HW chapt. 30

30.2: B vs. t is shown in Fig. 30.38

loop: $r = 12\text{cm}$ $R = 8.5\,\Omega$

loop plane is $\perp$ to $\vec{B}$, $\Rightarrow$ $\vec{A} \parallel \vec{B}$ $\quad \vec{B}\cdot\vec{A} = BA\cos\theta = BA\cos 0 = BA$.

a) $\mathcal{E} = ?$ $0 \le t \le 2\,s$

$$\mathcal{E} = -\frac{d\phi_B}{dt} = -\pi r^2 \underbrace{\frac{dB}{dt}}_{\text{slope}} = -\pi(0.12)^2\left(\frac{0.5}{2}\right) = \boxed{-0.0113\,V}$$

b) $\mathcal{E} = ?$ $2 \le t \le 4\,s$

$B = \text{Constant} \Rightarrow \boxed{\mathcal{E} = 0}$

c) $\mathcal{E} = ?$ $4 \le t \le 6\,s$

$$\mathcal{E} = -\pi(0.12)^2\left(-\frac{0.5}{2}\right) = \boxed{0.0113V}$$

30.4: $r = 12\text{ cm}$ $B = 0.8\,T$ $\frac{dr}{dt} = 75\text{ cm/s}$

$$\mathcal{E} = -\frac{d\phi_B}{dt} = -B\frac{dA}{dt} = -B\frac{d}{dt}(\pi r^2) = -B\pi\frac{dr^2}{dt} = -2\pi r B\frac{dr}{dt}$$

$$\mathcal{E} = -2\pi(0.12)(0.8)(-0.75) = \boxed{0.45\,V}$$

30.10: B vs. t is given in Fig 30.43 b and q vs. t is given in Fig 30.43 c

loop $A = 8\times10^{-4}\,m^2$ $\quad \vec{A}\cdot\vec{B} = AB\cos 0 = AB$

Resistance of the loop $R = ?$

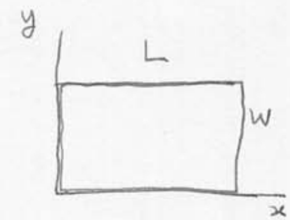
$$\mathcal{E} = -\frac{d\phi_B}{dt} = -A\underbrace{\frac{dB}{dt}}_{\text{slope}} = -(8\times10^{-4})\times\frac{(9\times10^{-3})}{3} = -24.\times10^{-6}V$$

$R = \frac{|\mathcal{E}|}{i}$ where $i = \frac{dq}{dt} = \text{slope} = \frac{6\times10^{-3}}{3} = 2\times10^{-3}\,A$

$$R = \frac{|\mathcal{E}|}{i} = \frac{2.4\times10^{-6}}{2\times10^{-3}} = 1.2\times10^{-3}\Omega = \boxed{1.2\times10^{-3}\,\Omega}$$

30.18:

$L = 40\text{ cm} \quad W = 25\text{ cm}$



$\vec{A} = A\,\hat{k}$

a) $\vec{B} = 4\times10^{-2} y\,\hat{k}$

$\mathcal{E} = -\frac{d\phi_B}{dt} = -A\frac{dB}{dt} = -A\,(4\times10^{-2})\,\frac{dy}{dt} = \boxed{0}$

b) None

c) $\vec{B} = 6\times10^{-2} t\,\hat{k}$ (B increasing with time)

$\mathcal{E} = -A\frac{dB}{dt} = -(0.4\times0.25)\,(0.06) = \boxed{-6\times10^{-3}\text{ V}}$

d) Clockwise

e) $\vec{B} = 8\times10^{-2} y\,t\,\hat{k}$ (B non uniform and increasing with time)

$\mathcal{E} = -\frac{d\phi}{dt}$

$\phi = \int_0^W B\,dA = (0.08\times0.4)\,t\int_0^{0.25} y\,dy \quad (dA = L\,y\,dy)$

$\phi = 0.032t\,\frac{y^2}{2}\Big|_0^{0.25} = 1\times10^{-3}t \qquad \mathcal{E} = -\frac{d\phi}{dt}$

$\boxed{\mathcal{E} = -1\times10^{-3}\text{ V}}$

f) clockwise

g) $\vec{B} = 3\times10^{-2} x\,t\,\hat{j} \qquad \vec{B}\cdot\vec{A} = 0$

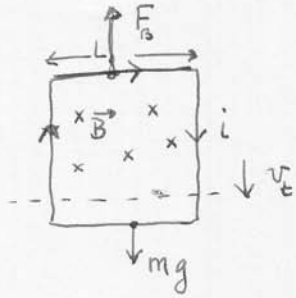
$\phi = 0$ h) $\mathcal{E} = 0$ None

i) $\vec{B} = 5\times10^{-2} y\,t\,\hat{i} \qquad \vec{B}\cdot\vec{A} = 0$

$\mathcal{E} = 0$

j) None

30.34:



$$F_{net} = F_B - mg = 0 \quad (V = \text{const.})$$

$$iLB = mg$$

$$i = \frac{mg}{LB} = \frac{|\mathcal{E}|}{R} = \frac{BLv_t}{R}$$

$$\Rightarrow \boxed{v_t = \frac{mgR}{B^2L^2}} \quad R: \text{resistance}$$

30.36:

