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# Factors affecting pseudo relative permeability curves

Saud S. Al-Otaibi, Abdulaziz A. Al-Majed *

*Petroleum Engineering Department, King Fahd University of Petroleum and Minerals, Box 1944, Dhahran, 31261, Saudi Arabia*

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**Abstract**

A large number of grid cells are needed to simulate very large field reservoirs such as those in the Middle East. Pseudo relative permeability curves are used to reduce the dimensionality of reservoir simulation models and also to account for intra-cell rock property variations. Identifying the parameters affecting pseudo relative permeability curves is very important so that the model cells can be grouped in different categories in which appropriate pseudo relative permeabilities are generated for each group. The objective of this study is to identify the factors affecting pseudo relative permeability curves. Different reservoir rock and fluid properties were studied for an homogeneous 2D reservoir model. Validation of some curves is also done. The most pronounced effect on pseudo relative permeability curves was found to be caused by the reservoir dip angle. Layer thickness and PVT properties have some effect. Production rate and vertical to horizontal permeability ratio have no significant effect within the parameter ranges studied. While absolute permeability on low dip angle cases has no effect, absolute permeability effects on high dip angle cases needs consideration. © 1998 Elsevier Science B.V. All rights reserved.

*Keywords:* Pseudo relative permeability curves; Simulation; Parameters; PVT properties; Production rate; Absolute permeability; Dip angle layer thickness; Vertical to horizontal permeability ratio

## 1. Introduction

The most reliable source of relative permeability data is from laboratory measurements performed on cores obtained from the reservoir of interest. Relative permeability data are not always available for each rock type, and there is a little reason to believe that a single, average relative permeability curve will be representative of the behavior of each layer or the whole reservoir. For this reason, different approaches were proposed in reservoir simulation studies for better history matches.

Pseudo relative permeability curves were initially proposed by Jacks et al. (1973), instead of rock relative permeability curves, in an effort to accommodate a third dimension in numerical simulators. A pseudo relative permeability curve uses weighted averages of the rock and fluid properties, or combined rock–fluid properties. The average is done over a representative volume of the reservoir.

Pseudo relative permeability curves are needed not only to substitute three-dimensional models with two-dimensional models, but also for three-dimensional models in order to reduce the number of model cells. Pseudo relative permeability curves are then used to account for the fine-grid structure.

* Corresponding author

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Modeling reservoirs that are complex and highly stratified like those found in Saudi Arabia can require dozens or even hundreds of sets of pseudo relative permeability functions.

To compensate for intra-cell variations of rock properties and fluid saturations, a refined grid can be used. Achieving adequate accuracies can be very costly, even with super-computers, especially if the study is being performed for a large size multi-layer reservoir.

The main purpose of this work is to study the effect of different fluids, rock and structure properties on pseudo relative permeability curves in an attempt to reduce the number of curves necessary for such reservoirs. Typical Saudi sandstone reservoir fluid and rock properties were used in this study. The Kyte and Berry (1975) method of calculating dynamic pseudo relative permeability functions was followed mainly because it is the most widely used method in the industry at present.

## 2. Background

Pseudo functions were introduced to upscale relative permeability and capillary pressure data from laboratory measurements as input for reservoir simulation. Coats et al. (1967, 1971) achieved this initial form of upscaling through the vertical equilibrium concept.

With the advent of multi-million cell geological models upscaling has taken a more general form and includes each term of the 'constant' as well as the variable part of the transmissivity coefficient used in reservoir simulators.

