

A Laboratory Manual for the

**ME 316: Thermo-Fluids
Laboratory**

(Part b: Heat Transfer)

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CLEANLINESS AND SAFETY

Cleanliness

There are certain rules that the user of the laboratory should be aware of and abide by. Equipment in the lab is delicate and each piece is used extensively for 2 or 3 weeks per semester. During the remaining time, each apparatus just sits there, literally collecting dust. University housekeeping staff is not required to clean and maintain the equipment. Instead, there is a technician who will work on the equipment when it needs repair, and when he is notified that a piece of equipment needs attention. It is important, however, that the equipment stay clean, so that dust will not accumulate too badly. The Heat Transfer Laboratory contains equipment that uses water or air as the working fluid. In some cases, performing an experiment will inevitably allow water to get on the equipment and/or the floor. If no one cleaned up his working area after performing an experiment, the lab would not be a comfortable or safe place to work in. No student appreciates walking up to and working with a piece of equipment that another student or group of students has left in a mess. Consequently, students are required to clean up their area at the conclusion of the performance of an experiment. Cleanup will include removal of spilled water (or any liquid), and wiping the tabletop on which the equipment is mounted (if appropriate). The lab should always be as clean or cleaner than it was when you entered. Cleaning the lab is your responsibility as a user of the equipment. This is an act of courtesy that students who follow you will appreciate, and that you will appreciate when you work with the equipment.

Safety

The layout of the equipment and storage cabinets in the Heat Transfer Laboratory involves resolving a variety of conflicting problems. These include traffic flow, emergency facilities, environmental safeguards, exit door location, etc. The goal is to implement safety requirements without impeding egress, but still allowing adequate workspace and necessary informal communication opportunities. Distance between adjacent pieces of equipment is determined by locations of water supply valves, drains, electrical outlets, and by the need to allow enough space around the apparatus of interest. Immediate access to the Safety Cabinet is also considered. Emergency facilities such as showers, eye wash fountains, spill kits; fire blankets and the like are not available in the lab. We do not work with hazardous materials and such safety facilities are not necessary.

Safety Procedures

There is unmistakably only one, clearly marked exit in this laboratory. It has a single door and leads directly to the hallway on the ground floor of building # 1. In case of fire, exit the lab to the hallway. After closing the door, turn right, and leave the building. There is a safety cabinet attached to the wall of lab adjacent to the door. In case of personal injury, the appropriate item should be taken from the supply cabinet and used in the recommended fashion. If the injury is serious enough to require professional medical attention, the student(s) should contact the University Clinic, Extension 3333. Every effort has been made to create a positive, clean, safety

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conscious atmosphere. Students are encouraged to handle equipment safely and to be aware of, and avoid being victims of, hazardous situations.

Safety Clothing

Students should be properly dressed for the laboratory work. Wearing a shirt, pants and shoes is mandatory for attending the laboratory session. It is important to note that students will not be permitted to the lab sessions unless they are properly dressed.

THE CODE OF STUDENT CONDUCT

The King Fahd University of Petroleum & Minerals (KFUPM) students are citizens of the Kingdom of Saudi Arabia, and of the KFUPM academic community. They are, therefore, expected to conduct themselves as law-abiding members of each community at all times. Admission to the University carries with it special privileges and imposes special responsibilities apart from those rights and duties enjoyed by non-students. In recognition of this special relationship that exists between the institution and the academic community, which it seeks to serve, the KFUPM administration has instructed to take such action as may be necessary to maintain campus conditions and to preserve the integrity of the institution and its educational environment. The following regulations (known as the Code of Student Conduct) have been developed by a committee made up of faculty, students, and staff utilizing input from all facets of the University Community in order to provide a secure and stimulating atmosphere in which individual and academic pursuits may flourish. Students are, however, subject to all Kingdom laws and ordinances. If a student's violation of such laws or ordinances also adversely affects the University's pursuit of its educational objectives, the University may enforce its own regulations regardless of any proceeding instituted by other authorities. Additionally, violations of any section of the Code may subject a student to disciplinary measures by the University whether or not such conduct is simultaneously violation of Kingdom laws. The term "academic misconduct" includes, but is not limited to, all acts of **cheating** and **plagiarism**. The term "**cheating**" includes, but is not limited to:

- Use of any unauthorized assistance in taking quizzes, tests, or examinations;
- Dependence upon the aid of sources beyond those authorized by the instructor in writing papers, preparing reports, solving problems, or carrying out other assignments;
- The acquisition, without permission, of tests or other academic material before such material is revealed or distributed by the instructor;
- The misrepresentation of papers, reports, assignments or other materials as the product of a student's sole independent effort, for the purpose of affecting the student's grade, credit, or status in the University;
- Failing to abide by the instructions of the proctor concerning test-taking procedures; examples include, but are not limited to, talking, laughing, failure to take a seat assignment, failing to adhere to starting and stopping times, or other disruptive activity;
- Influencing, or attempting to influence, any University official, faculty member, graduate student or employee possessing academic grading and/or evaluation authority or responsibility for maintenance of academic records, through the use of bribery, threats, or any other means or coercion in order to affect a student's grade or evaluation;

- Any forgery, alteration, unauthorized possession, or misuse of University documents pertaining to academic records, including, but not limited to, late or retroactive change of course application forms (otherwise known as “drop slips”) and late or retroactive withdrawal application forms. Alteration or misuse of University documents pertaining to academic records by means of computer resources or other equipment is also included within this definition of “**cheating**.” The term “**plagiarism**” includes, but is not limited to, the use, by paraphrase or direct quotation, of the published or unpublished work of another person without full or clear acknowledgment. It also includes the unacknowledged use of materials prepared by another person or agency engaged in the selling of term papers or other academic materials.

Course Policy

Academic misconduct (acts of cheating and of plagiarism) ***will not be tolerated***. The University policy is quite specific regarding the course of action to be taken by an instructor in cases where academic misconduct may be an issue.

REPORT WRITING

All reports in the Thermofluids Laboratory require a formal laboratory report unless specified otherwise. The report should be written in such a way that anyone could duplicate the experiment and obtain the same results as the originator. The reports should be simple and clearly written. Reports are due one week after the experiment was performed, unless specified otherwise. The report should communicate several ideas to the reader. First the report should be carefully done. The experimenter is in effect trying to convince the reader that the experiment was performed in a straightforward manner with great care and with full attention to detail. A poorly written report might instead lead the reader to think that just as little care went into performing the experiment. Second, the report should be well organized. The reader should be able to easily follow each step discussed in the text. Third, the report should contain accurate results. This will require checking and rechecking the calculations until accuracy can be guaranteed. Fourth, the report should be free of spelling and grammatical errors. The following format is to be used for formal Laboratory Reports.

Sample Title Page

Experiment Number

TITLE OF THE EXPERIMENT

Name of the Author

Name of Partner #1

Name of Partner #2

etc.

Date the Experiment was Performed

Due date for this Report

The King Fahd University of Petroleum & Minerals
Mechanical Engineering Department

ABSTRACT

This report was designed to contain the instructions on how to write a report, and to serve as an example of the format and style expected in all reports. It was based on the style and format of standard engineering reports used in the writing of professional

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engineering publications. The Title Page and Abstract are the first two components of the report.

The Abstract summarizes the information in the report. It provides a brief summary of the objective of the experiment, the procedures, the results, conclusions and recommendations. It should not reference any tables, figures or appendices. A short abstract may appear on the title page as in this example. A longer abstract would appear on the sheet following the Title Page.

The Abstract allows the reader to determine whether to read the report. It is written in the past tense, except for the recommendations, which may be written in the present or future tense.

