Chapter 9

Problem 4.

3 M \((0, 0)\)

M left \((-\frac{1}{2}, -\frac{1}{2})\)

M right \(\left(\frac{1}{2}, \frac{1}{2}\right)\)

\[
X_{cm} = \frac{0 \cdot 3M + \left(-\frac{1}{2} \cdot M\right) + \left(\frac{1}{2} \cdot M\right)}{5M} = 0
\]

\[
Y_{cm} = \frac{0 \cdot 3M + \left(-\frac{1}{2} \cdot M\right) + \left(-\frac{1}{2} \cdot M\right)}{5M} = -\frac{1}{4}
\]

Coordinates of the center of mass are \((0, -\frac{1}{4})\).

Problem 5.

Consider two pieces

\(\text{1. one which is a square of dimensions 6x6 m}^2\) and mass \(M\)

\(\text{2. one which is a square of dimensions 2x2 m}^2\) and mass \(m\).

\[
X_{cm} = \frac{0 \cdot M + 2 \cdot (-m)}{9m \cdot (-m)} = \frac{-2m}{8m} = -\frac{1}{4} = -0.25m
\]

\[
Y_{cm} = \frac{0 \cdot M + 0 \cdot (-m)}{9m \cdot (-m)} = 0
\]

Notice that \(\frac{M}{m} = \frac{36}{4} = 9 \Rightarrow M = 9m\).
Pb#8.

#1 \( M \ (20, 0, 20) \)
#2 removed
#3 \( M \ (20, 40, 20) \)
#4 \( M \ (20, 20, 0) \)
#5 \( M \ (40, 20, 20) \)
#6 \( M \ (0, 20, 20) \)

\[
X_{cm} = \frac{M_1 	imes (20 \times 3 + 40)}{5M} = \frac{40 + 60}{5} = \frac{100}{5} = 20 \text{ cm}
\]

\[
Y_{cm} = \frac{M_1 	imes (20 \times 8 + 40)}{5M} = \frac{100}{5} = 20 \text{ cm}
\]

\[
Z_{cm} = \frac{M_1 	imes (20 \times 8)}{5M} = \frac{80}{5} = 16 \text{ cm}
\]

\( Cm \ (20, 20, 16) \text{ cm} \)

Pb#11.

\[
V_{cm} = \frac{\sum m_i v_i}{\sum m_i} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2} = \frac{2400 \times 80 + 1600 \times 60}{2400 + 1600} = 72 \text{ Km/h}
\]

Pb#13.

\[
t_1 = 3 \times 10^{-3} \text{ s} \quad y_1 = \frac{1}{2} g t_1^2 = 0.44 \text{ m}
\]

\[
t_2 = 2 \times 10^{-3} \text{ s} \quad y_2 = \frac{1}{2} g t_2^2 = 0.20 \text{ m}
\]

\[
y_{cm} = \frac{\sum m_i y_i}{\sum m_i} = \frac{m_1 y_1 + m_2 y_2}{m_1 + m_2} = 0.38 \text{ m}
\]
b) \( v_1 = g t_1 = 9.8 \times 0.94 \text{ m/s} \)
\( v_2 = g t_2 = 9.8 \times 1.96 \text{ m/s} \)
\[ \frac{\sum m_i v_i}{\sum m_i} = 2.29 \text{ m/s} \]

Pb #14.

\( m_1 = 1000 \text{ kg} \quad a_1 = 4 \text{ m/s}^2 \)
\( m_2 = 2000 \text{ kg} \quad v_2 = 8 \text{ m/s} \)
\( t = 3 \text{ sec} \quad v_i = a_1 t = 12 \text{ m/s} \)

\[ x_1 = \frac{1}{2} a_1 t^2 = 18 \text{ m} \]
\[ x_2 = \frac{1}{2} a_2 t^2 + v_2 t = 24 \text{ m} \]
\[ X_{cm} = \frac{\sum x_i m_i}{\sum m_i} = \frac{x_1 m_1 + x_2 m_2}{m_1 + m_2} = \frac{22}{3.2} \text{ m} \]
\[ V_{cm} = \frac{v_1 m_1 + v_2 m_2}{m_1 + m_2} = \frac{12 \times 1000 + 8 \times 2000}{3000} \]
\[ = \frac{9.3 \text{ m/s}}{} \]
\[ a_{cm} = \frac{a_1 m_1 + a_2 m_2}{m_1 + m_2} = \frac{4 \times 1000}{3000} \]
\[ = \frac{4}{3} = 1.33 \text{ m/s}^2 \]
Pb # 22.

