Chapter 5
Nuclear Mass

5.1 Introduction

In this chapter we discuss nuclear mass, the factors which control its value in the range of nuclei from the very light to the very heavy and its relevance to several nuclear processes. However, as explained in section 3.2, it is the normal practice in nuclear physics to use not the mass of the bare nucleus but rather the mass of the nucleus with its full complement of orbital electrons, i.e. we normally use the mass of the neutral atom.

5.2 The experimental determination of mass values

Experimental information concerning mass values is available from two sources. Firstly it may be derived from the field of mass spectrometry. This technique had its birth in the investigations of canal rays by J. J. Thomson in 1913, and has now been applied to elements throughout the periodic table. The ion of an atom, or more usually of a molecule, is sent through a system of deflecting electric and magnetic fields and from its trajectory its charge-to-mass ratio is measured. Then, providing its charge is known, its mass may be calculated. Usually the mass difference between two ions known to be of almost the same mass value is measured. For example, the molecular ion $^{16}O_2$ and the atomic ion ^{32}S form a *doublet* suitable for a measurement of this kind. A determination of the mass difference then enables the mass of ^{32}S to be accurately related to the mass of ^{16}O.

Secondly, relationships between mass values, of accuracy comparable to that obtained by mass spectrometry, are available from the study of nuclear reactions. For example the measurement of the threshold energy for the photodisintegration of ^{2}H into a proton and a neutron enables us, invoking the conservation of mass–energy, to relate the mass of the deuteron to the masses of the proton and neutron. We recollect that in α- and β-decay a measurement of the particle energies, assuming the distintegration scheme to be known, leads to an estimate of the decay energy from which a relationship between the masses of the parent and daughter nuclei follows.

5.3 Atomic mass unit

For the purpose of discussing absolute masses as distinct from mass differences it is convenient to introduce an *atomic mass unit* rather than to use submultiple

units of the gramme or kilogramme. Several such atomic mass units have been proposed and for periods used. The obvious choice of the mass of ^{1}H as one atomic mass unit has the disadvantage that on this scale the mass values of heavy nuclei no longer have the appropriate A-values as the nearest integers. For example, on such a scale ^{208}Pb would have a mass of 206·36 units. This can be avoided by a different choice of unit mass. For many years, the *chemical scale* was based on the natural isotopic mixture of oxygen being sixteen atomic mass units by definition. Alongside this, there previously existed a *physical scale* based on the ^{16}O isotope being defined to be sixteen atomic mass units. Since 1960, an attempt has been made to replace these scales by a new scale based on the isotope ^{12}C being twelve atomic mass units, and this *carbon scale*, or *unified mass scale*, is now customarily used in nuclear physics.

5.4 Binding energy

It is frequently convenient to think in terms of *binding energy* rather than the mass of the nuclear system. The two quantities are of course related by the equation

$$M(Z, A) = ZM_H + NM_n - B(Z, A), \qquad 5.1$$

where $M(Z, A)$ is the mass of the atom whose nucleus contains Z protons and N neutrons, M_H the mass of the hydrogen atom and M_n the mass of the neutron. $B(Z, A)$ is clearly the energy necessary to dissociate the nucleus into its A components. We shall usually express $B(Z, A)$ in units of millions of electronvolts, and we note that the conversion factor to atomic mass units (on the ^{12}C = 12 a.m.u. scale) is 1 MeV = $1{\cdot}07356 \times 10^{-3}$ a.m.u.

A study of the distribution of the stable nuclei on the nuclear chart described in section 1.4 enables two important deductions to be made about nuclear binding energy. Firstly we note that, considering the light nuclei, the stable isotopes are grouped closely along the line of slope 45°, i.e. they tend to have $Z = N$. We have seen that the stability of these nuclei against β-decay means that their masses are less than the masses of the neighbouring isobars. It follows from equation 5.1 that, if the mass difference exceeds $M_n - M_H$ (i.e. about 0·75 MeV), which is usually the case, then the binding energy of a nucleus with $Z = N$ is greater than the binding energies of the neighbouring isobars. We deduce that equality of proton and neutron numbers enhances binding energy. As we proceed to heavier stable nuclei we notice that N increases more rapidly than Z and must suppose that other factors enter leading to an excess of neutrons. By the time $^{238}_{92}U$ is reached we note that this excess is 54, i.e. more than 50 per cent of the total proton number.

Table 1

A even		*A odd*	
Z even *N even*	*Z odd* *N odd*	*Z odd* *N even*	*Z even* *N odd*
163	4	49	54

The second important fact emerges from a consideration of the evenness or oddness of the Z- and N-numbers. Table 1 gives the number of stable nuclei (listed in Appendix A) in the four categories arising from the different combinations of evenness and oddness. It is immediately clear that the binding energy in the case of the odd-A nuclei is not affected by whether the odd, or unpaired, nucleon is a proton or a neutron. It is also clear that for even-A nuclei the binding energy is very much affected by the existence of two unpaired nucleons as opposed to the alternative of a complete pairing of nucleons of both kinds. This suggests that there is a pairing energy involved between nucleons of the same kind. We note further that the four exceptions to the general rule that nuclei with Z odd and N odd - which we shall term (odd, odd) nuclei - are not stable are ${}^{2}_{1}\mathrm{H}$, ${}^{6}_{3}\mathrm{Li}$, ${}^{10}_{5}\mathrm{B}$ and ${}^{14}_{7}\mathrm{N}$, the four lightest members of the set of nuclei which have both $Z = N$ and Z and N odd. This we can interpret in terms of the increase in the binding energy resulting from the equality of Z and N being more than sufficient to compensate for the loss of binding energy arising from the existence of two unpaired nucleons. For $Z > 7$ this apparently no longer holds.