This report was proofread by

(Signature of proofreader)

INTRODUCTION

This section tells the reader what the experiment is about. It begins with a description of the problem that is being investigated. It includes the background and refers to related experiments and publications. When the work of others is quoted, it is done by appropriately referencing their work. For example, the report will contain a list of references near the end, and are listed alphabetically by the author's last name. If the reference has two authors, include the last names of each author in the citation. If the reference has three or more authors, use the first author's last name and the abbreviation "et al."

Examples:

Callinan (1992) provides an example of report writing, and has found that it improves the writing skills of the students. A detailed report writing method was found to improve the writing skills of the students

In 1992, Callinan showed that a detailed report writing method improved the writing skills of the students.

Çengel and Boles (1998) provide a derivation of the energy equation.

Paré et al. (1997) investigated the graphical solution method for a number of descriptive geometry problems.

Subheadings

The beginning of this section showed how a major heading should appear in a report: all capital letters, boldface type, and left justified. It may be necessary to use subheadings, and the format for these is shown at the beginning of this paragraph. Note also, that the beginning of a paragraph is not indented, but instead is preceded by a blank line.

Write for the Reader

Consider that the report is written for a technically competent person who is unfamiliar with the specific subject matter, but will be after he reads the report. Also consider that the reader is not as closely associated with the test as you are. Check grammar and spelling. Check continuity of page, figure, and table numbers. Have an associate who did not perform the experiment with you, but who has technical competency, proofread your report.

Report Preparation

Reports must be composed on a word processor. Use white paper and black text. Use only one side of a page. All margins should be 1 in. Do not right justify the text. Each section does not need to begin on a new page. Each page is to be numbered with an Arabic numeral centered at the bottom of the page. Do not number the title page. Begin numbering with page 2.

Figures should be numbered sequentially using Arabic numbers. Each figure is to have a descriptive title. Figures should be drawn using a computer and a drawing program, or use the figures available with the lab manual. Figures are to be located near the place in the text where they are first referred. Figures should be centered left-to-right either on the page (single column) or within the column (two or more columns). The figure number and title should appear centered just below the figure itself.

Tables should contain as much information as possible. They are to be enclosed in a border. They can be placed in the text or at the end of the section where they are first referred. Tables are to be numbered consecutively with Arabic numerals. An acceptable table format is as follows:

TABLE 1 *Reduced data for heat transferred past a flat plate*

Run	Velocity (V), in m/s	Heat Transferred (Q), in W	Temperature (T), in °C
1	0.5	13.6	74.6
2	1.0	16.3	75.5
3	1.5	17.4	73.4
4	2.0	18.8	77.8

Note carefully the following features regarding this table:

- The first column is “trial” or “run.”
- Each column heading is of a parameter, followed by the symbol and the unit.
- Each column heading is centered within the column.
- The table is centered left to right within the page or column.
- A border has been placed around the table, and around each cell.
- The font size is smaller than that used for the text in the report.
- “**TABLE 1**” is in all capital letters, and the actual title is in italics.

Graphs In many instances, it is essential to compose a plot in order to graphically present the results. Graphs must be drawn carefully following a specific format. Figure R.1 shows an acceptable graph prepared using a computer. There are many PC-based programs that have graphing capabilities. An acceptable drawn graph has several features of note. These features are summarized next to the Figure. Graphs especially should have descriptive titles. A graph of temperature versus time, for example, should *not* have a title of:

FIGURE 1. *Temperature versus time.*

The reader can see by looking at the graph that this is so. A better title would be:

FIGURE 1. *Temperature variation with time for a brass sphere cooling in air.*

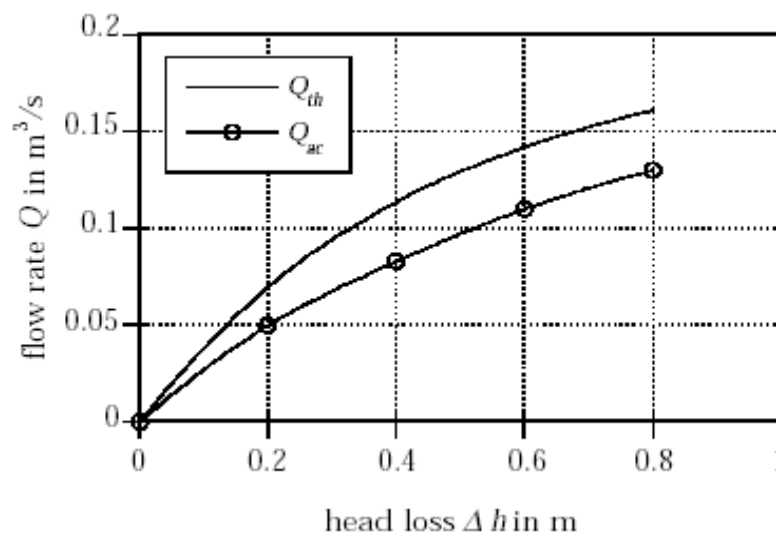


FIGURE R.1. *Theoretical and actual volume flow rate through a Venturi meter as a function of head loss.*

Note that “**FIGURE 1**” is in all capital letters, and the actual title is in italics.

Important features to note

- Border is drawn about the entire graph.
- Axis labels defined with symbols and units.
- A grid is drawn using major axis divisions.
- Each line is identified using a legend.
- Data points are identified with a symbol on the Q_{ac} line to denote data points obtained by experiment.
- The line representing the theoretical results has no data points represented.
- Nothing is drawn freehand.
- Title is descriptive, rather than something like Q vs. h .

Writing Style

Use simple words with exact meanings. Use technical terms to express a precise technical meaning. Do not use a large and unusual term to add false importance to the report or to yourself. Do not use slang words or expressions.

Use simple sentences that have a subject and a predicate. Add adjectives as required. Avoid extra long sentences. Never use “I” regardless of what you were taught in any previous courses. “We” is acceptable. Insert only one space between a period and the beginning of the next sentence.

Do not bind the report in a folder. Staple the pages together in the upper left-hand corner.

The introduction section should conclude with a brief statement of what the objective of the experiment is.

The Introduction is written in the past or present tense.

The report can be written using 1 column or two columns per page.

THEORY AND ANALYSIS

This section explains the theory associated with this experiment. The theory should be discussed in much greater detail in this section than in the introduction. It should contain an explanation of the theoretical model. For example, if an experiment was performed with a fan, then include a brief derivation of the mathematical model of a fan. Put the significant portions of the derivation in this section. Cite references using the guidelines previously described. Include simple sketches or diagrams to help the reader visualize the physical phenomenon being studied.

If there is little or no theory involved in this experiment, include the theory with the Introduction section.

All equations in the report should be indented and numbered consecutively with Arabic numerals. Each symbol in the equations should be named and its dimensional unit given.

An example:

“Newton’s Second Law of Motion (Resnick and Halliday, 1966) is:

$$F = ma \tag{1}$$

where F is the unbalanced external force in N, m is the mass of the block in kg, and a is the acceleration in m/s^2 .”

There are several important details associated with this example, specifically in the way equations and units are written. Note that the letters used in the equation are in italics. Every reference to force, for instance, is in italics. The units used for each variable are in normal type (e.g., non italics). Numerical subscripts and superscripts

are in normal type as well. However, subscripts and superscripts that are variables are italicized.

When a number is written with a unit, a space should separate the two. For example, 5 N, or 17.3 kPa. Numbers are written in normal type.

These features are very important in report writing. It is these features that will make a well-done technical report appear professional in every way.

The theory section in its entirety is written in the past or present tense.

PROCEDURE

This section describes the equipment used in the experiment and the test procedures. The equipment setup should be shown in a figure. The test equipment and instrumentation used should be listed with model and/or serial numbers, and the expected instrument precision. Figures of specific components should be provided if necessary to help the reader to better understand the test procedure.

Briefly describe the steps of the experimental procedure in the order in which they were conducted. Include sufficient detail in this section such that the reader could repeat the experiment.

The procedure is written in the past tense.

