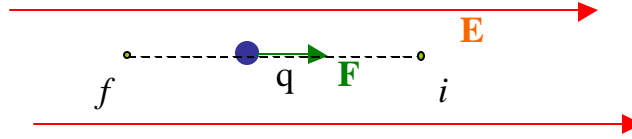


1. An *electric force* F acts on a point charge q in an external *electric field* E as shown below.



This force is conservative $\Rightarrow W = -DU$

2. We define the electric potential V at a point in the electric field as the potential energy per unit charge, that is

$$V = \frac{U}{q}$$

So the *electric potential difference* between points i and f is

$$\Delta V = V_f - V_i = \frac{\Delta U}{q}$$

The unit of the potentials energy is Joule and the unit of the electric potential is Joule/Coulomb = Volt.

The potential is a scalar.

Another unit of potential energy is used, that is electron-volt (eV);

$$1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$$

The work done by an external agent in moving the charge from i to f is equal to the negative of the work done by the electric force; therefore;

$$W = -q DV \quad (\text{work done by the electric force})$$

$$W_{app} = q DV \quad (\text{work done by the applied force})$$

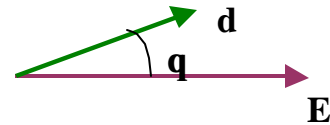
3. The electric potential difference can be calculated from

$$\Delta V = V_f - V_i = -\int \vec{E} \cdot d\vec{s}$$

If E is *uniform*, then

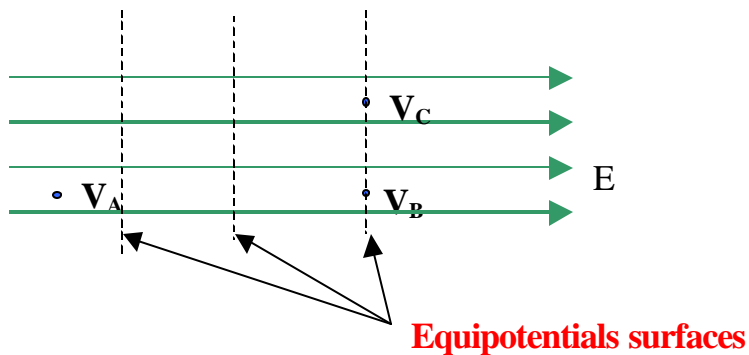
$$\Delta V = -\vec{E} \cdot \vec{d} = -E d \cos\theta$$

where θ is the angle between E and d (displacement).



Special cases:

- If $\theta = 0$, $\Delta V = V_f - V_i = -E d < 0$ *The potential decreases.*
- If $\theta = 180^\circ$, $\Delta V = V_f - V_i = E d > 0$ *The potential increases.*
- If $\theta = 90^\circ$, $\Delta V = V_f - V_i = 0$ *The potential remains constant.*



$$\Rightarrow V_A > V_B = V_C$$

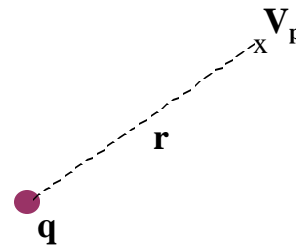
4. The change in electric potential energy, ΔU , of a charge moving from point i to point f in a uniform electric field is given by;

$$\Delta U = q(V_f - V_i) = -q E d \cos \theta$$

- *We can see from this formula that if q is positive, ΔU will be negative. A positive charge will lose potential energy when it moves in the direction of the electric field ($\theta = 0$) and at the same time will gain kinetic energy, because the total energy is conserved.*
- *On the other hand, if the charge is negative, ΔU will be positive. A negative charge will gain potential energy when it moves in the direction of the electric field ($\theta = 0$) and at the same time will lose kinetic energy.*

5. The electric potential due to a point charge q a distance r away from the point charge is

$$V_p = k \frac{q}{r}$$



If $q > 0$, then $V > 0$, and if $q < 0$, then $V < 0$.

Note: The electric potential is taken to be zero at infinity.

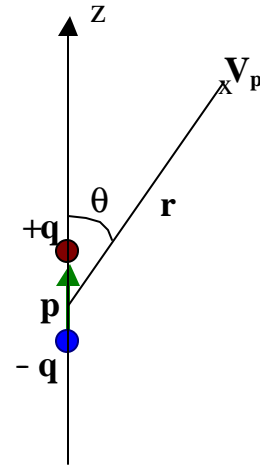
6. The electric potential at a point p due to a group of charges; $q_1, q_2, q_3, \dots, q_n$ is given by

$$V_p = k \left(\frac{q_1}{r_1} + \frac{q_2}{r_2} + \frac{q_3}{r_3} + \dots + \frac{q_n}{r_n} \right)$$

7. The potential due to an electric dipole is

$$V_p = k \frac{p \cos \theta}{r^2}$$

$p = q d$ is the dipole moment.

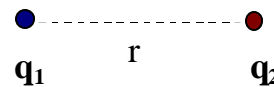


8. The electric field E can be evaluated if the electric potential V is known;

$$E_x = -\frac{\partial V}{\partial x}; E_y = -\frac{\partial V}{\partial y}; E_z = -\frac{\partial V}{\partial z}$$

9. The potential energy of a pair of charges separated by a distance r is given by;

$$U = k \frac{q_1 q_2}{r}$$



Important: THIS ENERGY REPRESENTS THE WORK done by an external agent to assemble the charges from infinity to their position at r.

To assemble three charges; q_1, q_2, q_3 , the potential energy (or work required) will be;

$$U = k \left(\frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}} \right)$$

If $U > 0$, an external agent do positive work to assemble the charges, and
if $U < 0$, the electric field does the work.

10. The electric potential of a charged isolated conductor

- The surface of a charged conductor is an *equipotential surface*.
- Since the electric field inside a conductor is zero, *the potential* is therefore *constant inside a charged conductor* and equal to that at the surface.
- For a conducting sphere of radius R and charge q , the electric potential is

- $V_{in} = \frac{kq}{R}$ (inside and on the surface of the conductor)

- $V_{out} = \frac{kq}{r}$ (outside)