H.W. Solution Chapter 39

Phys 201 Term 112

39.4 $L = 100 \text{ pm} = 0.1 \text{ nm}$

$$E_n = \frac{h^2}{8 m_p L^2} n^2 \qquad n = 1, 2, 3, \ldots$$

$$E_n = \frac{(hc)^2}{8 (m_p c^2) L^2} n^2$$

Ground state $n=1$

$$E_1 = \frac{(1240 \text{ eV.nm})^2}{8 (938 \text{MeV}) (0.1 \text{ nm})^2}$$

$$= \frac{(1240 \text{ eV.nm})^2}{8 (938 \times 10^6 \text{ eV}) (0.1 \text{ nm})^2}$$

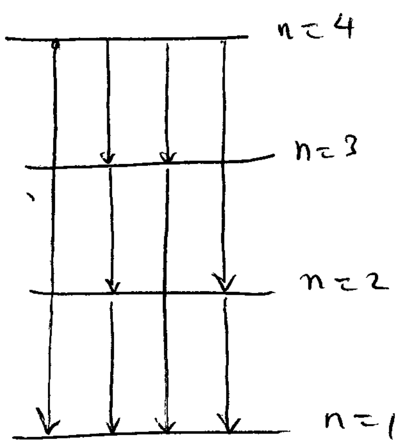
$$= \underline{\underline{0.0205 \text{ eV}}}$$

39.11 $L = 250 \text{ pm} = 0.25 \text{ nm}$ $\qquad n = 2 \rightarrow n = 4.$

a) $$\Delta E = E_4 - E_2 = \frac{h^2}{8 m L^2} (16 - 4) = \frac{12 (hc)^2}{8 (mc^2) L^2}$$

$$= \frac{12 (1240)^2}{8 (0.511 \times 10^6) (0.25)^2} = \underline{\underline{72.2 \text{ eV}}}$$

b)



various ways of de-excitation!

The shortest wavelength is for $4 \to 1$

$$\frac{hc}{\lambda_{4\to1}} = 72.2\,eV \Rightarrow \lambda_{4\to1} = \frac{1240}{\Delta E_{4\to1}} =$$

$$\Delta E_{4\to1} = 15 \underbrace{\frac{(hc)^2}{8(mc^2)L^2}}_{6.02\,eV} = 90.3\,eV$$

$$\lambda_{4\to1} = \frac{1240}{90.3} = \underline{\underline{13.7\,nm}}$$

The second shortest is for $4 \to 2$

$$\lambda_{4\to2} = \frac{1240}{\Delta E_{4\to2}} = \frac{1240}{72.2} = \underline{\underline{17.2\,nm}}$$

The longest is for $2 \to 1$

$$\Delta E_{2\to1} = 3 \times 6.02\,eV = 18.06\,eV$$

$$\lambda_{2\to1} = \frac{1240}{18.06} = \underline{\underline{68.7\,nm}}$$

The second longest for $3 \to 2$

$$\Delta E_{3\to2} = 5 \times 6.02\,eV = 30.1\,eV$$

$$\lambda_{3\to2} = \frac{1240}{30.1} = \underline{\underline{41.2\,nm}}$$

$$\lambda = 29.4\,nm = \frac{hc}{\Delta E} = \frac{1240}{\Delta E} \Rightarrow \Delta E = 42.1\,eV$$

$$\Delta E_{4\to3} = 7 \times 6.02 = 42.1\,eV$$

$\lambda = 29.4$ corresponds to $4 \to 3$ transition

the electron can go $3 \to 2 \to 1$ or $3 \to 1$

The longest wavelength is $2 \to 1$ $\quad \underline{\lambda_{2\to 1} = 68.7\,nm}$

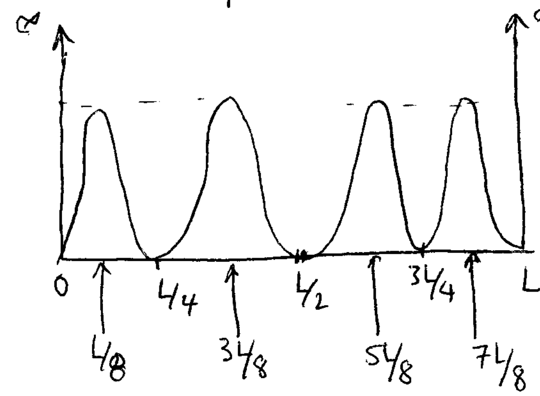
The shortest " is $3 \to 1$ $\quad \lambda_{3\to 1} = \dfrac{hc}{\Delta E_{3\to 1}}$

$$\Delta E_{3\to 1} = 8 \times 6.02 = 48.2\,eV$$

$$\lambda_{3\to 1} = \frac{1240}{48.2} = \underline{\underline{25.7\,nm}}$$

39.15 $\quad L = 0.2\,nm \quad n = 4 \quad \Delta x = 2\,pm = 0.002\,nm$

a) detector can be at $\dfrac{L}{8}$ or $\dfrac{3L}{8}$ or $\dfrac{5L}{8}$ or $\dfrac{7L}{8}$!