RESULTS AND DISCUSSION

The section should discuss the results. Summarize your outcome in the topic sentence, and then support that summary with the results. Use graphs and tables to concisely present the results. Do not draw conclusions in this section, only list and discuss results. This is also the section where a comparison of results with referenced values should be presented.

A sample calculation should also be provided. Start with raw data obtained while performing the experiment, and show the calculations involved in finding one of the numbers in this section.

The Results and Discussion section should be written in the past tense.

CONCLUDING REMARKS

This section is a clear and concise qualitative and quantitative summary of the experiment and results. It includes conclusions, observations, trends, and recommendations. Recommendations are especially valuable if the experiment failed or was impaired. Do not refer to tables or figures in this section. Coordinate the material in this section with the Introduction section. If there was a clear objective in this experiment, state whether the objective was reached. Make recommendations regarding the experiment.

Do not use sentences such as “We learned a lot in this experiment.” Remember that your perspective is that of an engineer writing a technical report to others who are technically minded. It is not that of a student writing to a lab report grader.

The conclusions should be written in the past or present tense, except for the recommendations, which are in the future tense.

APPENDICES

The Appendix section contains its own title page, with a list of what the reader will find inside.

References.

This portion of the appendix lists references used in the preparation of the report. You must cite the source publication for the work of all others, which you include. This gives them due credit for their work, and shows the research effort you put into your report. Do not list the lab manual as a reference. An example of an alphabetical Reference list follows:

Bannister, L. (1991). *University Style Manual*, Fourth edition, Los Angeles: Loyola Marymount University, pp. 36–39 and pp. 58–61.

Kovarik, M. (1989). “Optimal Heat Exchangers,” *Journal of Heat Transfer*, **111**, pp. 287–293.

Main, B. W. and A. C. Ward. (1992). “What do Design Engineers Really Know About Safety?” *Mechanical Engineer*, **114**(8), pp. 44–51.

Resnick, R. and D. Halliday. (1966) *Physics*. New York: John Wiley.

Jeter, S., and J. Donnell. (2004). *Writing Style and Standards in Undergraduate Report*, Virginia: College Publishing.

Original Data Sheet

The data sheet completed when the experiment was conducted is included here.

Calibration Curves

If provided by the instructor or the manufacturer of the lab equipment, calibration curves for each meter used should be included in this section.

SHORT FORM REPORT FORMAT

Once in a while the experiment requires not a formal report but an informal report. An informal report includes the **Title Page, Experiment Objective, Procedure, Results, and Conclusions**. Other portions may be added at the discretion of the instructor or the writer.

SUMMARY OF WRITING THE LAB REPORT

TITLE PAGE	Experiment Number, Title of the Experiment, Name of the Author, Name of Partners, Date the Experiment was Performed, Due date for this Report, The University of Petroleum & Minerals, Mechanical Engineering Department, proofreader's signature.
ABSTRACT	It's a brief summary of the objective of the experiment, the procedures, the results, conclusions and recommendations. Present or future tense.
INTRODUCTION	Description of the problem, references, objective. Past tense.
THEORY AND ANALYSIS	Theory associated with this experiment, equation derivation. Past or present tense.
PROCEDURE	Describes the equipment used, equipment setup, model and/or serial numbers, experimental procedure. Past tense.
RESULTS AND DISCUSSION	Summarize your outcome, graphs and tables, sample calculation. Past tense.
CONCLUSIONS	Conclusions, observations, trends, and recommendations. Past or present tense; recommendations in future tense.
APPENDICES	Title page, references, original data sheet, and calibration curves.

EXPERIMENT 1

Thermocouple Experiment

Objective

To experience and understand, the operation and use of thermocouples for temperature measurements.

Background

One of the most important activities in experimental thermo-fluid sciences is the measurement of temperature. The temperature of a surface, fluid, or solid body will provide much of the information concerning the heat transfer processes for a given thermal system. There are many ways to measure temperature. These include, to mention only a few, thermocouples, thermometers, resistance temperature detectors (RTDs), and thermistors. In this experiment we will work basically with thermocouples.

A thermocouple consists of two wires of two different materials that are joined at each end. When these two junctions are kept at different temperatures a small electric current is induced. Due to the flow of current a voltage drop occurs. This voltage drop depends on the temperature difference between the two junctions. This phenomenon is called Seebeck effect. The measurement of the voltage drop (or Emf) can then be correlated to this temperature difference.

It is important to note that a thermocouple does not measure the temperature, but rather the temperature difference between the two junctions. In order to use a thermocouple to measure temperature directly, one junction must be maintained at a known temperature. This junction is commonly called the reference junction having the reference temperature. The other junction, which is normally placed in contact with the body of unknown temperature, is called the measurement junction.

In experimental heat transfer we often encounter problems in which the temperature in the vicinity of a thermocouple is changing continuously. Since a thermocouple has finite mass and thus finite heat capacity, it cannot respond instantaneously to a temperature change. Considering a lumped capacitance model, the conservation of energy (or the first law of thermodynamics) for this process can be represented by the following differential equation [1-3],

$$MC_p \frac{dT}{dt} = hA_s(T_o - T) \quad (1.1)$$

where

- M is the mass of thermocouple (measurement junction)
C is the specific heat of thermocouple (measurement junction)
H is the heat transfer coefficient
 A_s is the surface area of thermocouple
T is the measurement junction temperature
 T_o is the surrounding temperature in the vicinity of the thermocouple.

Introducing dimensionless temperature difference

$$\theta = \frac{T - T_o}{T_i - T_o} \quad (1.2)$$

where T_i is the initial measurement junction temperature. The solution of Equation (1.1), is given by

$$\theta = e^{-t/\tau} \quad (1.3)$$

where

$$\tau = \frac{MC_p}{hA_s} \quad (1.4)$$

In the above equation τ is defined as time constant for this process. It is important to note that the time-dependent response of a thermocouple can be quantified by this time constant, as will be explained later in this handout.

Finally, there are three additional laws dealing with thermocouples.

- **Law of Homogeneous Metals:** A thermoelectric circuit cannot be sustained in a circuit of a single homogeneous material, however varying in cross section, by the application of heat alone. That is, two different materials are required for any thermocouple circuit.
- **Law of Intermediate Metals:** A third homogeneous material can always be added in a thermocouple circuit with no effect on the net emf of the circuit provided that the extremities of the third material are at the same temperature.
- **Law of Successive or Intermediate Temperatures:** If two dissimilar homogeneous metals produce a thermal emf of E_1 , when the junctions are at temperatures T_1 and T_2 , and a thermal emf of E_2 , when the junctions are at T_2 and T_3 , the emf generated when the junctions are at T_1 and T_3 , will be $E_1 + E_2$.

Experimental Procedure

The experiment you will be conducting in laboratory consists of three parts:

- (A) Fabrication of thermocouples
- (B) Calibration of thermocouples
- (C) Time response of thermocouples.

(A) Thermocouple Fabrication

Thermocouples can be composed of many different pairs of metals and the junctions can be formed in many different ways. For a variety of reasons, different pairs of metal are used for different applications. For our experiment we will use the following two types of thermocouples:

Copper/Constantan or Type T

It should be noted that copper has blue insulation and is the positive lead, while the constantan has red insulation and is the negative lead.

Iron/Constantan or Type J

In this arrangement iron has white insulation and is the positive lead, while the constantan has red insulation and is the negative lead.

We will fabricate thermocouples by the following methods

- Mechanical tying
- Soldering
- Spot welding

It is expected that each experimental group will construct six thermocouples: 4 - Type J (two by mechanical tying, one by soldering, and one by spot welding); 2 - Type T (one by mechanical tying and one by soldering).

