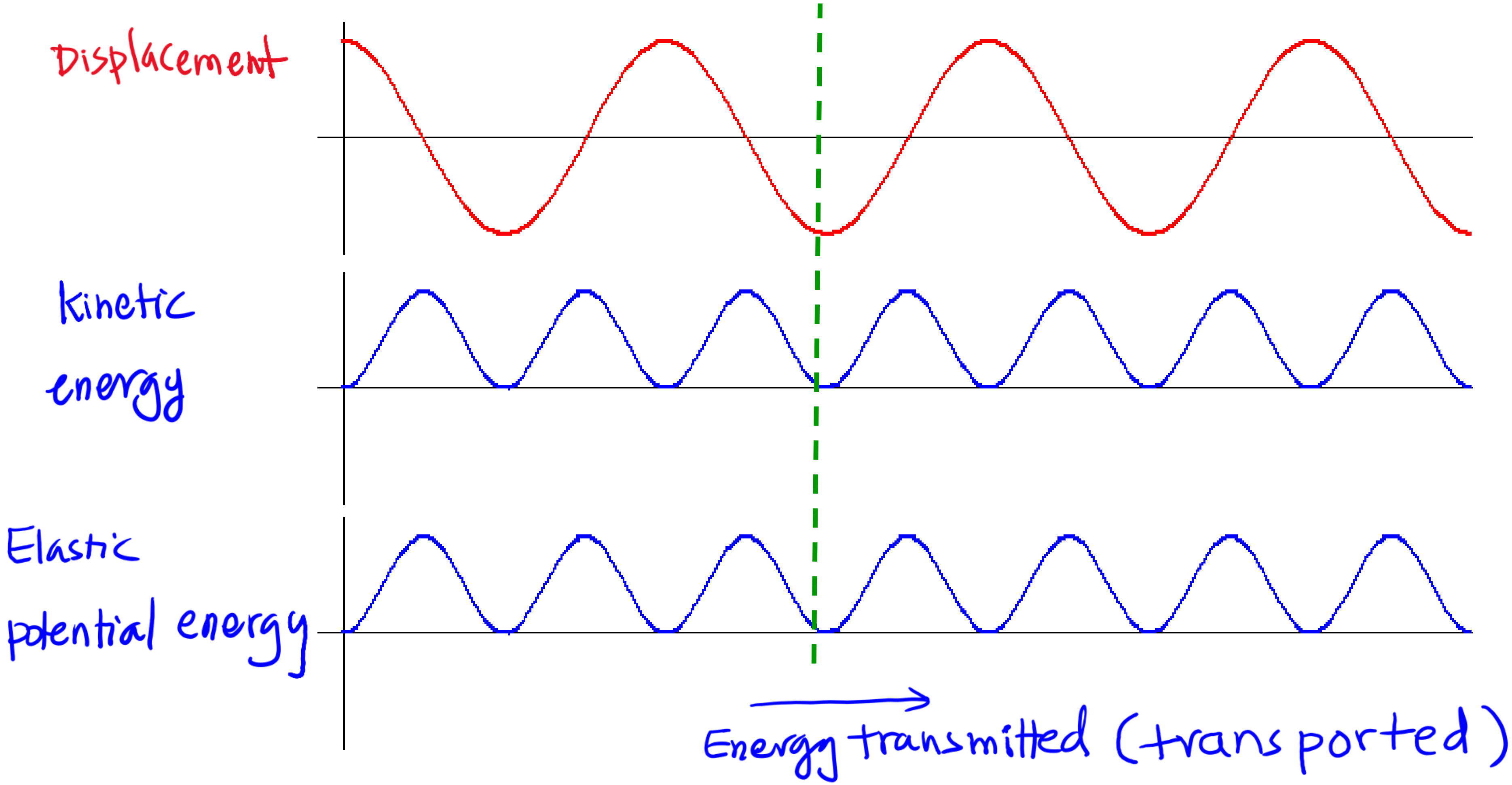
At zero displacement

Maximum transverse velocity

$\Rightarrow$ maximum kinetic energy

Maximum stretch

$\Rightarrow$ Maximum elastic potential energy



Average rate at which energy is transported =

$$\text{Average power transported} = \frac{\text{Energy transported in one period}}{\text{one period}}$$

