H.W chapter 31 Phys 201 Term 211

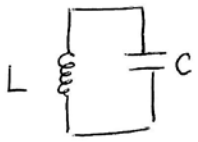
31.4 $f = 200\ kHz$ $t=0$ Q_{max}

a) See Fig. 31.1 Earliest time is $t = T = \frac{1}{f} = \frac{1}{200\times10^3} = 5\mu s$

b) See Fig. 31.1 Earliest time is $t = \frac{T}{2} = 2.5\mu s$

c) See Fig 31.1 Earliest time is $t = \frac{T}{4} = 1.25\mu s$

31.17 LC circuit



$C = 64\ \mu F$

$i(t) = 1.6 \sin(2500t + 0.68)$

$i(t) = I_{max} \sin(\omega t + \phi)$

See Fig 31.1

a) i is max whe $\sin(\omega t + \phi) = 1 \Rightarrow \omega t + \phi = \frac{\pi}{2}$

$\Rightarrow 2500t + 0.68 = \frac{\pi}{2} \Rightarrow t = 3.56\times10^{-4}\ s$

b) $\omega = \frac{1}{\sqrt{LC}} \Rightarrow L = \frac{1}{\omega^2 C} = \frac{1}{(2500)^2 \times (64\times10^{-6})} = 2.5\times10^{-3}\ H$

c) $U_{total} = \frac{1}{2} L I_{max}^2 = \frac{1}{2}(2.5\times10^{-3})(1.6) = 3.2\times10^{-3}\ J$

31.20

$I_{max} = 7.5\ mA$ $V_{max} = 250\ mV$ $C = 220\ nF$

a) $T = \frac{2\pi}{\omega}$

$I_{max} = \omega Q$

$V_{max} = \frac{Q}{C}$

$\Rightarrow \frac{I_{max}}{V_{max}} = \frac{\omega Q}{\frac{Q}{C}} = \omega C$

$\Rightarrow \omega = \frac{I_{max}}{C V_{max}} = \frac{7.5\times10^{-3}}{250\times10^{-3}\times 220\times10^{-9}}$

$\omega = 1.3636 \times 10^5$ rad/s $\Rightarrow T = \frac{2\pi}{1.3636 \times 10^5} = \underline{4.6 \times 10^{-5} s}$

$= \underline{46\ \mu s}$

b) $U_{E_{max}} = \frac{1}{2} C V_{max}^2 = \frac{1}{2} (220 \times 10^{-9})(250 \times 10^{-3})^2$

$= \underline{6.88 \times 10^{-9} J}$

~~a) $U_E = \frac{Q^2}{2C} \Rightarrow Q = \sqrt{2CU_E} =$~~

c) $U_{B_{max}} = \frac{1}{2} L \cdot I^2 = U_{E_{max}} = \underline{6.88 \times 10^{-9} J}$

d) $\left(\frac{di}{dt}\right)_{max} = ?$

$i = -Q\omega \sin(\omega t + \phi) = -I \sin(\omega t + \phi)$

$\frac{di}{dt} = -\underbrace{Q\omega^2}_{\left(\frac{dI}{dt}\right)_{max}} \cos(\omega t + \phi)$

$\left(\frac{di}{dt}\right)_{max} = Q\omega^2 = \sqrt{2CU_{E_{max}}}\ \omega^2 = 1023\ A/sec$

e) $U_B = \frac{1}{2} L i^2$

$\frac{dU_B}{dt} = L i \frac{di}{dt} = L I^2 \omega \sin\omega t \cos\omega t$

$= \frac{1}{2} L I^2 \omega \sin 2\omega t$ $\Big\}\ LI\omega = V$

$\left(\frac{dU_B}{dt}\right)_{max} = \frac{1}{2} L I^2 \omega = \frac{1}{2}(7.5 \times 10^3)(250 \times 10^{-3}) = 0.938 \times 10^{-3}$ W

$= \frac{1}{2}(LI\omega) I$

$\uparrow$

V

31.24

$$q = Q e^{-\frac{R}{2L}t} \cos(\omega' t + \phi) = q_{max} \cos(\omega' t + \phi)$$

$t = 0 \quad q = Q \Rightarrow \phi = 0$

$$q_{max} = Q e^{-\frac{R}{2L}t}$$

$$U = \frac{1}{2}\frac{Q^2}{C} \Rightarrow \frac{1}{2}\left(\frac{1}{2}\frac{Q^2}{2C}\right) = \frac{q_{max}^2}{2C} = \frac{Q^2 e^{-\frac{R}{L}t}}{2C}$$

$$\frac{1}{2} = e^{-\frac{R}{L}t} \qquad \ln\left(\frac{1}{2}\right) = -\frac{R}{L}t$$

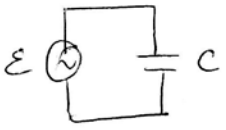
$$t = \frac{L}{R}\ln 2 = 0.693\frac{L}{R}$$

$$\boxed{t = 0.693\frac{L}{R}}$$

31.32

$\mathcal{E} = \mathcal{E}_m \sin \omega_d t \qquad \mathcal{E}_m = 25\,V \qquad \omega_d = 377\ rad/s$