The step-by-step procedure that you are supposed to follow in the laboratory is given below:

- (1) Check out thermocouple wire, a pair of pliers, and wire strippers from your instructor.
- (2) Strip approximately 1/2 inch of the leads from both ends of the wire.
- (3) For two iron/constantan wire pairs and one copper/constantan pair twist together the wires at one end. You have now made your mechanically tied thermocouples.
- (4) For the soldered thermocouples form the wires at one end into an oval shape so that the two wires nearly touch at a single point. Next, form a small pool of solder on the soldering plate. Keeping the pool liquid with the soldering iron, dip the thermocouple into the pool so that the solder will form a bridge between the two wires.

- (5) For the spot welded thermocouple, overlap the two wires at one end and flatten the wires at their point of crossing. Place this junction on the welding plate. Turn the spot welder on and set the power and timing switches. Carefully take the electrode end of the welder and press it to the junction until the welder fires. You may have to attempt this several times, varying the power and time until a good weld is achieved.

(B) Calibration of thermocouples

The thermocouples you have constructed must now be calibrated before it is used in any experiment. To calibrate, we measure the thermocouple voltage at various known temperatures, so as to develop a correlation between thermocouple voltage and thermocouple temperature. This correlation may be represented by a graph similar to that shown below.

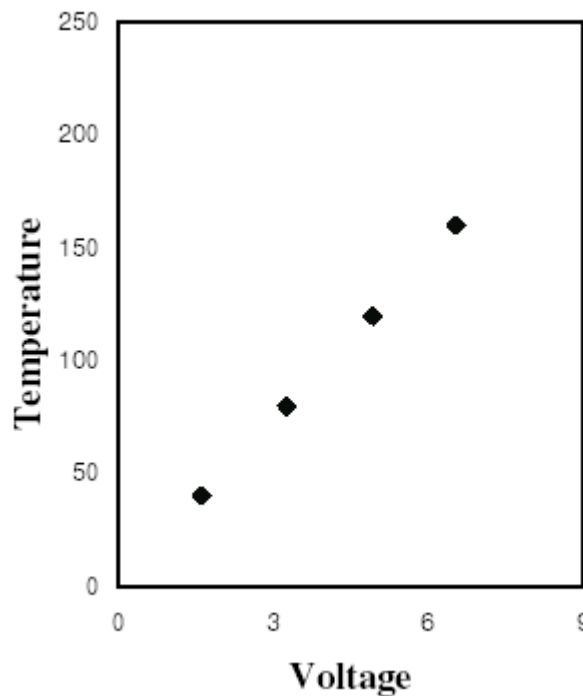


Figure 1.1: Sample of thermocouple calibration curve

The calibration is achieved with the use of a small block furnace that serves as the constant temperature heat reservoir. Your lab instructor will describe the operation of this device.

- (1) Attach the loose end of each thermocouple to the rotary selection switch. When attaching the thermocouples, note the polarity of the poles on the rotary selection switch. Since this is the first point in the circuit where the thermocouple will "see" dissimilar metals, it will serve as the reference

junction. Hence the temperature of the rotary selection switch must be measured for each thermocouple reading. To determine this temperature, a mechanically tied thermocouple of each type is employed. These thermocouples are inserted into an ice point calibration cell which maintains the temperature at 0°C , $\pm 0.1^{\circ}\text{C}$. Thus, for these two thermocouples (called the ice point thermocouples), the reference junction is in the ice point calibration cell and the measurement junction is at the rotary selection switch (which is the reference junction for the other four thermocouples).

- (2) With the furnace set at approximately 50°C , insert the remaining four thermocouples (called the calibration thermocouples) into the core and record the readings from the digital multi-meter. You also need to record the readings for the ice point thermocouples. Repeat this procedure at approximately 100°C and 150°C . At 150°C the calibration procedure may be suspended and the time response tests are then conducted. After the time response tests, the temperature of the furnace is increased to 180°C and the final calibration point is taken.
- (3) It will prove useful to record the data on an Excel spreadsheet. Set-up a spreadsheet of the form described below.

Table 1.1: Thermocouple experiment: raw and calibrated data

	Calibration Thermocouples (raw data)				Ice-Point Thermocouples		Calibration Thermocouples (ice-point corrections)			
T($^{\circ}\text{C}$)	TC # 1 (mV)	TC # 2 (mV)	TC # 3 (mV)	TC # 4 (mV)	TC # 5 (mV)	TC # 6 (mV)	TC # 1 (mV)	TC # 2 (mV)	TC # 3 (mV)	TC # 4 (mV)

The shaded regions on the spreadsheet indicate cells where the students will make entries, while blank cells require an equation (or numbers) to be inputted. In this case the equation will be the subtraction of the voltage of the appropriate ice point thermocouple, either TC # 5 or TC # 6, from the measured voltage of the calibration thermocouple.

You will be comparing your calibration to standard tables, which are determined for a reference junction at 0°C . Subtract the voltage reading of the appropriate ice point junction thermocouple from the calibration thermocouple reading to compare our calibration to the standard tables. Say our ice point thermocouple has a voltage reading of -0.935 milli Volts. Let the furnace be set at 150°C and we record a voltage reading for a calibration thermocouple in the furnace as 5.134 milli Volts. Then for a reference junction at 0°C and a measurement junction at 150°C the corresponding voltage would be the difference of these two readings ($5.134 - (-0.935)$), or 6.069 mV.

To obtain an overall perspective of the calibration apparatus a sample simple sketch is provided below for your understanding.

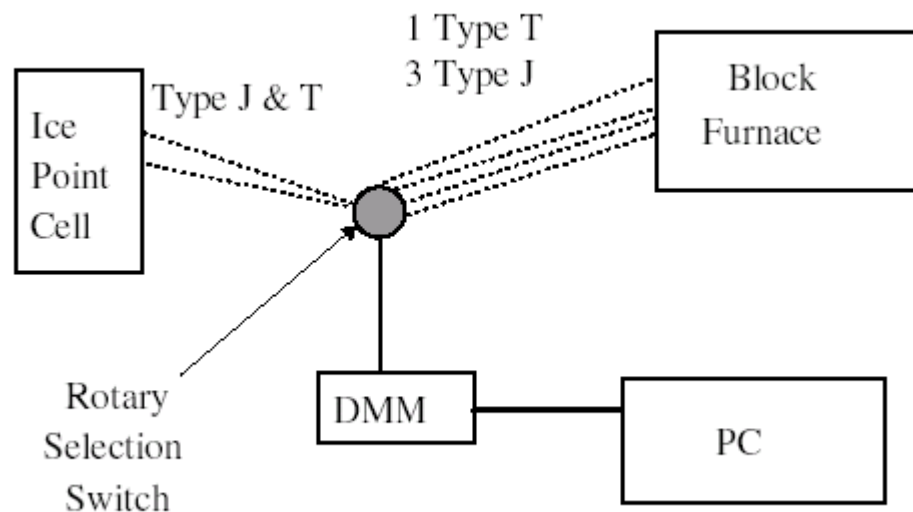


Figure 1.2: Layout of experimental apparatus

(B) Calibration of thermocouples

When the environs of a thermocouple change in temperature, the thermocouple reading is expected to show a response to this change. The speed of this response can be quantified in terms of a time constant. You will determine the time constant for each calibration thermocouple using the following procedure.

- Have the calibration thermocouples in the block calibration furnace at a steady state temperature of approximately 150°C.
- Initialise the data acquisition system. Your laboratory instructor will help you with this set-up.
- Start the data acquisition system and remove a calibration thermocouple from the furnace core.
- Allow the data acquisition system to record temperature data as the thermocouple cools in still air until the thermocouple approaches ambient temperature. Once a steady state is reached the data acquisition may be stopped.

