Solution

Quiz #1 Ch.#16 T122 Phys101.28-30-v1

Student ID:..... ..... Student Name:..... ..... Section # .....

Q1: A transverse wave on a string with a linear density of 0.200 kg/m is described by the following equation: y = 0.005 sin(419 t − 21.0 x), where x and y are in meters and t is in seconds. What is the tension in the string?

$$v = \sqrt{\frac{\tau}{\mu}} = \frac{\omega}{k} \Rightarrow \tau = \mu v^2 = \mu\left(\frac{\omega^2}{k^2}\right) = 0.2 \times \left(\frac{419}{21}\right)$$

$$\tau = 79.6\ N$$

2

Q#2: Two identical waves, moving in the same direction, have a phase difference of Pi/2. The amplitude of each of the two waves is 0.10 m. If they interfere, then the amplitude of the resultant wave is?

$$y' = 2 y_m \cos\left(\frac{\phi}{2}\right)$$

$$= 2 \times 0.1 \times \cos(45)$$

$$= 0.14\ m$$

Q#3: A wave of speed 20 m/s on a string, fixed at both ends, has an equation for a standing wave given by:y(x,t) = 0.05 sin(k x) cos(30 t), where x and y are in meters and t is in seconds. What is the distance between two consecutive nodes?

Distance between two consecutive nodes $D = \frac{\lambda}{2}$

$$D = \frac{\lambda}{2}, \quad v = \lambda f, \quad \omega = 2\pi f \Rightarrow f = \frac{\omega}{2\pi}$$

$$\lambda = \frac{v}{f} = \frac{2\pi v}{\omega}, \quad v = 20\ m/s$$

$$D = \frac{\lambda}{2} = \frac{2\pi \times 20}{2 \times 30} = \frac{2\pi}{3} = 2.09\ m$$

4

Quiz #1 Ch.#16 T122 Phys101.28-30-v2

Student ID:..... ..... Student Name:..... ..... Section # .....

Q#1: A stretched string has a length of 2.0 m and a mass of 2.5 g. A sinusoidal transverse wave traveling on the string is described by the equation: $y(x,t) = 0.010 \sin(3.0x - 75t)$, where x and y are in meters and t is in seconds. What is the tension in the string?

$$v = \frac{\omega}{k} = \sqrt{\frac{\tau}{\mu}} \Rightarrow \tau = \mu v^2$$

$$\tau = \mu\left(\frac{\omega}{k}\right)^2 = \frac{2.5\times10^{-3}}{2} \times \left(\frac{7.5}{3}\right)^2 = 0.78\ N$$

Q#2Two waves with the same amplitude y_m and wavelength λ are moving on a string. One of the waves is shifted relative to the other by a distance of 5.5 λ. The amplitude of the resultant wave is?

$$\frac{\phi}{2} = 5.5\lambda = [\text{illegible}] = 1980^\circ = 5.5 \times 360^\circ$$

$$y_m' = 2 y_m \cos\left(\frac{\phi}{2}\right) = 2 y_m \cos(1980) = 2 y_m$$

Q#3: A 40 cm string of linear mass density 8.0 g/m is fixed at both ends. The string is driven by a variable frequency audio oscillator ranged from 300 Hz to 800 Hz. It was found that the string is set in oscillation only at the frequencies 440 Hz and 660 Hz. What is the tension in the string?

$$f_{n+1} - f_n = f_1 = \frac{v}{2L}, \quad v = \sqrt{\frac{\tau}{\mu}}; \quad f_1 = \frac{1}{2L}\sqrt{\frac{\tau}{\mu}}$$

$$\tau = \mu\, 4L^2 f_1^2 \quad ; \mu = 8\times10^{-3}\ kg/m; \quad f_1 = 660 - 440 = 220\ Hz$$

$$= 8\times10^{-3} \times 4 \times (0.4)^2 \times (220)^2 = 247.8\ N$$

Quiz #1 Ch.#16 T122 Phys101.28-30-v3

Student ID:..... Student Name:..... Section # .....

