# Summary of Chapter 16

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**1.** Wave is the motion of a disturbance. In order to produce a wave, we need a source of disturbance, a medium and the medium should be continuous.

**2.** **There are two (2) types of waves:**

**Transverse Waves**: ***The particles of the disturbed medium move perpendicular to the wave velocity (example: waves on a string).***

**Longitudinal Waves**: ***The particles of the disturbed medium move parallel to the wave velocity (example: sound waves).***

**3.** For a **sinusoidal wave** the transverse displacement $y$ at position $x$ at time t is given by

$$y(x,t) = y_m \sin(kx \pm wt)$$

This equation represents the displacement of the particles of the medium as a function of $x$ and $t$.

where $y_m$ is the amplitude of the wave in (m), $k$ is the wave number in (rad/m), $\omega$ is the angular frequency in (rad/s).

$$k = \frac{2p}{l}; \quad w = 2pf = \frac{2p}{T}$$

The expression $(kx \pm wt)$ is called the phase angle and its units is **radians**.

The plus (+) sign means the wave is traveling to the left.

The minus (–) sign means the wave is traveling to the right.

- A wave is said to be stationary if is **<u>time independent</u>**, i.e., y (x) = f (x).
- A travelling wave can be longitudinal or transverse.

  - **The transverse velovity (*the velocity of the particles*) is given by ;**

$$u(x,t) = \frac{dy(x,t)}{dt} = \pm y_m \omega \cos(kx \pm \omega t)$$

The maximum transverse velocity is $u_{max} = y_m\ \omega$ (m/s)

  - **The transverse acceleration (*the acceleration of the particles*) is given by**;

$$a\ (x,t) = \frac{du(x,t)}{dt} = \pm y_m \omega^2 \sin(kx \pm \omega t)$$

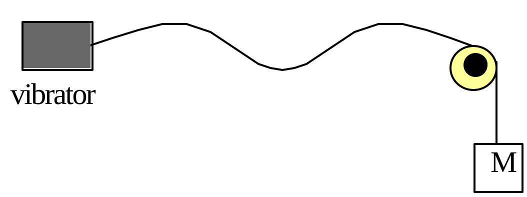
The maximum transverse acceleration is $a_{max} = y_m\ \omega^2$ (m/s$^2$)

**4.** The ***speed of the wave*** is constant and given by $v = \frac{\omega}{k} = \lambda\ f$

✓ The wavelength $\lambda$ is the **distance** between **two** successive maxima (crests) or minima (throughs) or any two identical points on the wave.

✓ The frequency of the wave is also related to the period T in (s) by $f = \frac{1}{T}$

**5.** In the case of a ***stretched string*** as in the figure, the wave speed is given by: $v = \sqrt{\frac{T}{m}}$




where $T$ is the tension in the string in (N), $m$ is the linear mass density (kg/m) and v is the speed of the wave (m/s).

**6.** The power transmitted by a harmonic wave moving in a stretched string is given by; $P = \frac{1}{2} m v w^2 y_m^{\,2}$ with units of *Watt*

## 7. Superposition of Waves

Suppose two waves $y_1(x,t)$ and $y_2(x,t)$ are moving through a medium, the total displacement of the medium $y'(x,t)$ at a point $x$ at time $t$ is

$$y'(x,t) = y_1(x,t) + y_2(x,t)$$

$y'(x,t)$ is called *the resultant wave*.

## 8. Interference of Waves

The net displacement of two equal sinusoidal waves (the resultant) traveling in the **same** direction equals the algebraic sum of the displacement of the two waves.

$$y_1 = y_m \sin(k x - w t)$$
$$y_2 = y_m \sin(k x - w t + f)$$

The resultant wave function is $y' = y_1 + y_2 = \underbrace{[2 y_m \cos f/2]}_{\text{Amplitude of the resultant wave}} \sin(k x - w t + f/2)$

**Amplitude of the resultant wave**

3 Cases:

a) The amplitude of the resultant wave is maximum (constructive interference) when $\phi = 0, 2\pi, 4\pi, 6\pi, \ldots$

b) The amplitude of the resultant wave is zero (destructive interference) when $\phi = \pi, 3\pi, 5\pi, \ldots$

c) Intermediate interference when $0 < \phi < \pi$ etc, ..

## 9. Standing Waves

If two equal sinusoidal waves travel in **opposite** directions along a stretched string, their interference with each other produce a standing wave.

$$y_1 = y_m \sin(k\,x - \omega\,t) \text{ and } y_2 = y_m \sin(k\,x + \omega\,t)$$

$$y' = y_1 + y_2 = \underbrace{[\,2\,y_m \sin(k\,x)]}_{\text{Amplitude of the resulting wave}} \cos(\omega\,t)$$

**2 Cases**:

a) position of the ***nodes*** or **minimum** amplitude:

$kx = n\pi,$ for $n = 0, 1, 2, 3, \ldots$

but $k = 2\pi/\lambda$

$$\Rightarrow x = \lambda/2,\ \lambda,\ 3\lambda/2, \ldots = n\lambda/2 \quad \text{for } n = 0, 1, 2, 3, \ldots$$

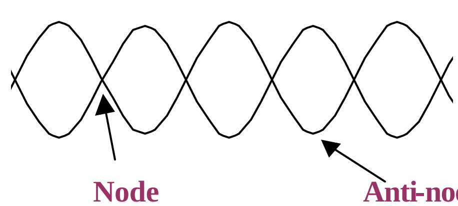
b) position of the ***anti-nodes*** or **maximum** amplitude:

$k\,x = \pi/2, 3\pi/2, 5\pi/2, \ldots = n\pi/2$ for $n = 1, 3, 5, \ldots$

but $k = 2\pi/\lambda$

$$\Rightarrow x = \lambda/4, 3\lambda/4, 5\lambda/4, \ldots = n\,\lambda/4 \quad \text{for } n = 1, 3, 5\ldots$$

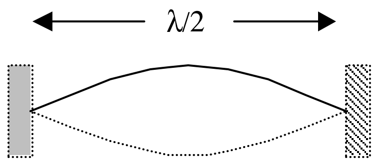
**A standing Wave**



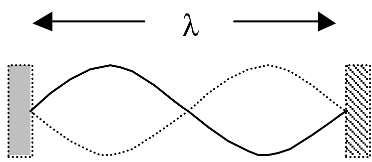
## 10. Standing Waves and Resonance in a stretched string

In the case of a stretched string resonance occur and standing wave patterns are formed at certain freqencies called **resonant frequencies**.

The standing wave patterns in the case of a stretched string are as follows:




***Fundamental mode*** or ***first harmonic***




***Second harmonic***

The relation between the wavelength ($l$) and the length of the string ($L$) is:

$$L = n\frac{l}{2} \qquad \text{or} \qquad l = \frac{2L}{n} \qquad \text{for } n = 1, 2, 3, \ldots$$

As you can see from the figures, n represents the number of loops or segments.

The resonant frequencies are given by ($v = \lambda f$)

$$f = \frac{v}{l} = n\frac{v}{2L} \qquad \text{for } n = 1, 2, 3, \ldots$$

In this case we can **fix the tension** and varie the frequency to obtain the various harmonics or **fix the frequency** and change the tension to obtain the various harmonics. ***This latter case was done in the Phys. 102 lab. (Standing waves in a string).***