Exam Problems from Chapter 11

A cord is wrapped around the rim of a flywheel 0.50 m in radius. A constant force of 50.0 N is applied on the cord, as shown in the figure. The wheel is mounted on frictionless bearings on a horizontal shaft through its center. The moment of inertia of the wheel is 4.00 kg . $\left(\mathrm{m}^{\star *} 2\right.$ ). If the wheel starts from rest, find its angular velocity after 4.00 seconds.
(A) 25.0 radians/second
B. 20.0 radians/second
C. 40.0 radians/second
D. 35.0 radians/second
E. 30.0 radians/second

$$
\begin{array}{ll}
\omega_{0}=0 & t=4 \mathrm{sec} \\
\omega=\omega_{0}+\alpha t & I=4 \mathrm{kgm}^{2}
\end{array}
$$


$\tau=I \alpha$

$$
\begin{aligned}
& I \alpha \\
& F R=I \alpha \Rightarrow \alpha=\frac{F R}{I}=\frac{(50)(0.5)}{4}=62 . \frac{5 \mathrm{rad}}{\mathrm{~s}^{2}}
\end{aligned}
$$

$$
\omega=(6.2 .5)(4)=250 \frac{\mathrm{rad}}{\mathrm{~s}}
$$

A torque of $0.80 \mathrm{~N} . \mathrm{m}$ applied to a pulley increases its angular speed fy 45.0 revolutions/minute to 180 revolutions/min in 3.00 seconds. Find $t$ moment of inertia of the pulley.
A. $\quad 0.11 \mathrm{~kg}^{*}\left(\mathrm{~m}^{* *} 2\right)$
B. $0.21 \mathrm{~kg}^{*}\left(\mathrm{~m}^{* *} 2\right)$
C. $0.17 \mathrm{~kg}^{*}\left(\mathrm{~m}^{* *}\right)$
E. $\quad 0.30 \mathrm{~kg}^{\star}\left(\mathrm{m}^{* *} 2\right)$

$$
\begin{gathered}
\tau=I \alpha \quad \Rightarrow I=\frac{\tau}{\alpha} \\
\omega=\omega_{0}+\alpha t \Rightarrow \alpha=\frac{\omega-\omega_{0}}{t}=4.7 \mathrm{rad} / \mathrm{s}^{2} \\
\omega_{0}=45 \frac{22 \pi}{60}=4.7 \mathrm{rad} / \mathrm{s} \\
\omega=18.8 \mathrm{rad} / \mathrm{s} \\
I=\left(\frac{4.7}{0.8}\right)^{-1}=\frac{0.8}{4.7}=0.17 \mathrm{~kg} \mathrm{~m}^{2}
\end{gathered}
$$

A disk of radius 10 cm is free to rotate about a frictionless axle perpendicular to the disk and situated at a point on its rim. The disk is released from a position where its center is at the same height as the axle (see figure). Find the velocity of the center at its lowest position.

A. $\quad 3.21 \mathrm{~m} / \mathrm{s}$
B. $\quad 1.41 \mathrm{~m} / \mathrm{s}$
C. $21.30 \mathrm{~m} / \mathrm{s}$
D. $0.76 \mathrm{~m} / \mathrm{s}$
(E.) $1.14 \mathrm{~m} / \mathrm{s}$

$$
\begin{gathered}
\Delta K+\Delta u_{g}=0 \\
\left(\frac{1}{2} I \omega_{f}^{2}-0\right)+(-m g h)=0 \\
h=R \Rightarrow \frac{1}{2} I \omega_{f}^{2}=m g R \\
I=I_{\tau_{m}}+m d^{2} \\
I=\frac{1}{2} m R^{2}+m R^{2}=\frac{3}{2} m R^{2} \\
\frac{3}{4} m R^{2} \omega_{f}^{2}=m g R \\
\omega_{f}=\sqrt{\frac{4}{3} \frac{g}{R}}=1.14 \mathrm{rad} / \mathrm{s}
\end{gathered}
$$

