Phys 212
Term 102

HW#5
Solution

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#2.

Since the kinetic energy is much less than mc^2, we Can easily use non-relativistic formula for the momentum - $\left(\gamma = \frac{K}{mc^2} + 1\right)$

a) $$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mK}} = \frac{h}{\sqrt{2meV}}$$

$$= \frac{hc}{\sqrt{2\,mc^2 eV}} = \frac{1240 \text{ eV.nm}}{\sqrt{2 \times 0.511 \times 10^6 \times 50\,(\text{eV})^2}}$$

$$= \boxed{0.173 \text{ nm}}$$

b) $$\lambda = \frac{1240 \text{ eV.nm}}{\sqrt{2 \times 0.511 \times 10^6 \times 50 \times 10^3\,(\text{eV})^2}} = \boxed{5.49 \times 10^{-3} \text{ nm}}$$

#4. $\lambda = 0.1 \text{ nm} = 1 \text{ Å}$

$$\lambda = \frac{h}{p} = \frac{h}{mv} \Rightarrow v = \frac{h}{\lambda m}$$

$$= \frac{6.63 \times 10^{-34}}{1 \times 10^{-10} \times 9.1 \times 10^{-31}} = \boxed{7.29 \times 10^{6} \text{ m/s}}$$

it is non-relativistic!

#5. Here we are not sure to use relativistic or non-relativistic formula for the kinetic energy.

but $K = E - mc^2 = \sqrt{p^2c^2 + (mc^2)^2} - mc^2$

$$= mc^2\left[1 + \frac{p^2c^2}{(mc^2)^2}\right]^{1/2} - mc^2$$

if $pc \ll mc^2$

$$K = mc^2\left[1 + \frac{1}{2}\frac{p^2c^2}{(mc^2)^2}\right] - mc^2$$

$$= \frac{1}{2}\frac{p^2c^2}{mc^2}$$

but $K = \frac{p^2}{2m} = \frac{p^2c^2}{2mc^2}$

So we use nonrelativistic K if $pc \ll mc^2$

a) $\lambda = 10\,nm$ $\quad pc = \frac{hc}{\lambda} = \frac{1240\ eV.nm}{10 \times 10^{-9}\,nm} = 124\ eV$

$pc = 124\ eV \ll 0.511\ MeV$

use non relativistic K

$$K = \frac{p^2}{2m} = \frac{p^2c^2}{2mc^2} = \frac{(124)^2}{2 \times 0.511 \times 10^6} = \boxed{0.015\,eV}$$

b) $\lambda = 0.1\,nm$ $\quad pc = 12.4\ keV \ll mc^2$

Use non relativistic K

$$K = \frac{p^2c^2}{2mc^2} = \frac{(12.4 \times 10^3)^2}{2 \times 0.511 \times 10^6} = \boxed{150\,eV}$$

c) $\lambda = 10 \times 10^{-15}$ m $\qquad pc = 1.240 \times 10^{8}$ eV $= 1240$ MeV

$pc \gg mc^2$

Use relativistic formula for K

$$K = \left[p^2c^2 + (mc^2)^2\right]^{1/2} - mc^2$$

$$= \left[(1240)^2 + (0.511)^2\right]^{1/2} - 0.511 \quad (\text{MeV})$$

$$\boxed{K = 1235 \text{ MeV}}$$

#14. a) Use Bragg's diffraction law $\quad n\lambda = d\sin\phi$

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mK}} = \frac{hc}{\sqrt{2mc^2K}}$$

$$\Rightarrow \frac{n\,hc}{\sqrt{2mc^2K}} = d\sin\phi$$

or $$\boxed{\sin\phi = \frac{nhc}{d\sqrt{2mc^2K}}}$$

b) $\phi = 24.1°$, $n = 1$ $\Rightarrow$ $d_1 = \dfrac{hc}{\sin\phi\sqrt{2mc}}$

$$= \frac{1240 \text{ eV.nm}}{\sin 24.1°\sqrt{2 \times 0.511 \times 10^6 \times 100}} = \boxed{0.3 \text{ nm}}$$

$\phi = 54.9°$, $n = 2$ $\Rightarrow$ $\boxed{d_2 = 0.3 \text{ nm}}$

17.

$$E^2 = p^2c^2 + (mc^2)^2$$

since $E = \hbar\omega$ and $p = \hbar k$

$$E = \sqrt{p^2c^2 + (mc^2)^2}$$

$$\hbar\omega = \sqrt{\hbar^2k^2c^2 + (mc^2)^2}$$

$$\omega(k) = \sqrt{c^2k^2 + \left(\frac{mc^2}{\hbar}\right)^2}$$

$$v_p = \frac{\omega}{k} = \sqrt{c^2 + \left(\frac{mc^2}{\hbar k}\right)^2} = c\sqrt{1 + \frac{m^2c^2}{\hbar^2k^2}} \gg c \text{ !!!}$$

$$v_g = \frac{d\omega}{dk} = \frac{1}{\cancel{2}}(\cancel{2}kc^2)\frac{1}{\sqrt{c^2k^2 + \left(\frac{mc^2}{\hbar}\right)^2}}$$

$$= \frac{kc^2}{\sqrt{c^2k^2 + \left(\frac{mc^2}{\hbar}\right)^2}} = \frac{c^2}{\sqrt{c^2 + \left(\frac{mc^2}{\hbar k}\right)^2}}$$

$$v_p v_g = \sqrt{c^2 + \left(\frac{mc^2}{\hbar k}\right)^2} \times \frac{c^2}{\sqrt{c^2 + \left(\frac{mc^2}{\hbar k}\right)^2}} = c^2 = \underline{\text{Constant}}$$

$$v_g = \frac{c^2}{v_p} \quad \text{but } v_p \gg c$$

$$\Rightarrow v_g < c \,!$$

20. Heisenberg uncertainty principle

$$\Delta x \cdot \Delta p \geq \frac{\hbar}{2}$$

$$p = \frac{h}{\lambda} \qquad dp = -\frac{h}{\lambda^2}\, d\lambda$$

$$\Rightarrow \quad \Delta p = \frac{h}{\lambda^2}\, \Delta\lambda$$

$$\Delta x_{min} = \frac{\hbar}{2\Delta p} = \frac{\hbar\, \lambda^2}{2\, \Delta\lambda\, h} = \frac{(6000 \times 10^{-10})^{\cancel{2}}}{4\pi \times 10^{-6} \times \cancel{6000 \times 10^{-10}}}$$

$$\boxed{\Delta X_{min} = 0.048\ m}$$

25. Heisenberg uncertainty principle

$$\Delta E \cdot \Delta t \geq \frac{\hbar}{2}$$

$$\Delta E_{min} = \frac{\hbar}{2\Delta t} = \frac{6.582 \times 10^{-16}\ eV.s}{2 \times (0.1 \times 10^{-9} s)} = \boxed{3.29 \times 10^{-6}\ eV}$$

The width is too small to be detected by a detector with resolution ± 5 eV.

28.

a) $$\lambda = \frac{h}{p} = \frac{h}{mv} = \frac{6.63 \times 10^{-34}}{1.67 \times 10^{-27} \times 0.4} = \boxed{9.93 \times 10^{-7} m}$$

b) See figure 5.28 in text book

$$D \sin\theta = \frac{\lambda}{2} \ (\text{minimum}) \Rightarrow \sin\theta \simeq \theta = \frac{\lambda}{2D} = 4.96 \times 10^{-4}$$

$$y \simeq L \sin\theta = L\theta = 10 \times 4.96 \times 10^{-4} = \boxed{4.96\ mm}$$