- The data acquisition system will provide the temperature-time data, which can be brought to an Excel spreadsheet file. To utilize this data for the prediction of a time constant, it will probably be necessary to edit the file. We first note that the temperature recorded by the data acquisition system is actually the temperature difference between the thermocouple and the environs ($T-T_o$), which we note as the numerator of θ in Equation (1.2). Since the data acquisition system is turned on prior to removal of the thermocouple from the furnace, the first few data points will be at the constant temperature of the block furnace. We will want to delete all of these except for the very last one. Similarly, the same is true at the end of the experiment, where we may need to delete some of the steady state temperature data. After these deletions, we will also want to correct the time, so that it begins at zero for the first data point retained. To calculate q at every time step we will need to take our measured temperature, $(T-T_o)$, and divide it by (T_i-T_o) . Notice that (T_i-T_o) is simply the measured temperature at the first time step.
- Repeat steps 2 and 3 for the remaining three calibration thermocouples.

In order to determine the time constant from experimental measurements of time and temperature we can take two approaches. One method is to plot $\ln(\theta)$ versus t . This should be a straight line with slope $-1/\tau$. This approach allows us to confirm the lumped capacitance model presented in the background. A second approach is a mathematical approach. In this method the time is equal to the time constant, we have

$$\theta = e^{-1} \approx 0.37 \quad \text{or} \quad \ln(\theta) = -1 \quad (1.5)$$

We can scan our data and find the experimental temperature that will give this value. The corresponding time must be the time constant of the thermocouple. You should use both approaches, and compare the results.

Data Analysis

- (1) On a single graph plot the calibration curves for the three type J calibration thermocouples and compare them to the standard calibration data provided in the attached table. On a second graph repeat this plot for the type T calibration thermocouple. For discussion purposes, it may also prove useful to graph the calibration data for the two soldered thermocouples on a third graph.
- (2) Plot the semi-log temperature history for at least one of the calibration thermocouples. Use a linear curve fit of this plot to determine the time constant by the first method as described above. Estimate the time constant of each calibration thermocouple using the second method (the e^{-1} method). Provide a table of the time constants for the four calibration thermocouples.
- (3) Report, to what precision (in milli Volts) is you reading the temperature?

Suggestions for Discussion

- (i) What effect does the method of junction have on the thermocouple calibration and time constant? What effect does theory tell us it should have?
 - (ii) What differences do we see between the iron/constantan and the copper-constantan thermocouples? Why?
 - (iii) Compare the two methods of estimating the time constant. Which one is better, and why?
 - (iv) What errors may be introduced by measuring temperature with a thermocouple?
 - (v) You may wish to consider the heat transfer modes acting on the thermocouple.
 - (vi) What role does the reference junction play in thermocouple readings?
-

References:

- [1] Incropera, F. P. and D. P. DeWitt, *Fundamentals of Heat and Mass Transfer, 4th edition*, John Wiley and Sons Inc., New York, 1996.
- [2] Çengel, Yunus A., *Heat Transfer: A Practical Approach, 2nd edition*, McGraw-Hill, New York, 2003.
- [3] Thomas, L.C., *Heat Transfer – Professional Version, 2nd edition*, Capstone Publishing Corp, 1999.

Type T Thermocouple Table¹
 Voltages are in mV

Temperature (°C)	0	1	2	3	4	5	6	7	8	9
0	0.0000	0.0388	0.0776	0.1165	0.1555	0.1946	0.2337	0.2729	0.3122	0.3516
10	0.3910	0.4305	0.4701	0.5097	0.5495	0.5893	0.6292	0.6692	0.7092	0.7494
20	0.7896	0.8299	0.8703	0.9108	0.9513	0.9920	1.0327	1.0735	1.1144	1.1554
30	1.1964	1.2376	1.2788	1.3201	1.3616	1.4030	1.4446	1.4863	1.5280	1.5699
40	1.6118	1.6538	1.6959	1.7381	1.7803	1.8227	1.8651	1.9076	1.9503	1.9929
50	2.0357	2.0786	2.1215	2.1646	2.2077	2.2509	2.2942	2.3375	2.3810	2.4245
60	2.4682	2.5119	2.5556	2.5995	2.6435	2.6875	2.7316	2.7758	2.8201	2.8645
70	2.9089	2.9534	2.9980	3.0427	3.0875	3.1323	3.1772	3.2222	3.2673	3.3125
80	3.3577	3.4030	3.4484	3.4939	3.5394	3.5851	3.6308	3.6766	3.7224	3.7683
90	3.8143	3.8604	3.9066	3.9528	3.9991	4.0455	4.0920	4.1385	4.1851	4.2318
100	4.2785	4.3253	4.3722	4.4192	4.4662	4.5133	4.5605	4.6078	4.6551	4.7025
110	4.7500	4.7975	4.8451	4.8928	4.9405	4.9883	5.0362	5.0842	5.1322	5.1803
120	5.2284	5.2767	5.3250	5.3733	5.4218	5.4703	5.5188	5.5675	5.6162	5.6649
130	5.7138	5.7627	5.8116	5.8606	5.9097	5.9589	6.0081	6.0574	6.1068	6.1562
140	6.2057	6.2552	6.3049	6.3545	6.4043	6.4541	6.5040	6.5539	6.6039	6.6540
150	6.7041	6.7543	6.8045	6.8548	6.9052	6.9557	7.0062	7.0567	7.1074	7.1580
160	7.2088	7.2596	7.3105	7.3614	7.4124	7.4635	7.5146	7.5658	7.6170	7.6683
170	7.7197	7.7711	7.8226	7.8741	7.9257	7.9774	8.0291	8.0809	8.1327	8.1846
180	8.2366	8.2886	8.3407	8.3928	8.4450	8.4973	8.5496	8.6020	8.6544	8.7069
190	8.7595	8.8121	8.8647	8.9174	8.9702	9.0231	9.0760	9.1289	9.1819	9.2350
200	9.2881	9.3413	9.3945	9.4478	9.5012	9.5546	9.6080	9.6615	9.7151	9.7687
210	9.8224	9.8761	9.9299	9.9838	10.038	10.092	10.146	10.200	10.254	10.308
220	10.362	10.417	10.471	10.525	10.580	10.634	10.689	10.743	10.798	10.853
230	10.907	10.962	11.017	11.072	11.127	11.182	11.237	11.292	11.347	11.403
240	11.458	11.513	11.569	11.624	11.680	11.735	11.791	11.846	11.902	11.958
250	12.013	12.069	12.125	12.181	12.237	12.293	12.349	12.405	12.461	12.518
260	12.574	12.630	12.687	12.743	12.799	12.856	12.912	12.969	13.026	13.082
270	13.139	13.196	13.253	13.310	13.366	13.423	13.480	13.537	13.595	13.652
280	13.709	13.766	13.823	13.881	13.938	13.995	14.053	14.110	14.168	14.226
290	14.283	14.341	14.399	14.456	14.514	14.572	14.630	14.688	14.746	14.804
300	14.862	14.920	14.978	15.036	15.095	15.153	15.211	15.270	15.328	15.386

¹ From Omega, Thermocouple Reference Tables, 1993.