$C = 4.15\,\mu F$



a) $i = \frac{dq}{dt}$ and $q = C\mathcal{E}$

$= C\mathcal{E}_m \sin \omega_d t$

$i = (\omega_d C \mathcal{E}_m)\cos \omega_d t = I_m \cos \omega_d t$

$I_m = \mathcal{E}_m C \omega_d = 25 \times (4.15\times10^{-6})(377) = \underline{\underline{3.91\times10^{-2}\,A}}$

b) when $i = I_m \Rightarrow \cos \omega_d t = 1 \Rightarrow \sin \omega_d t = 0$

$\Rightarrow \mathcal{E} = 0$ and $q = 0$

c) $\mathcal{E} = -\frac{\mathcal{E}_m}{2} = \mathcal{E}_m \sin \omega_d t \Rightarrow \sin \omega_d t = -\frac{1}{2}$

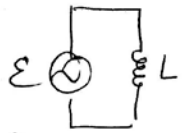
$\omega_d t = -\frac{\pi}{2}$

31.34

$\mathcal{E} = \mathcal{E}_m \sin \omega_d t$

$\mathcal{E}_m = 25\ V \qquad \omega_d = 377\ rad/s \qquad L = 12.7\ H$

a) $I_m = \dfrac{\mathcal{E}_m}{X_L} = \dfrac{\mathcal{E}_m}{\omega_d L} = 5.22 \times 10^{-3}\ A$



b) $L \dfrac{di}{dt} = \mathcal{E} \Rightarrow \dfrac{di}{dt} = \dfrac{\mathcal{E}}{L} \qquad i = \int \dfrac{\mathcal{E}}{L}\, dt$

$i = -\dfrac{\mathcal{E}_m}{L\omega_d} \cos(\omega_d t)$

$i = \dfrac{\mathcal{E}_m}{L\omega_d} \sin(\omega_d t - 90^\circ) = I_m \sin(\omega_d t - 90^\circ)$

$I_m = \dfrac{\mathcal{E}_m}{L\omega_d} = 5.22 \times 10^{-3}\ A$

b) $i = I_m \Rightarrow \sin(\omega_d t - 90^\circ) = 1$

$\Rightarrow \omega_d t - \dfrac{\pi}{2} = \dfrac{\pi}{2} \Rightarrow \omega_d t = \pi$

$\mathcal{E} = \mathcal{E}_m \sin(\omega_d t) = \mathcal{E}_m \sin \pi = 0$

32.36 driven RLC circuit

$I_s = 4\ A \qquad L = 200\ \mu H \qquad \mathcal{E}_m = 8\ V$

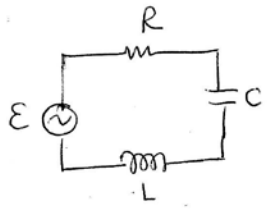
b) $I_s = \frac{\mathcal{E}_m}{R} \Rightarrow R = \frac{\mathcal{E}_m}{I_s} = \frac{8}{4} = \underline{2\ \Omega}$

a) $\omega_r = \frac{1}{\sqrt{LC}}$ at the resonance

$\omega_r = 25000\ rad/s \qquad C = \frac{1}{L\omega_r^2}$

$C = \frac{1}{200\times10^{-6}\times(25000)^2} = \underline{8\ \mu F}$

32.39



$R = 200\ \Omega \qquad C = 70\ \mu F$

$L = 230\ mH \qquad f_d = 60\ Hz$

$\mathcal{E}_m = 36\ V$

a) $Z = \sqrt{R^2 + (X_L - X_C)^2}$

$X_L = \omega_d L = 86.7\ \Omega \qquad X_L > X_C$ more inductive

$X_C = \frac{1}{\omega_d C} = 37.9\ \Omega$

$Z = \sqrt{(200)^2 + (86.7 - 37.9)^2} = \underline{205.9\ \Omega}$

b) $\tan\phi = \frac{X_L - X_C}{R} = 0.244 \Rightarrow \phi = \pm 0.239\ rad$

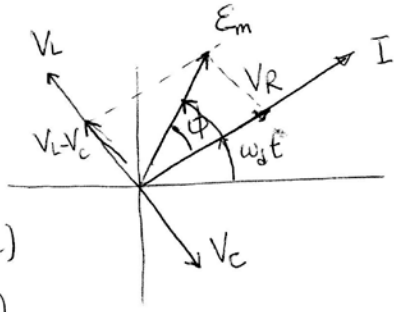
$= \pm 13.7°$

more inductive $\underline{\phi = +13.7°}$

c) $I = \frac{\mathcal{E}_m}{Z} = \frac{36}{205.9} = \underline{0.175\ A}$

$$P_\varepsilon = (1.93)(45)\sin\left(3000 \times 0.442 \times 10^{-3}\right)\sin\left(3000 \times 0.442 \times 10^{-3} - 46.5^\circ\right)$$

