Summary chapter 15

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- The **density,** ρ (rho), of any material (solid or fluid) is defined as the mass per unit volume. That is:

$$\rho = \frac{m}{V}$$

where m is the mass of the material and V its volume. The unit of the density is kg/m^3.

Note: $1 g/cm^3 = 1000\ kg/m^3$.

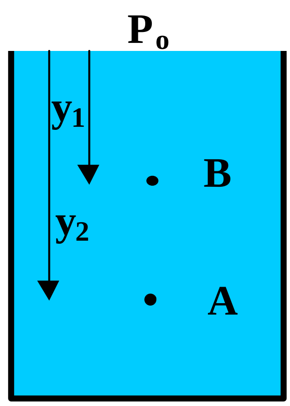
- The **pressure**, P, exerted by a fluid on a surface of area A is defined as:

$$P = \frac{F}{A}$$

where F is the force of the fluid and it is perpendicular to the surface A. The unit of pressure is N/m^2 or pascal (Pa).

Other units are: 1 atm.=1.013×10^5 Pa =760 Torr = 760 mm-Hg.

- **The pressure variation inside a fluid**:



$$\mathbf{P_A = P_B + \rho g(y_2 - y_1) = P_B + \rho g h}$$

h is the difference in height between point A and B and ρ is the density of the fluid.

The variation of pressure between the surface of the fluid and point A is:

$\mathbf{P_A = P_o + \rho g h}$, where P_o is the atmospheric pressure = 1.01×10^5 Pa

- **Pascal principle:**

*A **change in pressure** applied to an **<u>enclosed fluid</u>** is transmitted undiminished (with the same value) to every point of the fluid and the walls of the container.*

This principle is used in the hydraulic lift:

$$\frac{F_i}{A_i} = \frac{F_o}{A_o}$$

- **Archimedes principle:**

  The buoyant force acting on an object immersed in a fluid is equal to the weight of the fluid displaced by the object.

$$F_B = \rho_f gV = Mg$$

two cases:

✓ **<u>Submerged objects</u>**: The net force on the submerged object is:

$$F_{net} = F_B - W = (\rho_f - \rho_o)Vg$$

If $\rho_f > \rho_o$ then $F_{net} > 0$ and the object will **<u>accelerate upward.</u>**

If $\rho_f < \rho_o$ then $F_{net} < 0$ and the object will **<u>sink.</u>**

✓ **<u>Floating objects</u>:** The net force on the floating object is:

$$F_{net} = 0 \Rightarrow F_B = W$$

where $\mathbf{F_B} = \boldsymbol{\rho_f}\mathbf{gV}$ and $\mathbf{W} = \boldsymbol{\rho_o}\mathbf{gV_o}$

Be carful: ***V is the volume of the object beneath the fluid level*** (only the submerged volume).

In this case we have the following relationship:

$$\frac{\rho_o}{\rho_f} = \frac{V}{V_o}$$

The fraction of the object that lies above the fluid level is:

$$fraction = 1 - \frac{V}{V_o} = 1 - \frac{\rho_o}{\rho_f}$$

ρ_o is the density of the object and ρ_f is the density of the fluid.

- An ideal fluid has the following properties:
  - The flow is steady
  - The flow is incompressible
  - The flow is nonviscous and
  - The flow is irrotational

For and ideal fluid the **volume flow rate**, R, is constant, that is:

$$R = A_1 v_1 = A_1 v_2$$

where A is the cross section area and v is the speed of the fluid particles.

This equation is called the ***continuity equation***. The unit of the volume flow rate is m^3/s.

You can also calculate the **mass flow rate** as:

$$\rho A_1 v_1 = \rho A_2 v_2$$

The unit is kg/s.

- ***Bernoulli equation is a consequence of conservation of energy***.

It is written in this form:

$$P_1 + \rho g y_1 + \frac{1}{2}\rho v_1^{\,2} = P_2 + \rho g y_2 + \frac{1}{2}\rho v_2^{\,2}$$

Special case:

- If the fluid flow is leveled ($y_1 = y_2 = 0$) then,

$$P_1 + \frac{1}{2}\rho v_1^{\,2} = P_2 + \frac{1}{2}\rho v_2^{\,2}$$

- If the fluid is static ($v_1 = v_2 = 0$) then

$$P_1 + \rho g y_1 = P_2 + \rho g y_2$$

Note: ***In the case of a large container with a small hole, the speed v_1 is taken to be zero.***