5.5 Semi-empirical mass formula

The existence of an extensive set of measurements of mass values for both stable and unstable nuclei provides an incentive for the development of a mass formula to fit these experimental data. Weizsacker (1935) employing the analogy between the nucleus and a liquid drop suggested by Bohr, and proceeding on a semi-empirical basis, set up such a mass formula; this, with later modifications, still plays a vital role in systematizing mass values.

Two experimental facts encourage us to make the analogy between the nucleus and a liquid drop. Firstly, as discussed in section 3.14, the radius of a nucleus is with good accuracy found to be given by the formula

$$R = R_0 A^{\frac{1}{3}}.$$

This means that the number of nucleons per unit volume, which equals $A/\frac{4}{3}\pi R^3$, is a constant. The nucleon density thus behaves as does molecule density in a liquid drop, that is, it is independent of the size of the structure. Secondly, as was quickly realized in the early 1930s when the first results of accurate mass spectrometry appeared, the binding energy per nucleon is almost constant over a wide range of nuclei. This property is also shown by molecules in a liquid drop, as is evidenced by latent heat being a general property of the liquid independent of drop size. In both cases the property is interpreted as arising from the forces concerned being short range in nature, resulting in bonds being formed only between close neighbours. The forces are said to *saturate* since, once there is a sufficiency of close neighbours, the binding of one particular component particle is not altered by the existence or absence of more-distant neighbours. Each particle in the assembly thus makes a fixed contribution to the total energy of the system and so the total energy is proportional to the total number of particles in the assembly. If the force did not saturate, then each particle in an assembly of

say A particles might be assumed to form bonds with all the other $A - 1$ particles in the assembly. The total number of bonds formed would then be $\frac{1}{2}A(A - 1)$. Assuming a certain fixed energy to be attributable to each bond, the total energy would then be proportional to $A(A - 1)$. Consequently the binding energy per particle would be proportional to $A - 1$, that is, it would be expected to increase with A. This is at variance with the facts, which favour the saturation hypothesis for nuclear matter.

We therefore begin the construction of the mass formula by taking a main term in the binding energy proportional to A. This term we represent by $a_v A$, where a_v is a constant, and we refer to $a_v A$ as the *volume energy*. Now in any nucleus of a finite size some of the nucleons must lie on the surface and have a different arrangement of closest neighbours from those nucleons which lie in the interior. The same situation arises of course in the case of a liquid drop, where the fact that the molecules on the surface are differently arranged, with respect to nearest neighbours, from those in the volume of the drop gives rise to the phenomenon of surface tension. Under the action of surface tension drops take up a shape which minimizes the surface area and maximizes the total binding energy. In the nuclear case we thus have to correct the binding energy (in the belief that a nucleon on the surface will make a smaller contribution to the total binding energy) by an amount proportional to the surface area. As the radius is proportional to $A^{\frac{1}{3}}$, the surface area can be taken to be proportional to $A^{\frac{2}{3}}$ and the *surface energy* contribution we write as $-a_s A^{\frac{2}{3}}$.

So far it has been assumed that the nuclear system is held together by an attractive force between nucleons which acts irrespective of their identity as protons or neutrons. In addition to this cohesive nuclear force, there will be the Coulomb force acting between protons. This is a long-range repulsive force and hence reduces the total binding energy. The term to represent this effect, *the Coulomb energy*, can be calculated from the principles of elementary electrostatics if the spatial arrangement of protons in the nucleus is known. If we assume the protons to be uniformly distributed throughout the nuclear volume, we may then imagine the protons in the nucleus to be assembled in spherical layers. Assume that at an intermediate stage of formation the nucleus has radius r and a layer of thickness dr is brought up, proton by proton, from infinity. Let ρ_p be the number of protons per unit volume. Then the charge already in the partially assembled nucleus will be $\frac{4}{3}\pi r^3 \rho_p e$. The work done against the Coulomb force in bringing one additional proton from infinity will then be

$$\int_{\infty}^{r} \frac{4\pi r^3 \rho_p e^2}{3x^2}\, dx = \frac{4}{3}\pi r^2 e^2 \rho_p$$

in magnitude. In this layer there will be $4\pi r^2\, dr\, \rho_p$ protons. Thus the energy built into this layer is

$$\frac{16}{3}\pi^2 r^4 \rho_p^2 e^2\, dr.$$

We therefore see that the Coulomb energy built into the nucleus when it is assembled to a radius R will be

$$\int_0^R \tfrac{16}{3}\pi^2 r^4 \rho_p^2 e^2\, dr = \tfrac{16}{15}\pi^2 \rho_p^2 R^5 e^2.$$

Now $$\rho_p = \frac{Z}{\frac{4}{3}\pi R^3}.$$

Substituting this value into the above equation we have therefore the result

$$\text{Coulomb energy} = \frac{3}{5}\frac{Z^2 e^2}{R}$$

$$= \frac{3}{5}\frac{Z^2 e^2}{R_0 A^{\frac{1}{3}}},$$

for the uniform distribution assumed.

