2.4.2 *Long-lived parent*

If the parent has a much longer half-life than the daughter, then $\lambda_P \ll \lambda_D$. In this case, as t increases from zero, $\exp(-\lambda_D t)$ becomes quickly negligible compared with $\exp(-\lambda_P t)$. Also, we can neglect λ_P in the denominator of the right-hand side of equation 2.7 and write

$$n(t) = \frac{\lambda_P}{\lambda_D} N_0 e^{-\lambda_P t},$$

as the expression for $n(t)$ for large values of t. The activity of the daughter material is given as above by $\lambda_D n_t$, i.e.

$a_t = \lambda_P N_0 e^{-\lambda_P t}$.

This expression is of course also equal to A_t. Thus to the extent that the approximations are valid, the activities of daughter and parent materials are equal, and the rate of creation and rate of decay of the daughter material balance. It is not however an equilibrium condition as these rates are slowly changing with time. The term *secular equilibrium* is used to describe the situation in this radioactive context. When secular equilibrium has been attained, note that the ratio of the amounts of daughter and parent materials present is the ratio of their half-lives.

In Figure 3 the activities of parent and daughter, in the case when $\lambda_P = \frac{1}{10}\lambda_D$, are plotted as functions of time.

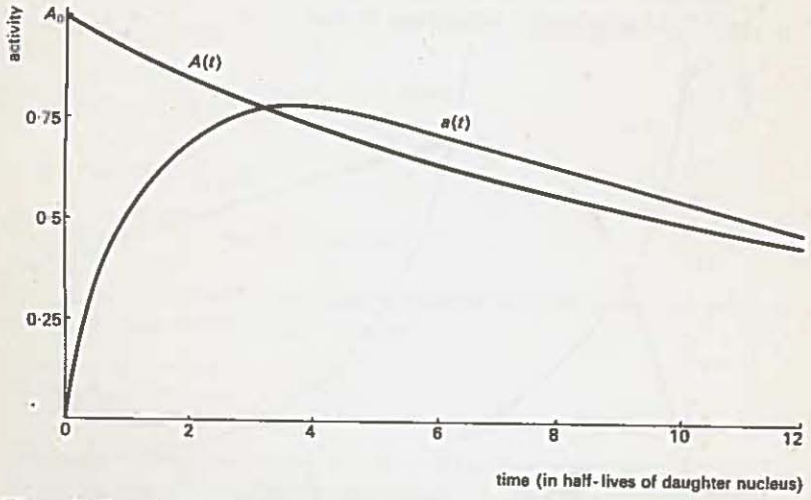

Figure 3 Plot of the variation with time of the radioactivities of long-lived parent and short-lived daughter materials in a source which initially contained only the parent material. The half-lives are taken to be in the ratio 10:1

2.5 Naturally occurring radioactive series

The theory developed in section 2.4 can be applied to the naturally occurring radioactive series. In the case of each of these series a very long-lived isotope (^{232}Th, half-life $1{\cdot}41 \times 10^{10}$ years in the case of the thorium series, ^{238}U, half-life $4{\cdot}51 \times 10^{9}$ years in the case of the uranium series and ^{235}U, half-life $7{\cdot}07 \times 10^{8}$ years in the case of the actinium series) constitutes the parent. From this parent stem between ten and twenty generations of radioactive descendants in each case. Above we have analysed the case involving only one descendant. However an equation similar to equation 2.6 can be formed for each succeeding generation. The mathematical analysis may then be carried out exactly as above and, on the basis that all half-lives are very much shorter than that of the parent material, secular equilibrium will be established. When sufficient time has elapsed for this equilibrium to be attained, all the members of the series have equal activities and the amount of material associated with any member is proportional to its half-life.

2.6 Definition of the curie

As a further application of these ideas, we consider the definition of the unit of activity named the *curie*. Originally the curie was defined as the activity of that amount of ^{222}Rn (radon; half-life 3·825 days) which is in secular equilibrium with one gramme of ^{226}Ra (radium; half-life 1622 years). The unit was so defined to permit sources of standard activity to be produced wherever a radium sample of known weight was available. It follows, since the radon is in secular equilibrium, that its activity will be the same as that of the gramme of radium. The curie is thus equal to

$$\lambda N_0 = \frac{0{\cdot}693}{T_{\frac{1}{2}}} \frac{N_A}{A},$$

where N_A is Avogadro's constant, A the atomic weight of radium and $T_{\frac{1}{2}}$ the half-life of radium. On substituting the numerical values for these quantities the curie may be seen to be $3{\cdot}61 \times 10^{10}$ disintegrations per second. However, to make the unit independent of the half-life of radium, redetermination of which had several times necessitated changing the curie as a practical unit, an internationally agreed definition of the curie as $3{\cdot}7 \times 10^{10}$ disintegrations per second is now accepted and the traditional unit abandoned.

2.7 Branching or parallel decay

It may happen that a nucleus can decay by either of two modes. If the probability per unit time that it will decay by mode one is λ_1 and by mode two is λ_2, then the probability that it decays by one *or* by two is $\lambda_1 + \lambda_2$. This latter quantity will be the decay constant on the usual definition. λ_1 and λ_2 are termed the *partial decay constants* and $\lambda = \lambda_1 + \lambda_2$ the *total decay constant*. These terms could in the obvious way be extended to more than two competing decay modes if necessary.

It is possible that the activity corresponding to decay by mode one can be measured without interference from the activity arising from mode two. For example, ^{64}Cu decays by one mode to ^{64}Ni and by another to ^{64}Zn. A detecting system can be set up to detect only the decays to ^{64}Ni. In this case, the activity measured $A_1(t)$ will be given by $\lambda_1 N(t)$. Now

$$dN(t) = -\lambda_1 N(t)\,dt - \lambda_2 N(t)\,dt = -(\lambda_1 + \lambda_2)N(t)\,dt = -\lambda N(t)\,dt,$$

from which it follows that $N(t) = N_0 e^{-\lambda t}$ and

$$A_1(t) = \lambda_1 N(t) = \lambda_1 N_0 e^{-\lambda t} = (A_1)_0 e^{-\lambda t}.$$

The half-life exhibited is therefore that corresponding to the total (not the partial) decay constant, and the material, in this case ^{64}Cu (despite its different possible decay modes), is still characterized by only one half-life. If it is desired to measure λ_1 or λ_2, then the fraction of the total number of decays proceeding by the mode in question must be determined.

2.8 Artificial radioactivity

It is possible to induce radioactivity in an initially nonradioactive sample by subjecting it to neutron irradiation in a reactor, or to particle bombardment in an accelerator. Let it be assumed that the production of the radioactive nuclei by one or other of these methods proceeds at a constant rate of S nuclei per unit time. If $N(t)$ is the total number of radioactive nuclei at time t, then

$$dN(t) = S\,dt - \lambda N(t)\,dt,$$

the right-hand side expressing the competition between increase due to production and decrease due to radioactive decay. This relation leads immediately to the differential equation

$$\frac{dN(t)}{dt} = S - \lambda N(t),$$

which on integration yields

$$N(t) = \frac{S}{\lambda} + \text{constant} \times e^{-\lambda t}.$$

If the starting conditions are that there is no activity in the sample, then $N(0) = 0$ and the constant of integration is such that

$$N(t) = \frac{S}{\lambda}[1 - e^{-\lambda t}].$$

The activity at any time is given by

$$A(t) = \lambda N(t) = S[1 - e^{-\lambda t}]. \qquad 2.11$$

As t gets very large, clearly the situation is reached of a balance between the activity, that is the decay rate, and S, the production rate. This means that the

activity produced can never exceed S and approaches it asymptotically. Note that one half-life from the beginning of the irradiation the activity reaches one half of its asymptotic value, after two half-lives it reaches three quarters. It is seldom economic to prolong irradiations beyond this stage in view of the diminishing increases in activity.

