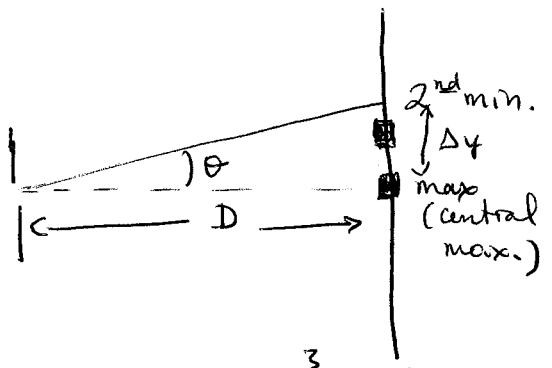
H.W. Chapter 36 Phys 201 Term 112

36.2 $\lambda = 441\ nm$ $\Delta y = 1.5\ cm$

$D = 2\ m$



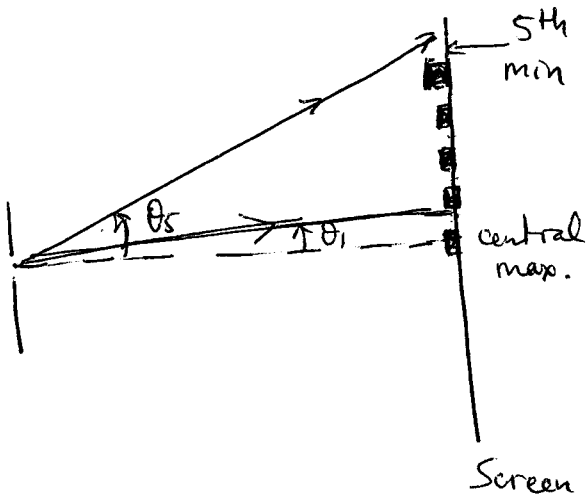
a) $\tan\theta = \frac{\Delta y}{D}$

$\theta = \tan^{-1}\left(\frac{\Delta y}{D}\right)$

$= \tan^{-1}\left(\frac{1.5\ cm}{200\ cm}\right) = \underline{0.43^\circ} = \underline{7.5\times10^{-3}\ rad}$

b) $a\sin\theta = 2\lambda \Rightarrow a = \frac{2\lambda}{\sin\theta} = \underline{0.118\ mm}$

36.7



$D = 40\ cm$

$\lambda = 550\ nm$

$\Delta y = 0.35\ mm$

a) $a\sin\theta_1 = \lambda$

$a\sin\theta_5 = 5\lambda$

$\sin\theta_5 - \sin\theta_1 = \frac{4\lambda}{a} = \frac{\Delta y}{D}$

$a = \frac{4\lambda D}{\Delta y} = \underline{2.5\ mm}$

b) $a\sin\theta_1 = \lambda$

$\theta_1 = \sin^{-1}\left(\frac{\lambda}{a}\right) = \sin^{-1}\left(\frac{550\ nm}{2.5mm}\right) = \underline{0.01^\circ}$

$= \underline{2.2\times10^{-4}\ rad}$

36.14 $\lambda = 610\,nm$

a) $\alpha = \frac{\pi a}{\lambda} \sin\theta$

Slope of α vs. $\sin\theta$ is $\frac{\pi a}{\lambda}$

from the graph slope = 12 rad

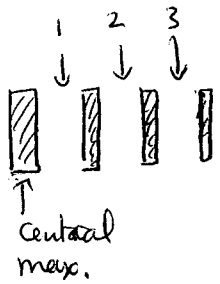
$\Rightarrow \frac{\pi a}{\lambda} = 12 \Rightarrow a = \frac{12\lambda}{\pi} = \underline{2.33\,\mu m}$

b) $a \sin\theta = m\lambda$

$m = \frac{a \sin\theta}{\lambda}$

$m_{max} = \frac{a (\sin\theta)_{max}}{\lambda}$ take $(\sin\theta)_{max} = 1$

$m_{max} = \frac{a}{\lambda} = 3.8$



m is integer $\Rightarrow m = 3$

total 6 minima!

least angle $m = 1$ $\quad a \sin\theta = \lambda \quad \theta = \sin^{-1}\left(\frac{a}{\lambda}\right) = 15°$

greatest angle $m = 3$ $\quad a \sin\theta = 3\lambda \quad \theta = \sin^{-1}\left(\frac{3a}{\lambda}\right) = 51.8°$

for $m = 4$ $\quad \sin\theta = \frac{4a}{\lambda} > 1$

$\sin\theta$ cannot be > 1

So $m = 4$ is excluded!