If the protons are not uniformly distributed, then the form of dependence on Z and A is the same but the numerical coefficient is different. For example, if all the protons were on the surface, the Coulomb energy would be that for a charge Ze on a conducting sphere of radius R. The sphere would have an electrical capacity equal to R and hence its energy when a charge Ze is placed on it is from elementary electrostatics given by $\frac{1}{2}Z^2e^2/R$. We therefore introduce in the general case a term, again negative since it represents a disruptive effect, into the binding energy equal to $-a_C Z^2/A^{\frac{1}{3}}$ to represent the Coulomb energy. In deriving this expression, it has been assumed that even within one proton there is a certain Coulomb energy associated with one 'part' of the basic charge interacting with another. Whether there is such a contribution to the energy or not is a basic assumption to be built into any nuclear model. If there is no such internal Coulomb energy to be associated with the proton, then from the above expression we must subtract a contribution $-a_C/A^{\frac{1}{3}}$, (obtained by putting $Z = 1$) for each proton in the assembly. On this assumption therefore, the Coulomb energy would be $-a_C Z(Z-1)/A^{\frac{1}{3}}$. The difference between this and the original expression becomes less important as Z increases and in what follows we retain the original expression $-a_C Z^2/A^{\frac{1}{3}}$ for the Coulomb energy.

So far we have considered the nucleus in terms of classical physics and on that basis there is no explanation for the equality of Z and N leading to particularly stable nuclear configurations. Rather, in view of the Coulomb energy, an excess of neutrons should result even in the case of light nuclei. The fact that light nuclei do not show neutron excess leads us to introduce into the binding energy, on an empirical basis, *an asymmetry energy* which is negative for $Z \neq N$ and zero for $Z = N$. A rough justification for this and an indication of the possible form of this term may be given by the following argument. The nucleons, which we assume to be obeying the laws of quantum mechanics, must be occupying states of definite

energy. In terms of the Pauli exclusion principle, only one nucleon of one kind can occupy one state. The lowest energy states will be filled first. In so far as we can neglect Coulomb effects, we may take the energy states to be similar for neutrons and protons. To go on adding particles of one kind in constructing heavier nuclei thus involves filling higher energy states appropriate to that particle while lower energy states appropriate to particles of the other kind remain vacant. Thus, if there is a neutron excess of $N - Z$, which we assume even, it means that there are $N - Z$ neutron states filled above the last filled proton state. If now the neutrons in the top half of this range of states, i.e. the top $\frac{1}{2}(N-Z)$ neutrons were to be transformed into protons then each could drop down $\frac{1}{2}(N-Z)$ states. If the states were evenly spaced and energy ϵ apart, the energy gained per nucleon would be $\frac{1}{2}(N-Z)\epsilon$ and the total energy gained would thus be $\frac{1}{4}(N-Z)^2\epsilon$. This substitution of protons for neutrons of course has the effect of increasing the Coulomb-energy term, and the N, Z values which lead to maximum binding energy will be determined by minimizing the net result of the opposed effects of asymmetry and Coulomb energy. It is believed that the average energy spacing ϵ for the last few nucleons is approximately proportional to $1/A$ and hence the asymmetry energy term is substituted into the binding energy in the form $-a_a(N-Z)^2/A$.

Finally we have to have regard to the stability of (even, even) as compared to (odd, odd) nuclei noted in section 5.4. The pairing energy which was suggested by the information in Table 1 is allowed for by introducing into the formula for the binding energy a term δ, which is taken positive for (even, even) nuclei, zero for (odd, even) and (even, odd), and negative for (odd, odd) nuclei. Pairing energy is a concept which is added for purely empirical reasons to the liquid-drop model and hence the model cannot pronounce on the form δ should take. Other models, for example the shell model, have been appealed to and various expressions for δ have been suggested. The form

$$\delta = a_P \frac{1}{A^{\frac{3}{4}}}$$

has been commonly used and we shall insert that form in what follows. Collecting together the various terms introduced above and substituting in equation 5.1 we have for the mass of a nucleus charge number Z, mass number A

$$M(Z, A) = ZM_H + NM_n - a_v A + a_s A^{\frac{2}{3}} + a_C \frac{Z^2}{A^{\frac{1}{3}}} + a_a \frac{(N-Z)^2}{A} \pm \delta$$

$$= AM_n + Z(M_H - M_n) - a_v A + a_s A^{\frac{2}{3}} + a_C \frac{Z^2}{A^{\frac{1}{3}}} + a_a \frac{(A-2Z)^2}{A} \pm \delta. \qquad 5.2$$