Average power transmitted

speed of the wave

$$P_{avg} = \frac{1}{2}\mu v \omega^2 y_m^2$$

amplitude

linear mass density

angular frequency

$$(P_{avg})_{kinetic} = (P_{avg})_{elastic} = \frac{1}{2} P_{avg}$$

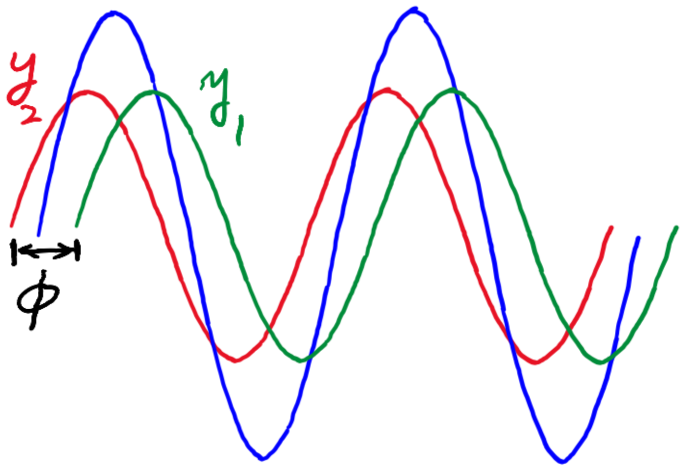
16.5 Interference of waves

Overlapping waves do not alter the travel of each other.
Wave 1 moves through wave 2 as if wave 1 were not present.

Principle of superposition

The displacement of the resultant wave
= displacement of wave 1 + displacement of wave 2

$$y'(x,t) = y_1(x,t) + y_2(x,t)$$



The two waves are moving in the same direction

same amplitude

same wavelength

same frequency

Wave 1 and wave 2 are out of phase by ϕ

Displacement of the resultant wave

$$\sin\alpha + \sin\beta = 2\cos\tfrac{1}{2}(\alpha-\beta)\,\sin\tfrac{1}{2}(\alpha+\beta)$$

$$
\begin{aligned}
y' &= y_1 + y_2 \\
&= y_m \sin\overbrace{(kx-\omega t)}^{\alpha} + y_m \sin\overbrace{(kx-\omega t+\phi)}^{\beta} \\
&= \underbrace{2y_m\cos\tfrac{1}{2}\phi}\,\sin(kx-\omega t+\tfrac{1}{2}\phi)
\end{aligned}
$$

amplitude $= 2y_m\left|\cos\frac{1}{2}\phi\right|$

The resultant wave has the same wavelength and frequency

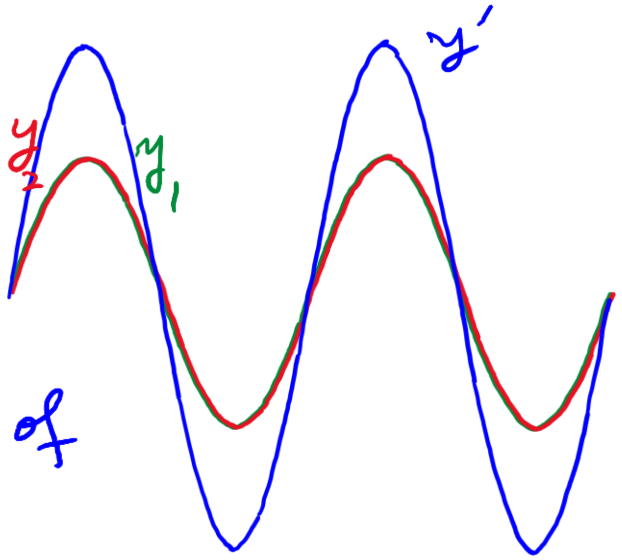
$$y'(x,t) = \boxed{2y_m \cos\tfrac{1}{2}\phi}\,\sin(kx-\omega t+\tfrac{1}{2}\phi)$$