Q#1: A uniform wire, having a mass of 0.4 kg and length of 6.5 m, is connected to a pulse generator. The tension is maintained in the wire by suspending a 3.5 kg mass on the other end. Find the time it takes a pulse to travel from a pulse generator to the other end?

$$v = \sqrt{\frac{T}{\mu}} \; ; \; T = mg = 3.5 \times 9.8 = 34.3\,N \; ; \; \mu = \frac{0.4}{6.5} = 0.0615\,kg/m$$

$$v = \sqrt{\frac{34.3}{0.0615}} = 23.6\,m/s$$

$$v = \frac{d}{t} \Rightarrow t = \frac{d}{v} = \frac{6.5}{23.6} = 0.28\,s$$

Q#2: Two identical traveling waves, with a phase difference φ, are moving in the same direction. If they are interfering and the combined wave has an amplitude 0.5 times that of the common amplitude of the two waves, calculate φ (in radians)?

$$y'_m = 2y_m \cos\left(\frac{\phi}{2}\right)$$

$$y'_m = 0.5\,y_m = 2y_m \cos\left(\phi/2\right)$$

$$\phi = 2\cos^{-1}\left(\frac{1}{4}\right) = 151^\circ = 2.64\,rad$$

Q#3: Two waves *A* and *B* are generated on a string of length 4.0 m to produce a four-loop standing wave. The wave speed is 1000 m/s. What is the frequency of each of the two waves *A* and *B*?

$$L = 4.0\,m = 4\lambda_n \frac{\lambda}{2} \Rightarrow \lambda = 2.0\,m$$

$$v = \lambda f_n$$

$$f_n = \frac{v}{\lambda_n} = \frac{1000}{2.0} = 500\,Hz$$

Quiz #1 Ch.#16 T122 Phys101.28-30-v4

Student ID:..... ..... Student Name:..... ..... Section # .....

Q#1: A stretched string of mass 2.0 g and length 10 cm, carries a wave having the following displacement wave: $y(x,t) = 0.05\sin(2\pi x - 400\pi t)$, where x and y are in meters and t is in seconds. What is the tension in the string?

$$v = \frac{\omega}{k} = \sqrt{\frac{\tau}{\mu}} \Rightarrow \tau = \mu\left(\frac{\omega}{k}\right)^2 ; \mu = \frac{2\times10^{-3}}{0.1}\ kg/m$$

$$\mu = 0.02\ kg/m$$

$$\tau = 0.02 \times \left(\frac{400\pi}{2\pi}\right)^2$$

$$= 0.02 \times (200)^2 = 800\,N$$

Q#2: A string, fixed at its ends, vibrates according to the equation $y = 0.5\sin(1.5\pi x)\cos(40\pi t)$ where x and are in meters and t is in seconds. What are the amplitude and velocity of the component waves whose superposition can give rise to this wave?

$$y = 2y_m \sin(kx)\cos(\omega t)$$

$$2y_m = 0.5\,m \Rightarrow y_m = \frac{0.5}{2} = 0.25\,m$$

$$v = \frac{\omega}{k} = \frac{40\pi}{1.5\pi} = 26.7\ m/s$$

Q3 A vibrator having a frequency of 200 Hz generates a standing wave of six loops with amplitude of in a string clamped at both side. If the speed of the wave on the string is 100 m/s, what is the length of the string ?

$$f_m = 200\,Hz, \quad L = 3 \times \frac{\lambda}{2} = 3\lambda$$

$$\lambda = \frac{v}{f} = \frac{100}{200} = 0.5\,m$$

$$L = 3\lambda = 3 \times 0.5 = 1.5\,m$$

Quiz #1 Ch.#16 T122 Phys101.28-30-v5

Student ID:..... Student Name:..... ..... Section # .....

Q1. A stretched string has a length of 2.00 m and mass of 1.56 g. A transverse sinusoidal wave is travelling on this string, and is given by: $y(x, t) = 0.100 \sin(3.00x - 144t)$ where x and y are in meters and t is in seconds. What is the magnitude of the tension in the string?

$$v = \sqrt{\frac{\tau}{\mu}} \Rightarrow \tau = \mu v^2 ; \mu = \frac{1.56\times10^{-3}}{2.0} = 0.78\times10^{-3}\ kg/m$$

$$\tau = \mu v^2 = \mu\left(\frac{\omega}{k}\right)^2 = 0.78\times10^{-3}\times\left(\frac{144}{3}\right)^2 = 1.797\ N$$

Q#2: Two identical sinusoidal traveling waves are sent along the same string in the same direction. What should be the phase difference between the two waves so that the amplitude of the resultant wave is equal to the amplitude of each wave?

$$y_m' = 2y_m\cos(\phi/2)$$

$$y_m' = y_m = 2y_m\cos(\phi/2)$$

$$\phi = 2\cos^{-1}(1/2) = 120^\circ$$

Q#3: For the superposition of the following two harmonic waves:
$Y_1(x,t) = 4.0$ m sin $[2\pi x - 4\pi t)$ and $Y_2(x,t) = 4.0$ m sin $[2\pi x + 4\pi t)$ where x is in meter and t is in second, the distance between any two successive nodes will be?