See figure 30.60 $\quad r_1 = 20\text{ cm} \quad r_2 = 30\text{ cm} \quad B_1 = 50\text{ mT} \quad B_2 = 75\text{ mT}$

a) For path 1

$$\oint_1 \vec{E}\cdot d\vec{s} = -\frac{d\phi_{B_1}}{dt} = +A_1\frac{dB_1}{dt} = +\pi r_1^2 \frac{dB_1}{dt}$$

$$\frac{dB}{dt} = -8.5\ \frac{\text{mT}}{\text{s}}$$

$$\oint \vec{E}\cdot d\vec{s} = +\pi(0.2)^2(-8.5\times10^{-3}) = \boxed{-1.07\times10^{-3}\text{ V}}$$

b) For path 2

$$\oint_2 \vec{E}\cdot d\vec{s} = -\frac{d\phi_{B_2}}{dt} = +A_2\frac{dB_2}{dt} = +\pi r_2^2\frac{dB_2}{dt}$$

$$= +\pi(0.3)^2(-8.5\times10^{-3}) = \boxed{-2.4\times10^{-3}\text{ V}}$$

c) For path 3

$$\oint_3 \vec{E}\cdot d\vec{s} = -\frac{d\phi}{dt} = \oint_1 \vec{E}\cdot d\vec{s} + \oint_2 \vec{E}\cdot d\vec{s}$$

$$= -1.07\times10^{-3} + 2.4\times10^{-3} = \boxed{1.33\times10^{-3}\text{ V}}$$

30.46:

$$|\mathcal{E}| = L\,\frac{di}{dt} \qquad \frac{di}{dt} \text{ is the slope of } i \text{ vs. } t \text{ graph.}$$

a) $0 \le t \le 2$ ms

$$|\mathcal{E}| = 4.6 \times \frac{7-0}{2\times10^{-3}} = \boxed{16{,}100\ V}$$

b) $2 \le t \le 5$ ms

$$|\mathcal{E}| = 4.6\ \frac{7-5}{3\times10^{-3}} = \boxed{3{,}066.7\ V}$$

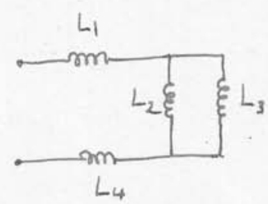
c) $5 \le t \le 6$ ms

$$|\mathcal{E}| = 4.6\ \frac{5-0}{1\times10^{-3}} = \boxed{23{,}000\ V}$$

30.49:

$L_1 = 30$ mH $\quad L_2 = 50$ mH

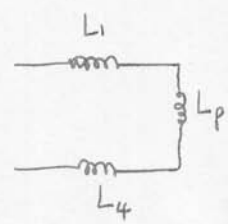
$L_3 = 20$ mH $\quad L_4 = 15$ mH



L_2 and L_3 are parallel

$$\frac{1}{L_p} = \frac{1}{L_2} + \frac{1}{L_3} \Rightarrow L_p = \frac{L_2 L_3}{L_2+L_3} = 14.3 \text{ mH}$$

L_1, L_p and L_4 are in series



$$L_s = L_1 + L_p + L_4$$

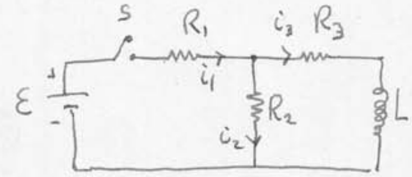
$$\boxed{L_s = 59.3 \text{ mH}}$$

30.56:

$\mathcal{E} = 100$ V

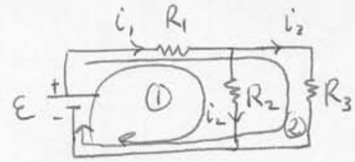
$R_1 = 10\,\Omega \quad R_2 = 20\,\Omega \quad R_3 = 30\,\Omega$