\[ p_i = m_i v_i = 0.7 \times 5 = 3.5 \text{ Kg m/s} \]

\[ p_f = m_f v_f = 0.7 \times (-2) = -1.4 \text{ Kg m/s} \]

\[ \Delta p = p_f - p_i = -1.4\hat{i} - 3.5\hat{i} = -4.9\hat{i} \text{ Kg m/s} \]

\[ |\Delta p| = 4.9 \text{ Kg m/s} \]

Pb # 24.

a) Since \( p_{x_f} = p_{x_f} \)

\[ m v_i \sin 30^\circ = m v_f \sin \theta \]

\[ \Rightarrow \theta = 30^\circ \]

b) \( \Delta p_x = m v_x - m v_{x_i} = m v \sin 30^\circ - m v \sin 30^\circ = 0 \)

\( \Delta p_y = m v_{y_f} - m v_{y_i} = m v \cos 30^\circ + m v \cos 30^\circ \)

\[ = -2 m v \cos 30^\circ = -0.57 \text{ Kg m/s} \]

\[ \Delta p = 0\hat{i} - 0.57\hat{j} \text{ Kg m/s} \]
Pb # 34.

\[ \vec{v}_c = ? \]

before

\[ \vec{p}_c = \vec{p}_3 \Rightarrow \vec{p}_{ci} = \vec{p}_{if} \]
\[ p_{yc} = p_{3y} \]

\[ M \cdot v_{cx} = 0 + m \cdot v_{2x} \cos 36^\circ \quad (1) \]

\[ m \cdot v_{cy} = m \cdot v_{1y} + m \cdot v_{2y} \sin 36^\circ \quad (2) \]

(1) \Rightarrow \quad 4 \cdot v_{cx} = 2 \times 5 \times \cos 36^\circ \Rightarrow v_{cx} = 2.16 \text{ m/s}

(2) \Rightarrow \quad 4 \cdot v_{cy} = 2 \times 3 + 2 \times 5 \times \sin 36^\circ \Rightarrow v_{cy} = 2.75 \text{ m/s}

velocity \quad \vec{v}_c = 2.16 \hat{i} + 2.75 \hat{j} \text{ m/s}

speed \Rightarrow |\vec{v}_c| = 3.5 \text{ m/s}
14P. Figure 10-29 shows an approximate plot of force magnitude versus time during the collision of a 58 g Superball with a wall. The initial velocity of the ball is 34 m/s perpendicular to the wall; it rebounds directly back with approximately the same speed, also perpendicular to the wall. What is \( F_{\text{max}} \), the maximum magnitude of the force on the ball from the wall during the collision?

![Graph of force versus time](image)

\[ \text{Impulse} = \int F \, dt = \Delta p \]

\[ \text{Area} = \frac{1}{2} \left( F_{\text{max}} \times 2 \times 10^{-3} \right) + F_{\text{max}} \times 2 \times 10^{-3} + \frac{1}{2} \left( F_{\text{max}} \times 2 \times 10^{-3} \right) \]

\[ = 4 F_{\text{max}} \times 10^{-3} \]

\[ \Delta p = p_f - p_i = 58 \times 10^{-3} \left( 34 - (-34) \right) = 3944 \times 10^{-3} \]

\[ \Rightarrow 4 F_{\text{max}} = 3944 \Rightarrow F_{\text{max}} = 986 \, \text{N} \]
A ball having a mass of 150 g strikes a wall with a speed of 5.2 m/s and rebounds with only 50% of its initial kinetic energy.

(a) What is the speed of the ball immediately after rebounding?

(b) What is the magnitude of the impulse on the wall from the ball?

(c) If the ball was in contact with the wall for 7.6 ms, what was the magnitude of the average force on the ball from the wall during this time interval?

\[ K_f = \frac{1}{2} K_i \]
\[ v_i = 5.2 \text{ m/s} \]
\[ K_i = \frac{1}{2} m v_i^2 = 2.028 \text{ J} \]
\[ K_f = 1.014 \text{ J} = \frac{1}{2} m v_f^2 \Rightarrow v_f = 3.7 \text{ m/s} \]

\[ J = \Delta p = p_f - p_i = m(v_f - v_i) \]
\[ = 0.15 \times (3.7 - (-5.2)) = 1.3 \text{ Kg.m/s} \]

\[ J = F \Delta t \Rightarrow F = \frac{J}{\Delta t} = \frac{1.3}{7.6 \times 10^{-3}} = 170 \text{ N} \]
The blocks in Fig. 10-37 slide without friction. (a) What is the velocity \( \vec{v} \) of the 1.6 kg block after the collision? (b) Is the collision elastic? (c) Suppose the initial velocity of the 2.4 kg block is the reverse of what is shown. Can the velocity \( \vec{v} \) of the 1.6 kg block after the collision be in the direction shown?

