Summary Chapter 21

I. objective:

① Understand the assumptions made in developing the molecular model of an ideal gas.

② The temperature of an ideal gas is proportional to the average molecular Kinetic energy.

③ State the theorem of equipartition of energy.

④ Recognize that the internal energy of an ideal gas is proportional to the absolute temperature and the specific heat at constant volume.

⑤ Define an idiabatic process and show that $PV^{\gamma} = \text{Const.}$

⑥ The total energy and specific heats associated with the possible degrees of freedom associated with a molecule.

II. Summary of major points:

① The pressure of N molecules of an ideal gas is given by;

$$P = \frac{2}{3}\frac{N}{V}\left(\frac{1}{2} m \overline{v^2}\right)$$

pressure (N/m^2); # of molecules; volume of the gas (m^3); average Kinetic energy. (J)

② The temperature (absolute) of an ideal gas is given by;

$$T = \frac{2}{3k}\left(\frac{1}{2} m \overline{v^2}\right)$$

temperature (K); Boltzmann Constant (J/K); average Kinetic energy (J)

③ * The average translational Kinetic energy per molecule of a gas is given by;

$$\frac{1}{2} m \overline{v^2} = \frac{3}{2} kT$$

* The rms speed of a molecule is given by; $v_{rms} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3kT}{m}}$

④ The total energy U (or internal energy) of N molecules of a gas (monoatomic) is proportional to the absolute temperature

$$\boxed{U = \frac{3}{2} NkT = \frac{3}{2} nRT}$$

The change in internal energy ΔU is given by;

Important: $\boxed{\Delta U = n C_v \Delta T}$ This is true **ALWAYS!**

For monoatomic Gas ⇒ $\boxed{C_v = \frac{3}{2} R}$ specific heat at constant volume.

Also $\boxed{C_p - C_v = R}$ — perfect gas constant. or; $\boxed{C_p = \frac{5}{2} R}$

(C_p: specific heat at const. pressure.)

⑤ For quasi-static, adiabatic expansion of an ideal gas;

This is true only for adiabatic process. ⟶ $\boxed{P V^{\gamma} = \text{Const}}$ ⇒ $\left[P_i V_i^{\gamma} = P_f V_f^{\gamma} \; ; \; T_i V_i^{\gamma - 1} = T_f V_f^{\gamma - 1} \right]$

(P: pressure of the gas; V: volume of the gas.)

$\gamma = \frac{C_p}{C_v}$ = specific heat ratio. (constant)

An adiabatic process is a process for which there is no heat exchange between the system (gas) and the surrounding.

⑥
* For translation only (monoatomic gas)

  $C_v = \frac{3}{2} R$ $\quad C_p = \frac{5}{2} R$ $\quad \boxed{\Delta U = \frac{3}{2} R n \Delta T}$

* For rotation + translation (diatomic molecule)

  $C_v = \frac{5}{2} R$ $\quad C_p = \frac{7}{2} R$ $\quad \boxed{\Delta U = \frac{5}{2} R n \Delta T}$

* For vibration + rotation + translation (diatomic molecule)

  $C_v = \frac{7}{2} R$ $\quad C_p = \frac{9}{2} R$ $\quad \boxed{\Delta U = \frac{7}{2} R n \Delta T}$