Constructive interference

y' has maximum amplitude $2y_m$

$\cos\frac{1}{2}\phi = \pm 1 \implies \phi = 0, \pm 2\pi, \ldots$

wave 1 and 2 are shifted by a distance of $0, \pm\lambda, \pm 2\lambda$

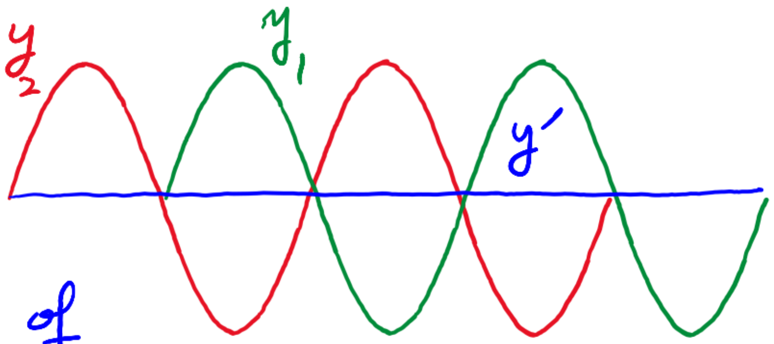


Destructive interference

y' has zero amplitude

$\cos\frac{1}{2}\phi = 0 \implies \phi = \pm\pi, \pm 3\pi$

wave 1 and 2 are shifted by a distance of $\pm\frac{\lambda}{2}, \pm\frac{3\lambda}{2}$



Intermidiate interference

$1 > |\cos\frac{1}{2}\phi| > 0$

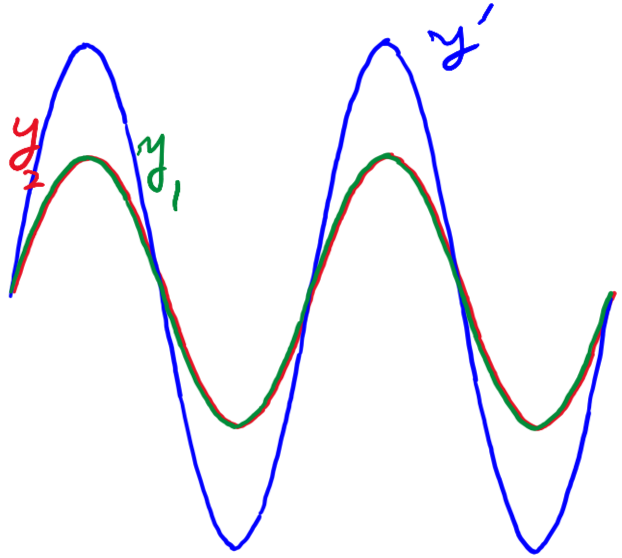
$$y'(x,t) = \boxed{2y_m \cos \tfrac{1}{2}\phi} \sin(kx - \omega t + \tfrac{1}{2}\phi)$$

Constructive interference

$\phi = 0, \pm 2\pi, \pm 4\pi, \ldots$

shifted by $0, \pm\lambda, \pm 2\lambda, \ldots$

Wave 1 and 2 are <u>in phase</u>

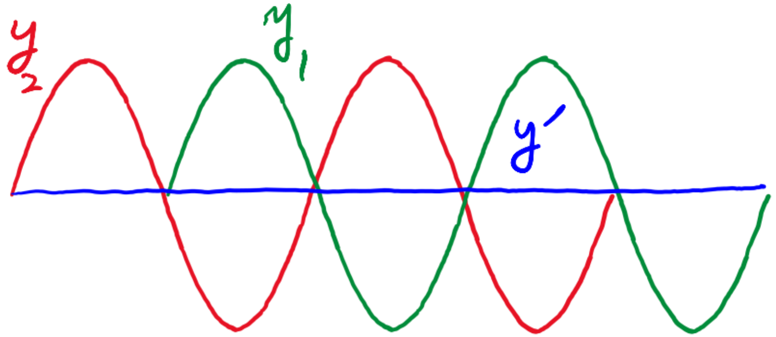


Destructive interference

$\phi = \pm\pi, \pm 3\pi, \pm 5\pi, \ldots$

shifted by $\pm\frac{\lambda}{2}, \pm\frac{3\lambda}{2}, \pm\frac{5\lambda}{2}, \ldots$

Wave 1 and 2 are <u>out of phase</u>



Example

What is the resultant amplitude if the two waves are shifted by 0.4λ ?