It is usual, instead of specifying S, to define the *yield* Y in an irradiation of this kind. The yield is the rate of increase of activity at time $t = 0$, i.e.

$$Y = \left[\frac{dA}{dt}\right]_{t=0}.$$

The activity-against-time curve, shown in Figure 4, thus starts off from a zero value at $t = 0$ with a tangent of slope Y. The value of Y is, from differentiating equation 2.11, given by $Y = \lambda S$ and equation 2.11 may then be written

$$A(t) = \frac{Y}{\lambda}[1 - e^{-\lambda t}].$$

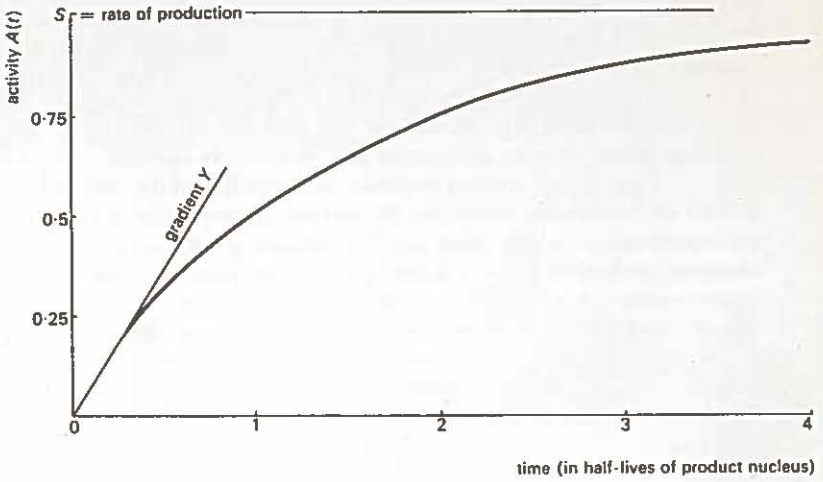

Figure 4 Artificially produced radioactivity as a function of time of bombardment; S is the rate of production, Y the yield

2.9 Summary

The assumption that a radioactive nucleus has a transformation probability per unit time which is independent of its previous history enabled a formula to be derived which related the activity of a simple source to the time of observation. This dependence, the 'exponential law', was found to be in strict agreement with experiment. Equally valid formulae were derived for the cases of series and parallel decay arising in the naturally occurring radioactive materials and for the case of the production of radioactivity in accelerators and nuclear reactors.

Chapter 3
Radioactivity: Alpha Decay

3.1 Fundamentals of alpha decay

Certain radioactive nuclei, on transforming, emit positively charged particles whose measured charge and e/m values indicate that their charge is twice, and their mass four times, that of the proton. These particles, on passing through a gas or into a solid material, expend their energy rapidly in the process of ionization (i.e. stripping electrons from the originally neutral atoms), thus leaving in their wake a short, dense track of positive ions and electrons, the latter remaining as free electrons or forming negative ions depending on the properties of the medium concerned. In the early days of the study of radioactivity these heavy doubly charged particles, because they produced concentrated ionization in the gas of electroscopes and because they were readily absorbed in thin foils, again by virtue of their ionizing properties, were the first of the 'radiations' to be studied and were named *α-particles*. Rutherford's pioneer work of 1909 established that α-particles, after they had been brought to rest, captured electrons and became atoms of helium gas. It is now known that the helium isotope concerned is ^{4}He and hence the α-particle, being the nucleus of this isotope, must be a cluster of two protons and two neutrons. Once this is appreciated it follows that in α-decay the parent nucleus, denoted by P, and the daughter nucleus, denoted by D, are related in A, Z and N value according to

$$^{A}_{Z}\mathrm{P}_{N} \rightarrow {}^{A-4}_{Z-2}\mathrm{D}_{N-2} + {}^{4}_{2}\mathrm{He}_{2}.$$

This relationship, which was in the early days of radioactivity referred to as the *displacement law*, may also be stated as $\Delta A = -4$, $\Delta Z = -2$, $\Delta N = -2$.

3.2 Mass-energy relations in alpha decay

The conservation of mass-energy must apply to the α-decay process. Hence

$$M'_{\mathrm{P}} = M'_{\mathrm{D}} + M'_{\alpha} + Q,$$

where the masses are the masses of the nuclei and Q is the energy shared by the products of the reaction. As in all equations of this type with which we shall be concerned, we can either write the quantities in the traditional mass units or write all quantities in energy units, mass and energy transforming according to $E = mc^2$.

For the process to be energetically possible Q must be positive and it therefore follows that

$$M'_{\mathrm{P}} > M'_{\mathrm{D}} + M'_{\alpha}.$$

Universally, nowadays, in the published mass tables, neutral-atom masses rather than nuclear masses are listed. The inequality is therefore customarily written as $M_{\mathrm{P}} > M_{\mathrm{D}} + M_{\mathrm{He}}$ where the masses are now neutral-atom masses. In taking this step it is however to be noted that something more than adding Z electrons to each side of the inequality is involved. $M_{\mathrm{P}} < M'_{\mathrm{P}} + Zm_{\mathrm{e}}$ by an amount equal to that mass which is equivalent to the binding energy of the Z electrons in the atom. For heavy nuclei such as those involved in α-decay, this total binding energy is of the order of several hundred thousand electronvolts. However, this is largely compensated by the total binding energy of the atoms involved on the other side of the inequality and the overall error introduced in substituting atomic for nuclear masses is usually negligible compared with the α-particle energies, which are in the range of a few million electronvolts.

From tables of atomic masses, it is found that for A-values from about 150 upwards very many nuclei should be unstable against α-decay. For reasons which are discussed in section 3.16 the mass condition being satisfied does not necessarily result in observable α-activity. The plot in Figure 5 of the distribution of α-emitters as a function of A-values indicates the extent to which the α mode of decay is limited to the heavy nuclides.

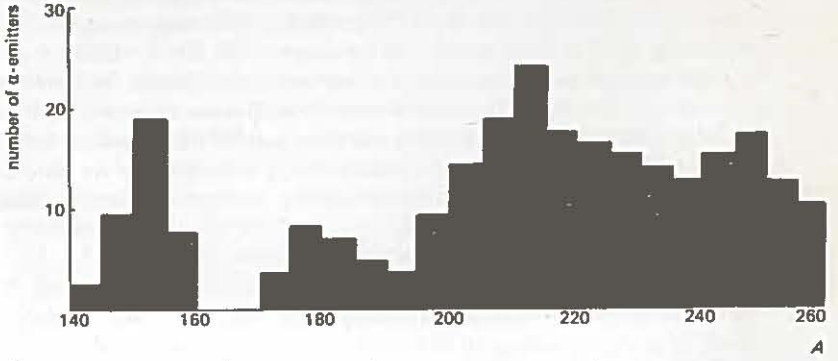

Figure 5 Histogram of the number of α-emitters in terms of their A-values (based on *Chart of the Nuclides,* 1966)

3.3 Alpha-particle fine structure

The kinetic energy of the α-particles emitted by a particular isotope can be measured accurately with a magnetic spectrometer. When such measurements are made, it is found that in some cases there is only one group of monoenergetic -particles; in other cases there are two or more such groups. When there is more than one group, the α-particle spectrum is said to exhibit *fine structure*, a phenomenon which is discussed in detail in section 3.15.

3.4 Recoil energy in alpha decay

The uniqueness of the α-particle energy in the general case points to the process being a 'two-body' process with the available energy Q shared by the α-particle and the daughter nucleus, which must recoil to conserve linear momentum. As the kinetic energies involved are no more than a few million electronvolts, whereas the rest masses are to be measured in thousands of millions of electronvolts, we may without appreciable error assume the formulae of Newtonian dynamics to apply. Thus, if M_D and M_α are the masses of the daughter nucleus and α-particle and T_D, T_α, V_D, V_α their kinetic energies and velocities, $M_D V_D = M_\alpha V_\alpha$ by the conservation of linear momentum. Therefore

$$T_D = \tfrac{1}{2} M_D V_D^2 = \frac{M_\alpha}{M_D} T_\alpha.$$

Now $$Q = T_D + T_\alpha = T_\alpha \left[1 + \frac{M_\alpha}{M_D}\right],$$

so we have $$T_\alpha = \frac{M_D}{M_D + M_\alpha} Q \simeq \frac{M_D}{M_P} Q \quad \text{and} \quad T_D \simeq \frac{M_\alpha}{M_P} Q. \qquad 3.1$$

3.5 Observed energies and half-lives of alpha emitters

The energies of α-particles emitted by different α-active nuclides range from 1·9 MeV for ^{144}Nd to 9·2 MeV for ^{213}At. The half-lives range from 2×10^{17} yr in the case of ^{209}Bi to $2{\cdot}9 \times 10^{-7}$ s in the case of ^{212}Po. Thus, whereas the energies are contained within a range of one order of magnitude, the half-lives range over more than thirty orders of magnitude. There is an apparent correlation between half-life (or decay constant) and the α-particle energy; a short half-life is associated with a high value of α-particle energy and vice versa. An attempt to fit the then known (1912) experimental α-decay constants and particle energies by an empirical formula of the form

$$\log \lambda = C_1 \log T_\alpha + C_2, \qquad 3.2$$

by Geiger and Nuttall, had limited success. While values of C_1 and C_2 could be found to give a satisfactory fit of the calculated decay constant with the measured decay constant within one natural radioactive series, the values of the constants had to be altered to maintain the goodness of the fit on going from one radioactive series to another. Insight into the correlation between half-life and α-particle energy had to await the development of the theory of α-decay, which we now proceed to discuss.

