Summary chapter 16

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- Oscillatory motion is the one that repeats itself.
- The number of oscillations per second is an important property of simple harmonic motion and is called **the frequency** f. Its unit is Hertz (Hz) or s^{-1}.
- **The period** of oscillations, T, is the time to complete on oscillation.
- Any motion that repeats itself with a certain period is called **periodic** or **harmonic** motion.
- **The displacement**, x(t) of the particle is given by:

$$x(t) = x_m \cos(wt + f)$$

where x_m is the **amplitude** of the motion (maximum displacement), w is the **angular frequency** or angular speed and f is the **phase constant**. The quantity ($wt+f$) is called the **phase angle** and has unit of **radian**.

- **The velocity** of the particle is given by:

$$v(t) = \frac{dx}{dt} = -wx_m \sin(wt + f)$$

The maximum velocity of the particle is $v_m = wx_m$. ***The particle has maximum speed at the origin.***

- The acceleration of the particle is given by:

$$a(t) = \frac{dv}{dt} = -w^2 x_m \cos(wt + f) = -w^2 x(t)$$

The maximum acceleration of the particle is $a_m = w^2 x_m$. ***The particle has maximum acceleration when it is at*** $x = \pm x_m$.

- The relationship between the frequency, the angular frequency and the period is:

$$\omega = 2\pi f = \frac{2\pi}{T}$$

<u>Important:</u> ***whenever the force acting on a particle is proportional to the negative of the displacement, then the motion is said to be simple harmonic motion.***

- Special case #1: **<u>The mass-spring system.</u>**

  In this case Hook s look gives F = – k*x*. Therefore the motion is harmonic.

  The angular frequency and the period of oscillations are:

$$\omega = \sqrt{\frac{k}{m}} \qquad \text{and} \qquad T = 2\pi\sqrt{\frac{m}{k}}$$

- Special case # 2: **<u>The simple pendulum.</u>**

  In this case the force acting the mass is $F = -(\frac{mg}{L})s$. Therefore the motion is also harmonic.

  The period of oscillations is given by:

$$T = 2\pi\sqrt{\frac{L}{g}}$$

- Energy in simple harmonic motion:

  Consider the mass-spring system.

The potential energy of the spring is:

$$U = \frac{1}{2}kx^2 = \frac{1}{2}kx_m{}^2 \cos^2(wt + f)$$

The kinetic energy of the block is:

$$K = \frac{1}{2}mv^2 = \frac{1}{2}mw^2 x_m{}^2 \sin^2(wt + f)$$

but $mw^2 = k$

then the total mechanical energy of the mass-spring system is :

E = K + U

$$E = \frac{1}{2}kx_m{}^2$$

We see that the total energy of the system does not depend on time and is therefore CONSTANT.

⇒ ***K+U has always the same value for all positions of the mass.***