KING FHAD UNIVERSITY OF PETROLEUM & MINERALS
DEPARTMENT OF PHYSICS

PHYSICS 102 (992)
SOLUTION (CHAPTER 21)

Q.1 (a) What is the number of molecules per cubic meter in air at 20 °C and at a pressure of 1.0 atm, (=1.10 × 10^5 Pa)? (b) What is the mass of this 1 m^3 of air? Assume that 75 % of the molecules are nitrogen (N_2) and 25 % are oxygen (O_2).

a) $PV = nRT$

$$\frac{n}{V} = \frac{P}{RT} = \frac{1.013\times10^5}{(8.31)(20+273)} = 41.60 \text{ moles}/m^3.$$

$$\frac{N}{V} = \frac{n}{V} \times 6.02\times10^{23} = 2.50\times10^{25} \text{ molecules}/m^3.$$

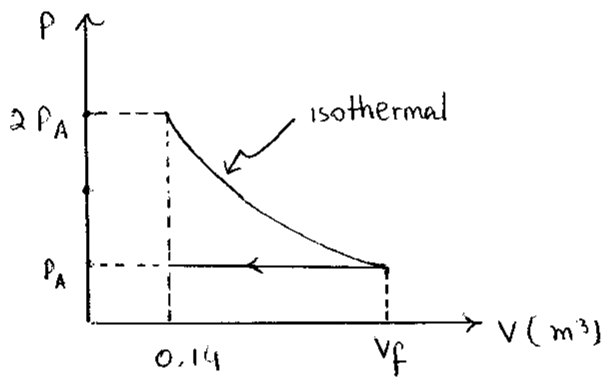
b) $$\text{mass of } N_2 \text{ molecule} = \frac{28\times10^{-3}}{6.02\times10^{23}} = 4.65\times10^{-26} \text{ kg}$$

$$\text{mass of } O_2 \text{ molecule} = \frac{32\times10^{-3}}{6.02\times10^{23}} = 5.32\times10^{-26} \text{ kg}$$

$$\text{mass of one } m^3 \text{ of air} = 4.65\times10^{-26}\times0.75\times2.50\times10^{25} + 5.32\times10^{-26}\times0.25\times2.50\times10^{25}$$

$$= 1.20 \text{ kg}$$

Q.2 Air that occupies 0.14 m^3 at 1.03×10^5 Pa gauge pressure is expanded isothermally to atmospheric pressure and then cooled at constant pressure until it reaches its initial volume. Compute the work done by the air.



$$P_{guage} = P_{Absolute} - P_A$$

$$P_{Absolute} = P = P_{guage} + P_A = 2\,P_A$$

$$W = nRT \ln \frac{V_f}{V_i} + (0.14 - V_f)\,P_A$$

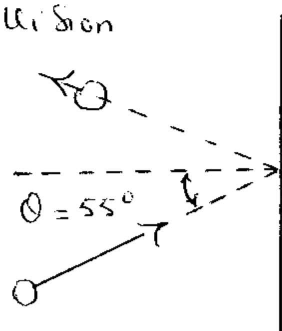
$$\left[\; 2P_A V_i = nRT = P_A V_f \,,\; V_f = 2V_i \;\right]$$

$$W = 2\,P_A V_i \ln \frac{2V_i}{V_i} + 1.013\times10^5 (0.14 - 2\times0.14)$$

$$\boxed{W = 5500\ \mathrm{J}}$$

Q.3 The mass H_2 molecule is 3.3×10^{24} g. If 10^{23} H_2 molecules per second strike 2.0 cm^2 of wall at an angle of 55° with the normal when moving with a speed of 1.0×10^5 cm/s, what pressure do they exert on the wall?

$v = 1.0\times10^5$ cm/s, elastic collision



$$p = \frac{\frac{\Delta(\text{momentum})}{\Delta t}}{A}$$

↑ Pressure ↑ Area

$$= \frac{2 m v \cos\theta}{A}$$

$$= \frac{2(3.3\times10^{-27})\times1.0\times10^{3}\times\cos 55^\circ}{2.0\times10^{-4}} = 1.9 \text{ kPa}$$

$$\boxed{p = 1.9 \text{ kPa}}$$

Q.4 Let 20.9 J of heat be added to a particular ideal gas. As a result, its volume changes from 50.0 cm^3 to 100 cm^3 while the pressure remains constant at 1.00 atm. (a) By how much did the internal energy of the gas change? If the quantity of gas present is 2.00×10^{-3} mol, find the molar specific heat at (b) constant pressure and (c) constant volume.

a. $\Delta U = Q - W = 20.9 - 1.013\times10^{5}(100-50)\times10^{-6}$

$$\boxed{\Delta U = 15.9\ J}$$

b. $C_p = \dfrac{20.9}{n\Delta T}$ [where n is the number of moles of the ideal gas, ΔT is the change in temp.