Type J Thermocouple Table²

Voltages are in mV

Temperature (°C)	0	1	2	3	4	5	6	7	8	9
0	0.0000	0.0504	0.1009	0.1514	0.2020	0.2527	0.3034	0.3541	0.4050	0.4558
10	0.5068	0.5578	0.6088	0.6599	0.7111	0.7623	0.8136	0.8649	0.9162	0.9677
20	1.0191	1.0707	1.1223	1.1739	1.2256	1.2773	1.3291	1.3809	1.4328	1.4847
30	1.5367	1.5887	1.6407	1.6928	1.7450	1.7972	1.8494	1.9017	1.9541	2.0064
40	2.0588	2.1113	2.1638	2.2164	2.2689	2.3216	2.3742	2.4269	2.4797	2.5325
50	2.5853	2.6382	2.6911	2.7440	2.7970	2.8500	2.9031	2.9562	3.0093	3.0625
60	3.1157	3.1689	3.2222	3.2755	3.3288	3.3822	3.4356	3.4890	3.5425	3.5960
70	3.6495	3.7031	3.7567	3.8103	3.8640	3.9177	3.9714	4.0252	4.0789	4.1327
80	4.1866	4.2404	4.2943	4.3483	4.4022	4.4562	4.5102	4.5642	4.6183	4.6724
90	4.7265	4.7806	4.8348	4.8890	4.9432	4.9974	5.0517	5.1059	5.1602	5.2146
100	5.2689	5.3233	5.3777	5.4321	5.4865	5.5410	5.5955	5.6500	5.7045	5.7591
110	5.8136	5.8682	5.9228	5.9774	6.0321	6.0867	6.1414	6.1961	6.2508	6.3056
120	6.3603	6.4151	6.4699	6.5247	6.5795	6.6343	6.6892	6.7440	6.7989	6.8538
130	6.9087	6.9637	7.0186	7.0736	7.1285	7.1835	7.2385	7.2936	7.3486	7.4036
140	7.4587	7.5137	7.5688	7.6239	7.6790	7.7341	7.7893	7.8444	7.8996	7.9547
150	8.0099	8.0651	8.1203	8.1755	8.2307	8.2859	8.3412	8.3964	8.4517	8.5069
160	8.5622	8.6175	8.6728	8.7281	8.7834	8.8387	8.8940	8.9494	9.0047	9.0601
170	9.1154	9.1708	9.2262	9.2815	9.3369	9.3923	9.4477	9.5031	9.5585	9.6139
180	9.6694	9.7248	9.7802	9.8356	9.8911	9.9465	10.002	10.057	10.113	10.168
190	10.224	10.279	10.335	10.390	10.446	10.501	10.557	10.612	10.668	10.723
200	10.779	10.834	10.890	10.945	11.001	11.056	11.112	11.167	11.223	11.278
210	11.334	11.389	11.445	11.501	11.556	11.612	11.667	11.723	11.778	11.834
220	11.889	11.945	12.000	12.056	12.111	12.167	12.222	12.278	12.334	12.389
230	12.445	12.500	12.556	12.611	12.667	12.722	12.778	12.833	12.889	12.944
240	13.000	13.056	13.111	13.167	13.222	13.278	13.333	13.389	13.444	13.500
250	13.555	13.611	13.666	13.722	13.777	13.833	13.888	13.944	13.999	14.055
260	14.110	14.166	14.221	14.277	14.332	14.388	14.443	14.499	14.554	14.609
270	14.665	14.720	14.776	14.831	14.887	14.942	14.998	15.053	15.109	15.164
280	15.219	15.275	15.330	15.386	15.441	15.496	15.552	15.607	15.663	15.718
290	15.773	15.829	15.884	15.940	15.995	16.050	16.106	16.161	16.216	16.272
300	16.327	16.383	16.438	16.493	16.549	16.604	16.659	16.715	16.770	16.825

² From Omega, Thermocouple Reference Tables, 1993.

EXPERIMENT 2

Conduction Through Copper Bars

Objective

To evaluate the thermal conductivity of copper experimentally using “Thermal Conduction System – Model 9051”, and then use the conductivity value to determine the conduction through constant and variable area copper bars.

Background

When a temperature gradient exists in a stationary medium, which may be a solid or a fluid, we use the term **conduction** to refer to the heat transfer that will occur across the medium. The *physical mechanism* of conduction involves concepts of atomic and molecular activity, which sustains the transfer of energy from the more energetic to the less energetic particles of a substance due to interactions between the particles.

Consider a *gas* occupying the space between two surfaces maintained at different temperatures and assume that there is *no bulk motion*. We associate the temperature at any point with the energy of the gas molecule. This energy is related to the random translational motion, as well as to the internal rotational and vibrational motions, of the molecules.

Higher temperatures are associated with higher molecular energies, and when neighbouring molecules collide, as they are constantly doing, a transfer of energy from the more energetic to the less energetic molecules must occur. In the presence of a temperature gradient, energy transfer by conduction must then occur in the direction of *decreasing* temperature. We may speak of the net transfer of energy by this molecular motion as a *diffusion* of energy. The situation is much the same in *liquids*, although the molecules are more closely spaced and the molecular interactions are stronger and more frequent. In a *solid*, conduction is attributed to atomic activity in the form of lattice vibrations and electron migration. We treat the conduction phenomena by Fourier’s law, which is defined in terms of an important material property, defined as thermal conductivity.

It is important to emphasize that the origin of Fourier’s law is phenomenological. That is, it is developed from observed phenomena—the generalization of extensive experimental evidence rather than being derived from first principles. Mathematically, it is defined as [1-3]

$$q_x = -kA \frac{\Delta T}{\Delta x} \quad (2.1)$$

where

- q_x is defined as the heat transfer rate, in Watts;
 A , is the heat transfer area normal to the direction of heat flow, in m^2 ;
 k , is the material property defined as thermal conductivity, in $W/m.K$;
 ΔT is the temperature difference, in K; and
 Δx is the rod length, in m.

The above expression defines the important material property, *thermal conductivity*, one of most important *transport properties* that you will encounter in performing conduction analyses. Tabulated values of the thermo-physical properties required for solution of heat transfer problems are provided in your textbook. A typical range of thermal conductivity values for various matter are shown in Fig. 2.1.

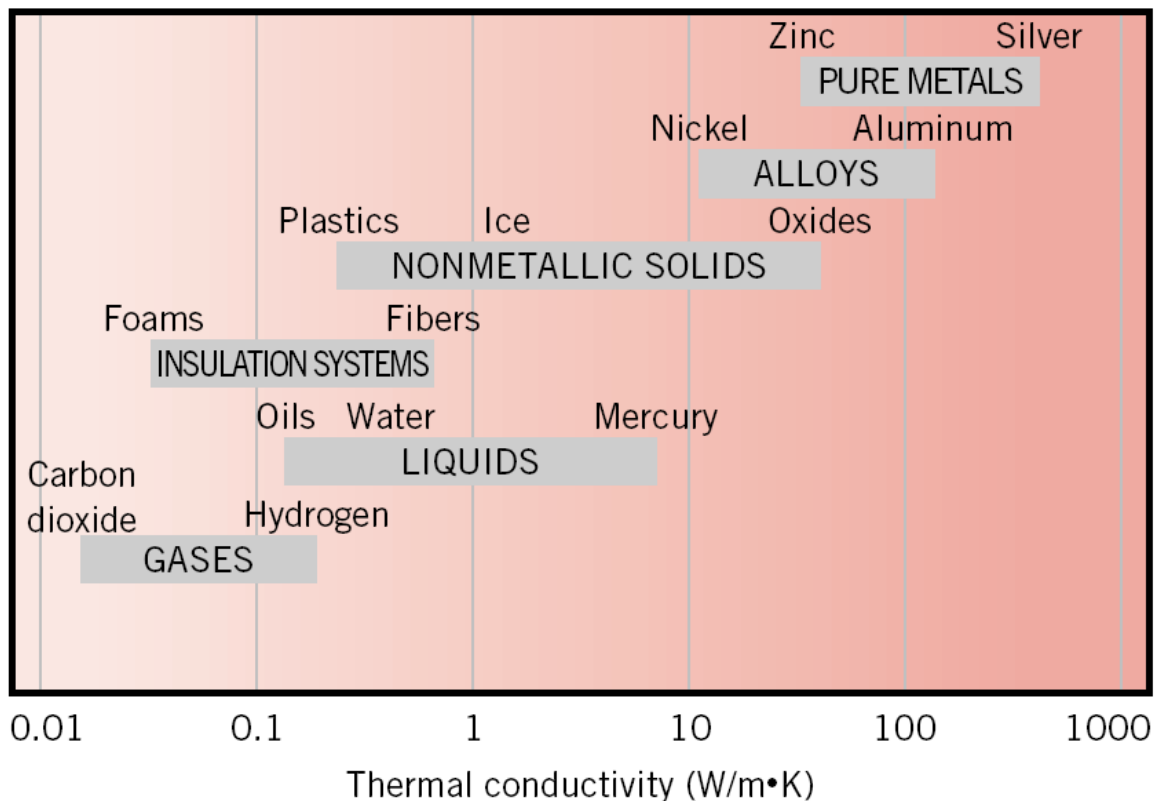


Figure 2.1: Range of thermal conductivity for various phases of matter at normal temperatures and pressure [1].