Let us take $\dfrac{3L}{8} = 0.075\,nm = x$

$$P = \frac{2}{L}\int \sin^2\left(\frac{4\pi}{L}x\right) dx$$

since dx is small $\quad P \simeq \dfrac{2}{L}\sin^2\left(\dfrac{4\pi}{\cancel{L}}\,\underset{\frac{3\cancel{L}}{8}}{x}\right)\Delta x$

$$P = \frac{2}{L}\sin^2\left(\frac{\pi}{4}\right)\Delta x$$

$$= \frac{2}{0.2}(0.5)^2(0.075) = \underline{\underline{0.1875}}$$

b) $n = NP = 1000 \times 0.1875 = \underline{\underline{18.75}} \simeq 18$ electrons!

39.23

$$E_{n_x n_y} = \frac{h^2}{8m}\left(\frac{n_x^2}{L_x^2} + \frac{n_y^2}{L_y^2}\right)$$

Ground state $n_x = n_y = 1$

$$E_{11} = \frac{(hc)^2}{8\,(mc^2)}\left(\frac{1}{L_x^2} + \frac{1}{L_y^2}\right)$$

$$= \frac{(1240)^2}{8\,(0.511\times10^6)}\left(\frac{1}{0.8^2} + \frac{1}{1.6^2}\right) = \underline{\underline{3.21\,eV}}$$

39.27

$$L_x = L_y = L_z = L \Rightarrow E_{n_x n_y n_z} = \frac{h^2}{8mL^2}\left(n_x^2 + n_y^2 + n_z^2\right)$$

a) Let us find the lowest 5 energy levels

$E_{111} = 3\dfrac{h^2}{8mL^2}$ ——— ground state

$E_{112} = E_{211} = E_{121} = 6\dfrac{h^2}{8mL^2}$ ——— 1st excited

$E_{221} = E_{122} = E_{212} = 9\dfrac{h^2}{8mL^2}$ ——— 2nd "

$E_{311} = E_{131} = E_{113} = 11\dfrac{h^2}{8mL^2}$ ——— 3rd "

$E_{222} = 12\dfrac{h^2}{8mL^2}$ ——— 4th "

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not included $E_{123} = E_{213} = \cdots = 14\dfrac{h^2}{8mL^2}$ ——— 5th "

$E_{222} \longrightarrow E_{111}$ $\quad f = \dfrac{\Delta E}{h} = (12-3)\dfrac{h}{8mL^2} = \dfrac{9h}{8mL^2}$ ✓

$E_{222} \longrightarrow E_{112}$ $\quad f = (12-6)\dfrac{h}{8mL^2} = \dfrac{6h}{8mL^2}$ ✓

$E_{222} \rightarrow E_{221}$ $\quad f = (12-9)\dfrac{h}{8mL^2} = \dfrac{3h}{8mL^2}$ ✓

$E_{222} \rightarrow E_{311}$ $\quad f = (12-11)\dfrac{h}{8mL^2} = \dfrac{h}{8mL^2}$ ✓

$E_{311} \rightarrow E_{111}$ $\quad f = (11-3)\dfrac{h}{8mL^2} = \dfrac{8h}{8mL^2}$ ✓ $\dfrac{h}{mL^2}$

$E_{311} \rightarrow E_{112}$ $\quad f = (11-6)\dfrac{h}{8mL^2} = \dfrac{5h}{8mL^2}$ ✓

$E_{311} \rightarrow E_{221}$ $\quad f = (11-9)\dfrac{h}{8mL^2} = \dfrac{2h}{8mL^2}$ ✓

$E_{221} \rightarrow E_{111}$ $\quad f = (9-3)\dfrac{h}{8mL^2} = \dfrac{6h}{8mL^2}$ ×

$E_{221} \rightarrow E_{112}$ $\quad f = (9-6)\dfrac{h}{8mL^2} = \dfrac{3h}{8mL^2}$ ×

$E_{112} \rightarrow E_{111}$ $\quad f = (6-3)\dfrac{h}{8mL^2} = \dfrac{3h}{8mL^2}$ ×

There are 7 frequencies as shown by ✓ mark!

b) c) d) e) Check the frequencies above and rank them from smallest to highest

39.29 $L = 150$ pm $\Delta x = \Delta y = 5$ pm

probe is at $(0.2L, 0.8L)$

In general $\psi_{n_x n_y} = \frac{2}{\sqrt{L_x L_y}} \sin\left(\frac{n_x \pi}{L_x} x\right) \sin\left(\frac{n_y \pi}{L_y} y\right)$

$P = \iint |\psi|^2 dx\, dy$ since Δx and Δy as small

$\Rightarrow P = |\psi|^2 \Delta x \Delta y = \frac{2^2 \Delta x \Delta y}{L_x L_y} \sin^2\left(\frac{n_x \pi}{L_x} x\right) \sin^2\left(\frac{n_y \pi}{L_y} y\right)$