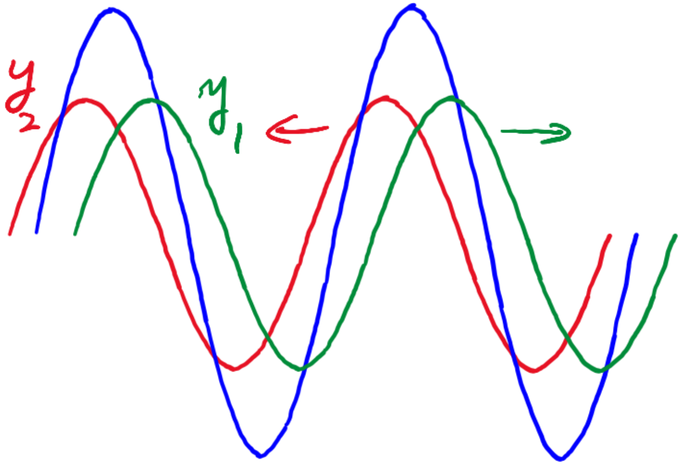
$$\text{Amplitude} = 2y_m \left| \cos \tfrac{1}{2}\phi \right|$$

$$\left.\begin{aligned} 2\pi &\equiv \lambda \\ \phi &\equiv \underset{\substack{\uparrow \\ 0.4\lambda}}{\Delta x} \end{aligned}\right\} \quad \frac{\phi}{2\pi} = \frac{\Delta x}{\lambda} = \frac{0.4\lambda}{\lambda} \Rightarrow \phi = 2\pi(0.4)$$

$$\text{Resultant amplitude} = 2y_m \left| \cos\left[\frac{1}{2}(2\pi)(0.4)\right] \right|$$

16.7 Standing waves

wave 1 and wave 2 are traveling in <u>opposite directions</u>



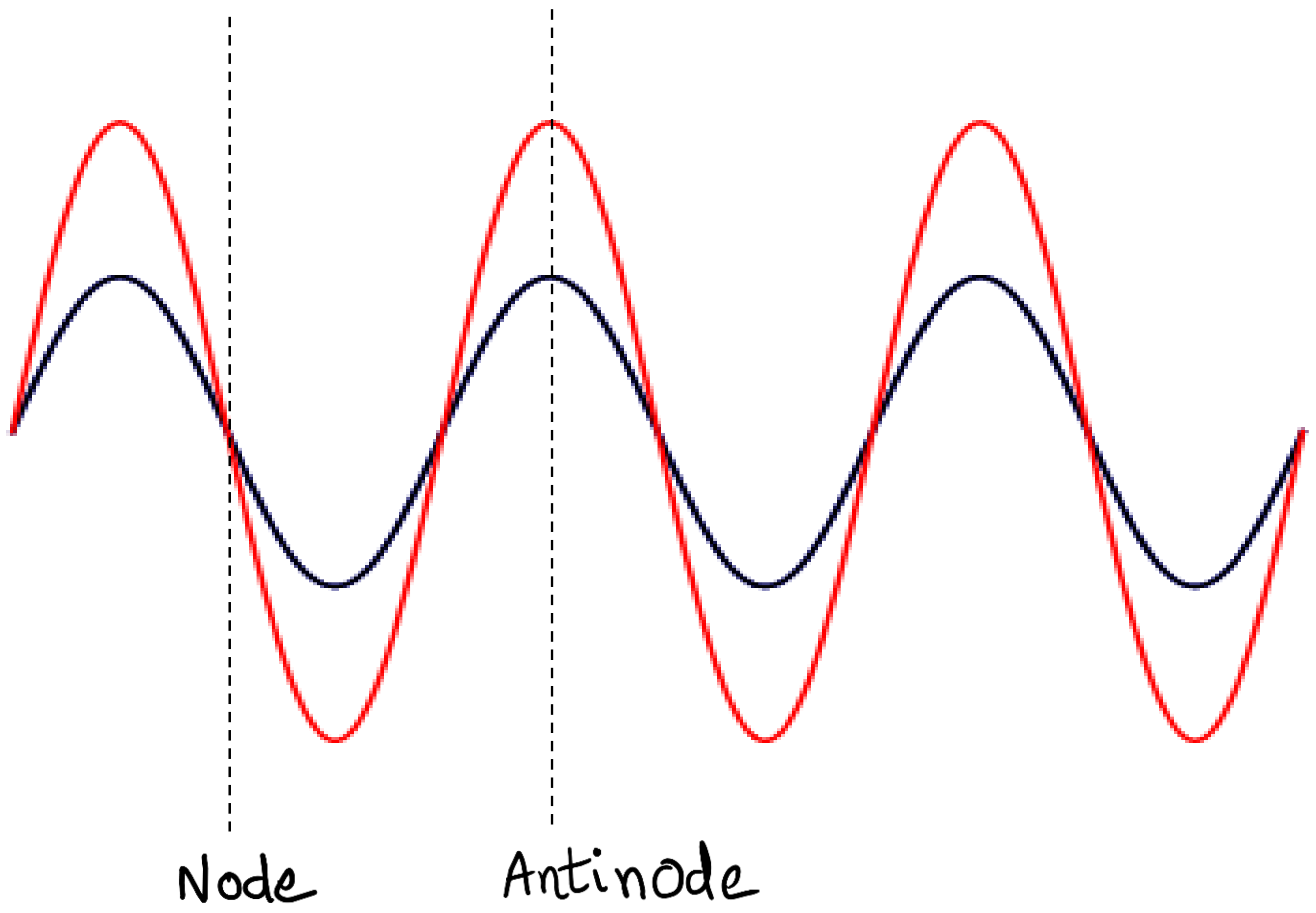
They have the same amplitude
same frequency
same wavelength