A mass of 5 kg is suspended from a light string which is wrapped around a flywheel of mass 5 kg and radius 0.5 m . The wheel is mounted on frictionless bearings on a horizontal shaft through its center (see figure). Find the acceleration of the mass and tension in the
 string. (take $I$ wheel $=1 / 2$ * $M^{*} R^{* *} 2$ ).
A. $\quad 10.80 \mathrm{~m} / \mathrm{s}^{\star \star} 2,26.54 \mathrm{~N}$
B. $\quad 3.75 \mathrm{~m} / \mathrm{s}^{* *} 2,49.00 \mathrm{~N}$
C. $\quad 5.35 \mathrm{~m} / \mathrm{s}^{* * 2}, 33.25 \mathrm{~N}$
(D) $6.53 \mathrm{~m} / \mathrm{s}^{* *} 2,16.33 \mathrm{~N}$
E. $\quad 2.25 \mathrm{~m} / \mathrm{s}^{* *} 2,10.21 \mathrm{~N}$
block: $m g-T=m a$
pulley: $T R=I \alpha$

$$
T=\frac{I \alpha}{R}=\frac{I a}{R^{2}}
$$

$$
m g-\frac{T a}{R^{2}}=m a
$$

$$
a\left(m+\frac{I}{R^{2}}\right)=m g \Rightarrow a=\frac{m g}{m+\frac{I}{R^{2}}}
$$

$$
I=\frac{1}{2} M R^{2} \Rightarrow a=\frac{m g}{m+\frac{M}{2}}=\frac{49}{7.5}=6.53!
$$

(1) $\Rightarrow T=m(g-a)=5(9.8-6.53)=16.3 \mathrm{~N}$

1 block of mass $m=5.00 \mathrm{~kg}$ slides down a surface inclined at 37 deg to the horizontal, as shown in the figure. The coefficient of sliding friction is 0.250 . A string attached to the block is wrapped around a flywheel on a fixed axis at 0 . The flywheel has an outer radius $R=0.200 \mathrm{~m}$, and a moment of inertia with respect to its axis of $0.200 \mathrm{~kg}{ }^{*} \mathrm{~m}^{* *} 2$. What is the acceleration of the block down the plane ?
(4.) $1.96 \mathrm{~m} / \mathrm{s} * * 2$
B. $2.05 \mathrm{~m} / \mathrm{s}^{* *} 2$
C. $\quad 0.00 \mathrm{~m} / \mathrm{s}^{* *} 2$
D. $1.80 \mathrm{~m} / \mathrm{s}^{\star *} 2$
E. $\quad 0.98 \mathrm{~m} / \mathrm{s}^{\star * 2}$

$m g \sin \theta-f_{k}-T=m a$ $f_{k}=\mu_{k} N=H_{k} m g \cos \theta$
$m g \sin \theta-\mu_{k} m g \cos \theta-T=m a-(1)$
$m g\left(\sin \theta-H_{k} \cos \theta\right)-\frac{I a}{R^{2}}=m a$

$$
\Rightarrow a\left(m+\frac{I}{R^{2}}\right)=m g\left(\sin \theta-\mu_{k} \cos \theta\right)
$$

$$
a=\frac{m g\left(\sin \theta-\mu_{k} \cos \theta\right)}{m+\frac{I}{R^{2}}}=\frac{19.6}{10}=1.96 \mathrm{~m} / \mathrm{s}^{2}
$$

A grinding wheel of moment of inertia $0.01 \mathrm{kg*}\left(\mathrm{~m}^{* *} 2\right)$ is brought to rest from an initial angular velocity of $3000 \mathrm{rev} / \mathrm{min}$. What is the average power dissipated if the wheel is brought to rest in 10 revolutions ? (Assume constant angular acceleration).
A. 5.55* $\left(10^{* *} 3\right)$ watts
B. 725
C. 923
(D) $1.23^{*}\left(10^{* * 3}\right)$ watts
E. 4.25* $\left(10^{* *} 3\right)$ watts watts watts

$$
\begin{aligned}
& \bar{P}=\tau \bar{\omega} \quad \bar{\omega}=\frac{\omega+\omega_{0}}{2} \\
& \omega_{0}=3000 \frac{\mathrm{rev}}{\min }=3000 \times \frac{2 \pi}{60} \frac{\mathrm{rad}}{\mathrm{~s}}
\end{aligned}
$$

$$
\omega_{0}=314.2 \frac{\mathrm{rad}}{\mathrm{~s}}
$$

$$
\tau=I \alpha
$$

$$
\omega^{2}=\omega_{0}^{2}+2 \alpha \theta
$$

$$
\Rightarrow \alpha=-785 \mathrm{rad} / \mathrm{s}^{2}
$$

$$
\Rightarrow \alpha=-\frac{\omega_{0}^{2}}{2 \theta}=-\frac{(314.2)^{2}}{2(10 \times 2 \pi)}
$$

$$
|\tau|=0.01 \times 785=7.85 \mathrm{~N} \cdot \mathrm{~m}
$$

$$
\bar{P}=\tau \bar{\omega}=7.85 \times \frac{314.2}{2}=1.23 \times 10^{3} \mathrm{~W}
$$