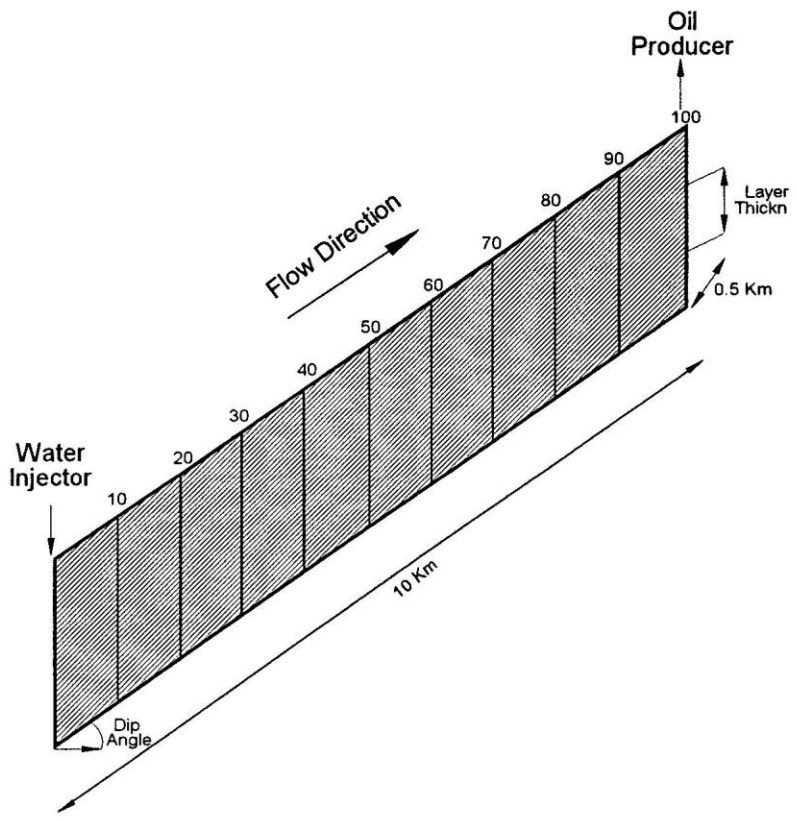


Fig. 1. Schematic diagram of a cross-section used for developing pseudo relative permeability curves.

To date, the method of Kyte and Berry (1975) and variations (Commercial simulator ECLIPSE...) are the most widely used to determine the dynamic pseudo relative permeability and capillary pressure functions. However, there is no concensus over the reliability of this method as reported by many authors, among them, Barker and Thibeau (1996).

This paper attempts to evaluate the specific impact of parameters such as the dip angle, production rate, absolute permeability,... on the pseudo relative permeability functions.

## 3. Literature review

The pseudo relative permeability curves were first introduced to the petroleum industry by Coats et al. (1967). They developed an analytical method to calculate pseudo relative permeability curves based on the assumption of gravity-capillary equilibrium in the vertical direction or for vertical equilibrium (VE). Poolen et al. (1968) proposed two methods to calculate the pseudo relative permeabilities at well locations.

Martin (1968) proposed a two dimensional simulator where the equations for three-phase, three-dimensional, compressible flow, including capillary effects, are reduced to a two-dimensional relation by partial integration. Martin also provided a firm theoretical foundation to Coats et al. (1967) assumption of vertical equilibrium by developing the equations mathematically from basic flow equations.

Coats et al. (1971) proposed a method wherein a full three dimensional mathematical model, that treats simultaneously both the areal and cross-sectional aspects of reservoir flow, is used in the simulator.

If the vertical sweep is dominated by viscous forces instead of gravity forces, another method proposed by Hearn (1971) can be used to calculate pseudo functions.

Jacks et al. (1973) proposed a procedure to generate dynamic pseudo relative permeability curves from cross-sectional models. Emanuel and Cook (1974) expanded the concept of vertical cross-section pseudo relative permeabilities to include vertical performance of individual wells. Kyte and Berry (1975) improved the Jacks et al. procedure to allow the calculation of dynamic pseudo capillary pressure, and also to compensate for differences between grid sizes in the areal and cross-sectional models. Woods and Khurana (1977) proposed a procedure to obtain

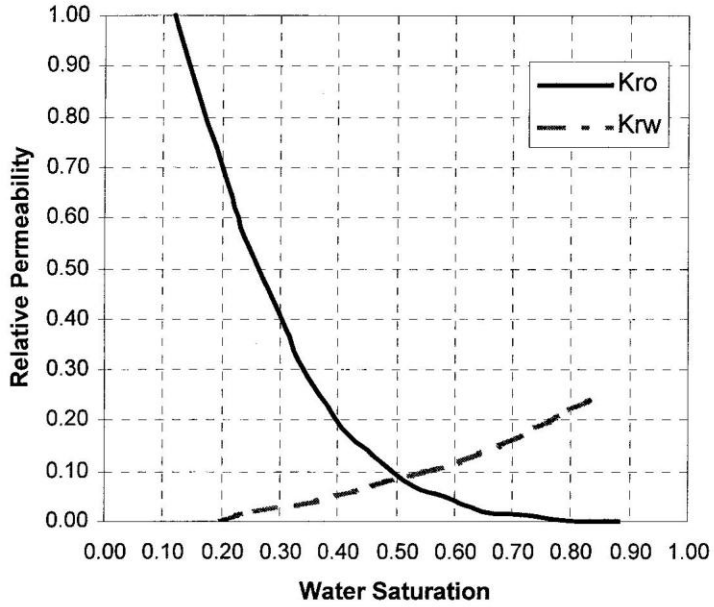


Fig. 2. Rock relative permeability curved used in all fined-grid cross-section models to generate pseudo relative permeability curves.

the pseudo functions for wells by the partial integration of flow equations in order to include water coning in three dimensional reservoir models.

Killough and Foster (1979) discussed the use of pseudo relative permeability in three-dimensional models. Andrew and Koederitz (1982) developed a technique to account for the effect of subordinate phase production in cross-flow or non-wetting phase production after breakthrough in non-communicating layers. This adoption is an improvement to the original method introduced by Hearn (1971).

Thomas (1983) examined the various pseudo function concepts then available in the technical literature. Al-Yousef (1985) developed a technique to improve the procedure to calculate the pseudo relative permeabilities from cross-sectional or three-dimensional models. Starley (1986) described a material balance method for deriving interblock water/oil pseudo functions for application in coarsely gridded areal reservoir models. Stone (1991) proposed a method for calculating two or three phase pseudo functions from fine grid simulation in either two or three dimensions.