Distance between two successive nodes $d = \frac{\lambda}{2}$

$$\lambda = \frac{2\pi}{k} = \frac{2\pi}{2\pi} = 1.0\ m$$

$$d = \frac{\lambda}{2} = \frac{1}{2}\ m = 0.5\ m$$

Quiz #1 Ch.#16 T122 Phys101.28-30-v6

Student ID:..... ..... Student Name:..... ..... Section # .....

Q#1: A sinusoidal string wave traveling in the negative x direction has an amplitude of 0.20 mm and a frequency of 10 Hz. If the string has a linear mass density 0.50 kg/m and is under a tension of 10 N, what is the equation of the wave?

$y_m = 0.20\ mm = 0.2 \times 10^{-3}\ m$; $f = 10\ Hz$

$\omega = 2\pi f = 2\pi \times 10 = 20\pi\ rad/s$

$v = \sqrt{\frac{\tau}{\mu}} = \frac{\omega}{k} \Rightarrow k = \omega\sqrt{\frac{\mu}{\tau}} = 20\pi \times \sqrt{\frac{0.5}{10}} = 4.5\pi$

$k = 4.5\pi$

$k = 20\pi \times 0.224$

$= 14.05\ m^{-1}$

Q#2: Two identical sinusoidal waves, each having amplitude *ym*, are traveling in the same direction on the same stretched string. What phase difference between them will give a resultant wave whose amplitude is 0.5 *ym*?

$y_m' = 0.5\, y_m = 2 y_m \cos(\phi/2)$

$\phi = 2\cos^{-1}\left(\frac{0.5}{2}\right) = 151^\circ$

Q#3: A standing wave is set up on a string that is fixed at both ends. The standing wave has four loops and a frequency of 600 Hz. The speed of waves on the string is 400 m/s. What is the length of the string?

$L = 4 \times \frac{\lambda}{2} \Rightarrow L = 2\lambda$; $\lambda = \frac{v}{f} = \frac{400}{600} = 0.67\ m$

$L = 2\lambda = 2 \times 0.67 = 1.34\ m$

Quiz #1 Ch.#16 T122 Phys101.28-30-v7

Student ID:..... ..... Student Name:..... ..... Section # .....

Q#1: A wave with an amplitude of 1.0 cm and wavelength 2.5 m is generated on a string with a linear density of 20 g/m that is under a tension of 5.0 N. What is the maximum transverse speed of a point on the string?

$$v_m = y_m \omega = y_m \times k \times v \;,\; k = \frac{2\pi}{\lambda} = \frac{2\pi}{2.5} = 2.51\ m^{-1}$$

$$v_m = 0.01 \times k \times \sqrt{\frac{T}{\mu}} = 0.01 \times 2.51 \times \sqrt{\frac{5}{20\times10^{-3}}}$$

$$v_m = 0.397\ m/s$$

Q2.Two identical traveling waves of amplitude 10.0 cm, moving in the same direction, are out of phase by $\pi/4$ rad. Find the amplitude of the resultant wave?

$$y_m' = 2 y_m \cos\left(\frac{\phi}{2}\right)$$

$$= 2 \times 0.1 \times \cos\left(\frac{\pi}{8}\right) = 0.185\ m$$

$$= 18.5\ cm$$

Q#3: Standing waves are produced on a string at the two consecutive resonant frequencies 120 and 160 Hz. If the string has a length of 3.0 m, what is the distance between two adjacent nodes at the resonant frequency 240 Hz?

$$f_1 = f_{n+1} - f_n = 160 - 120 = 40\ Hz\ ,\ L = 3.0\ m$$

distance between two adjacent nodes $d = \frac{\lambda}{2}$

$$d = \frac{\lambda}{2}\ ;\ f_1 = \frac{v}{2L} \Rightarrow v = 2Lf_1 = 2 \times 3 \times 40 = 240\ m/s$$

$$\lambda' = \frac{v}{f_1} = \frac{240}{240} = 1.0\ m$$

$$d = \frac{\lambda'}{2} = \frac{1}{2} = 0.5\ m$$

Quiz #1 Ch.#16 T122 Phys101.28-30-v8

Student ID:..... ..... Student Name:..... ..... Section # .....