It is thus important to know thermal conductivity of various conducting and insulating materials that are used in many industrial processes. In this regard, we will conduct an experiment to determine heat transfer rate through constant and variable solid areas. First, we will determine thermal conductivity of copper bars and then use simple analysis to calculate heat

transfer rates through these bars.

Apparatus

The objectives of this experiment are achieved through the use of Thermal Conduction System, Model 9051 (refer to Figures 2.2 and 2.3) available in our laboratory. The system consists of two hot plate type heat sources copper bars (unit 3 and unit 4) and 10 thermocouple junctions on each bar. Unit 3 has a tapered bar and Unit 4 has a cylindrical bar as indicated in the figure.

It should be noted that these units provide vertical heat flux paths. Referring to Figure 2.3, the one at the left is a cylindrical bar while the one at the right is a tapered bar. Each bar is in contact at its lower end with its own hot plate. Contact for the tapered bar is at the smaller end. The maximum electrical input through the plate is 750 Watts. The surface temperature can be modulated between 5 °F above the room temperature to 400 °F. A metal plate attached to the actual heater plate functions as a heat source, concentrating the heat flux concentrically into the test bar.

Both bars are of the same diameter at the upper end and in contact with a non-immersion type fluid-cooled heat sink. Instrumentation and control of coolant flow through these heat sinks is provided to monitor and control the heat flow rates through the bars.



Figure 2.2: Thermal Conduction System Model 9051

Conduction Theory

Under steady-state condition, heat flux (in W/m^2) through the constant cross-section cylindrical bar is constant over the entire length. As a result, the heat transfer rate along

the cylindrical bar, since it is insulated on its sides, is given by Equation (3.1), while the heat flux by

$$q_x'' = \frac{q}{A} = k \frac{\Delta T}{\Delta x} \quad (2.2)$$

The quantity of heat, which is conducted through the rod, is transferred to the cooling water. Therefore, the heat transferred to the cooling water can be expressed as

$$q_x = -kA \frac{\Delta T}{\Delta x} = \dot{m} C_p (T_{w,out} - T_{w,in}) \quad (2.3)$$

Now from the above equation, we can express thermal conductivity, k as

$$k = - \frac{\dot{m} C_p (T_{w,out} - T_{w,in})}{A (\Delta T / \Delta x)} \quad (2.4)$$

where

- \dot{m} is defined as the mass flow rate of water, in kg/s;
- C_p , is the specific heat of water, in J/kg.K;
- $T_{w,out}$, is the outlet temperature of water, in °C;
- $T_{w,in}$, is the inlet temperature of water, in °C;
- ΔT is the temperature difference, in K; and
- Δx is the rod length, in m.
- A , is the cross-sectional area of the bar, in m²;
- $\Delta T / \Delta x$ is the temperature gradient obtained from T versus x plot.

It is important to recognize that in the tapered bar, the heat flux is not constant along it. In fact, it is the heat transfer rate that remains constant, while the flux increases with the decrease of cross-sectional area (refer to Equation (2.2)). The temperature distribution through the bar can be calculated by using Equation 2.1 in the limiting condition; i.e., when $\Delta x \rightarrow 0$. This gives after separating the variables [1-3],

$$- \frac{q_x dx}{kA(x)} = dT \quad (2.5)$$

Integrating, the left hand side from x_1 to x , and the right hand side from T_1 to T , we have

$$- \frac{q_x}{k} \int_{x_1}^x \frac{dx}{A(x)} = \int_{T_1}^T dT \quad (2.5)$$

Notice that $A(x)$ can be expressed in terms of x and $d(x)$. From the geometry of the tapered bar, $d(x)$ can be expressed as

$$d(x) = \frac{x(d_L - d_0)}{L} + d_0 \quad (2.6)$$

where

d_L is defined as diameter at $x = L$, $d_L = 50.8$ mm

d_0 is defined as diameter at $x = 0$, $d_0 = 25.4$ mm

L is the length of the bar, $L = 30$ cm

T_1 is the temperature recorded at $x = x_1$

The desired temperature distribution can be obtained from Equation (2.5) and using (2.6).

Experimental Procedure

The following experimental procedure should be followed while conducting this experiment:

- Establish constant and steady cooling water flow of about 400 mL/min
- Turn on heaters to Units 3 and 4 - set each one to 500 W. Allow the system to reach steady-state conditions.
- Start recording temperatures using a digital thermometer. Note that two selector switches on the apparatus allow you to select whatever thermocouple you wish to read.
- Measure cooling water flow rates using the flow rate-measuring device provided by your instructor.
- Record data at least once every 15 minutes, and continue until steady conditions have reached. You may have to wait for about an hour after setting up the apparatus to allow the unit to reach the desired state conditions.

Data Analysis

Notice that ten thermocouples, located at the center of each bar and positioned along it, enable the student to measure temperature under both dynamic and stable conditions. The electrical input is determined by measuring (with laboratory meters) voltage and current. The heat flux through the bar as well as the heat loss through the insulation should be calculated. In your report, you are required to present the following:

- (1) On a single graph plot the temperature versus position along the bar length for both the bars.
- (2) Using data for constant cross-sectional area bar, calculate the thermal conductivity for the copper bar and compare with the value given in your heat transfer text.

- (3) For the tapered bar, derive an equation that can be used to predict temperature distribution as a function of x .
- (4) Plot the temperature distribution from your equation and the one obtained from your experiment for the tapered bar.
- (5) Carry out sensitivity analysis of k in terms of input (measured) variables.

Suggestions for Discussion

- (i) What effect does the heat input have on the value of thermal conductivity? What effect does theory tell us it should have?
 - (ii) What differences do we see between the constant cross section and variable cross section bars in terms of heat flux?
 - (iii) What errors may be introduced by measuring thermal conductivity by this method?
 - (iv) Is it important to consider heat losses from the insulation?
 - (v) What would you recommend to improve the reliability of this experiment?
-

References:

- [1] Incropera, F. P. and D. P. DeWitt, *Fundamentals of Heat and Mass Transfer, 4th edition*, John Wiley and Sons Inc., New York, 1996.
 - [2] Çengel, Yunus A., *Heat Transfer: A Practical Approach, 2nd edition*, McGraw-Hill, New York, 2003.
 - [1] Thomas, L.C., *Heat Transfer – Professional Version, 2nd edition*, Capstone Publishing Corp, 1999.
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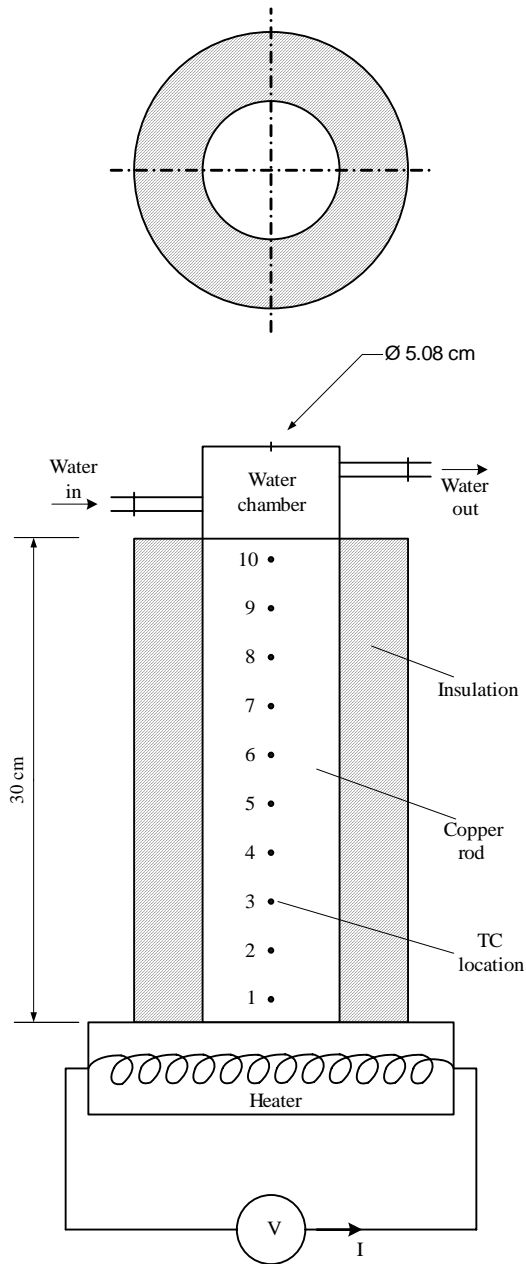