Pickup and Sorbie (1996) addressed the issue of cross-flow when scaling up two-phase flow away from a source/sink. This approach (Pickup and Sorbie, 1996) considers the full permeability tensor and a 9-point finite difference scheme for a 2D model.

Tan (1995) presented a method to generate 2D and 3D pseudo relative permeability functions with nonlinear regression. In this paper, a single pseudo relative permeability is generated to represent a fine (3D model) with a coarse 3D model through automatic history matching.

The question: how accurate are dynamic pseudos outside their parent case was discussed by Ekrann and Mykkeltveit (1995). This issue has been addressed before and to some extent, by Thomas (1982) at least with respect to the Kyte and Berry's approach.

Saad et al. (1995) discussed various approaches for the determination and assignment of effective relative permeability and capillary pressure on different scales of heterogeneity. The methods presented in this paper, for calculating effective relative permeability, are labeled: Steady-state simulation technique, analytic method and end-point simulation technique.

Barker and Thibeau (1996) presented a critical review of the use of pseudo relative permeability for upscaling. The properties and limitations of a number of different dynamic pseudo relative permeability methods are discussed and summarized. Barker and Thibeau recognize, as do many others, that dynamic pseudo relative permeability and pseudos in general cannot at present be reliably used to upscale from a geological model to a fluid flow model. Nor is it likely that this will be possible in the near future.

Guzman et al. (1994) presented an evaluation of the reliability of various methods used to generate pseudo functions to upscale flow properties in reservoir simulation. Theoretical and actual field applications were used to evaluate three methods namely: Kyte and Berry, flux weighted potential and Stone. Guzman et al. (1994) reported that their simulator does not allow for unrestricted and non-monotonic relative permeability function (a known limitation of Kyte and Berry's approach) which introduces a bias. At any rate, the authors wisely conclude that each of the methods evaluated can be superior to the others under specified flow conditions. Moreover, these

Table 1
Reservoir properties used in the cross-section (fixed parameters)

| | |
|---|---|
| Location of Water–Oil Contact (Subsea) | 5515 ft |
| Datum depth (Subsea) | 5275 ft |
| Pressure at Datum Depth | 2642 psi |
| Layer Width | 1460 ft (0.5 km) |
| Porosity (Fraction) | 0.28 |
| Connate Water Saturation (Fraction) | 0.12 |
| Horizontal Permeability | 5000 md |
| Rock Compressibility | $15.6\times10^{-1}$ psi$^{-1}$ |
| Water Compressibility | $2.28\times10^{-1}$ psi$^{-1}$ |
| Water Density | 1.1143 cP |
| Water Viscosity | 0.645 cP |
| Water Formation Volume Factor | 1.0 RB/STB |
| Oil Production Rate First Cell | 1–1.2% OOIP/year |
| Depth ($I=1$) Angle 1° | |
| Thickness 110′ (Subsea) First Cell | 5489 ft |
| Depth ($I=1$) Angle 4° | |
| Thickness 110′ (Subsea) First Cell | 6414 ft |
| Depth ($I=1$) Angle 1° | |
| Thickness 50′ (Subsea) First Cell | 5102 ft |
| Depth (I = 1) Angle 4° | |
| Thickness 50′ (Subsea) | 6574 ft |

authors recognize that the upscaling problem is far from being solved and the limitations of current dynamic pseudo functions may warrant the need for a different methodology to upscale relative permeability and capillary pressure.

### 3.1. Parameters affecting pseudo relative permeabilities

Dynamic pseudo relative permeability curves are often derived from detailed vertical cross-sections. These cross-sections should represent the reservoir under conditions to be expected in the areal or three-dimensional simulation model where these pseudo relative permeability curves are to be used. The laboratory derived rock curves are used in these cross-sections among other reservoir representative parameters.

In this study, the PVT properties, ratio of vertical to horizontal permeabilities, layer thickness, production rate, reservoir dip angle and absolute permeability were investigated.

### 3.2. Model description

A vertical fine-gridded cross-section consisting of 3000 cells ($100 \times 30$) was used to study the factors affecting pseudo relative permeability curves. The cross-section can be divided into three identical full-field layers. It is designed to try to eliminate most of the boundary condition effects, especially in the area where the pseudo relative permeability curves are

