#6.

Free particle

$$\psi(\vec{r},t) = \psi_1(x)\,\psi_2(y)\,\psi_3(z)\,\phi(t)$$

$$-\frac{\hbar^2}{2m}\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)\psi(\vec{r},t) = i\hbar\frac{\partial}{\partial t}\psi(\vec{r},t)$$

dividing by $\psi(\vec{r},t)$ gives

$$-\frac{\hbar^2}{2m\psi_1(x)}\frac{\partial^2\psi_1(x)}{\partial x^2}-\frac{\hbar^2}{2m\psi_2(y)}\frac{\partial^2\psi_2(y)}{\partial y^2}-\frac{\hbar^2}{2m\psi_3(z)}\frac{\partial^2\psi_3(z)}{\partial z^2} = -\frac{i\hbar}{\phi(t)}\frac{\partial\phi(t)}{\partial t}$$

$$-\frac{\hbar^2}{2m}\frac{\partial^2\psi_1(x)}{\partial x^2} = E_1\psi_1(x) \Rightarrow \frac{\partial^2\psi_1}{\partial x^2}+k_1^2\psi_1 = 0 \quad -(1)$$

$$-\frac{\hbar^2}{2m}\frac{\partial^2\psi_2(y)}{\partial y^2} = E_2\psi_2(y) \Rightarrow \frac{\partial^2\psi_2}{\partial y^2}+k_2^2\psi_2 = 0 \quad -(2)$$

$$-\frac{\hbar^2}{2m}\frac{\partial^2\psi_3(z)}{\partial z^2} = E_3\psi_3(z) \Rightarrow \frac{\partial^2\psi_3}{\partial z^2}+k_3^2\psi_3 = 0 \quad -(3)$$

$$i\hbar\frac{\partial\phi(t)}{\partial t} = E\phi(t) \Rightarrow \frac{\partial\phi}{\partial t}+\frac{iE}{\hbar}\phi = 0 \quad -(4)$$

$$E = E_1+E_2+E_3$$

$$(4) \Rightarrow \phi(t) = C e^{-\frac{iE}{\hbar}t} = C e^{-i\omega t}$$

$$(1) \Rightarrow \psi_1(x) = A_1\sin k_1x + B_1\cos k_1x$$

$$\psi_2(y) = A_2\sin k_2y + B_2\cos k_2y$$

$$\psi_3(z) = A_3\sin k_3z + B_3\cos k_3z$$