KING FHAD UNIVERSITY OF PETROLEUM & MINERALS
DEPARTMENT OF PHYSICS

PHYSICS 102 (992)
SOLUTION (CHAPTER 22)

Q.1 In a hypothetical nuclear fusion reactor, the fuel is deuterium gas at a temperature of about 7×10^{8} K. If this gas could be used to operate an ideal heat engine with $T_c = 100\ ^0C$, what would be the engine's efficiency?

$$e = 1 - \frac{100+273}{7\times10^{8}}$$

$$\boxed{e = 99.99995\ \%}$$

Q.2 One mole of an ideal monatomic gas is taken through the cycle in Figure 22.2. (a) How much work is done by the gas in going from state a to state c along path abc? What are the changes in internal energy and entropy in going (b) from b to c and (c) through one complete cycle? Express all answers in terms of the pressure p_o, volume V_o, and temperature T_o of state a.

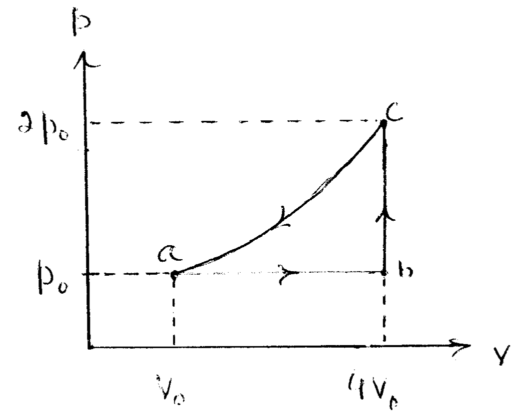

FIGURE 22.2.

(a) $W_{ac} = W_{ab} + W_{bc} = p_o(4V_o - V_o) = 3p_oV_o$

(b) $\Delta U_{bc} = nC_v\Delta T = n\frac{3}{2}R\Delta T = \frac{3}{2}(nR\Delta T)$

$\Delta U_{bc} = \frac{3}{2}V\Delta P \qquad [pV = nRT,\ V\Delta P = nR\Delta T]$

$\Delta U_{bc} = \frac{3}{2}(4V_o)(P_o) = 6p_oV_o$

$\Delta S_{bc} = nC_v\int_{T_b}^{T_c}\frac{dT}{T} = nC_v\ln\frac{T_c}{T_b} = \frac{3}{2}nR\ln 2$

$\Delta S_{bc} = \frac{3}{2}nR\ln 2$

$$\left[\begin{array}{c} P_o(4V_o) = nRT_b \\ 2p_o(4V_o) = nRT_c \\ \frac{T_c}{T_b} = 2 \end{array}\right]$$

(c) $\Delta S = 0,\ \Delta U = 0$

Q.3 One mole of an ideal monatomic gas is taken through the cycle shown in Figure 22.3. Assume that $p = 2p_o$, $V = 2V_o$, $p_o = 1.01 \times 10^5$ Pa, and $V_o = 0.0225\ m^3$. Calculate (a) the work done during the cycle, (b) the heat added during stroke abc, and (c) the efficiency of the cycle. (d) What is the efficiency of an ideal engine operating between the highest and lowest temperatures that occur in the cycle? How does this compare to the efficiency calculated in (c)?

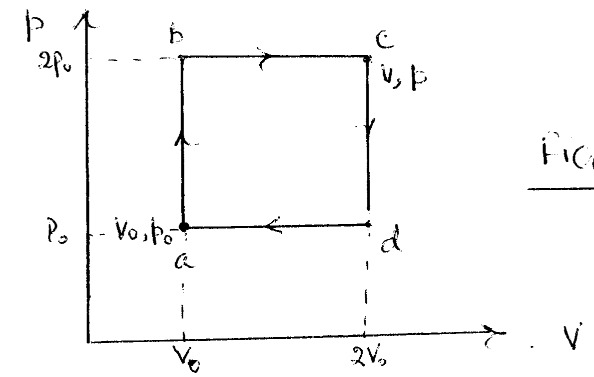

FIGURE 22.3

$n = 1$ mole , $p = 2p_o$, $V = 2V_o$, $P_o = 1.013\times10^{5}$ Pa

$V_o = 0.0225\ m^3$

(a) $W_{net} =$ Area enclosed by the path in P-V diagram

$= p_o V_o = (1.013\times10^5)(0.0225) = 2.27$ kJ

Heat added:

(b) $Q_{abc} = Q_{ab} + Q_{bc} = nC_v \Delta T_{ab} + nC_p \Delta T_{bc}$

$= n\left(\frac{3}{2}R\right)\frac{V \Delta P_{ab}}{nR} + n\left(\frac{5}{2}R\right)\frac{p\,\Delta V_{ab}}{nR}$

$= \frac{3}{2}V_o p_o + \frac{5}{2}(2p_o)(V_o) = \frac{13}{2}p_o V_o$

$Q_{abc} = \frac{13}{2}(1.013\times10^5)(0.0225) = 1.48\times10^4$ J

(c) $e = \frac{W}{Q_h} = \frac{2.27}{14.8}\times 100 = 15.4\%$

(d) $e_c = 1 - \frac{T_c}{T_h}$

$PV = nRT$

$T_c = \frac{p_0 V_0}{nR}$ (lowest temp.)