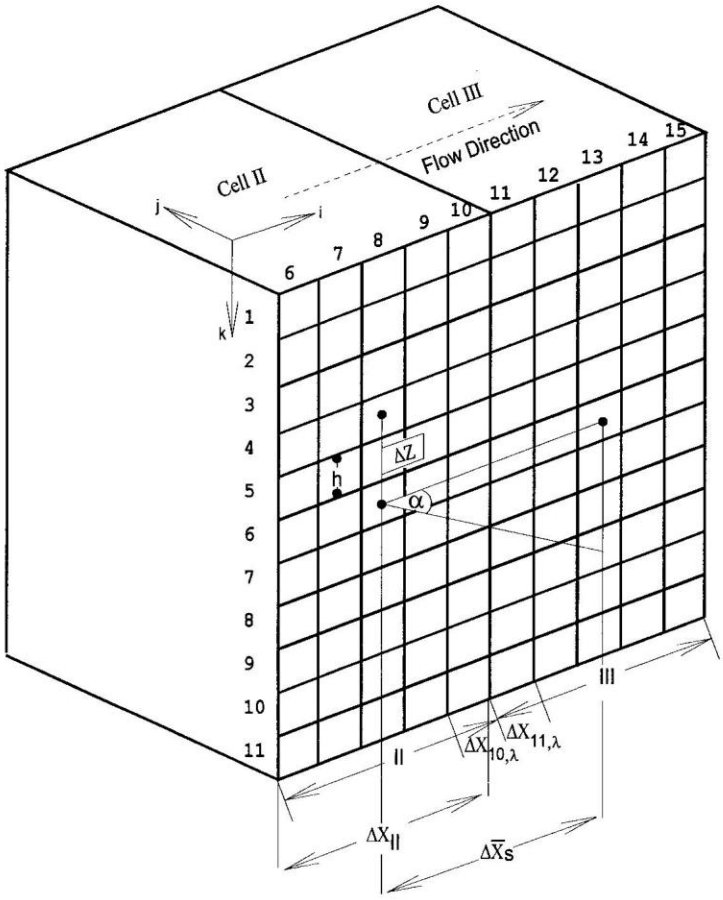


Fig. 3. Schematic diagram showing two adjacent model cells where the pseudo relative permeability curve parameters are calculated.

generated and to have a complete sweep of the area of interest in order to obtain the end-points of studied pseudo relative permeability curves.

Fig. 1 shows a schematic diagram of this cross-section. The rock permeability curve used in all the cross-sectional models is shown in Fig. 2. Table 1 shows the reservoir parameters used in the vertical fine-gridded cross-sectional models. The average oil production rate per year used for all cases is about 1.1% of original oil-in-place which is assumed to be the scaled rate for the historical performance of the reservoir.

### 3.3. Pseudo curves validation

A commercial simulator was used in running the cross-sectional models using a CRAY2 computer. Each coarse grid of the two-dimensional model is equal to five columns of the fine grid. The procedure followed to construct the pseudo relative permeability curves was:

1. A suitable fine grid cross-section is built for the property to be studied,
2. The reservoir flow behavior for the minimum value of the property is simulated by the fine grid cross-sectional model as a function of time, and the run is analyzed,
3. The parameters used in the pseudo relative permeability calculations are obtained,
4. Pseudo relative permeability curves are then calculated using Kyte and Berry (1975) method, and
5. Steps 2 to 4 above are repeated for the maximum value of the property being studied and the results are then compared to assess the impact of this property on the generated pseudo relative permeability.

Fig. 3 shows how a fine grid cross-sectional model is reduced to the corresponding coarse grid cross-sectional model.

Table 2
Oil PVT properties (PVT No. 1)

| Pressure (psi) | $B_o$ (RB/STB) | Density (g/cc) | Viscosity (cP) | Solution gas (SCF/STB) |
|---|---|---|---|---|
| 100.0 | 1.0715 | 2.5147 | 35.06 | 23.0 |
| 300.0 | 1.0903 | 0.9835 | 11.45 | 69.0 |
| 500.0 | 1.1090 | 0.8759 | 6.74 | 115.0 |
| 937.0 (BPP) | 1.1501 | 0.8136 | 3.45 | 215.0 |
| 1000.0 | 1.1490 | 0.8136 | 3.489 | 215.0 |
| 1200.0 | 1.1468 | 0.8136 | 3.560 | 215.0 |
| 1400.0 | 1.1446 | 0.8136 | 3.632 | 215.0 |
| 1600.0 | 1.1425 | 0.8136 | 3.703 | 215.0 |
| 1800.0 | 1.1403 | 0.8136 | 3.775 | 215.0 |
| 2000.0 | 1.1381 | 0.8136 | 3.846 | 215.0 |
| 2200.0 | 1.1360 | 0.8136 | 3.918 | 215.0 |
| 2400.0 | 1.1338 | 0.8136 | 3.989 | 215.0 |
| 2600.0 | 1.1316 | 0.8136 | 4.061 | 215.0 |
| 2800.0 | 1.1295 | 0.8136 | 4.132 | 215.0 |
| 3000.0 | 1.1273 | 0.8136 | 4.204 | 215.0 |
| 3200.0 | 1.1251 | 0.8136 | 4.276 | 215.0 |
| 3400.0 | 1.1230 | 0.8136 | 4.347 | 215.0 |
| 3600.0 | 1.1208 | 0.8136 | 4.419 | 215.0 |
| 3800.0 | 1.1186 | 0.8136 | 4.490 | 215.0 |
| 4000.0 | 1.1165 | 0.8136 | 4.562 | 215.0 |
| 4200.0 | 1.1143 | 0.8136 | 4.633 | 215.0 |
| 4400.0 | 1.1121 | 0.8136 | 4.705 | 215.0 |
| 4600.0 | 1.1100 | 0.8136 | 4.776 | 215.0 |
| 4800.0 | 1.1078 | 0.8136 | 4.848 | 215.0 |
| 5000.0 | 1.1056 | 0.8136 | 4.919 | 215.0 |
| 5200.0 | 1.1035 | 0.8136 | 4.991 | 215.0 |