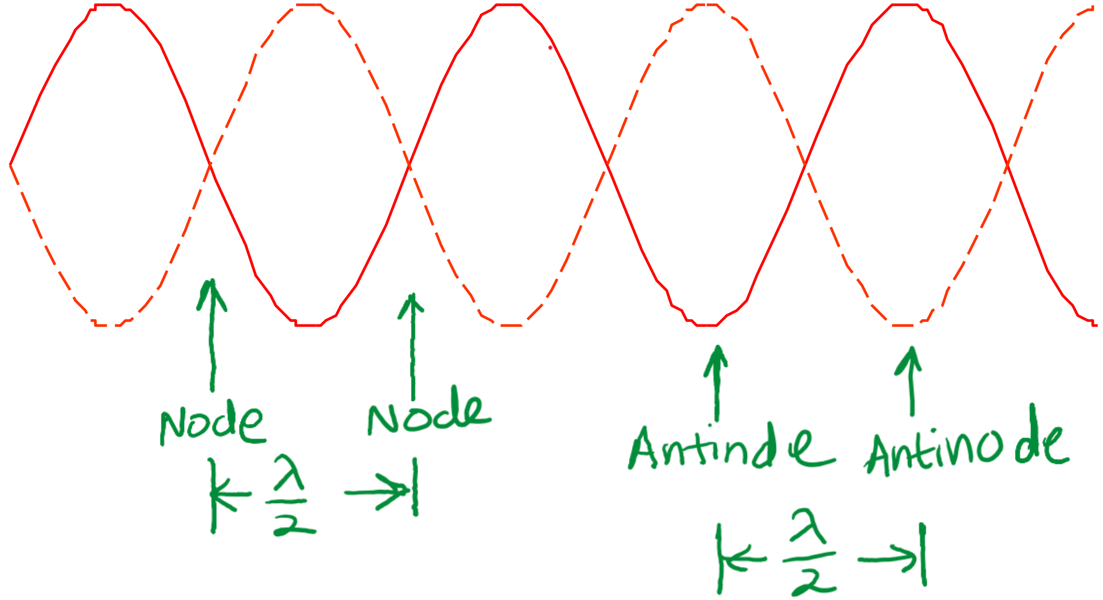
$$\sin\alpha + \sin\beta = 2\cos\tfrac{1}{2}(\alpha-\beta)\,\sin\tfrac{1}{2}(\alpha+\beta)$$

$$\begin{aligned} y' &= y_1 + y_2 \\ &= y_m \sin(\overbrace{kx-\omega t}^{\alpha}) + y_m \sin(\overbrace{kx+\omega t}^{\beta}) \\ &= \underbrace{2y_m \sin kx}_{\text{Amplitude}}\ \underbrace{\cos \omega t}_{\text{Oscillating term}} \end{aligned}$$

Not a traveling wave
It is a standing wave.