We note that in this expression there are five adjustable constants. In principle these can be found from five simultaneous equations formed by substituting five known mass values. The usefulness of the formula and the validity of the physical arguments employed in its construction are then to be judged by how well it predicts the hundreds of other mass values which have been measured and by how well it predicts the mass differences involved, and hence energy released, in very many nuclear reactions. Various sets of values for the constants have been suggested, differing slightly depending on the range of nuclei which were under investigation. We shall follow R. D. Evans (*The Atomic Nucleus*, McGraw-Hill, 1955) in taking for the constants the following set of values which result in reasonably good agreement with measured mass values over the whole range of A-values:

$$a_v = 14{\cdot}1 \text{ MeV}; \quad a_s = 13 \text{ MeV}; \quad a_C = 0{\cdot}595 \text{ MeV};$$

$$a_a = 19 \text{ MeV}; \quad \delta = \frac{33{\cdot}5}{A^{\frac{3}{4}}} \text{ MeV}.$$

5.6 Binding energy per nucleon

An impression of the relative importance of the contributions to the binding energy of a nucleus made by the various terms in the mass formula, and the change in their relative importance as we proceed from light to heavy nuclei, is obtained from Table 2 and from Figure 14. As the surface-energy contribution falls, it is largely compensated by increased Coulomb energy. The important observed fact that middle-weight nuclei have a slightly greater binding energy per nucleon than either heavier or lighter nuclei is reproduced by the formula. Arising from the interplay of the surface and Coulomb terms, it is seen that energy may be released by fusing lighter nuclei or by dividing heavier nuclei. The effect of the Coulomb barrier however inhibits these processes, otherwise all material would tend to transform so as to end in middle-weight elements.

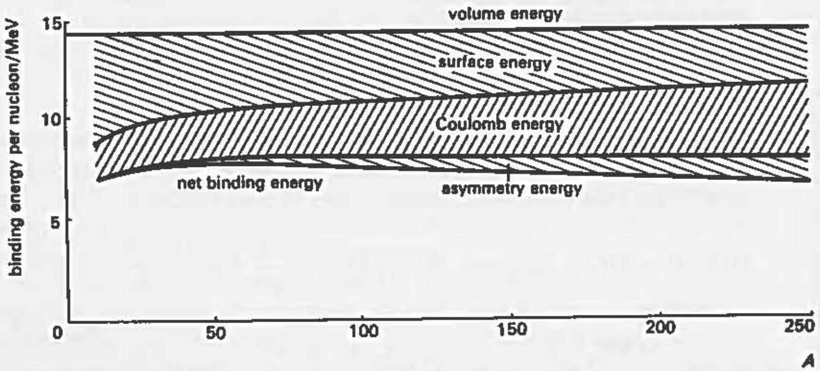

Figure 14 The different negative contributions to the binding energy per nucleon successively subtracted from a constant-volume energy per nucleon to leave the net binding energy per nucleon, all plotted as a function of the mass number A

Table 2

	Z	A	Volume B_v/A $a_v = 14{\cdot}1$ MeV	Surface B_s/A $a_s = 13$ MeV	Coulomb B_C/A $a_C = 0{\cdot}595$ MeV	Asymmetry B_a/A $a_a = 19$ MeV	B/A (*calculated*) /MeV	B/A (*experimental*) /MeV
O	8	17	14·1	5·06	0·87	0·07	8·10	7·75
S	16	33	14·1	4·05	1·44	0·02	8·59	8·50
Mn	25	55	14·1	3·42	1·78	0·16	8·74	8·75
Cu	29	65	14·1	3·23	1·91	0·22	8·74	8·75
I	53	127	14·1	2·59	2·62	0·52	8·37	8·43
Pt	78	195	14·1	2·24	3·20	0·76	7·90	7·92
Bk	97	245	14·1	2·08	3·65	0·82	7·55	7·52

5.7 **Mass surface**

Since $A = Z + N$, we can regard equation 5.2 as expressing the mass of a nucleus in terms of the two parameters (Z, N). If now, on the plot of the nuclei which has N along the x-axis and Z along the y-axis, we imagine verticals erected along the z-axis of length proportional to $M(Z, N)$, then the end points of these verticals will define a surface. This we refer to as the *mass surface.*

5.8 **Mass excess**

The masses, as was noted in section 5.3, are in all cases quite close to integral values when expressed in atomic mass units. However, in nuclear radioactive transformations and in nuclear reactions it is mass differences we are concerned with, and the important information then lies entirely in the amounts by which the masses depart from integral values. It is consequently convenient to work with the quantity $M(Z, N) - A$, where the mass is in atomic mass units and this quantity is referred to as the *mass excess.* By virtue of the relationship $E = mc^2$ between energy and mass, the quantity can also be expressed in energy units.

It may easily be confirmed that in equation 5.2 we may substitute on the left-hand side the mass excess for $M(Z, A)$ providing we replace the mass M_H by the mass excess of the hydrogen atom and M_n by the mass excess of the neutron.

5.9 **Mass parabolas**

We now consider the relationship that equation 5.2 predicts between the masses of isobars. To do so, we eliminate N through the relationship $N = A - Z$ and regard, for a particular set of isobars, A as a constant. We then have that

$$M(Z, A) = B + CZ + DZ^2, \qquad 5.3$$

where $B = AM_n - a_v A + a_s A^{\frac{2}{3}} + a_a A \pm a_P \dfrac{1}{A^{\frac{3}{4}}}$,

$$C = M_H - M_n - 4a_a,$$

$$D = \frac{a_C}{A^{\frac{1}{3}}} + \frac{4a_a}{A}.$$

The section of the mass surface taken through isobars is therefore seen to be parabolic in shape.