$$P_\varepsilon = 41.6\ \text{W}$$



b) $$v_c = V_c \sin\left(\omega_d t - \phi - \frac{\pi}{2}\right)$$

$$= I X_C \sin\left(\omega_d t - \phi - \pi/2\right)$$

$$= -\frac{I}{\omega_d C}\cos\left(\omega_d t - \phi\right)$$

$$P_c = \frac{d}{dt}\left(\frac{q^2}{2C}\right) = \left(\frac{q}{C}\right)\frac{dq}{dt} = v_c\, i$$

$$= -\frac{I}{\omega_d C}\cos(\omega_d t - \phi)\; I \sin(\omega_d t - \phi)$$

$$P_c = -\frac{I^2}{2\omega_d C}\sin\left[2(\omega_d t - \phi)\right]$$

at $t = 0.442\text{ms}$

$$P_c = -\frac{(1.93)^2}{2\times(3000)(31.2\times10^{-6})}\sin\left[2\left(3000\times 0.442\times10^{-3} - 0.81\right)\right]$$

$$= -17.1\ \text{W}$$

c) $$P_L = \frac{d}{dt}\left(\frac{1}{2}Li^2\right) = L\, i\, \frac{di}{dt}$$

$$i = I\sin(\omega_d t - \phi)$$

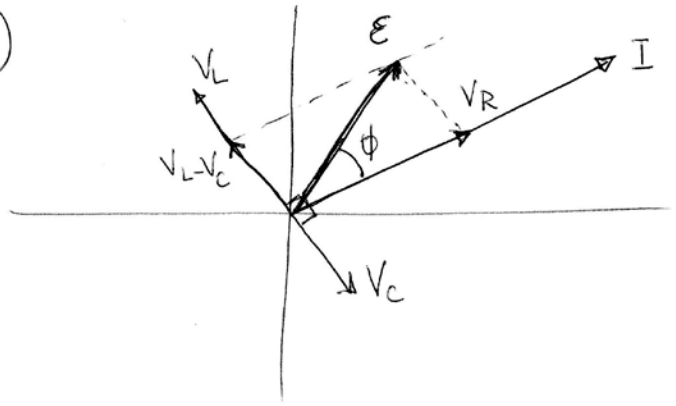
$$\frac{di}{dt} = I\omega_d \cos(\omega_d t - \phi)$$

$$P_L = L I^2 \omega_d \sin(\omega_d t - \phi)\cos(\omega_d t - \phi)$$

$$= \frac{1}{2} L I^2 \omega_d \sin\left[2(\omega_d t - \phi)\right]$$

$$= \frac{1}{2}\left(9.2\times10^{-3}\right)(1.93)^2(3000)\sin\left[2\left(3000\times0.442\times10^{-3} - 0.8\right.\right.$$

d)



31.42

$\mathcal{E} = 6\ V \qquad \phi = 30^\circ$ more inductive

According to previous figure

$$\sin\phi = \frac{V_L - V_C}{\mathcal{E}} \Rightarrow V_L - V_C = \mathcal{E}\sin\phi$$

$$V_L - V_C = 6 \sin 30^\circ = 3\ V$$

when $V_C = 5\ V \Rightarrow V_L = V_C + 3 = \underline{\underline{8\ V}}$

31.58

RLC circuit

$R = 16\ \Omega \qquad C = 31.2\ \mu F \qquad L = 9.2\ mH$

$\mathcal{E} = \mathcal{E}_m \sin\omega_d t \qquad \mathcal{E}_m = 45\ V \qquad \omega_d = 3000\ rad/s$

a) $P_\mathcal{E} = i(t)\,\mathcal{E}(t) = I\,\mathcal{E}_m \sin\omega_d t \sin(\omega_d t - \phi)$

Calculate I and ϕ

$$I = \frac{\mathcal{E}_m}{Z} \qquad Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{(16)^2 + (27.6 - 10.7)^2}$$

$$= 23.3\ \Omega$$

$$I = \frac{45}{23.3} = \underline{1.93\ A}$$

$$\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right) = 0.81\ rad = 46.5^\circ$$

$P_L = \underline{44.1\ W}$

d) $P_R = i^2 R = I^2 R \sin^2(\omega_d t - \phi)$

$= (1.93)^2 \times (16) \sin^2(3000 \times 0.442 \times 10^{-2} - 0.81)$

$= \underline{14.5\ W}$

$P_L + P_C + P_R \stackrel{?}{=} P_\mathcal{E}$

$44.1 - 17.1 + 14.5 = 41.5\ W = P_\mathcal{E}$!!!

31.63

$N_p = 500$ $\quad V_p = 120\ V$ This is a step down transformer.

$N_s = 10$ $\quad V_s = ?$

a) $\frac{V_s}{V_p} = \frac{N_s}{N_p} \Rightarrow V_s = V_p \frac{N_s}{N_p} = 120 \times \frac{10}{500} = \underline{2.4\ V}$

b) $I_s = \frac{V_s}{R} = \frac{2.4}{15} = \underline{0.16\ A}$

$\frac{I_s}{I_p} = \frac{N_p}{N_s} \Rightarrow I_p = I_s \frac{N_s}{N_p} = \underline{3.2 \times 10^{-3}\ A}$