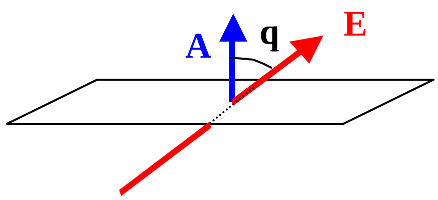
Summary of chapter 23

Prepared by Dr. A. Mekki

1. The ***electric flux*** through a planar surface in a uniform electric field $\vec{E}$ is;

$$\Phi = E\ A\ \cos q$$

where the angle θ is between the vector E and the vector A. The vector A is always perpendicular (normal) to the surface in consideration.



- If $\theta = 0 \quad \Rightarrow \quad \phi = E\ A$
- If $\theta = 90^{o} \quad \Rightarrow \quad \phi = 0$
- If $\theta = 180^{o} \quad \Rightarrow \quad \phi = -E\ A$
- In general $\phi = E\ A \cos\theta$

If the electric field **E** is **not uniform** then we must perform the integration to calculate the electric flux using the relation

$$\Phi = \int \vec{E}.d\vec{A}$$

If the surface is ***closed***, then the electric flux is given by;

$$\Phi = \oint \vec{E}.d\vec{A}$$

2. The electric flux through a ***closed surface*** is also given by;

$$\Phi = \frac{q_{enc}}{e_o}$$

where q_{enc} is the charge ***inside*** the closed surface.

3. ***Gauss' law*** states that Φ, the electric flux through a closed surface, is given by

$$\Phi_c = \oint \vec{E}.d\vec{A} = \frac{q_{inc}}{e_o}$$

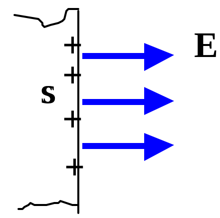
where q_{inc} is the charge enclosed by the ***Gaussian surface*** and ε_o is a the permitivity of free space equal to 8.85×10^{-12} $C^2/N.m^2$.

Note: *Gauss law is generally used to calculate* ***the electric field*** *for a highly symmetric charge distribution.*

4. Examples of the use of Gauss law in calculating ***the electric field***.

(i) The electric field near the surface (outside) of a charged **conductor** is

$$E_{out} = \frac{s}{e_o}$$

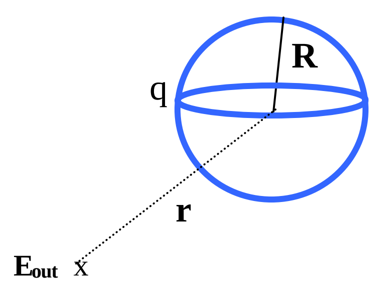


σ is the ***surface charge density*** = q / A (C/m^2).

The electric field inside any conductor is zero, since the charge always moves to the surface of the conductor and remains there. So there is NO charge inside a conductor.

$$E_{in} = 0$$

(ii) The electric field due to charged spherical shell of radius R, whether it is insulator or conductor, is:



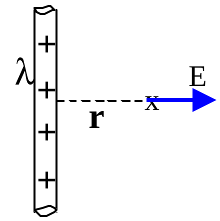
Since there is no charge inside the shell $\Rightarrow E_{in} = 0$

The electric field outside the shell is $E_{out} = k\frac{q}{r^2}$

where r is the distance from the ***center*** *of the shell to the point outside the shell.*

(iii) The electric field due to an infinitely long, straight line charge, at a point that is a radial distance r from the line is

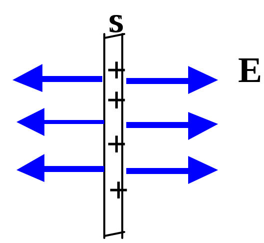
$$E = \frac{\lambda}{2\pi \varepsilon_o r}$$



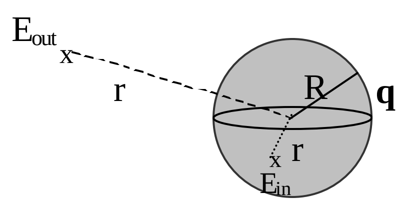
λ is the linear charge density = q/L (C/m)

(iv) The electric field outside an ***thin infinite*** ***non-conducting*** ***plane sheet*** of charge density σ is

$$E = \frac{\sigma}{2\varepsilon_o}$$



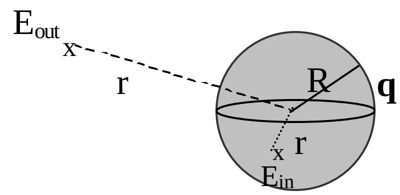
(v) The electric field due to a non-conducting (insulating) solid sphere of charge q distributed uniformly thought the sphere of radius R is



$$E_{in} = k\frac{q}{R^3}r$$

and $$E_{out} = k\frac{q}{r^2}$$

(vi) The electric field due to a *conducting solid sphere* of charge q and radius R.



Since this is a conductor, the charge q will be distributed ONLY on the surface of the sphere and NOT inside it!

$$E_{in} = 0$$

and

$$E_{out} = k\frac{q}{r^2}$$

r is the distance from the center of the sphere to the point outside the sphere where the electric field is to be evaluated.

Summary

Charge Distribution	E(N/C)	Location
Conductor of charge density s	0 $\frac{s}{e_o}$	inside outside
Infinite thin line charge carrying a charge density l	$2k\frac{l}{r}$	
Non-conducting infinite thin plane sheet carrying a charge density s	$\frac{s}{2e_o}$	
Spherical shell of radius R and charge q **(Conducting and non-conducting)**	0 $k\frac{q}{r^2}$	r < R (inside) r > R (outside)
Non-conducting solid sphere of radius R and charge q	$\frac{kq}{R^3}r$ $\frac{kq}{r^2}$	r < R (inside) r > R (outside)
Conducting solid sphere of radius R and charge q	0 $\frac{kq}{r^2}$	r < R (inside) r > R (outside)

$l = \frac{q}{L}$ is the linear charge density (C/m)

$s = \frac{q}{A}$ is the surface charge density (C/m^2)

$r = \frac{q}{V}$ is the volume charge density (C/m^3)