First we consider isobars corresponding to odd values of A. For these $a_P = 0$. The coefficients in equation 5.3 are then single valued and the nuclear-mass values lie on one parabola. Having regard to the conditions for β-decay, we see that there will be only one stable nucleus in this set; it will be the nucleus with the smallest mass value and therefore the nucleus whose mass value is closest to the vertex of the parabola. All other members of the set have mass relationships with respect to a neighbour which permit β-decay (or electron capture) to that neighbour. The vertex of the parabola will be given by $Z = Z_A$, say, where

$$\left[\frac{\partial M}{\partial Z}\right]_{Z=Z_A} = C + 2DZ_A = 0.$$

i.e. $$Z_A = -\frac{C}{2D} = \frac{(M_n - M_H) + 4a_a}{2(a_C/A^{\frac{1}{3}} + 4a_a/A)}.$$

From the mass tables we have $M_n - M_H = 0{\cdot}7824$ MeV. Using this, together with the values of a_a and a_C quoted above, we have

$$Z_A = \frac{76{\cdot}7824}{2(0{\cdot}595/A^{\frac{1}{3}} + 76/A)}. \qquad 5.4$$

We now take as an example the case of isobars with $A = 141$. For this value of A, equation 5.4 gives Z_A as 58·76. From the table of isotopes it will be found that the stable member of this set of isobars is $^{141}_{59}Pr$, which corresponds to the mass value closest to the vertex of the parabola. In Figure 15 the experimental masses of the other members of this isobaric set are plotted together with the section of the mass surface.

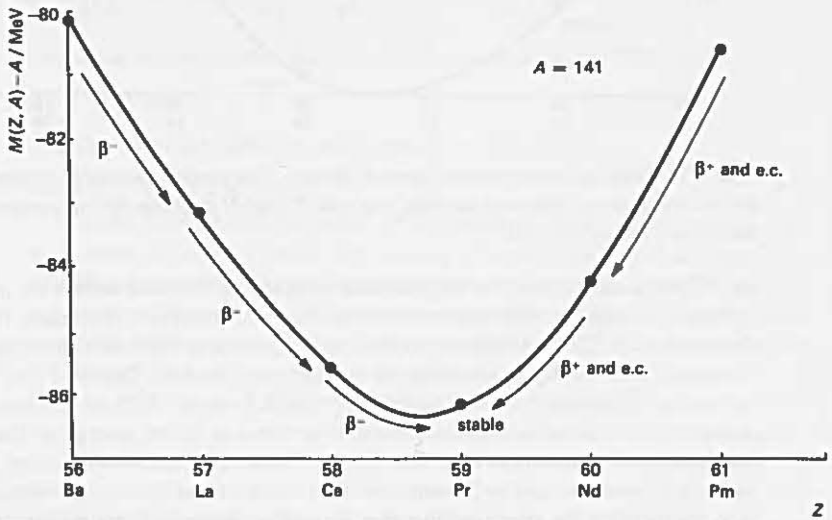

Figure 15 Mass excess of a set of odd-A isobars defining a single sheet of the mass surface when plotted against the charge number Z

Turning to isobars with even A-values, we no longer have $a_P = 0$. In equation 5.3 B has then two values, giving rise to two parabolas, an upper parabola corresponding to (odd, odd) nuclei and a lower corresponding to (even, even) nuclei. Note that the vertices of the two parabolas have the same value of Z. The (even, even) nucleus whose mass number is closest to Z_A should be stable. We take as an example the case of $A = 134$ giving $Z_A = 56{\cdot}17$. $^{134}_{56}Ba$ is in fact found to be stable in accordance with this prediction. However, a second member of the