Q#1: The function y(x,t) = 15.0 cos (πx- 20 πt) with x and y in meters and t in seconds, describes a wave on a taut string. What is the mass of one meter of the string if the tension in the string is 40.0 N?

$$v = \sqrt{\frac{\tau}{\mu}} = \frac{\omega}{k} \Rightarrow \mu = \tau \cdot \left(\frac{k}{\omega}\right)^2$$

$$\mu = \tau \left(\frac{k}{\omega}\right)^2 = 40 \times \left(\frac{\pi}{20\pi}\right)^2 = 40 \times \left(\frac{1}{20}\right)^2 = 0.1 \text{ kg/m}$$

mass of one meter = 0.1 kg = 100 g

Q#2: What phase difference (in wavelength λ) between two identical traveling waves, moving in the same direction along a stretched string, results in the combined wave having an amplitude 1.75 times that of the common amplitude of the two combined waves?

$$y'_m = 2 y_m \cos(\phi/2)$$

$$1.75\, y_m = 2 y_m \cos(\phi/2)$$

$$\phi = 2\cos^{-1}\left(\frac{1.75}{2}\right) = 57.9^\circ = 1.01 \text{ rad} = \frac{1.01}{2\pi} \times \lambda$$

$$= 0.16\,\lambda$$

Q#3: A 40 cm string of linear mass density 8.0 g/m is fixed at both ends. The string is driven by a variable frequency audio oscillator ranged from 300 Hz to 800 Hz. It was found that the string is set in oscillation only at the frequencies 440 Hz and 660 Hz. What is the tension in the string?

$$f_1 = f_{n+1} - f_n = 660 - 440 = 220 \text{ Hz}$$

$$f_1 = \frac{v}{2L} = \frac{1}{2L}\sqrt{\frac{\tau}{\mu}}$$

$$\tau = 4L^2 f_1^2 \mu = 4 \times (0.4)^2 \times (220)^2 \times 8 \times 10^{-3} = 247.8 \text{ N}$$

Quiz #1 Ch.#16 T122 Phys101.28-30-v9

Student ID:..... ..... Student Name:..... ..... Section # .....

Q#1: When a wave travels through a medium, individual particles execute a periodic motion given by the equation:

$y = 4.0 \sin[\pi/4(2t+x/8)]$

where $\hat{x}$ and $\hat{y}$ are in meters and t is in seconds. The phase difference (in degrees) at any given instant between two particles that are 20.0 m apart is?

$\Delta\phi = \frac{\Delta L}{\lambda} \times 2\pi \text{ rad.}$

$\lambda = \frac{2\pi}{k} = \frac{2\pi}{\pi/8 \times 4} = 64.0 \text{ m}$

$\Delta\phi = \frac{20}{64} \times 2\pi = 1.963 \text{ rad} = 112.5^{0}$

Q#2: Two identical sinusoidal traveling waves are sent along the same string in the same direction. What should be the phase difference between the two waves so that the amplitude of the resultant wave is equal to the amplitude of each wave?

$y_m' = 2y_m \cos(\phi/2)$

$y_m = 2y_m \cos(\phi/2)$

$\phi = 2\cos^{-1}(\frac{1}{2}) = 120^{0}$

Q#3: What is the third lowest frequency for standing waves on a 10.0 m long wire with 2.0 g mass and stretched under a tension of 200 N?

$f_3 = 3f_1 = 3 \times \frac{v}{2L} = \frac{3 \times 1}{2L} \times \sqrt{\frac{\tau}{\mu}}$

$= \frac{3}{2L}\sqrt{\frac{\tau}{\mu}} = \frac{3}{2 \times 10}\sqrt{\frac{200}{2 \times 10^{-3}/10}} = \frac{3}{2 \times 10}\sqrt{10^{6}}$

$f_3 = \frac{3 \times 10^{3}}{20} = 150 \text{ Hz}$