3.6 Rutherford scattering: theoretical treatment

To be in a position to consider a 'model' in terms of which to picture α-decay we require quantitative information about the electrostatic field around the nucleus. This can be obtained from the study of scattering in this field of incoming α-particles.

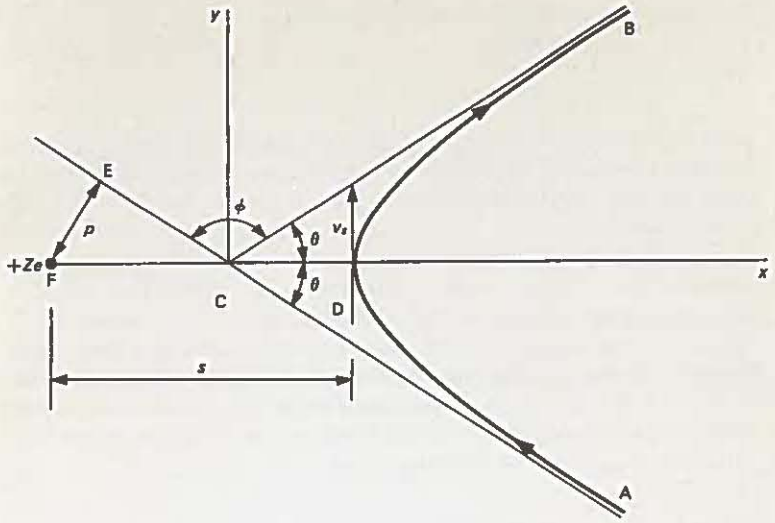

Figure 6 Scattering of an α-particle by a nucleus, charge number Z, with an impact parameter p

We begin by considering the motion of a single α-particle in the Coulomb field of a single nucleus which is assumed to be fixed in position. In Figure 6 the incoming α-particle is directed along AC. Under the influence of the Coulomb field associated with the charge Ze on the nucleus situated at the point F, the particle will experience a repulsive force whose line of action passes through F and whose magnitude is inversely proportional to the square of the distance of the particle from F. This is completely analogous to planetary motion under gravity.† The trajectory is a conic section, in this case a branch of the hyperbola with the nucleus at a focus F. The α-particle will be scattered through an angle ϕ and goes off finally along CB.

Let A and B be the lengths of the principal semi-axes of the hyperbola. Referred to the principal axes Cx and Cy, the trajectory has the equation

$$\frac{x^2}{A^2} - \frac{y^2}{B^2} - 1,$$

and the asymptotes AC and BC have the equations

$$\frac{x}{A} \pm \frac{y}{B} = 1.$$

From the gradient of these lines, it follows that

$$\tan \theta = \frac{B}{A}. \qquad 3.3$$

† For gravity, as for charged particles of opposite sign, the force is attractive, and the other branch of the hyperbola is followed.

The eccentricity of the hyperbola, ϵ, is given by

$$\epsilon^2 = \frac{A^2 + B^2}{A^2}. \qquad 3.4$$

From the geometry of the hyperbola CF = ϵA and CD = A. Thus s, the distance of closest approach to the nucleus for the trajectory under consideration is given by s = FD = $A(1 + \epsilon)$. From equations 3.3 and 3.4 we see that $\epsilon = \sec\theta$ and hence $s = A(1 + \sec\theta)$.

It is customary to specify the 'closeness' of the collision by the *impact parameter* which is p, the length of the perpendicular from the scattering centre on to the original direction of travel of the scattered particle. A 'head-on' collision corresponds to $p = 0$, the collision getting more 'distant' as p increases. We wish to derive a relation between the angle of scatter ϕ and the impact parameter p. We start by equating the total energy (i.e. kinetic plus potential) at the point D on the trajectory with the kinetic energy at infinity, where the potential energy is zero by definition. Thus

$$\tfrac{1}{2}M_\alpha V_s^2 + \frac{2Ze^2}{s} = \tfrac{1}{2}M_\alpha V_0^2. \qquad 3.5$$

The line of action of the force passing through F, the angular momentum of the particle about an axis through F must stay constant throughout the motion. Thus

$$M_\alpha V_0 p = M_\alpha V_s s.$$

From this it follows that

$$V_s = \frac{p}{s} V_0.$$

On substituting this value of V_s into equation 3.5 we have

$$\tfrac{1}{2}M_\alpha V_0^2 \left(\frac{s^2 - p^2}{s}\right) = 2Ze^2.$$

It is convenient to introduce b, the distance of closest approach in a head-on collision. By equating the potential energy at a separation b, at which the particle is instantaneously at rest, to the kinetic energy at infinity, we have

$$b = \frac{4Ze^2}{M_\alpha V_0^2}.$$

Hence $p^2 = s(s - b)$. 3.6

Now we wish to find a relation between s and θ. From the triangle FCE, we see that $\sin\theta = p/A\epsilon$. From the values of ϵ and A given above it follows that

$$\epsilon = \sec\theta \quad \text{and} \quad A = \frac{s}{1 + \sec\theta}.$$

Hence $\sin\theta = \dfrac{p(1 + \sec\theta)}{s \sec\theta}$

and $s = p \cot \tfrac{1}{2}\theta$. 3.7

When this is substituted in equation 3.6 we find

$$p = \tfrac{1}{2}b \tan\theta.$$

Since $\phi = \pi - 2\theta$ we have finally

$$p = \tfrac{1}{2}b \cot \tfrac{1}{2}\phi. \qquad 3.8$$

We now pause in the discussion to examine the values of s and b arising in the experimental situation and to establish their magnitudes relative to atomic dimensions. Suppose an α-particle of energy 4·2 MeV, emitted by a $^{238}_{92}$U nucleus to be scattered by another $^{238}_{92}$U nucleus. In this case

$$b = \frac{2 \times 92 \times (4{\cdot}8 \times 10^{-10})^2}{4{\cdot}2 \times 1{\cdot}6 \times 10^{-6}} \text{ m} = 63 \text{ fm},$$

the fermi, defined as 10^{-15} m, being a suitable unit of length in a nuclear context. Using this value in equation 3.8 we see that for scattering angles in excess of 20°, the impact parameter must be less than 180 fm. Hence, from equation 3.7, s must be less than 215 fm. The radius of the orbit of the two innermost electrons (i.e. the K-electrons) in the uranium atom is about 600 fm. Thus we see that the α-particle in the energy range being used experiences the full effect of the nuclear charge during the important part of the collision.