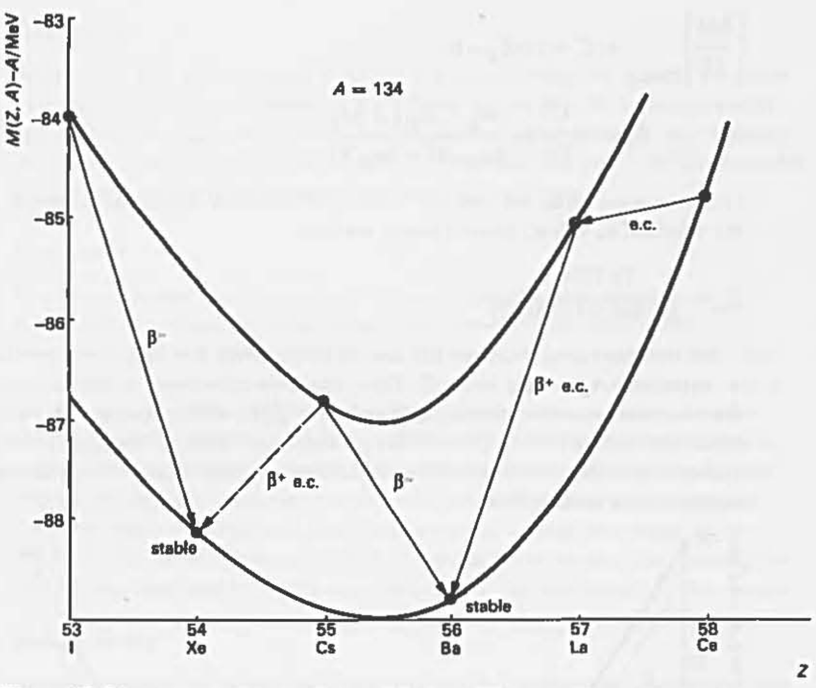

Figure 16 Mass excess of a set of even-A isobars. The even-Z, even-N nuclides define one sheet of the mass surface, the odd-Z, odd-N nuclides define a separate sheet lying above the first

set, $^{134}_{54}$Xe, is also stable. The experimental mass values for these isobars are plotted in Figure 16 together with the two sections of the mass surface. The reason for the stability of $^{134}_{54}$Xe, which lies on the lower parabola, is there seen to be due to the position of its neighbours which lie on the upper parabola. Decay of this isotope to $^{134}_{56}$Ba would only be possible by double β-decay which, it was noted in section 4.17, is likely to be unobservable by virtue of the very long half-life involved. It is interesting to note that $^{134}_{55}$Cs satisfies the conditions for decay both by β^- emission and by β^+ emission. In the case of this nucleus, β^- emission is so favoured by the selection rules that it alone is observed. There are however other examples of nuclei similarly situated with respect to neighbouring isobars which do exhibit β^- emission as well as β^+ emission and electron capture. $^{64}_{29}$Cu is a well-known example of such a nucleus.

5.10 Stability against alpha decay

The mass formula may be used to decide whether or not a nucleus is stable against transformation by α-particle emission. It follows from the discussion of section 3.2 that the stability is determined by the algebraic sign of

$$Q = M_{\text{parent}} - M_{\text{daughter}} - M_{\text{He}}$$
$$= M(Z, A) - M(Z-2, A-4) - M(2, 4).$$

Treating Z and A as continuous variables, we have

$$M(Z, A) - M(Z-2, A-4) = \frac{\partial M}{\partial Z}\Delta Z + \frac{\partial M}{\partial A}\Delta A, \qquad 5.5$$

where on the right-hand side Z and A are to be averaged as between the parent and daughter values.

Thus, from equations 5.2 and 5.7, and taking nuclei with odd A-values so that the pairing energy δ may be ignored, we have

$$Q = \frac{4a_C Z}{A^{\frac{1}{3}}} - 4a_v + \frac{8}{3}a_s A^{-\frac{1}{3}} - \frac{4}{3}a_C\frac{Z^2}{A^{\frac{4}{3}}} - 4a_a\frac{(A-2Z)^2}{A^2} + B(^4\text{He}).$$

Substituting the values quoted above for the constants and introducing the experimental value for the binding energy of the α-particle, we have

$$Q = \frac{Z}{A^{\frac{1}{3}}}\left(2{\cdot}38 - 0{\cdot}793\frac{Z}{A}\right) + \frac{34{\cdot}67}{A^{\frac{1}{3}}} - 76\left(1 - \frac{2Z}{A}\right)^2 - 28{\cdot}1.$$

If into this expression we substitute the Z- and A-values for $^{141}_{59}$Pr, then

$Q = -0{\cdot}24$ MeV.

If the values for $^{145}_{60}$Nd are used, then $Q = +0{\cdot}92$ MeV.

For $^{191}_{77}$Ir, $Q = +2{\cdot}62$ MeV.

Thus the formula with the values of the constants chosen above predicts that for A-values in excess of about 145, α-decay is becoming energetically possible. Note that the term making the largest contribution to the change in the Q-value as we proceed to heavier nuclei is that associated with the Coulomb energy, that is, it is the electrostatic repulsion between protons that is leading to α-instability. It does not follow that if α-decay is energetically possible it will be experimentally observed. With very low Q-values, the outgoing α-particle will require to penetrate a very wide barrier and, as was discussed at length in section 3.13, this will give rise to very long half-lives. We note that this general prediction concerning the onset of α-decay as we proceed up the periodic table is in line with the observed distribution of α-emitters plotted in Figure 5.

5.11 Stability of nuclei against spontaneous symmetric fission

As a further example of the application of the mass formula, we consider the stability of a nucleus against undergoing *spontaneous symmetric fission.* This process involves a nucleus (Z, A) splitting into two identical nuclei, called *fission fragments*, each having charge number $\frac{1}{2}Z$ and mass number $\frac{1}{2}A$, Z and A both being assumed even. The Q-value for such a transformation will be given by

$$Q = M(Z, A) - 2M(\tfrac{1}{2}Z, \tfrac{1}{2}A),$$

and the transformation will be energetically possible if $Q > 0$.