$= \dfrac{20.9}{0.610}$

$PV = nRT$, $P\Delta V = nR\Delta T$

$$\boxed{C_p = 34.3\ J/mole\ k}$$

$n\Delta T = \dfrac{P\Delta V}{R} = \dfrac{1.01\times10^{5}\times50\times10^{-6}}{8.31}$

$n\Delta T = 0.610$]

c. $C_p - C_v = R$

$C_v = C_p - R = 34.3 - 8.31 = 26.0\ J/mole\,k$

$$\boxed{C_v = 26.0\ J/mol\ K}$$

Q.5 One mole of an ideal monatomic gas traverses the cycle shown in Figure 21.5. Process 1 → 2 takes place at constant volume, process 2 → 3 is adiabatic, and process 3 → 1 takes place at constant pressure. (a) Compute the heat Q, the change in internal energy ΔU, and the work done W, for each of the three processes and for the cycle as a whole. (b) If the initial pressure at point 1 is 1.00 atm, find the pressure and the volume at points 2 and 3. Use 1.00 atm = 1.013×10^5 Pa and R = 8.314 J/mol.K.

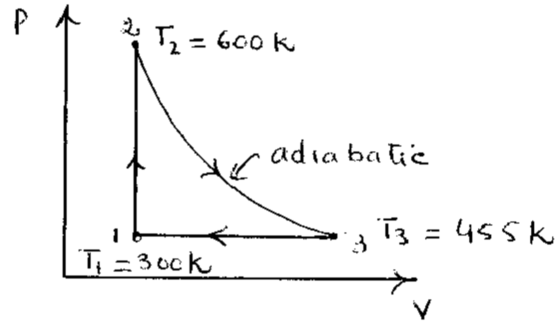


FIGURE 21.5

(a)

$\Delta U = Q - W$

Process	Q	W	ΔU
1 → 2	$nC_V \Delta T = (1)(\frac{3}{2}R)(600-300)$ $= 3.74$ kJ	0	$nC_V\Delta T = \frac{3}{2}(1)(8.31)(600-300)$ $= 3.74$ kJ
2 → 3	0	$-\Delta U = -nC_V\Delta T$ $= (1)(\frac{3}{2}R)(455-600)$ $= 1.81$ kJ	$nC_V\Delta T = \frac{3}{2}(1)(8.31)(455-600)$ $= -1.81$ kJ
3 → 1	$nC_p\Delta T = (1)(\frac{5}{2}R)(300-455)$ $= -3.22$ kJ	$P\Delta V = nR\Delta T$ $= (1)(8.31)(300-455)$ $= -1.29$ kJ	$nC_V\Delta T = \frac{3}{2}(1)(8.31)(300-455)$ $= -1.93$ kJ
Total for the Complete Cycle.	0.52 kJ	0.52 kJ	0

b) $P_1 V_1 = nRT_1$

$$V_1 = \frac{nRT_1}{P_1} = \frac{(1)(8.31)(300)}{1.013 \times 10^5} = 0.0246 \text{ m}^3$$

$$P_2 = \frac{nRT_2}{V_2}, \quad V_2 = V_1 = 0.0246 \text{ m}^3$$

$$P_2 = \frac{(1)(8.31)(600)}{0.0246} = 2.026 \times 10^5 \text{ Pa}.$$

$$= \frac{2.026 \times 10^5}{1.013 \times 10^5} \text{ atm} = 2.0 \text{ atm}.$$

$$V_3 = \frac{nRT_3}{P_3}, \quad P_3 = P_1 = 1 \text{ atm}$$

$$V_3 = \frac{(1)(8.31)(455)}{1.013 \times 10^5} = 0.0373 \text{ m}^3$$