We now resume the general discussion of scattering by considering the experimental arrangement sketched in Figure 7. A well-collimated beam of α-particles falls at right angles on a thin foil, thickness t, containing n scattering centres (say nuclei with charge $+Ze$) per unit volume. The foil is assumed to be so thin that the loss of α-particle energy by ionization is negligible. In these

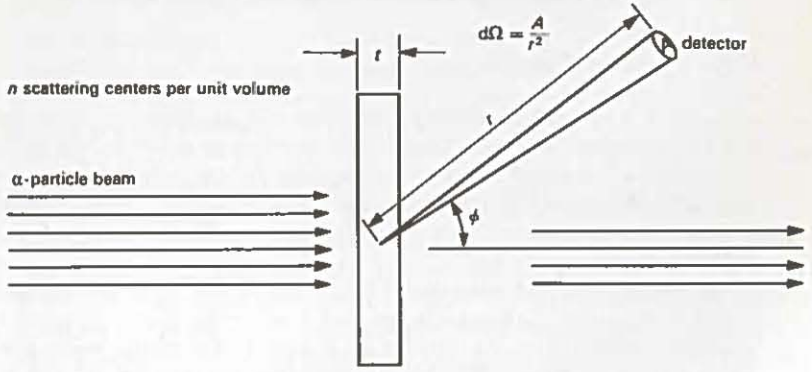

Figure 7 The geometry of the scattering of an α-particle beam by a thin foil

circumstances, the probability of an α-particle experiencing two large-angle-scattering collisions is also negligible. The spot on which the α-particles are incident is viewed at an angle ϕ to the beam by an α-particle detector subtending a solid angle of $d\Omega$ for scattered particles. We now consider how the rate of detection of α-particles will be expected to vary with the angle ϕ.

The probability that an α-particle be scattered through an angle lying between ϕ and $\phi + d\phi$ will equal the probability of finding a scattering centre at a distance between p and $p + dp$ from the α-particle trajectory. This probability is to be measured by the average number of nuclei (necessarily very much less than unity) contained in the volume of a hollow cylinder of radius p, thickness dp and length t. This volume is equal to $2\pi pt\,dp$ and hence the required probability is $2\pi pnt\,dp$. If A is the number of incident α-particles per unit time, the number scattered through an angle between ϕ and $\phi + d\phi$ per unit time is $2\pi Apnt\,dp$. These particles will be scattered into a hollow cone having an inside semi-angle equal to ϕ and an outside semi-angle equal to $\phi + d\phi$ with its axis along the direction of the incident α-particles. The scattered particles are thus contained in a total solid angle $2\pi \sin\phi\,d\phi$. The fraction of the scattered particles which enter the detector is then the ratio of the solid angles, namely

$$\frac{d\Omega}{2\pi \sin\phi\,d\phi}.$$

Therefore the number of particles detected per unit time is equal to

$$C = \frac{2\pi Apnt\,dp\,d\Omega}{2\pi \sin\phi\,d\phi}. \qquad 3.9$$

But, from equation 3.8,

$$\left|\frac{dp}{d\phi}\right| = \tfrac{1}{4}b \operatorname{cosec}^2 \tfrac{1}{2}\phi.$$

Substituting this value in equation 3.9 we have

$$C = A\frac{b^2}{16} nt \operatorname{cosec}^4 \tfrac{1}{2}\phi\,d\Omega. \qquad 3.10$$

If now it is found in a scattering experiment with α-particles of a given energy that the dependence of C on ϕ is accurately described by equation 3.10 for angles greater than a few degrees, then we can conclude that the potential down to a separation distance of b is accurately proportional to $1/r$, i.e. it is the Coulomb potential. Any other dependence of potential on r would necessarily lead to a different angular distribution.

We now note that the derivation of equation 3.10, above, rests on the total charge of the nucleus Ze being effective, and therefore the impact parameter must not exceed 600 fm, the radius of the K-shell. This restriction on p-value means that a restriction on the ϕ-value follows from equation 3.8. It means that in the case of uranium ϕ must be greater than 6°. It is also to be noted that the foil thickness permitted, having regard to the requirement to limit the scattering

to 'pure single scattering', is given by the condition that $2\pi pnt\,dp$ is very much less than unity. This means that t must be very much less than

$$\frac{1}{2\pi pn\,dp} = \frac{4}{2\pi pnb \operatorname{cosec}^2 \tfrac{1}{2}\phi\,d\phi}.$$

Assuming there to be about 5×10^{19} scattering centres per millimetre cubed and that the detector accepts an angular range of scattered particles of about a tenth of a radian, then the single scattering condition will be satisfied up to a foil thickness of 10^{-2} mm.

It should be noted that we have neglected in the above derivation the effect of the recoil of the scattering nucleus. This is equivalent to assuming that the nuclear mass is infinite. A more general treatment by C. Darwin (1914) shows that formula 3.10 holds for finite nuclear mass provided:

(a) that the α-particle mass used to evaluate b is replaced by the expression

$$\frac{m_\alpha\, m_{\text{nucleus}}}{m_\alpha + m_{\text{nucleus}}},$$

which is referred to as the *reduced mass*, and

(b) that the angle ϕ, the scattering angle measured in the laboratory reference frame, is replaced by the scattering angle measured in a frame of reference travelling with the centre-of-mass of the α-particle and nucleus. This frame of reference will have a constant velocity

$$\frac{m_\alpha}{m_\alpha + m_{\text{nucleus}}} V_\alpha,$$

with respect to the frame of reference fixed in the laboratory, and we refer to the moving frame as the *centre-of-mass system*.

3.7 Rutherford scattering: experimental results

Geiger and Marsden (1913), by scattering α-particles of 7·68 MeV in gold films typically 3×10^{-4} mm thick, confirmed formula 3.10 over a range of values of ϕ from 5° to 150°. The distance of closest approach for an α-particle of this energy to a gold nucleus, in the event of a scatter of 150°, is, from equations 3.8 and 3.7, 30 fm. It was thus established that from the dimensions of the radius of the K-shell down to 30 fm the law of force is accurately that for the Coulomb field surrounding a point charge. It was on the basis of this observation and by the reasoning given above that Rutherford proposed the atomic model that now bears his name. The gold foil of the thickness used by Geiger and Marsden is seen, in the light of the above discussion, to be such that the probability of double scattering can be neglected. Also the energy loss of the α-particle by ionization in passing through a foil of this thickness is a negligibly small fraction of its kinetic energy, and hence the α-particle energy may be assumed constant throughout its passage through the foil.

Later experiments with uranium foils gave angular distributions in agreement with the Rutherford formula and showed that in the case of uranium the Coulomb law held down to at least a separation of thirty fermis.

3.8 Alpha-decay paradox

The α-particle-scattering experiments showed that the field around the nucleus, $Z = 90$, was accurately the Coulomb field of a point charge down to thirty fermis from the centre of the nucleus. Any α-particle emitted by radioactive decay must therefore originate from a point closer to the nuclear centre than thirty fermis. Hence it must emerge with at least the electrostatic potential energy a doubly charged particle would have at that distance from the nucleus. This is about 8·6 MeV. However, α-particles of about 4 MeV are observed to be emitted from nuclei with $Z = 92$. This paradox could not be resolved within the framework of classical physics; understanding had to await the proposal of Gamow (and independently of Condon and Gurney) in 1928, who suggested abandoning the classical description and substituting one in terms of wave mechanics.

3.9 Nuclear potential barrier

Let us consider the potential energy of an α-particle as a function of its separation, measured centre-to-centre, from a heavy nucleus. Scattering experiments of the type discussed above have now been extended to higher energies and have established that, up to α-particle energies of 28 MeV, formula 3.10 holds. Thus for separations in excess of ten fermis the potential is that of the Coulomb force falling off as $1/r$. At about a distance of ten fermis the Rutherford scattering formula breaks down. This is due to forces between nucleons in the nucleus and the nucleons in the passing α-particle coming into play. These forces, which are believed to dominate inside the nucleus and which hold the nucleons together, the so called *strong-interaction* forces, are attractive and only operate for very short distances of separation. They cause a rapid fall in the potential when the α-particle gets within their short range. We make the simplifying assumption that the potential falls infinitely fast as the nuclear surface is crossed to reach a constant value inside the nucleus of $-V_0$, where $V_0 \ll B$ the maximum value the potential reaches. This potential curve is drawn in Figure 8. If we now think of this curve as the potential of the daughter nucleus, then the parent nucleus may be described by adding an α-particle which moves in the region $r \ll R$ with a kinetic energy $T_\alpha + V_0$ where T_α is the kinetic energy on emission (i.e. at infinite separation). The α-particle is represented in *total energy* (i.e. kinetic + potential) by the line ABCD on the diagram. On the basis of classical theory, the kinetic energy of the α-particle reaches zero at B and becomes negative between B and C. This is therefore a physically forbidden region which the α-particle cannot enter and hence an α-particle, once in the central well, is trapped forever in the central region. The potential is thus said to constitute a barrier which prevents the escape of the α-particle. In the wave-mechanical treatment however, the α-particle has a small but finite probability of penetrating into the region CB and in fact of succeeding in 'tunnelling' through the barrier to the region BA.