$n_x = 1$ $n_y = 3$

$$P = \frac{2^2 (5\text{ pm})^2}{(150)^2} \sin^2\left(\frac{\pi}{\cancel{150}} \times 0.2 \times \cancel{150}\right) \sin^2\left(\frac{3\pi}{\cancel{150}} \times 0.8 \times \cancel{150}\right)$$

$$= 4.4\times10^{-3} \sin^2(0.2\pi) \sin^2(2.4\pi) = 4.4\times10^{-3} \times 0.345 \times 0.904$$

$\underline{\underline{P = 1.37\times10^{-3}}}$

39.31

$\psi(r) = \frac{1}{\sqrt{\pi}\, a^{3/2}} e^{-r/a}$ radial wavefunction

a) $\psi^2(r) = \frac{1}{\pi a^3} e^{-2r/a}$ $r = a \Rightarrow \psi^2(a) = \frac{1}{\pi a^3} e^{-2}$

$a = 0.053$ nm (Bohr radius)

$\psi^2(a) = \frac{1}{\pi (0.053\text{nm})^3} e^{-2} = \underline{\underline{291 \text{ nm}^{-3}}}$

b) $P(r) = \frac{4}{a^3} r^2 e^{-2r/a}$ $P(a) = \frac{4}{a^3} a^2 e^{-2} = \frac{4}{a} e^{-2} = \underline{\underline{10.2\text{nm}^{-1}}}$

39.37

Balmer series $n_f = 2$ $n_i = 3, 4, 5, 6, \ldots \infty$

Shortest wavelength $n_i = \infty \rightarrow n_f = 2$

$$\Delta E = hf = \frac{hc}{\lambda} = -13.6\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right) eV$$

$$= \underline{13.6}\left(\frac{1}{n_i^2} - \frac{1}{n_f^2}\right)$$

$$\frac{1}{\lambda} = \frac{13.6}{hc}\left(\frac{1}{4} - \cancel{\frac{1}{\infty}}^{0}\right) = \frac{13.6\ \cancel{eV}}{1240\ nm\ \cancel{eV}} \times \frac{1}{4}$$

$$\underline{\underline{\lambda = 364.7\ nm.}}$$

Lyman Series $n_f = 1$ $n_i = 2, 3, 4, \ldots \infty$

Shortest wavelength $n_i = \infty \rightarrow n_f = 1$

$$\Delta E = hf = \frac{hc}{\lambda} = 13.6\left(\frac{1}{n_i^2} - \frac{1}{n_f^2}\right)$$

$$\frac{1}{\lambda} = \frac{13.6}{hc}\left(1 - \cancel{\frac{1}{\infty}}^{0}\right) = \frac{13.6\ \cancel{eV}}{1240\ \cancel{eV}.nm} \Rightarrow$$

$$\underline{\underline{\lambda = 91.1\ nm}}$$

ratio $= \underline{\underline{4.0}}$

39.41

$$P(r) = 1 - e^{-2x}(1 + 2x + 2x^2)$$ see problem 39.8

$x = r/a$ when $r = a \Rightarrow x = 1$

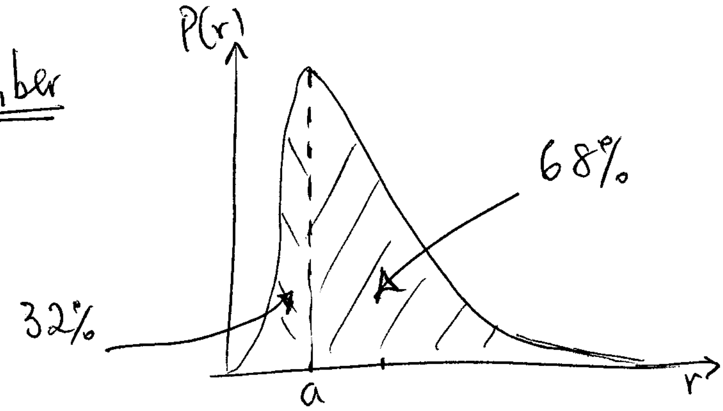
$$P(a) = 1 - e^{-2}(1+2+2) = 1 - 5e^{-2} = 0.32$$

This is the probability of finding the electron inside a sphere of radius a

We want the probability of finding the electron outside

Bohr radius $\Rightarrow$ $P = 1 - 0.32 = \underline{0.68}$

<u>remember</u>



39.47

$$P(r) = \frac{4}{a^3} r^2 e^{-2r/a}$$

$$\int_0^\infty P(r)\, dr = \frac{4}{a^3} \int_0^\infty r^2 e^{-2r/a}\, dr$$

change variable $z = 2r/a$ $\quad r = \frac{a}{2} z \quad dr = \frac{a}{2} dz$

$$\frac{4}{a^3}\left(\frac{a}{2}\right)^3 \underbrace{\int_0^\infty z^2 e^{-z}\, dz}_{2! = 2\times 1} = \frac{4}{a^3}\, \frac{a^3}{8} \times 2 = 1 \quad !!!$$