The use of pseudo relative permeability in coarse grid models is designed to reproduce the water saturation movement in the model without the use of fine grid cross-sectional model. When rock relative permeability curves are used rather than the pseudo relative permeability curves, the water front moves faster which makes the coarse model or full-field model results significantly different, due to the effects of numerical dispersion. In this way, pseudo functions can be used to control the effects of numerical dispersion.

## 4. Results and discussion

In the following discussions, the parameters used are specified and their effect is shown for all cases studied. Individual pseudo relative permeability curves for all the cases is studied in detail by Al-Otaibi (1992).

### 4.1. Reservoir PVT properties

The two sets of PVT data shown in Tables 2 and 3 were used for this reservoir. Each of the two PVT data sets was used in all sets of cross-sections. The comparison of the resultant pseudo relative permeability curves are shown in Figs. 4 and 5. The PVT properties have an effect on pseudo relative permeability curves, which is minimal in the cross-sections where the thickness is large and the dip angle is low (i.e., 1°). The maximum effect is noticed for the smaller thickness (i.e., 50 ft.), and a dip angle higher than 4°. This effect is shown for the heavier oil but in a lower scale compared to the lighter oil.

Table 3
Oil PVT properties (PVT No. 2)

| Pressure (psi) | $B_o$ (RB/STB) | Density (g/cc) | Viscosity (cP) | Solution gas (SCF/STB) |
|---|---|---|---|---|
| 100.0 | 1.0715 | 2.5147 | 35.06 | 23.0 |
| 300.0 | 1.0903 | 0.9835 | 11.45 | 69.0 |
| 500.0 | 1.1090 | 0.8759 | 6.74 | 115.0 |
| 1000.0 | 1.1560 | 0.8094 | 3.21 | 229.0 |
| 1200.0 | 1.1747 | 0.7993 | 2.63 | 275.0 |
| 1389 (BPP) | 1.1925 | 0.7925 | 2.23 | 318.0 |
| 1400.0 | 1.1924 | 0.7925 | 2.236 | 318.0 |
| 1600.0 | 1.1901 | 0.7925 | 2.307 | 318.0 |
| 1800.0 | 1.1879 | 0.7925 | 2.379 | 318.0 |
| 2000.0 | 1.1856 | 0.7925 | 2.451 | 318.0 |
| 2200.0 | 1.1834 | 0.7925 | 2.522 | 318.0 |
| 2400.0 | 1.1811 | 0.7925 | 2.594 | 318.0 |
| 2600.0 | 1.1789 | 0.7925 | 2.665 | 318.0 |
| 2800.0 | 1.1766 | 0.7925 | 2.737 | 318.0 |
| 3000.0 | 1.1744 | 0.7925 | 2.808 | 318.0 |
| 3200.0 | 1.1721 | 0.7925 | 2.880 | 318.0 |
| 3400.0 | 1.1699 | 0.7925 | 2.951 | 318.0 |
| 3600.0 | 1.1676 | 0.7925 | 3.023 | 318.0 |
| 3800.0 | 1.1654 | 0.7925 | 3.094 | 318.0 |
| 4000.0 | 1.1632 | 0.7925 | 3.166 | 318.0 |
| 4200.0 | 1.1609 | 0.7925 | 3.237 | 318.0 |
| 4400.0 | 1.1587 | 0.7925 | 3.309 | 318.0 |
| 4600.0 | 1.1564 | 0.7925 | 3.380 | 318.0 |
| 4800.0 | 1.1542 | 0.7925 | 3.452 | 318.0 |
| 5000.0 | 1.1519 | 0.7925 | 3.523 | 318.0 |
| 5200.0 | 1.1497 | 0.7925 | 3.595 | 318.0 |