Now, substituting the appropriate values of Z and A into equation 5.2 leads to

$$Q = a_s A^{\frac{2}{3}}(1-2^{\frac{1}{3}}) - a_C \frac{Z^2}{A^{\frac{1}{3}}}(1-2^{-\frac{2}{3}})$$

$$= -3{\cdot}38\, A^{\frac{2}{3}} + 0{\cdot}22 \frac{Z^2}{A^{\frac{1}{3}}}. \qquad 5.6$$

Q will thus be positive for nuclei satisfying

$$\frac{Z^2}{A} > \frac{3{\cdot}38}{0{\cdot}22} = 15{\cdot}36.$$

This inequality is satisfied for $Z = 35, A = 79$ (i.e. ^{79}Br) and for heavier nuclei. Once again we note that the process being energetically possible does not mean that it is observed. The fission fragments are very highly charged and we must have regard to the Coulomb barrier that will exist. To see how to make allowance for this imagine the process to be reversed, i.e. imagine the fission fragments, each having a charge $\frac{1}{2}Ze$ and being spherical with a radius $R_0(\frac{1}{2}A)^{\frac{1}{3}}$, brought towards each other from infinite separation. As they approach, the energy of the system is increased by virtue of the work done against the Coulomb repulsion. If we ignore any deformation of shape of the fission fragments and assume that the only forces involved until the spherical nuclei touch are the Coulomb forces, then the work done against the Coulomb forces in bringing the fission fragments from infinite separation until they are in contact will be

$$\frac{(\frac{1}{2}Ze)^2}{2R_0(\frac{1}{2}A)^{\frac{1}{3}}} = 0{\cdot}1532 \times \frac{Z^2}{A^{\frac{1}{3}}}\ \text{MeV}.$$

It is assumed that the short-range nuclear forces take over when the distance between centres is less than twice the fission-fragment radius and that the potential energy will drop as the fragments coalesce. The value to which the potential energy drops will be the fission energy given by Q in equation 5.6. Now, if the potential barrier is not penetrable and the fission is to be truly spontaneous, the peak potential energy reached, namely that at separation $2R_0(\frac{1}{2}A)^{\frac{1}{3}}$, should equal the fission energy, i.e. there should be no drop in potential energy as the fragments coalesce. Thus Q should be greater than $0{\cdot}1532 \times Z^2/A^{\frac{1}{3}}$ MeV and not simply greater than zero. Incorporating this more stringent condition, then from equation 5.6 we have

$$\frac{Z^2}{A} > \frac{3{\cdot}38}{0{\cdot}0668} = 50{\cdot}6.$$

For the lead isotopes Z^2/A is approximately 32, for uranium it is 36 and for lawrencium it is 41. Thus the prediction of the mass formula, with the choice of values of constants made above, is that spontaneous symmetric fission will not take place for nuclei with a Z-value less than about 110. It is to be noted however that there is the possibility of spontaneous *asymmetric* fission and the possibility

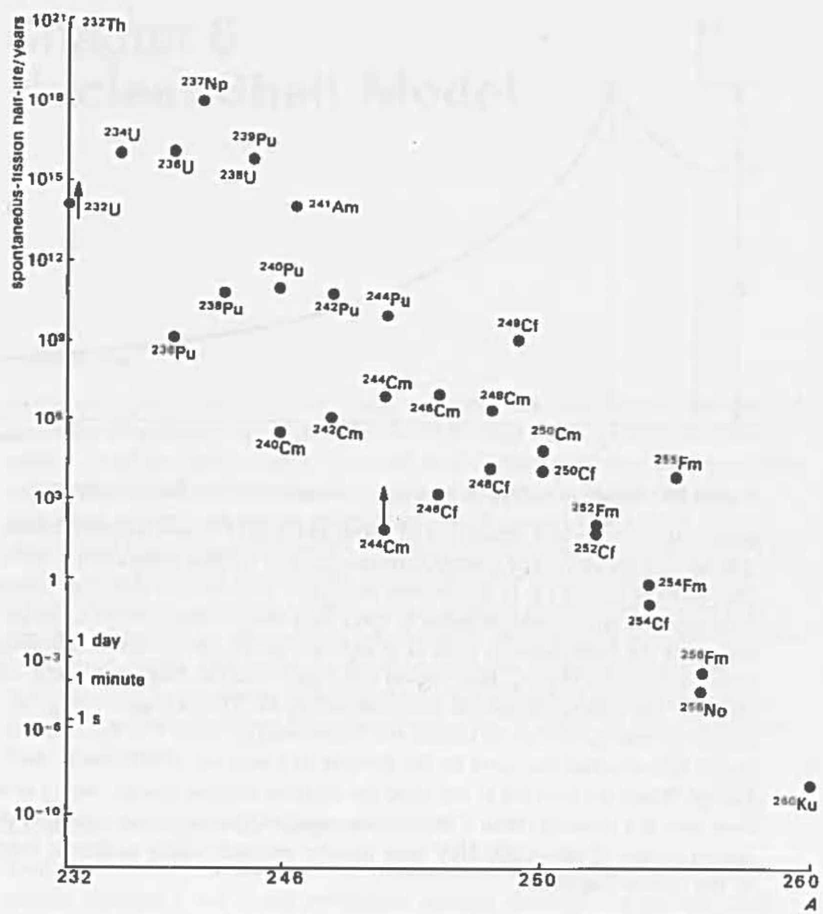

Figure 17 Logarithmic plot of spontaneous fission half-life against mass number, based on data of Figure 86

of barrier penetration to be taken into account. We shall not attempt to treat these in terms of the simple model here developed, but note that spontaneous fission is observed for Z-values of 92 and upwards. In Figure 17 the half-life against spontaneous fission is plotted as a function of A. It is clear that the probability of spontaneous fission is one of the factors setting a limit to an extension of the observed heavy nuclei to even higher A-values.

