Summary of chapter 18

I. Objective:

1. Write the wave function, which represents the superposition of two sinusoidal waves of **equal** amplitude, wavelength, and frequency traveling in opposite direction.

2. Identify the angular frequency, maximum amplitude, and determine the value of x which correspond **nodes** and **anti-nodes** for standing waves.

3. Calculate the normal mode frequencies for string under tension, and for open and closed air columns (pipe).

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II. Summary of major points:

1. Superposition of harmonic waves

The net displacement of two or more waves (the resultant) equals the algebraic sum of the displacement of all waves.

$$y_1 = A_o \sin(kx - \omega t)$$

$$y_2 = A_o \sin(kx - \omega t)$$

The resultant wave function is $y_T = y_1 + y_2 = [2Ao \cos \phi/2] \sin(kx - \omega t - \phi/2)$

3 Cases:

a) The amplitude of the resultant wave is maximum (constructive interference) when $\phi = 0, 2\pi, 4\pi, 6\pi, \ldots$

b) The amplitude of the resultant wave is zero (destructive interference) when $\phi = \pi, 3\pi, 5\pi, \ldots$

c) Partially destructive interference when $0 < \phi < \pi$ etc, ..

Interference of sound waves **:**

2 Cases:

a) **Constructive interference** (maximum sound) $\Delta r = 0, \lambda, 2\lambda, 3\lambda \ldots$

$$\Delta r = n\lambda \;;\; n = 0, 1, 2, 3, 4, \ldots$$

b) **Destructive interference** (minimum sound) $\Delta r = 0, \lambda/2, \lambda, 3\lambda/2 \ldots.$

$$\Delta r = n\frac{\lambda}{2}; n = 1, 3, 5, 7, \ldots$$

Δr is the difference in path between the two sound waves

The relationship between the difference in path and the phase difference between the two waves at the location of the listener is

$$\Delta r = \frac{\lambda}{2\pi}\phi$$

2. Standing Waves in a string

We have standing waves when two equal harmonic waves are traveling in positive directions,

$$y_1 = A_o \sin(kx - \omega t) \text{ and } y_2 = A_o \sin(kx + \omega t)$$

$$y_T = y_1 + y_2 = 2A_o \sin(kx) \cos(\omega t)$$

2 Cases:

a) position of the **anti-nodes** or **maximum** amplitude:

$$kx = \pi/2, 3\pi/2, 5\pi/2, \ldots \qquad \text{but } k = 2\pi/\lambda$$

$$\Rightarrow x = \lambda/4, 3\lambda/4, 5\lambda/4, \ldots = n\lambda/4 \text{ where } n = 1, 3, 5, 7, \ldots$$

b) position of the **nodes** or **minimum** amplitude:

$$k_{x} = \frac{2p}{l}$$

$$kx = 0, \Rightarrow x = \lambda/2, \lambda, 3\lambda/2, \ldots = n\lambda/2 \text{ where } n = 1, 2, 3, 4, 5, \ldots$$

In the case of a stretched string

Normal modes wavelength: $\lambda_n = 2Lv/n$ where $n = 1, 2, 3, \ldots$
L: Length of the string and v is the speed of the wave in the string.

Normal modes frequencies: $f_n = (n/2L) \sqrt{T/\mu}$ where $n = 1, 2, 3, \ldots$.

3. Standing Waves in air columns (pipes)

Sound sources can be used to produce longitudinal standing waves in air columns.

2 Cases:

a) ***pipe open at both ends***: The resonance occur for

(length of pipe) $L = n\,\lambda/2$ where $n = 1, 2, 3, 4, \ldots$..

since $v = \lambda f \Rightarrow f_n = (nv)/2L$ where v is the speed of sound waves

b) ***pipe closed at one end***: the resonance occur when

(length of pipe) $L = n\ \lambda/4$ where $n = 1, 3, 5, 7, \ldots\ldots$

or: $f_n = nv/4L$ where $n = 1, 3, 5, 7, \ldots\ldots$