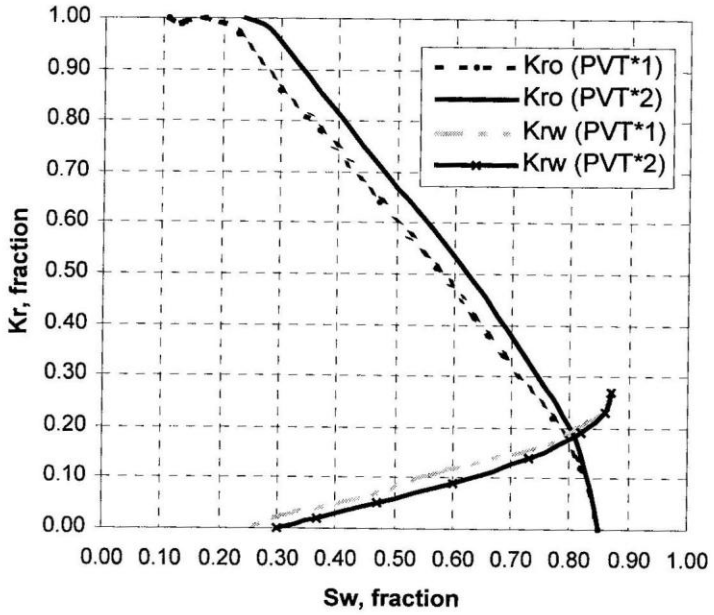


Fig. 4. PVT data effect for the cross-section—thickness = 110′, layers = 11–20, Kv/Kx = 0.7, cells = 71–80, angle 4°.

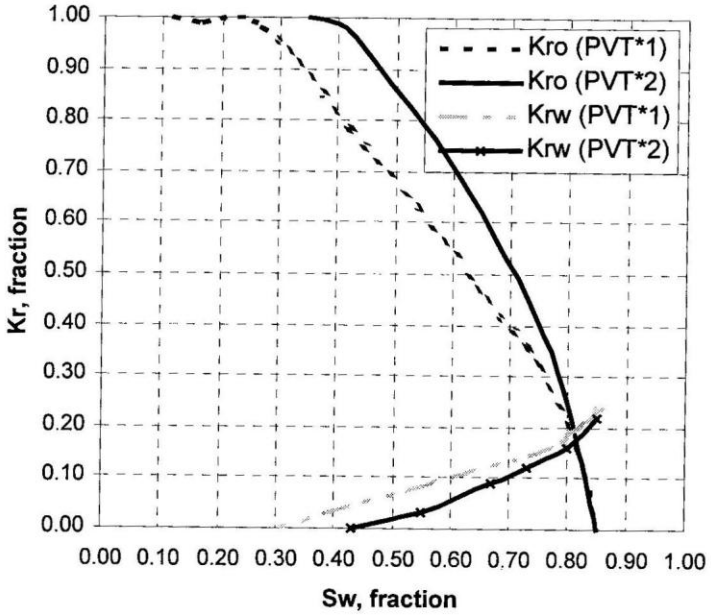


Fig. 5. PVT data effect for the cross-section—thickness = 50′, layers = 11–20, Kv/Kx = 0.7, cells = 71–80, angle 4°.

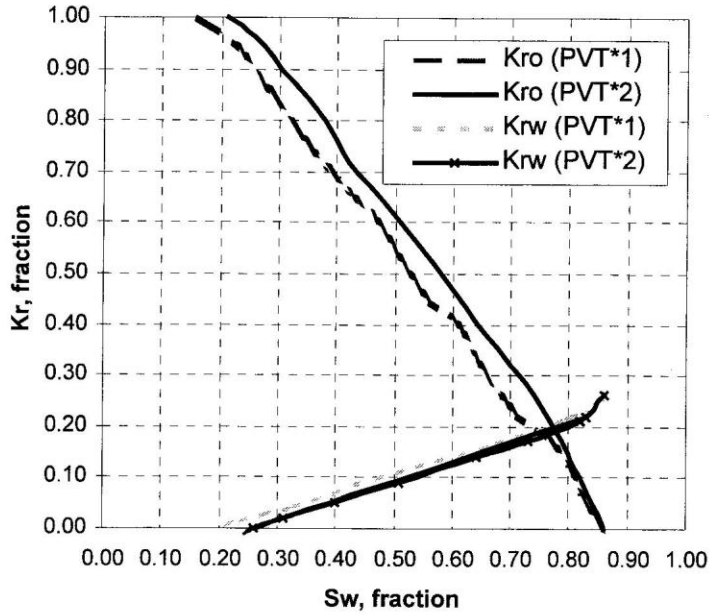


Fig. 6. Layer thickness effect for the cross-section—angle 1°, layers = 11–20, Kv/Kx = 0.7, cells = 71–80, PVT No. 2.

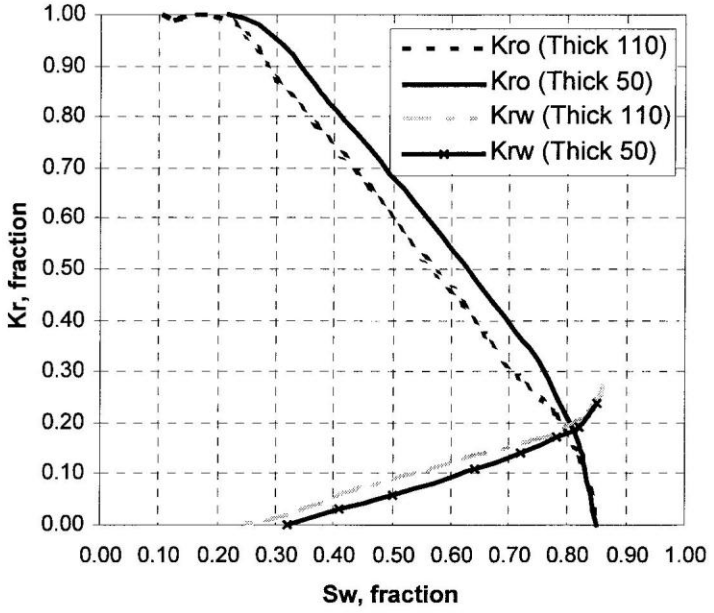


Fig. 7. Layer thickness effect for the cross-section—angle 4°, layers = 11–20, Kv/Kx = 0.7, cells = 71–80, PVT No. 1.