$L = 2$ H



a) and b) $\quad i_1 = i_2 = \dfrac{\mathcal{E}}{R_1+R_2} = \dfrac{100}{30} = \boxed{3.33\,A}$

c) Apply Kirchhoff to



loop 1

$$\mathcal{E} - i_1 R_1 - i_2 R_2 = 0 \quad \text{— ①}$$

$$i_1 = i_2 + i_3 \Rightarrow i_3 = i_1 - i_2$$

loop 2

$$\mathcal{E} - i_1 R_1 - i_3 R_3 = 0$$

or $\quad \mathcal{E} - i_1 R_1 - (i_1 - i_2) R_3 = 0 \quad$ -②

$$① \Rightarrow i_1 = \frac{\mathcal{E} - i_2 R_2}{R_1}$$

$$② \Rightarrow \mathcal{E} - \left(\frac{\mathcal{E} - i_2 R_2}{\cancel{R_1}}\right)\cancel{R_1} - \left(\frac{\mathcal{E} - i_2 R_2}{R_1}\right) R_3 + i_2 R_3 = 0$$

$$\cancel{\mathcal{E}} - \cancel{\mathcal{E}} + i_2 R_2 - \mathcal{E}\left(\frac{R_3}{R_1}\right) + i_2 \frac{R_2}{R_1} R_3 + i_2 R_3 = 0$$

$$i_2\left(\frac{R_1 R_2}{\cancel{R_1}} + \frac{R_2}{\cancel{R_1}} R_3 + \frac{R_1 R_3}{\cancel{R_1}}\right) = \mathcal{E}\left(\frac{R_3}{\cancel{R_1}}\right)$$

$$\boxed{i_2 = \frac{\mathcal{E} R_3}{R_1 R_2 + R_1 R_3 + R_2 R_3}} \qquad \boxed{i_2 = 2.73\,A}$$

$$i_1 = \frac{\mathcal{E}}{R_1} - \frac{\mathcal{E} R_2 R_3}{(R_1 R_2 + R_1 R_3 + R_2 R_3) R_1}$$

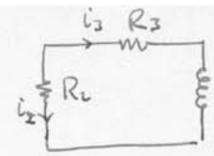
$$= \frac{\mathcal{E}(R_1 R_2 + R_1 R_3 + R_2 R_3) - \mathcal{E} R_2 R_3}{R_1(R_1 R_2 + R_1 R_3 + R_2 R_3)}$$

$$= \frac{\mathcal{E}(R_1 R_2 + R_1 R_3)}{R_1(R_1 R_2 + R_1 R_3 + R_2 R_3)} = \frac{\mathcal{E}(R_2 + R_3)\cancel{R_1}}{\cancel{R_1}(R_1 R_2 + R_1 R_3 + R_2 R_3)}$$

$$\boxed{i_1 = \frac{\mathcal{E}(R_2 + R_3)}{R_1 R_2 + R_1 R_3 + R_2 R_3}} \qquad \boxed{i_1 = 4.55\,A}$$

$$\boxed{i_3 = 1.82\,A}$$

e) $i_1 = 0$



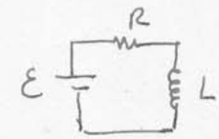
f) $i_2 = -i_3 = -1.82\,A$

g) $i_1 = 0$

h) $i_2 = 0$

30.64:

$L = 2\,H \quad R = 10\,\Omega \quad \mathcal{E} = 100\,V$



$$i(t) = i_{max}\left(1 - e^{-t/\tau_L}\right)$$

$$= \frac{\mathcal{E}}{R}\left(1 - e^{-t/\tau_L}\right) \qquad \tau_L = \frac{R}{L}\ (\text{sec})$$

a) rate of energy stored $\frac{dU_B}{dt} = \frac{d}{dt}\left(\frac{1}{2}L\,i^2\right) = L\,i\,\frac{di}{dt}$

$$\frac{di}{dt} = +\frac{\mathcal{E}}{R}\frac{1}{\tau_L}e^{-t/\tau_L} = \frac{\mathcal{E}}{L}e^{-t/\tau_L}$$

$$P_B = \frac{dU_B}{dt} = \frac{\mathcal{E}^2}{R}\left(1 - e^{-t/\tau_L}\right)\left(e^{-t/\tau_L}\right)$$

$$\tau_L = \frac{2}{10} = 0.2\,s$$

at $t = 0.1\,s$ $\quad \frac{dU_B}{dt} = \frac{(100)^2}{10}\left(1 - e^{-\frac{1}{2}}\right)e^{-1/2} = \boxed{238.7\ W}$

b) $P_{ther} = i^2R = \frac{\mathcal{E}^2}{R^{\cancel{2}}}\left(1 - e^{-t/\tau_L}\right)^2\cancel{R} = \frac{\mathcal{E}^2}{R}\left(1 - e^{-t/\tau_L}\right)^2$

at $t = 0.1\,s$ $\quad P_{ther.} = \frac{(100)^2}{10}\left(1 - e^{-0.5}\right)^2 = \boxed{154.8\,W}$

c) Energy delivered by battery $= \mathcal{E}\,i = \frac{(100)^2}{10} \times \left(1 - e^{-0.5}\right) = \boxed{393W}$

Note $\quad$ c) = a) + b) = 238.7 + 154.8 =