36.19

$$\frac{y}{D} = \theta_R = 1.22\frac{\lambda}{d}$$

$$D = \frac{y\,d}{1.22\lambda} = 1.6\times10^{6}\,m = \underline{1600\ km}$$

36.24

$$\frac{y}{D} = \theta_R = 1.22\frac{\lambda}{d}$$

$$y = \frac{1.22\lambda D}{d} = \underline{53\ m}$$

36.35

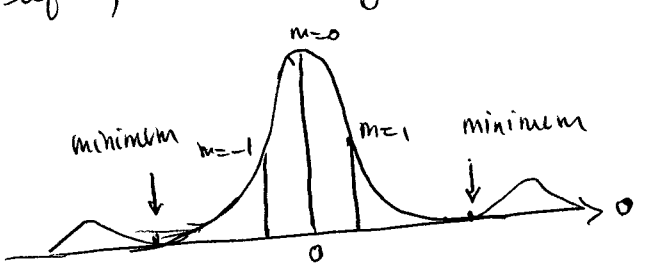
$d = 2a$

$a\sin\theta = \lambda$ first dark fringe diffraction

$d\sin\theta = m_2\lambda$ m_2 # of bright fringes (interference) inside the ~~first~~ central bright fringe (diffraction)

$\frac{d}{a} = m_2 = 2 \leftarrow$ missing

one bright fringe on left, one on right and one m=0



There are 3 bright fringes from the interference.

36.39

a) $m_2 = \frac{d}{a} = \frac{0.15\,nm}{30\,\mu m} = 5 \leftarrow$ missing

$m_2 = -4, -3, -2, -1, 0, 1, 2, 3, 4$ 9 bright fringe

b) $I_p = I_m \cos^2\beta \frac{\sin^2\alpha}{\alpha^2}$

$\beta = \frac{\pi d}{\lambda} \sin\theta$

$\alpha = \frac{\pi a}{\lambda} \sin\theta$

third bright fringe $d \sin\theta = 3\lambda$

$\sin\theta = \frac{3\lambda}{d}$

$\beta = \frac{\pi d}{\lambda} \frac{3\lambda}{d} = 3\pi$ rad

$\alpha = \frac{\pi a}{\lambda} \frac{3\lambda}{d} = \frac{3\pi a}{d} = 0.6$ rad

$\frac{\sin^2\alpha}{\alpha^2} = 0.255$ $\cos^2\beta = 1$

$\underline{I_p = 0.255\, I_m}$

36.44 $d = \frac{W}{N} = \frac{1\text{ mm}}{315\text{ rulings}} = 3.17\times10^{-6}\text{ m} = 3.17\,\mu\text{m}$

$d \sin\theta = 5\lambda$ $\sin\theta = \frac{5\lambda}{d} \leq 1$

$\lambda \leq \frac{d}{5} = 6.34\times10^{-7}\text{ m}$

$\lambda_{longest} = 6.34\times10^{-7}\text{ m} = \underline{634\text{ nm}}$

36.50

a) $d \sin\theta = m\lambda$ max is when $\sin\theta = 1$

$$m_{max} = \frac{d}{\lambda} = \frac{900\text{ nm}}{600\text{ nm}} = 1.5$$

$\Rightarrow m = 1$ $\quad m = -1, 0, 1$ 3 maxima.

b) $$\Delta\theta_{HW} = \frac{\lambda}{Nd\cos\theta}$$

$d\sin\theta = \lambda \Rightarrow$

$$\Rightarrow \Delta\theta_{HW} = \frac{d\sin\theta}{Nd\cos\theta} = \frac{\tan\theta}{N}$$

$$d\sin\theta = \lambda \Rightarrow \theta = \sin^{-1}\left(\frac{\lambda}{d}\right) = \sin^{-1}\left(\frac{600}{900}\right) = 41.8°$$

$$\Delta\theta_{HW} = \frac{\tan 41.8}{1000} = 8.9 \times 10^{-4}.$$

36.58

a) $\lambda_1 = 589\text{ nm}$ $\quad \lambda_2 = 589.6\text{ nm}$

$\Delta\lambda = 0.6\text{ nm}$ $\quad \lambda_{avg} = 589.3\text{ nm}$

$$R = \frac{\lambda_{avg}}{\Delta\lambda} = Nm \Rightarrow N = \frac{\lambda_{avg}}{m\Delta\lambda}$$

$$m = 2 \Rightarrow N = \frac{589.3}{2 \times 0.6} = 491 \text{ lines}.$$

36.64

a) $\Delta\theta_{HW} \cdot R = \frac{\lambda}{N d \cos\theta} \cdot N m = \frac{m\lambda}{d\cos\theta} = \frac{d\sin\theta}{d\cos\theta} = \tan\theta$

b) For first order $m=1$ $\quad d\sin\theta = \lambda$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\sin\theta}{\sqrt{1-\sin^2\theta}} = \frac{1}{\sqrt{\frac{1}{\sin^2\theta}-1}}$$

but $\sin\theta = \frac{\lambda}{d}$

$$\Rightarrow \tan\theta = \frac{1}{\sqrt{\left(\frac{d}{\lambda}\right)^2-1}} = \frac{1}{\sqrt{\left(\frac{9}{6}\right)^2-1}} = \underline{\underline{0.894}}$$

$$\Rightarrow \quad \Delta\theta_{HW} \cdot R = \underline{\underline{0.894}}$$

36.68

$$\lambda = 30\times10^{-12}\,m \qquad d = 0.3\times10^{-9}\,m$$

$\theta_{small} = ?$

Bragg's law for diffraction is $2d\sin\theta = m\lambda$

smallest angle $\Rightarrow m=1$ $\quad 2d\sin\theta = \lambda$

$$\theta = \sin^{-1}\left(\frac{\lambda}{2d}\right) = \underline{\underline{2.86^\circ}}$$