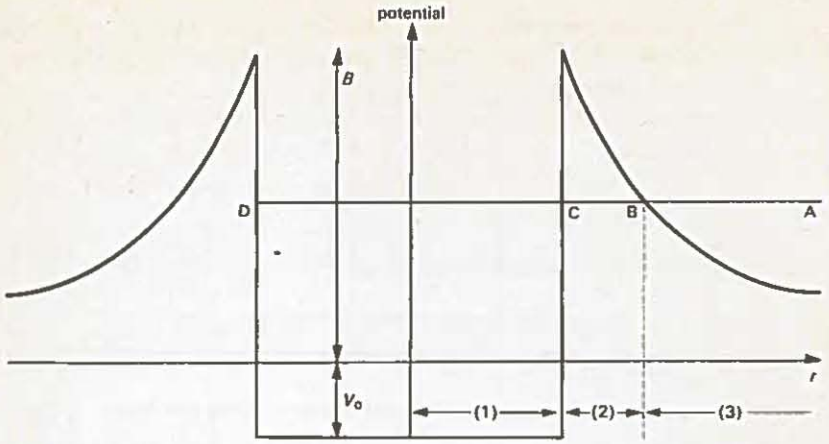

Figure 8 Potential well arising from short-range nuclear force combined with Coulomb potential due to long-range electrostatic force

Before treating a barrier in the shape of that in Figure 8 we consider a mathematically simpler but physically similar situation, namely a rectangular potential barrier in one dimension.

3.10 Rectangular potential barrier: one-dimensional wave-mechanical treatment

Consider a beam of particles incident on the barrier illustrated in Figure 9. In the wave-mechanical treatment (see, for example, R. M. Eisberg, *Fundamentals of Modern Physics*, Wiley, 1961, p. 212) the beam has an associated wave function Ψ, which is a function of x and t in the one-dimensional case. Ψ is the product of a time-dependent factor, which in the present problem is of the form $\exp(i2\pi\nu t)$ throughout, and a space dependent factor which we denote by $\psi(x)$. In any region of space $\psi(x)$ must satisfy the time-independent Schrödinger equation

$$\frac{d^2\psi}{dx^2} + \frac{2M(W-V)}{\hbar^2}\psi = 0, \qquad 3.11a$$

where M is the mass of the particles in the beam, V is the potential energy in the region and $W = T + V$, where T is the kinetic energy in the same region.

We find it convenient to write Schrödinger's equation as

$$\frac{d^2\psi}{dx^2} + k^2\psi = 0, \qquad 3.11b$$

with $k^2 = \dfrac{2M(W-V)}{\hbar^2}$.

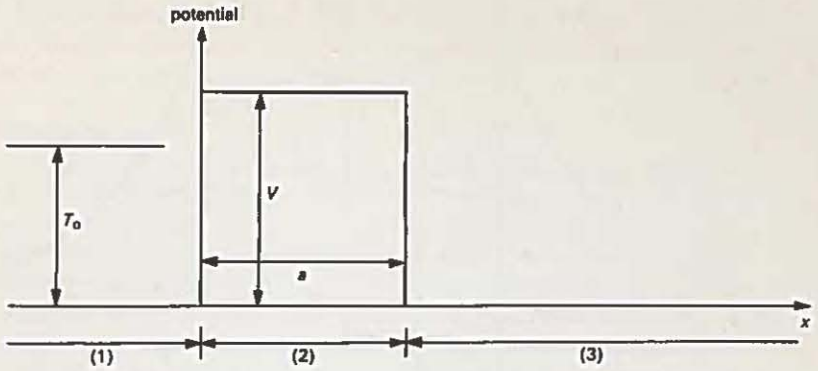

Figure 9 Idealized rectangular potential barrier of finite thickness

We now treat the three regions in Figure 9 in turn. In region (1) $V = 0$ and $W = T_0$. Therefore

$$k_1^2 = \frac{2MT_0}{\hbar^2} > 0.$$

The general solution of equation 3.11 then takes the form

$$\psi_1(x) = A_1 e^{ik_1 x} + B_1 e^{-ik_1 x}.$$

Taken in conjunction with the time-dependent factor $\exp(i2\pi\nu t)$, these terms then correspond to an incident wave travelling along the positive direction of the x-axis and a reflected wave travelling in the opposite direction.

In region (2) W is unchanged and therefore must still be taken as equal to T_0, and, as we are assuming that $T_0 < V$, it follows that $2M(T_0 - V)/\hbar^2$ is less than zero. Let it equal $-k_2^2$ where k_2 is real. Thus in region (2) equation 3.11b becomes

$$\frac{d^2\psi}{dx^2} - k_2^2\psi = 0,$$

which has the general solution

$$\psi_2(x) = A_2 e^{-k_2 x} + B_2 e^{k_2 x}.$$

Taken in conjunction with the factor $\exp(i2\pi\nu t)$ this is seen to correspond to standing waves.

In region (3) the situation with respect to W, V and T_0 is the same as in region (1). In this case however we need only consider the wave propagated along the positive direction of the x-axis as there is assumed to be no further potential discontinuities to cause reflections. Hence the solution in region (3) is

$$\psi_3(x) = A_3 e^{ik_1 x}.$$

The five constants A_1, B_1, A_2, B_2 and A_3 must now be chosen to achieve the correct conditions at the boundaries between the regions. We note that both

$W - V$ and the wave function, since it is related to the particle density in the beam, must everywhere have finite values. It follows from equation 3.11a that $d^2\psi/dx^2$ is everywhere finite. Hence $d\psi/dx$ cannot undergo sudden changes in value. In turn it can be argued that ψ cannot change discontinuously. Across the boundaries, ψ and $d\psi/dx$ must therefore both be continuous.

$\psi_1(0) = \psi_2(0)$ yields $A_1 + B_1 = A_2 + B_2$.

$$\left[\frac{d\psi_1}{dx}\right]_{x=0} = \left[\frac{d\psi_2}{dx}\right]_{x=0} \quad \text{yields} \quad i_1 k_1 A_1 - i_1 k_1 B_1 = -k_2 A_2 + k_2 B_2.$$

From $\psi_2(a) = \psi_3(a)$ we have $A_2 e^{-k_2 a} + B_2 e^{k_2 a} = A_3 e^{ik_1 a}$,

and from

$$\left[\frac{d\psi_2}{dx}\right]_{x=a} = \left[\frac{d\psi_3}{dx}\right]_{x=a}$$

we have $-k_2 A_2 e^{-k_2 a} + k_2 B_2 e^{k_2 a} = ik A_3 e^{ik_1 a}$.

From these four equations B_1, A_2 and B_2 may be eliminated to give

$$\frac{A_1}{A_3} = \left[\frac{1}{2} + \frac{1}{4}\left(\frac{k_2}{k_1} - \frac{k_1}{k_2}\right)\right] e^{(ik_1 + k_2)a} + \left[\frac{1}{2} - \frac{1}{4}\left(\frac{k_2}{k_1} - \frac{k_1}{k_2}\right)\right] e^{(ik_1 - k_2)a}.$$

The flux of particles in the beam is given by the density of particles in the beam multiplied by the particle velocity. Since velocities in regions (1) and (3) are the same, the ratio of the fluxes in these regions will simply be in the ratio of the particle densities. This in turn, from wave-mechanical theory, is given by $|A_1/A_3|^2$. This may also be written $(A_1/A_3)^*(A_1/A_3)$, where $(A_1/A_3)^*$ is the complex conjugate of A_1/A_3. Taking the complex conjugate and carrying out the multiplication, we have

$$\left[\frac{A_1}{A_3}\right]^* \left[\frac{A_1}{A_3}\right] = \left[\frac{1}{4} + \frac{1}{16}\left(\frac{k_2}{k_1} - \frac{k_1}{k_2}\right)^2\right][e^{2k_2 a} + e^{-2k_2 a}] + \frac{1}{2} - \frac{1}{8}\left[\frac{k_2}{k_1} - \frac{k_1}{k_2}\right]^2.$$

Using the identity

$$\sinh^2 k_2 a = \tfrac{1}{4}(e^{2k_2 a} + e^{-2k_2 a}) - \tfrac{1}{2},$$

and simplifying, we can write

$$\left[\frac{A_1}{A_3}\right]^* \left[\frac{A_1}{A_3}\right] = 1 + \frac{1}{4}\left[2 + \left(\frac{k_2}{k_1}\right)^2 + \left(\frac{k_1}{k_2}\right)^2\right] \sinh^2 k_2 a. \qquad 3.12$$

But $\left[\frac{k_2}{k_1}\right]^2 = \frac{V - T_0}{T_0}$.