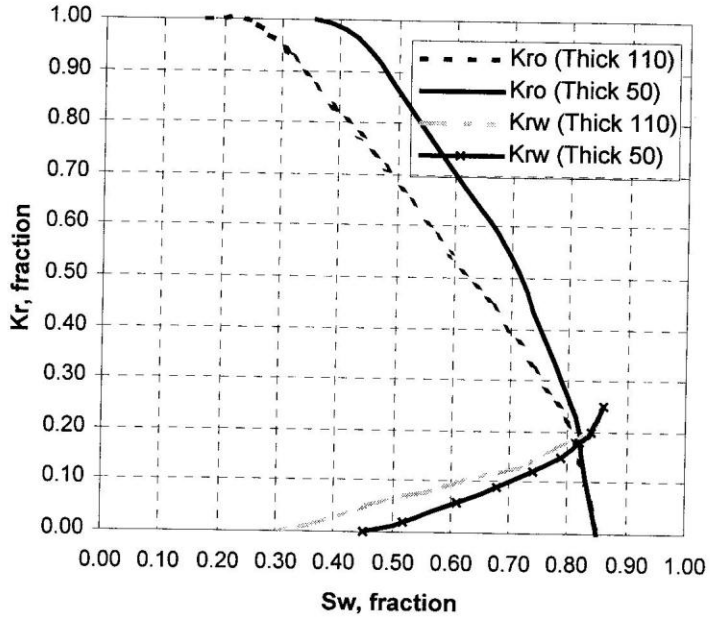


Fig. 8. Layer thickness effect for the cross-section—thickness = 110′, angle 4°, layers = 11–20, Kv/Kx = 0.7, cells = 71–80, PVT No. 2.

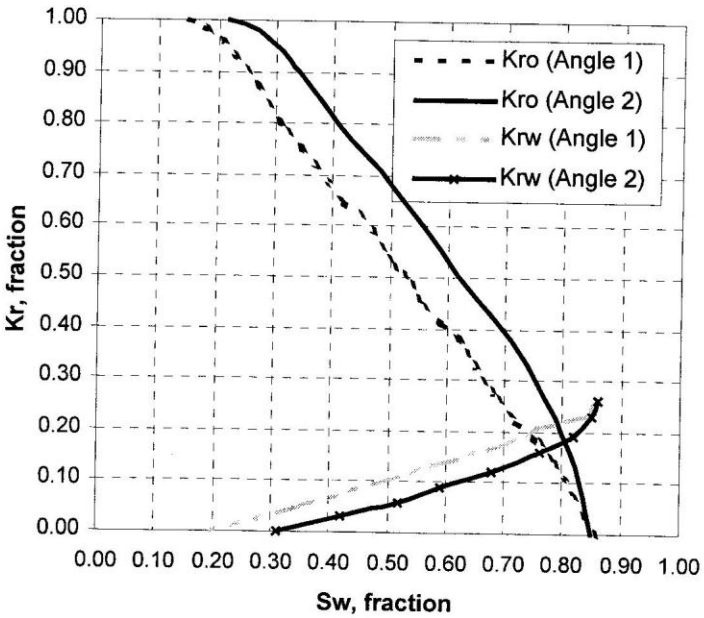


Fig. 9. Dip angle effect for the cross-section—thickness = 110′, layers = 11–20, Kv/Kx = 0.7, cells = 71–80, PVT No. 2.

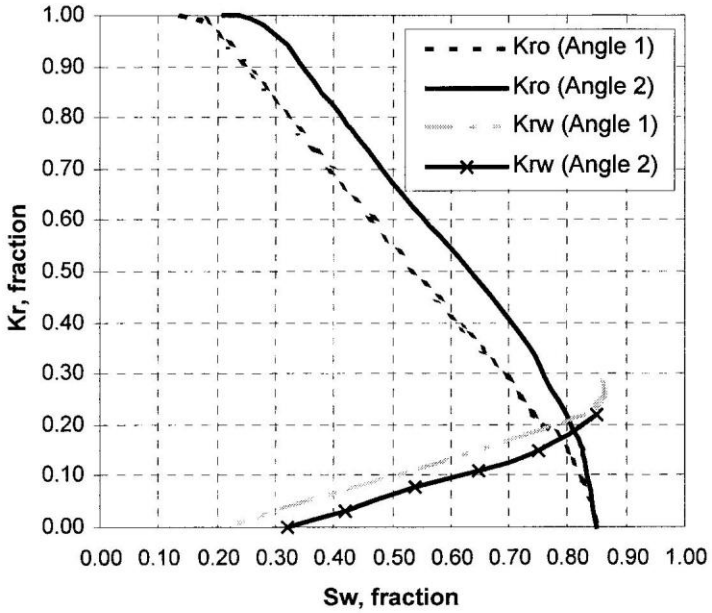


Fig. 10. Dip angle effect for the cross-section—thickness = 50′, layers = 11–20, Kv/Kx = 0.7, cells = 71–80, PVT No. 1.