A wheel with a moment of inertia of $86 \mathrm{~kg} . \mathrm{m}^{* *} 2$ is rotating with an angular velocity of $17 \mathrm{rad} / \mathrm{s}$. If an accelerating torque of $90 \mathrm{~N} . \mathrm{m}$ is applied tc this wheel for 7.0 s , what is the final angular velocity of the wheel: Neglect any frictional effects.
A. $145 \mathrm{rad} / \mathrm{s}$
B. $\quad 64.7 \mathrm{rad} / \mathrm{s}$
C. $\quad 16.3 \mathrm{rad} / \mathrm{s}$
(D) $24.3 \mathrm{rad} / \mathrm{s}$
E. $\quad 35.6 \mathrm{rad} / \mathrm{s}$

$$
\begin{gathered}
\tau=I \alpha \Rightarrow 90=86 \alpha \Rightarrow \alpha=\frac{90}{86} \\
\alpha=1.04 \mathrm{rad} / \mathrm{s}^{2}
\end{gathered}
$$

$$
w=\omega_{0}+\alpha t=17+1.04 \times 7=24.3 \mathrm{rad} / \mathrm{s}
$$

The second hand (arm) of a watch is 2 cm long. Find the linear speed of the tip of this hand.
A. $0.05 \mathrm{~cm} / \mathrm{sec}$
B. $\quad 0.84 \mathrm{~cm} / \mathrm{sec}$
C. $0.21 \mathrm{~cm} / \mathrm{sec}$
D. $\quad 0.42 \mathrm{~cm} / \mathrm{sec}$
E. $19.09 \mathrm{~cm} / \mathrm{sec}$

A turntable is initially rotating at $33.33 \mathrm{rev} / \mathrm{min}$. When the power to the turntable is switched off, the turntable slows down at a constant rate of $0.20 \mathrm{rad} / \mathrm{s}^{* *} 2$. How many revolutions will the turntable make before stopping?
A. 1.08 rev
B. $\quad 5.37 \mathrm{rev}$
C. 2.13 rev
D. 3.21 rev
(E.) 4.85 rev

$$
\begin{gathered}
\omega_{0}=33.3 \text { rev/min } \quad \omega_{+}=0 \quad \alpha=0.2 \mathrm{rad} / \mathrm{s}^{2} \\
\omega_{f}=\omega_{0}^{2}+2 \alpha \theta \\
\theta=\frac{\omega_{0}^{2}}{2 \alpha}=\frac{\left(33 . \frac{3 \times 2 \pi}{60}\right)^{2}}{2 \times 0.2}=30.4 \mathrm{rad} \\
\theta=\frac{30.4}{2 \pi}=4.8 \mathrm{rev} .
\end{gathered}
$$

A disk 6 cm in radius rotates at a constant rate of $1200 \mathrm{rev} / \mathrm{min}$ about its axis. Find the radial acceleration of a point on the outer edge of the disk.
A. $0 \mathrm{~m} / \mathrm{s}^{* *} 2$
B. $7200 \mathrm{~m} / \mathrm{s}^{* *} 2$
C. $126 \mathrm{~m} / \mathrm{s} * * 2$
D. $200 \mathrm{~m} / \mathrm{s}^{* *} 2$
(E.) $947 \mathrm{~m} / \mathrm{s}^{\star * 2}$

$$
\begin{aligned}
R=6 \mathrm{~cm} \quad \omega & =1200 \frac{\mathrm{rev}}{\min }=1200 \times \frac{2 \pi}{60}=125.7 \mathrm{rad} / \mathrm{s} \\
a_{r}=\frac{v^{2}}{R} & =\frac{\omega^{2} R^{2}}{R}=\omega^{2} R=(125.7)^{2}(0.06) \\
a_{r} & =947.5 \mathrm{~m} / \mathrm{s}^{2} \sim 948 \mathrm{~m} / \mathrm{s}^{2}
\end{aligned}
$$

note that since $\omega=$ constant $\Rightarrow a_{t}=0$