Substituting this in equation 3.12 we find

$$\left|\frac{A_1}{A_3}\right|^2 = 1 + \frac{1}{4}\frac{V^2}{T_0(V-T_0)}\sinh^2 k_2 a.$$

The probability that an incident α-particle will penetrate the barrier is called the *penetration factor* and is equal to $|A_3/A_1|^2$. The ratio of the transmitted flux to the incident flux, which in the general case is given by

$$\left|\frac{A_3}{A_1}\right|^2 \frac{V_3}{V_1},$$

is called the *transmission coefficient.* As $V_3 = V_1$, these quantities are equal in the present case and we can write both equal to

$$\left|\frac{A_3}{A_1}\right|^2 = \left[1 + \frac{1}{4}\frac{V^2}{T_0(V-T_0)}\sinh^2 k_2 a\right]^{-1}. \qquad 3.13$$

We recall that

$$k_2 = \frac{1}{\hbar}\sqrt{[2M(V-T_0)]}.$$

If $k_2 a \gg 1$,

$$\sinh k_2 a = \frac{e^{k_2 a} - e^{-k_2 a}}{2} \simeq \frac{e^{k_2 a}}{2},$$

and hence

$$\sinh^2 k_2 a \simeq \frac{e^{2k_2 a}}{4}.$$

Also we note that

$$\frac{V^2}{4T_0(V-T_0)} \simeq 1$$

for $T_0 = \frac{1}{2}V$ and increases as T_0 decreases. We therefore proceed to ignore the first term in the bracket on the right-hand side of equation 3.13 and write

$$T = 16\frac{T_0}{V}\left[1 - \frac{T_0}{V}\right]e^{-2k_2 a},$$

where T is the transmission coefficient. The approximations made mean that this result is valid for a wide barrier which is high compared to the incident kinetic energy. Further, for the range of values of T_0/V normally of interest, which is T_0/V not too close to zero nor too close to unity, the factor

$$16\frac{T_0}{V}\left[1 - \frac{T_0}{V}\right]$$

lies between 1 and 4, and the value of T is dominated by the exponential factor. Therefore without serious error we can simply write

$$T = e^{-2\gamma},$$

where $\gamma = k_2 a$.

3.11 One-dimensional Coulomb barrier

We now consider the case of the one-dimensional potential illustrated in Figure 8. In region (2) we take the potential to be proportional to $1/x$ as for the Coulomb field. We assume that in this region the eigenfunction can be taken to be

$$\psi_2(x) = A_2 e^{-\gamma(x)} + B_2 e^{\gamma(x)}.$$

This is a generalization of the result for the rectangular barrier of constant height. In the present case the height is varying with distance through the barrier and the exponent cannot be assumed to be a linear function of x. We shall assume below that, since V is a slowly varying function of x, $\gamma(x)$ will also be slowly varying and $d^2\gamma/dx^2$ will consequently be very small. $\psi_2(x)$ must satisfy Schrödinger's equation, which in region (2), since $V = 2Ze^2/x$, will take the form

$$\frac{d^2\psi}{dx^2} + \frac{2M}{\hbar^2}\left[W - \frac{2Ze^2}{x}\right]\psi = 0. \qquad 3.14$$

Now $$\frac{d^2\psi_2}{dx^2} = \psi_2(x)\left[\frac{d\gamma}{dx}\right]^2,$$

where a term involving $d^2\gamma/dx^2$ has been neglected. Substituting into equation 3.14 we have

$$\left[\frac{d\gamma}{dx}\right]^2 + \frac{2M}{\hbar^2}\left[W - \frac{2Ze^2}{x}\right] = 0.$$

It follows that $$\frac{d\gamma}{dx} = \sqrt{\left[\frac{2M}{\hbar^2}\left(\frac{2Ze^2}{x} - W\right)\right]}$$

and so $$\gamma(x) = \frac{1}{\hbar}\sqrt{(4MZe^2)}\int_R^x \sqrt{\left(\frac{1}{x} - \frac{1}{b}\right)}\,dx,$$

where $$b = \frac{2Ze^2}{T_0}.$$

Note that the lower limit of integration has been chosen so that $\gamma(R) = 0$ and the boundary conditions at $x = R$ are as for the rectangular barrier. This integral can be evaluated by a change of variable to θ, where

$$\frac{x}{b} = \cos^2\theta.$$

Thus $$\gamma(b) = \frac{2b}{\hbar}\sqrt{(2MT_0)} \int_0^{\cos^{-1}\sqrt{(R/B)}} \sin^2\theta\, d\theta$$

$$= \frac{b}{\hbar}\sqrt{(2MT_0)}\left[\cos^{-1}\sqrt{\frac{R}{b}} - \sqrt{\frac{R}{b}}\sqrt{\left(1-\frac{R}{b}\right)}\right].$$

If now we assume $R \ll b$,

$$\cos^{-1}\sqrt{\frac{R}{b}} = \frac{\pi}{2} - \sin^{-1}\sqrt{\frac{R}{b}} = \frac{\pi}{2} - \sqrt{\frac{R}{b}}.$$

Hence $$\gamma(b) = \frac{b}{\hbar}\sqrt{(2MT_0)}\left(\frac{\pi}{2} - 2\sqrt{\frac{R}{b}}\right). \qquad 3.15$$

The discussion can then proceed as in 3.10 with $\gamma(b)$ substituted for $k_2 a$ at the second boundary.

The transmission coefficient (assumed in this case to be equal to the penetration factor, see 3.10) is therefore given by

$$T = e^{-2\gamma(b)}.$$

3.12 Nuclear Coulomb barrier

The nuclear potential barrier must of course be considered in three dimensions. We assume spherical symmetry by taking V, the potential, to be a function only of r, the distance from the nuclear centre. Figure 8 can then be taken to be a section through the three-dimensional barrier.

We can then carry out the same analysis as in the one-dimensional case but Schrödinger's equation takes the three-dimensional form

$$\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} + \frac{\partial^2\psi}{\partial z^2} + \frac{2M(W-V)}{\hbar^2}\psi = 0.$$

To take advantage of the spherical symmetry of the problem we transform from Cartesian coordinates to r, θ, ϕ, the usual spherical polar coordinates, and Schrödinger's equation then becomes

$$\left[\frac{\partial^2}{\partial r^2} + \frac{2}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial\theta^2} + \frac{\cot\theta}{r^2}\frac{\partial}{\partial\theta} + \frac{1}{r^2\sin^2\theta}\frac{\partial^2}{\partial\phi^2}\right]\psi + \frac{2M}{\hbar^2}[W - V(r)]\,\psi = 0. \qquad 3.16a$$

Next, a solution of the form $\psi(r,\theta,\phi) = R(r)Y(\theta,\phi)$ is assumed. When this is substituted into equation 3.16a we find that all of the terms are either functions of r alone or functions of θ and ϕ. The r-dependent terms are taken to one side of the equation, the terms depending on θ and ϕ to the other side. Each side of the equation must then be a constant independent of r, θ and ϕ. We set each side equal to a separation constant $l(l+1)$. Carrying out this procedure we find that

$$\frac{1}{r^2}\frac{d}{dr}\left[r^2\frac{dR}{dr}\right] + \frac{2M}{\hbar^2}\left[W - V(r) - \frac{l(l+1)\hbar^2}{2Mr^2}\right]R = 0.$$

This constitutes *the radial wave equation.* If a function $G(r) = r\,R(r)$, the so-called *modified radial wave function* be now introduced, the radial wave equation can be written

$$\frac{d^2G}{dr^2} + \frac{2M}{\hbar^2}\left[W - V(r) - \frac{l(l+1)\hbar^2}{2Mr^2}\right]G = 0. \qquad 3.16b$$

We note that $|R|^2$ is to be associated with a flux density of particles, that is, the number of particles crossing unit area per unit time. The total number of particles crossing a spherical surface of radius r per unit time will therefore be associated with $4\pi r^2\,|R|^2$. This, the particle flux, will be proportional to $|G|^2$. If $l = 0$, we note that equation 3.16b is exactly equivalent to the one-dimensional case analysed above, $G(r)$ playing the role of $\psi(x)$. The particle flux in the spherical case was noted to be proportional to $|G|^2$; in the one-dimensional case it is proportional to $|\psi(x)|^2$. We can therefore, relying on the analogy of G with $\psi(x)$, assume that the penetration factor in the spherical case is given, as in the one-dimensional case, by $e^{-2\gamma}$.