$T_h = \frac{(2p_0)(2V_0)}{nR}$ (highest temp.)

$e_c = 1 - \frac{T_c}{T_h} = 1 - \frac{(p_0 V_0)/nR}{(2p_0)(2V_0)/nR} = 1 - \frac{1}{4} = 0.75$

$e_c = 75\%$

Q.4 An ideal engine has efficiency ε. Show that if you run it back as an ideal refrigerator, the coefficient of performance will be K = (1- ε)/ε.

$$\epsilon_c = \frac{W}{Q_h} = \frac{Q_h - Q_c}{Q_h} = 1 - \frac{Q_c}{Q_h} \quad \text{—} (1)$$

[Carnot efficiency
max efficiency]

$$COP_{refrig.} = \frac{Q_c}{W} = \frac{Q_c}{Q_h - Q_c} = \frac{Q_c/Q_h}{1 - \frac{Q_c}{Q_h}} \quad \text{—} (2)$$

From (1) $\frac{Q_c}{Q_h} = 1 - e$, substitute in (2)

$$COP_{refrig.} = \frac{1-e}{1-(1-e)} = \frac{1-e}{e}$$

Q.5 Find (a) the heat absorbed and (b) the change in entropy of a 2.00 kg block of copper whose temperature is increased reversibly from 25 °C to 100 °C. The specific heat of copper is 386 J/kg.K.

(a) $Q = (2.00)(386)(100 - 25) = 5.79 \times 10^4$ J

(b) $\Delta S = \int_{25+273}^{100+273} mc \frac{dT}{T} = (2.00)(386) \ln \frac{373}{298}$

$\Delta S = 173$ J/K.

Q.6 A 10g ice cube at $-10\ ^{o}C$ is placed in a lake whose temperature is $15\ ^{o}C$. Calculate the change in entropy of the cube lake system as the ice cube comes to thermal equilibrium with the lake. The specific heat of ice is 2220 J/kg.K. (Hint : Will the ice cube effect the temperature of the lake?)

$$m = 10g, \quad T_{ice} = -10 + 273 = 263K$$

$$T_{lake} = 15 + 273 = 288\ K$$

$$\Delta S = \Delta S_{ice} + \Delta S_{lake}$$

$$\Delta S_{ice} = (10\times 10^{-3})(2220)\ln\frac{273+0}{273+(-10)} + \frac{(10\times 10^{-3})(3.33\times 10^{5})}{273+0}$$

$$+ (10\times 10^{-3})(4186)\ln\frac{273+15}{273+0} = 0.828 + 12.2 + 2.24$$

$$\Delta S_{ice} = 15.3\ J/K$$

$$\Delta S_{lake} = \frac{-[(10\times 10^{-3})(2220)(10) + (10\times 10^{-3})(3.33\times 10^{5}) + (10\times 10^{-3})(4186)(15)]}{15+273}$$

$$\Delta S_{lake} = -\frac{222+3330+628}{288} = -\frac{4180}{288} = -14.5\ J/K$$

$$\Delta S = 15.3 - 14.5 = 0.79\ J/K.$$

Q.7 The motor in a refrigerator has a power of 200 W. If the freezing compartment is at 270 K and the outside air is at 300 K, assuming ideal efficiency, what is the maximum amount of heat that can be extracted from the freezing compartment in 10.0 min?

$P = 200\ W$

$T_c = 270\ K,\ T_h = 300\ K$

The max. efficiency when $\dfrac{Q_c}{Q_h} = \dfrac{T_c}{T_h}$

$$COP = \frac{Q_c}{W} = \frac{Q_c}{Q_h - Q_c} = \frac{1}{\frac{Q_h}{Q_c} - 1} = \frac{1}{\frac{T_h}{T_c} - 1}$$

$$\frac{Q_c}{W} = \frac{1}{\frac{300}{270} - 1} = \frac{1}{3/27} = \frac{27}{3} = 9$$

$$\frac{Q_c}{W} = 9 \rightarrow Q_c = 9W = 9\,(200)\,(10)\,(60) = 1.08 \times 10^{6}\ J$$

$\left[power = \dfrac{work}{time} \right]$