![Diagram](image)

\( 5.5 \text{ m/s} \)  \( \longrightarrow \)  \( 2.5 \text{ m/s} \)  \( \longrightarrow \)

Before collision

\( \vec{v} \)  \( \longrightarrow \)  \( 4.9 \text{ m/s} \)  \( \longrightarrow \)

After collision

Fig. 10-37 Exercise 35.

\[ \text{a) Conservation of momentum} \]

\[ m_1 \vec{v}_{1i} + m_2 \vec{v}_{2i} = m_1 \vec{v}_{1f} + m_2 \vec{v}_{2f} \]

\[ \vec{v}_{1f} = \frac{1.6 \times 5.5 + 2.4 \times 2.4 - 4.9 \times 4.9}{1.6} = 1.9 \text{ m/s} \]

\[ \text{b) } K_i = \frac{1}{2} m_1 \vec{v}_{1i}^2 + \frac{1}{2} m_2 \vec{v}_{2i}^2 = 31.7 \text{ J} \]

\[ K_f = \frac{1}{2} m_1 \vec{v}_{1f}^2 + \frac{1}{2} m_2 \vec{v}_{2f}^2 = 31.7 \text{ J} \]

Since \( K_i = K_f \Rightarrow \text{the collision is elastic} \).

\[ \text{c) } \vec{v}_{2i} = -2.5 \text{ m/s} \]

\[ \vec{v}_{1f} = \frac{1.6 \times 5.5 + (2.4)(-2.5) - 2.4 \times 4.9}{1.6} = -5.6 \text{ m/s} \]

\[ \begin{array}{c}
\text{5.6 m/s} \\
\text{1.6 kg} \\
\text{2.4 kg}
\end{array} \]

\[ \rightarrow 4.7 \text{ m/s} \]

after collision
Two 2.0 kg bodies, A and B, collide. The velocities before the collision are $v_A = 15i + 30j$ and $v_B = -10i + 5.0j$. After the collision, $v'_A = -5.0i + 20j$. All speeds are given in meters per second. (a) What is the final velocity of B? (b) How much kinetic energy is gained or lost in the collision?

a) Conservation of momentum (two dimensions)

$$\vec{P}_i = \vec{P}_f$$

$$m_A \vec{v}_{A_i} + m_B \vec{v}_{B_i} = m_A \vec{v}_{A_f} + m_B \vec{v}_{B_f}$$

Since $m_A = m_B \Rightarrow \vec{v}_{B_f} = \vec{v}_{A_i} + \vec{v}_{B_i} - \vec{v}_{A_f}$

$$\vec{v}_{B_f} = 10i + 15j \text{ m/s}$$

b) $K_i = \frac{1}{2} m v_{A_i}^2 + \frac{1}{2} m v_{B_i}^2 = \frac{1}{2}(2) [1125 + 125] = 1250J$

$$K_f = \frac{1}{2} m v_{A_f}^2 + \frac{1}{2} m v_{B_f}^2 = \frac{1}{2}(2) [425 + 325] = 750J$$

$$\Delta K = K_f - K_i = -500J \text{ kinetic energy lost}$$
An alpha particle collides with an oxygen nucleus that is initially at rest. The alpha particle is scattered at an angle of 64.0° from its initial direction of motion, and the oxygen nucleus recoils at an angle of 51.0° on the opposite side of that initial direction. The final speed of the nucleus is $1.20 \times 10^5$ m/s. Find (a) the final speed and (b) the initial speed of the alpha particle. (In atomic mass units, the mass of an alpha particle is 4.0 u, and the mass of an oxygen nucleus is 16 u.)

(a) \[ \vec{p}_i = \vec{p}_f \]

\[ P_{x_i} = P_{x_f} \Rightarrow m_i v_{i_x} = m_i v_{f_x} \cos \Theta_1 + m_2 v_{f_y} \cos \Theta_2 \quad (1) \]

\[ P_{y_i} = P_{y_f} \Rightarrow 0 = -m_i v_{i_y} \sin \Theta_1 + m_2 v_{f_y} \sin \Theta_2 \quad (2) \]

(1) \[ v_{f_y} = \frac{m_2 v_{f_x} \sin \Theta_2}{m_i \sin \Theta_1} = \frac{16u \times 1.2 \times 10^5 \sin 51}{4u \times \sin 64} = 4.2 \times 10^5 \text{ m/s} \]

(b) \[ v_{i_x} = \frac{m_1 v_{i_x} \cos \Theta_1 + m_2 v_{f_x} \cos \Theta_2}{m_i} \]

\[ = \frac{4u \times 4.2 \times 10^5 \cos 64^\circ + 16u \times 1.2 \times 10^5 \cos 51^\circ}{4u} \]

\[ = 4.86 \times 10^5 \text{ m/s} \]