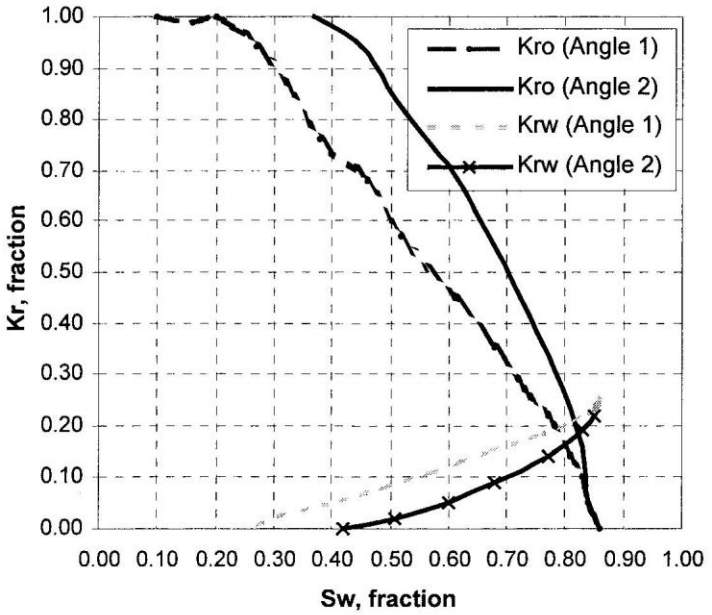


Fig. 11. Dip angle effect for the cross-section—thickness = 50′, layers = 11–20, Kv/Kx = 0.7, cells = 71–80, PVT No. 2.

### 4.2. Ratio of vertical permeability to horizontal permeability

The ratio of vertical to horizontal permeability (over the range studied: 0.3–0.7) does not have a significant effect on this set of data, as a homogeneous reservoir was studied.

### 4.3. Effect of layer thickness

Two layer thicknesses of 50 ft. and 110 ft. were used. The pseudo relative permeability curves generated from the cross-sectional models for these parameters are shown in Figs. 6–8. The layer thickness always affects the pseudo relative permeability curves. The most significant effect is caused by the combination of the smallest thickness and the highest dip angle of 4°. A minimum effect is noticed for the minimum dip angle and the highest thickness which is 110 ft.

### 4.4. Production rate

The production rates used have an effect only for the case having the smallest layer thickness of 50 ft. and the highest dip angle of 4°.

### 4.5. Reservoir dip angle

Two reservoir dip angles of 1° and 4° were investigated. The resulting pseudo relative permeability curves are shown in Figs. 9–11. Among the factors studied, the dip angle has the most significant influence on pseudo relative permeability curves. The higher the angle, the more hold-up is noticed because the tilt of the water front is lower and the resulting sweep is better. The dip angle effect becomes more pronounced for the small thickness cases and the maximum effect is seen for the case where the dip angle is the maximum and the layer thickness is the smallest.

### 4.6. Absolute permeability

A dip angle of 4° has a noticeable effect on absolute permeability; but for the cases where the 1° angle is used, the effect is almost negligible. This observation suggests that for horizontal reservoirs or reservoirs with low dip angles, the absolute permeability is not a factor to be considered in the generation of pseudo relative permeability curves for these reservoirs. The maximum effect is seen in the case with the lowest layer thickness and the highest dip angle.

## 5. Conclusions

Based on the discussion presented in this paper, the following conclusions are drawn with regard to the factors affecting pseudo-relative permeability curves.

(1) The PVT data have an effect on pseudo relative permeability curves. The effect becomes more pronounced for the smaller thickness and higher dip angle.

(2) Vertical to horizontal permeability ratio (over the range studied: 0.3–0.7) has minimal effect on pseudo relative permeability curves regardless of reservoir dip angle or reservoir layer thickness.

(3) The model layer thickness has an effect on pseudo relative permeability curves in all cases. The smaller the layer thickness, the better the sweep.

(4) The effect of production rate on the pseudo relative permeability curves was minimal for the range of data used in this study.

(5) The most pronounced effect on the pseudo relative permeability curves is caused by the reservoir dip angle.

(6) Absolute permeability has an effect only in the cases where the dip angle is high regardless of the layer thickness changes. However, the extremely large reservoir permeability considered may mask any small variation.

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