Amplitude
$2y_m|\sin kx|$


Node
Antinode



$$y' = \boxed{2y_m \sin kx} \cos \omega t$$

Nodes $\quad \sin kx = 0 \Rightarrow kx = n\pi \qquad n = 0, \pm 1, \pm 2, \dots$

$$\frac{2\pi}{\lambda} x = n\pi \Rightarrow x = \frac{n}{2}\lambda$$

Antinodes $\quad \sin kx = \pm 1 \Rightarrow kx = (2n+1)\frac{\pi}{2}$

$$\frac{2\pi}{\lambda} x = (2n+1)\frac{\pi}{2} \Rightarrow x = \left(n + \frac{1}{2}\right)\frac{\lambda}{2}$$

Question

$y' = 4\sin(3x-2t)$ traveling wave $\longrightarrow$

$y' = 4\sin(3x+2t)$ traveling wave $\longleftarrow$

$y' = 4\sin(3x)\cos(2t)$ standing wave

Reflection at boundaries

Hard reflection

Wall

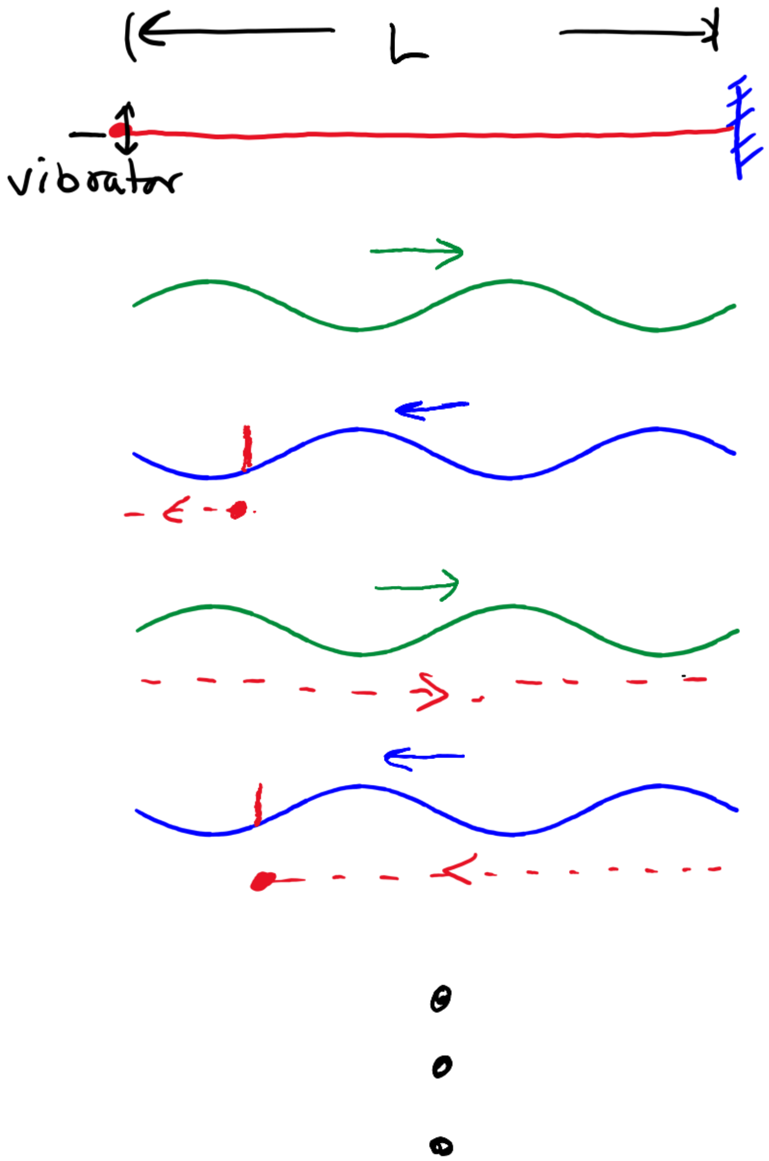
Inverted

Soft reflection

rod and ring

Not inverted

Resonance



Resonance occurs when the left-going (or the right-going) waves are in phase

change in position $= n\lambda$ $\quad n = 1, 2, \cdots$

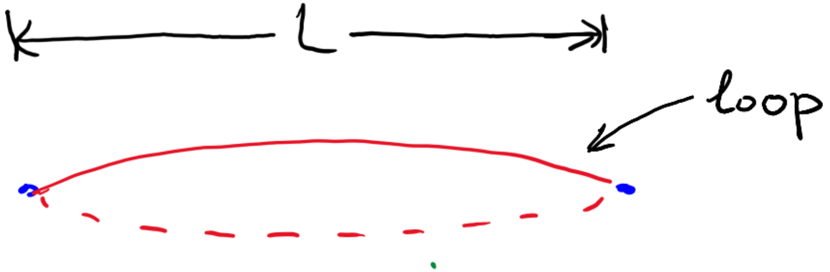
$$2L = n\lambda$$

Resonant wavelength $\quad \lambda_n = \frac{2L}{n}$

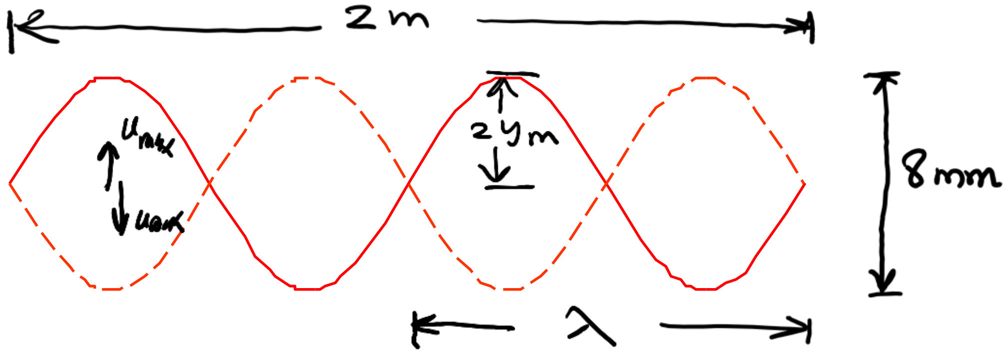
