KING FHAD UNIVERSITY OF PETROLEUM & MINERALS
DEPARTMENT OF PHYSICS

PHYSICS 102 (992)
SOLUTION (CHAPTER 16)

Q.1 The equation of a transverse wave traveling along a very long string is given by y = 6.0sin(0.020πx + 4.0tπ), where x and y are expressed in centimeters and t is in seconds. Determine (a) the amplitude, (b) the wavelength, (c) the frequency, (d) the speed, (e) the direction of propagation of the wave, and (f) the maximum transverse speed of a particle in the string (g) What is the transverse displacement at x –3.5 cm when t = 0.26 s?

$$y = 6.0 \sin(0.02\pi x + 4.0\pi t)$$

compare with $y = A \sin(kx \pm \omega t)$

(a) $A = 6.0$ cm.

(b) $k = \frac{2\pi}{\lambda} = 0.020\pi$, $\lambda = \frac{2\pi}{0.020\pi} = 100$ cm.

(c) $\omega = 2\pi f = 4.0\pi$, $f = 2$ Hz.

(d) $v = \frac{\omega}{k} = \frac{4.0\pi}{0.02\pi} = 200$ cm/s

(e) to the left along the x-axis.

(f) $v_y = \frac{\partial y}{\partial t} = (6)(4\pi)\cos(0.020\pi x + 4.0\pi t)$

$(v_y)_{max} = 24\pi$ cm/s

(g) $y = 6.0 \sin(0.020\pi(3.5) + 4\pi(0.26)) = 6.0 \sin(1.11\pi)$

$y = -2.03$ cm

Q.2 A sinusoidal wave is traveling along a string toward decreasing x. Figure 16.2 shows a plot of the displacement as a function of position at time t = 0. The string tension is 3.6 N, and its linear density is 25 g/m. Find (a) the amplitude, (b) the wavelength, (c) the wave speed, and (d) the period of the wave. (e) Find the maximum speed of a particle in the string. (f) Write an equation describing the traveling wave.

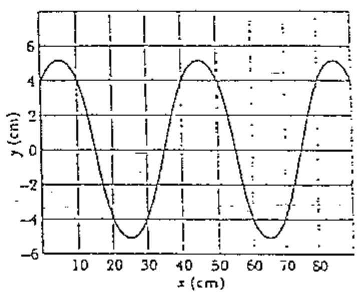

Figure 16.2

From the Fig 16.2.

(a) $A = 5\text{ cm}$

(b) $\lambda = 40\text{ cm}$

(c) $v = \sqrt{\frac{F}{\mu}} = \sqrt{\frac{3.6}{25\times10^{-3}}} = \sqrt{\frac{36\times10^{2}}{25}} = \frac{6}{5}\times10 = 12\text{ m/s}$

$k = \frac{2\pi}{\lambda} = \frac{2\pi}{0.40} = 15.7\text{ m}^{-1}$

(d) $\frac{\lambda}{T} = v$, $T = \frac{\lambda}{v} = \frac{0.40}{12} = 0.0335$, $\omega = \frac{2\pi}{T} = \frac{2\pi}{0.0333} = 190\ \text{s}^{-1}$

(e) $A\omega = A\left(\frac{2\pi}{T}\right) = 5\left(\frac{2\pi}{0.033}\right) = 942\text{ cm/s} = 9.42\text{ m/s}$

(f) $y = A\sin(kx + \omega t + \phi) = 5.0\sin(16x + 190t + \phi)$

apply the condition at $x = 0$, $t = 0$

$y = 5\sin(16(0) + 190(0) + \phi) = 5\sin\phi = 4$

$\phi = \sin^{-1}\left(\frac{4}{5}\right) = 53.1^\circ = 0.93\text{ Rad}.$

$y = 5\ \cancel{\sin}(16x + \cancel{190t + 0.93})$ where x is in m, t in s and y in cm.

Q.3 A stretched string has a mass per unit length of 5.0 g/cm and a tension of 10 N. A sinusoidal wave on this string has amplitude of 0.12 mm and a frequency of 100 Hz and is traveling toward decreasing x. Write an equation for this wave.

$$\mu = 5.0 \ g/cm$$

$$v = \sqrt{\frac{10}{5 \times 10^{-3}/10^{-2}}} = 4.47 \ m/s$$

$$F = 10 \ N, \quad f = 100 \ Hz$$

$$v = \frac{\omega}{k}, \quad \omega = 2\pi f = 200\pi \ s^{-1}$$

$$k = \frac{\omega}{v} = \frac{200\pi}{4.47} = 141 \ m^{-1}$$

$$A = 0.12 \ mm$$

$$y = 0.12 \sin(141x + 200\pi t)$$, where x is in m, t is s, y in mm.

Q.4 Energy is transmitted at rate P_1 by a wave of frequency f_1 on a string with tension τ_1. What is the new energy transmission rate P_2 in terms of P_1 (a) if the tension of the string is increased to $\tau_2 = 4\tau_1$ and (b) if, instead, the frequency of the wave is decreased to $f_2 = f_1/2$?

$$P = \frac{1}{2}\mu v A^2 \omega^2$$

(a) $$P_1 = \frac{1}{2}\mu\sqrt{\frac{\tau_1}{\mu}}\,A^2\omega^2 = a\sqrt{\tau_1}\quad, \quad a \equiv \text{Constant}$$

$$P_2 = a\sqrt{4\tau_1} = 2\left(a\sqrt{\tau_1}\right) = 2P_1$$

(b) $$P = b\omega^2 = cf^2, \quad b \,\&\, c \text{ are Constants}$$

$$P_1 = cf_1^2$$

$$P_2 = c\left(\frac{f_1}{2}\right)^2 = \frac{1}{4}\left(cf_1^2\right) = \frac{P_1}{4}$$

Q.5 Show that $y = y_m \sin(kx - \omega t)$ may be written in the alternative forms

$$y = y_m \sin k(x - vt), \qquad y = y_m \sin 2\pi(x/\lambda - ft),$$

$$y = y_m \sin \omega(x/v - t), \qquad y = y_m \sin 2\pi(x/\lambda - t/T).$$

$y = y_m \sin(kx - \omega t)$

Take K out side

$y = y_m \sin K\left(x - \frac{\omega t}{K}\right)$, $\frac{\omega}{K} = v$

$$\boxed{y = y_m \sin K(x - vt)}$$

In $y = y_m \sin K(x - vt)$ substitute $K = \frac{2\pi}{\lambda}$ & take λ inside

$y = y_m \sin 2\pi\left(\frac{x}{\lambda} - \frac{v}{\lambda}t\right)$

$$\boxed{y = y_m \sin 2\pi\left(\frac{x}{\lambda} - ft\right)}, \quad f = \frac{v}{\lambda}$$

In $y = y_m \sin(Kx - \omega t)$ take ω out side, use

$v = \omega/K$

$y = y_m \sin \omega\left(\frac{K}{\omega}x - t\right)$

$$\boxed{y = y_m \sin \omega\left(\frac{x}{v} - t\right)}$$

In $y = y_m \sin \omega \left(\frac{x}{v} - t \right)$

substitute $\omega = 2\pi f$, take f inside, use $\lambda = \frac{v}{f}$

and $f = \frac{1}{T}$

$$\boxed{y = y_m \sin 2\pi \left(\frac{x}{\lambda} - \frac{t}{T} \right)}$$