5.12 Induced fission

When the potential energy of the fission fragments is considered, as it was above, as a function of their separation, the energy at zero separation, while below the

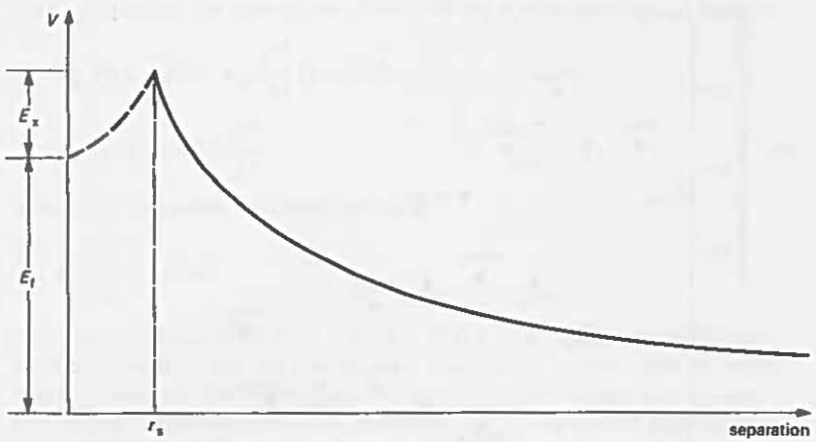

Figure 18 Potential energy as a function of separation of fission fragments

peak which occurs at a separation of about twice the fission-fragment radius, may still be very far above the energy corresponding to infinite separation. This is illustrated in Figure 18. If the original nucleus is disturbed so that the fragments only separate within certain limits ($r < r_s$) then the nucleus will not undergo fission. If the disturbance is such as to separate the fragments instantaneously by a distance greater than r_s, then fission will result with the fragments going off to infinity. The energy released in that case will be E_f. Thus a trigger energy or excitation energy of E_x can release the fission energy of E_f. For the heaviest nuclei this situation can arise by the capture of a neutron of effectively zero energy. When the neutron is captured the neutron binding energy, which as we have seen is a little less than 8 MeV, then provides this excitation and the full fission energy of about 200 MeV may then be released mainly as kinetic energy of the fission fragments.

5.13 Summary

By appeal to a simple classical model with an empirical overlay of quantum-mechanical effects, the semi-empirical formula arrived at permits, in terms of five adjustable parameters, a description to be given of a mass surface which, with exceptions to be noted in the next chapter, gives a satisfactory fit to the experimental values for many hundreds of nuclei. Using the formula, predictions can be made concerning the stable members to be expected in a set of isobaric nuclei. Criterion of stability against α-decay and spontaneous fission can be arrived at which lead to an explanation of why these processes are limited to particular ranges of A-values. Also some insight is given into the balance of the different contributions to binding energy and into the change in this balance as one proceeds from light to heavy nuclei.

Chapter 6
Nuclear Shell Model

6.1 Introduction

In Chapter 5 the liquid-drop model was developed as a basis for the discussion of a number of nuclear properties, in particular binding energy. This model will again be used in a later chapter to explain further nuclear properties, for example nuclear fission. However, there are certain properties, one of which is the important property of angular momentum, which cannot find a place in any elementary way in the framework of the liquid-drop model. We now proceed to outline a model which developed in parallel with the liquid-drop model and which plays a very important role in certain areas of nuclear physics. We shall see that it depends on assumptions which appear incompatible with those of the liquid-drop model. The reason for these two models, based on apparently contradictory assumptions, each having its areas of useful application, has for long been a central problem in nuclear physics and is a topic to which we return in a later chapter.

6.2 Experimental evidence for 'magic' numbers

Evidence from several different fields of study can be assembled to show that certain values of Z and N, the proton and neutron numbers of a nucleus, confer special properties. These Z- and N-values, which are referred to as the *'magic' numbers*, are 2, 8, 20, 28, 50, 82 and 126. We now collect some of the more important strands of the evidence for the existence of these magic numbers.

When the adjustable constants in the semi-empirical mass formula of section 5.5 are chosen for the best general fit to experimentally measured mass values, it is found that the greatest discrepancies are in regions corresponding to magic numbers of protons or neutrons. Whereas the formula reproduces the general trend of the mass surface to an accuracy of 1 or 2 MeV, in the neighbourhood of magic numbers the experimental mass values fall about 10 MeV below the mass-formula values. Thus the indications are that a nucleus with a magic number of neutrons or protons has an unusually large binding energy.

This high binding energy brings in its train several other effects. For example, an examination of the nuclear chart shows that the element with the largest number of isotopes is tin, for which $Z = 50$, while the neutron number corresponding to the greatest number of isotones is $N = 82$.