Resonant frequency $\quad f_n = \frac{v}{\lambda_n} = n\frac{v}{2L}$

Oscillation modes

$n = 1, 2, \ldots$

n	oscillation mode	$\lambda_n = \frac{2L}{n}$	$f_n = n\overbrace{\boxed{\frac{v}{2L}}}^{f_1}$
1	 The first harmonic (The fundamental mode)	$\lambda_1 = \frac{2L}{1}$	$f_1 = \frac{v}{2L}$
2	 The second harmonic	$\lambda_2 = \frac{2L}{2}$	$f_2 = 2f_1$
3	 The third harmonic	$\lambda_3 = \frac{2L}{3}$	$f_3 = 3f_1$

Example



$\tau = 100$ N

$m = 20$ g

$\mu = \frac{m}{L} = \frac{20 \times 10^{-3}}{2} = 0.01 \ \frac{kg}{m}$

What is

$\lambda = 1$ m

$n = 4$

$v = \sqrt{\frac{\tau}{\mu}} = \sqrt{\frac{100}{0.01}} = 100$ m/s

$f = \frac{v}{\lambda} = 100$ Hz

maximum transverse speed

Displacement: $y = 2y_m \sin kx \cos \omega t$

transverse velocity: $u = \frac{\partial y}{\partial t}$

$u = -2\omega y_m \sin kx \sin \omega t$

$u_{max} = 2\omega y_m = 2(2\pi(100))(2\times10^{-3})$

Displacement when a string element has maximum transverse speed

$\sin \omega t = \pm 1 \Rightarrow \cos \omega t = 0$

$\Rightarrow y = 0$