The separation constant was introduced in the somewhat artificial form above in order that l would be equivalent to the angular-momentum quantum number of early quantum theory. $l = 0$ therefore implies that the emitted particle has no angular momentum about the centre of the nucleus, that is, it is emitted radially. We note that if $l \neq 0$, there is an additional term in Schrödinger's equation which effectively adds to the height of the electrostatic potential barrier. This term is said to represent the *centrifugal barrier.* The centrifugal barrier does not depend on electric charge and is effective in all cases, including the case of the emitted particle being a neutron when, because of its electrical neutrality, the Coulomb barrier is not effective.

3.13 Gamow's theory of alpha decay

From the above discussion the penetration factor for a nuclear α-particle incident on the Coulomb potential barrier is $e^{-2\gamma}$, where γ is given by equation 3.15. When the values of the fundamental constants are substituted into this expression, we find that the exponent can be written

$$2\gamma = 3{\cdot}95\frac{Z}{\sqrt{T_0}} - 2{\cdot}97\sqrt{(ZR)},$$

T_0, the emergent energy of the α-particle, being in millions of electronvolts and R, the nuclear radius, being in fermis.

The penetration factor varies rapidly with α-particle energy. To see how rapid this variation is, we substitute $Z = 90$, with $T_0 = 4{\cdot}2$ MeV and $R = 10$ fm, the values corresponding to the α-decay of $^{238}_{92}$U. Then $2\gamma = 173{\cdot}4 - 89{\cdot}1 = 84{\cdot}3$ and the penetration factor is therefore $e^{-84{\cdot}3}$, which is equal to $10^{-36{\cdot}6}$. Had the

α-particle energy been 9 MeV, that is, about double its actual value, then we should have had $2\gamma = 118{\cdot}5 - 89{\cdot}1 = 29{\cdot}4$, in which case the penetration factor would have been $e^{-29{\cdot}4} = 10^{-12{\cdot}8}$. Thus for an approximate doubling of α-particle energy there would be a variation of about twenty-four orders of magnitude in the penetration factor.

We now have to consider how many α-particles per second are incident on the potential barrier. This will depend on two factors, namely the number of α-particles existing in the nucleus and the number of collisions each α-particle makes with the potential barrier per unit time.

The number of α-particles existing at any time in a heavy nucleus is probably somewhere between one and ten. These are transient groupings formed by the nucleons in their motion within the nucleus. Fortunately, as we shall see, the decay constant is not critically dependent on this number, which we now denote by P_α. If V_α be the α-particle velocity inside the nuclear potential well, then, the average distance travelled between collisions being of the order of the nuclear radius, the time between collisions will be R/V_α and hence the number of collisions per α-particle per unit time will be V_α/R. If the potential inside the nucleus is close to the value of the potential at infinity, then T_α in the well is equal to T_0, that is, it will be a few million electronvolts in value. V_α is then approximately $c/10 = 3 \times 10^7$ m s^{-1}. We can now write the probability λ that the nucleus will decay per unit time as the product of the number of α-collisions per second with the inside of the barrier and the probability of the α-particle tunnelling through the barrier, as discussed above. Hence

$$\lambda = P_\alpha \frac{V_\alpha}{R} e^{-\{3{\cdot}95Z/\sqrt{T_0} - 2{\cdot}97\sqrt{(ZR)}\}}, \qquad 3.17$$

or, taking logarithms,

$$\log \lambda = \log P_\alpha + \log \frac{V_\alpha}{R} - \left[3{\cdot}95\frac{Z}{\sqrt{T_0}} - 2{\cdot}97\sqrt{(ZR)}\right]\frac{1}{2{\cdot}3}.$$

The quantities inside the logarithmic terms do not of course affect the result critically. However, T_0 and R, which occur in the remaining terms, clearly have a very large effect on the value of the decay constant.

The theory gives, in a very satisfactory way, the variation of λ with T_0, although this is of different analytical form from the empirical expressions of Geiger and Nuttall.

3.14 Nuclear unit radius

As we have discussed in detail above, the energies of scattered α-particles at which departure from Rutherford scattering takes place may be used to deduce nuclear radii. This has now been done for nuclei ranging through the periodic table. It is found that the variation of radius as a function of A is very well described by $R = R_0 A^{\frac{1}{3}}$. We refer to R_0 as the *nuclear unit radius*. Once R_0 is known we can then evaluate the radius of any nucleus. The above theory of α-decay can be used to determine R_0 within close limits.

We can rewrite equation 3.17 in the form

$$\lambda = \lambda_0 e^{-(a - b\sqrt{R_0})},$$

regarding a and b as constants to be calculated from the Z, A, T_0 values for any particular α-emitting isotope. The uncertain quantities P_α and V_α are included in λ_0. We assume that these quantities, and hence λ_0, do not change appreciably as we go from one isotope to others with only slightly different Z- and N-values. A determination of λ and T_0 for two different isotopes will then provide two equations from which both λ_0 and R_0 can be found. Because of the occurrence of R_0 in the exponent, this is a very sensitive method for its determination, a variation of 10^{12} in half-life resulting from a change of a factor of two in the value of R_0. The best value of R_0 arrived at by this method is 1·48 fm.

3.15 Fine structure in alpha-particle spectra

It was noted in section 3.3 that α-particles emitted from a single nuclide do not always have the same energy. The spectrum of α-particles from ThC, shown in Figure 10, reveals that in the case of that nuclide there are five possible α-particle energies. If we have regard to equation 3.1 we see that different α-particle energies are only possible (assuming that α-decay is a two-body decay) if different values of Q are possible. These different Q-values in turn demand that the nuclear masses be multivalued. To explain this, the hypothesis is now made that, just as the atomic electron structure can have configurations of different energy content, so may the nucleus have different configurations each with its own associated energy. The lowest energy configuration we call the *ground state*, the others *excited states*. In conformity with mass–energy equivalence, the effective mass of

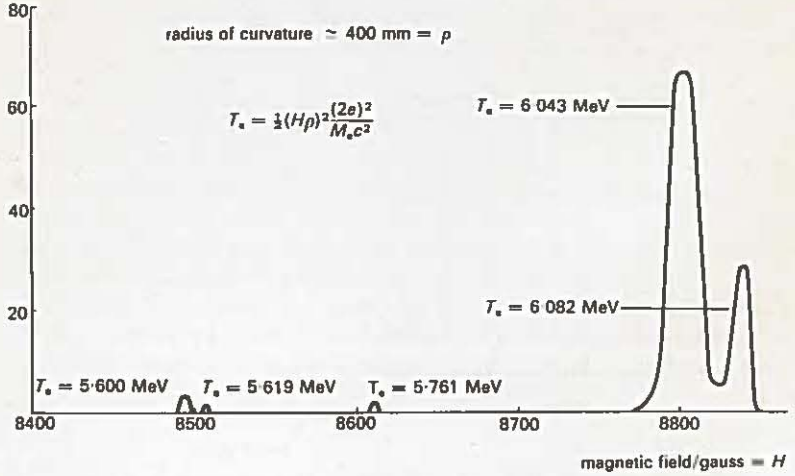

Figure 10 Spectrum of α-particles emitted by ThC (based on measurements by Rutherford and colleagues, 1933)

the nucleus when in an excited state will be the ground-state mass plus E_e/c^2, where E_e is the energy of excitation. Thus the radioactive transformation will have a value of Q which depends on the excited state involved.

There is no evidence for the existence of excited states of the emitted α-particles. However, there are examples of excited states of parent and of daughter nuclei. ThC, quoted above, is a case of the daughter nucleus being created either in the ground state or in an excited state. If the α-particle energies are substituted into equation 3.1, the Q-values may then be calculated. The highest Q-value will correspond to the formation of the ground state. The energies of the excited states, measured from the ground state as zero, will then be given by the amount by which the associated Q-value is less than the ground-state Q-value. The results of this calculation for ThC are shown in Figure 11 in which the states are represented as horizontal lines on a vertical scale of energy, the intervals being proportional to the energies associated with the states. Such a diagram is referred to as an *energy-level diagram*. The excited states may de-excite directly to the ground state, the energies of excitation being carried off by γ-rays (i.e. quanta of electromagnetic energy) or they may 'cascade' through lower states to the ground state with the emission of a series of γ-rays. The time for de-excitation is usually very much less than a nanosecond (10^{-9} s) and thus the

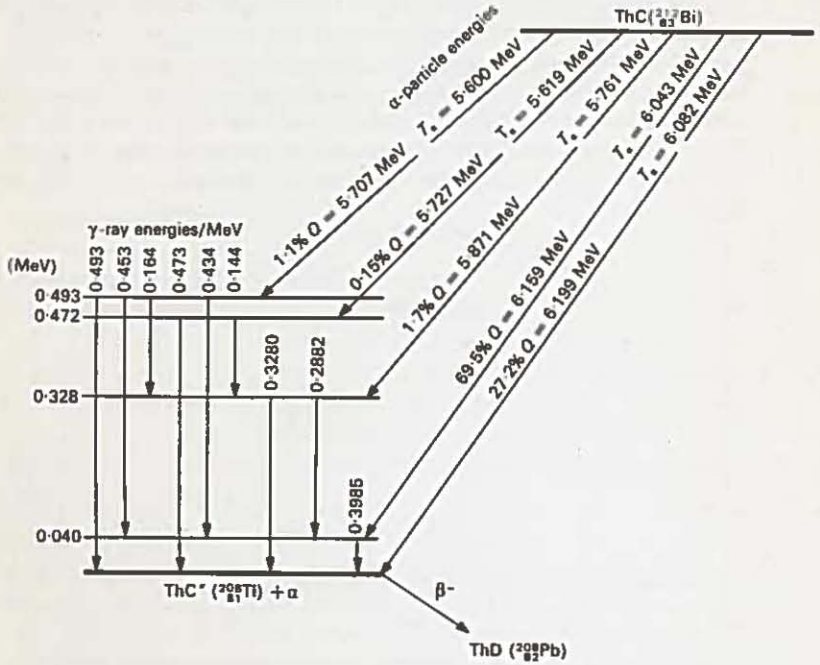

Figure 11 Energy-level diagram of ThC″ showing relationship of -particle and γ-ray spectra.

γ-rays will appear to be in good time coincidence with the α-particles associated with the creation of the excited state. The above interpretation of the decay of ThC can therefore be confirmed by searching for γ-rays, measuring the energy of these γ-rays to compare with the predicted energy from the level diagram and showing that they are in time coincidence with α-particles in the appropriate energy group.

As an example of an excited parent nucleus, we quote ThC′, which has a half life of only 0·3 μs following its formation by β-decay from ThC. It may be formed in its ground state or in one of a set of excited states. The half-life for α-particle emission is so short that α-decay from the excited states is not entirely insignificant compared with photon emission. In a small fraction of cases (approximately 10^{-4} for ThC′) an α-particle of more than normal energy is emitted in this way. Such α-particles are generally referred to as *long-range α-particles*. Compared with the α-particles emitted from the ground state, they carry excess energy of the order of one or two million electronvolts.

3.16 Alpha decay with extremely long half-life

The mass condition of 3.2 has necessarily to be satisfied if α-decay is to be energetically possible. There are instances however where the condition is satisfied but α-decay is not observed. This is always where the mass difference, and hence the energy released in the α-decay transformation, is very small. The α-particle, having low energy, has to penetrate a very wide barrier. As a consequence λ is small and $T_{\frac{1}{2}}$ is very long even if the α-particle has no angular momentum with respect to the centre of the daughter nucleus. If, because of angular-momentum conservation requirements, the α-particle has angular momentum relative to the daughter nucleus, then the centrifugal barrier will further decrease the decay probability. As a result the activity per unit mass of sample of material will be very small.

There is a limit to the mass of sample which can be under examination by a given detection system at any one time. Further, any detection system has a background counting rate due to cosmic rays and radioactive contamination in its structural materials and its surroundings. This background counting rate arising from random effects has statistical fluctuations which set a limit to the accuracy with which an activity can be measured. In the extreme case, it may be impossible to distinguish between a very weak activity and no activity.

For this reason, nuclei once believed to be stable may subsequently be found to be unstable. For example ^{142}Ce appeared, until recently, in nuclear data tables as a stable isotope. There is now evidence that it is α-unstable, emitting α-particles of about 1·5 MeV and having a half-life of 5×10^{15} years.

3.17 Summary

The use of the α-particle as a charge probe for the measurement of the electrostatic field within the atom led to the concept of the potential barrier and to an estimate of nuclear size. It also revealed a paradox with respect to the emission of α-particles with insufficient energy to have surmounted the potential barrier. The resolution of this paradox by the abandonment of classical dynamics in favour of wave mechanics led to an explanation of the observed relationship between half-life and α-particle energy. The measurements of these two quantities for two α-emitting nuclides were used to determine R_0, the nuclear unit radius which enters the formula for the nuclear radius, namely $R = R_0 A^{\frac{1}{3}}$. The interpretation of the fine structure observed in the energy spectra of α-particles established the existence of excited states of the nucleus and led to the introduction of energy-level diagrams.

Chapter 4
Radioactivity: Beta Decay

4.1 Introduction

Beta decay, the most generally occurring mode of radioactive transformation, takes place between neighbouring isobars (i.e. without change in A and with a change of one in Z). In contrast to α-decay, which is a phenomenon limited to nuclei with medium and high A-values, β-decay has been observed for nuclei with all A-values from one upwards. Essentially in β-decay a neutron switches into a proton or vice versa. When the switch occurs a β-particle, of negative sign of electric charge if a neutron switch, of positive sign if a proton has switched, is observed to be emitted. Careful experimentation has failed to reveal any difference between the physical properties of the negative β-particle and those of the electron of atomic structure, and we assume that these particles are identical. The positive β-particle, apart from the sign of its electric charge, has the same properties as the negative β-particle. The β-particles are sometimes named *negatron* (perhaps more properly, but less usually, *negaton*) and *positron* (or *positon*), *electron* then being available to apply generically to either.

4.2 Beta decay and the conservation laws

The measurement, by Chadwick in 1914, of the energy of β-particles emitted from a source containing a single isotopic species revealed a continuous spectrum of energy ranging from zero to a finite maximum value. If it is assumed that, as in α-decay, the parent and daughter nuclei have well-defined mass values, then the conservation of mass–energy and linear momentum requires that there be at least three 'products' of the decay, that is, one product in addition to the β-particle and the recoiling daughter nucleus. Careful measurement of the energy absorbed in massive calorimeters containing strong β-sources indicated an energy per decay corresponding to the mean β-energy, not to the maximum β-energy. Thus the third 'product', if such existed, did not deposit any energy in the material of the calorimeter (Ellis and Wooster, 1927).

In addition to the difficulty thus presented in respect of energy conservation, β-decay set a problem with respect to conservation of angular momentum. The simplest β-emitter is the free neutron, which, with a half-life of about thirteen minutes, decays to a proton. We start with a neutron which has intrinsic angular momentum of $\frac{1}{2}\hbar$. If we end with only a proton and electron, each having intrinsic angular momentum of $\frac{1}{2}\hbar$ and only permitted by the rules of quantum