Figure 2.3(a) Schematic of a thermal conduction system with a constant cross section copper bar.

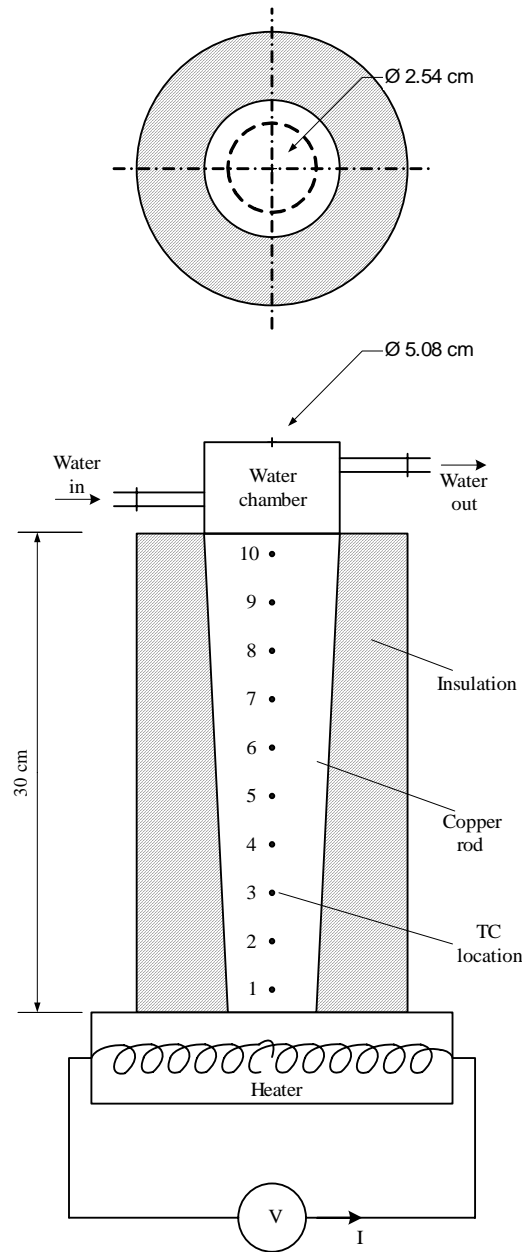


Figure 2.3(b) Schematic of a thermal conduction system with a variable cross section copper bar.

LAB DATA SHEET**Experiment 2: Conduction through Copper Bars**

Name : _____ Id# : _____ Date: _____

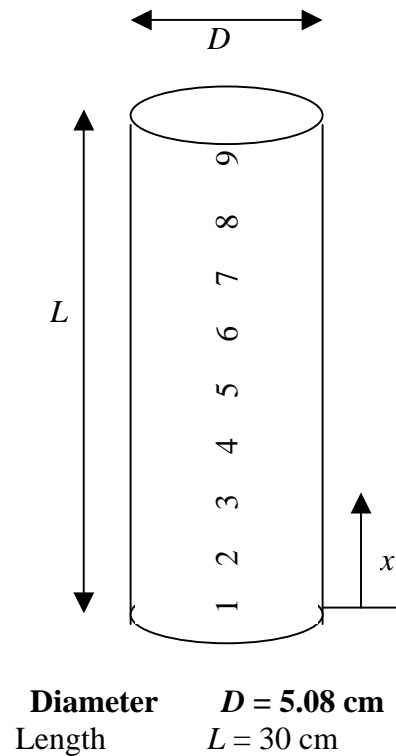
Note: It is important that the process has reached steady state before recording any readings.

Fluid Temperature and Properties			
	Temp (°C)	Density (kg/m ³)	Sp. Heat (J/kg.K)

Useful Information
1 liter = 0.001 m ³
1 liter/min = 1.667 x 10 ⁻⁵ m ³

Table 2.1 Cylindrical bar (Unit # 4)

TC #	$x(m)$	T(°C)
1	0.0493	
2	0.0739	
3	0.0986	
4	0.1229	
5	0.1476	
6	0.1722	
7	0.1969	
8	0.2215	
9	0.2461	
10	0.2708	

**Coolant: Water (Unit # 5)**

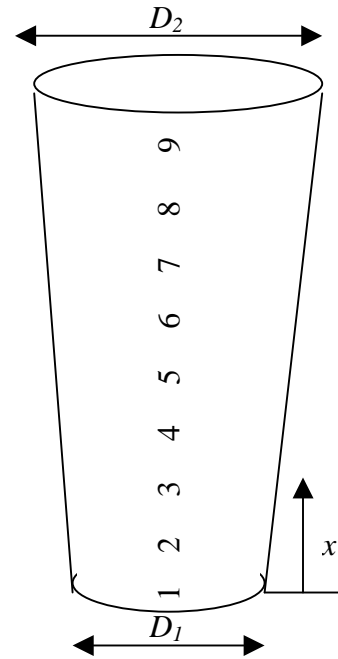
Parameter	TC #	
Outlet temperature	TC # 4	°C
Inlet temperature	TC # 5	°C
Flow rate	—	Lit/min

Table 2.2 Tapered bar (Unit # 3)

TC #	$x(\text{m})$	T(°C)
1	0.0493	
2	0.0739	
3	0.0986	
4	0.1229	
5	0.1476	
6	0.1722	
7	0.1969	
8	0.2215	
9	0.2461	
10	0.2708	

Coolant: Water (Unit # 5)

Parameter	Thermo-couple	
Outlet temperature	TC # 3	°C
Inlet temperature	TC # 5	°C
Flow rate	—	liter/min



Diameters $D_1 = 2.54 \text{ cm}$
 $D_2 = 5.08 \text{ cm}$
Length $L = 30 \text{ cm}$

EXPERIMENT 3

Forced Convection Heat Transfer

Objective

To evaluate the heat transfer coefficient for forced convection turbulent flow in a pipe with uniform cross-sectional area.

Background

The term **convection** refers to heat transfer (refer Figure 3.1) that will occur between a surface and a moving or stationary fluid when they are at different temperatures.

This mode of heat transfer comprises of *two mechanisms*. In addition to energy transfer due to *random molecular motion (conduction)*, energy is also transferred by the *bulk, or macroscopic, motion* of the fluid. This fluid motion is associated with the fact that, at any instant, large numbers of molecules are moving collectively or as aggregates. Such motion, in the presence of a temperature gradient, contributes to heat transfer. Because the molecules in the aggregate retain their random motion, the total heat transfer is then due to a superposition of energy transport by the random motion of the molecules and by the bulk motion of the fluid. It is customary to use the term *convection* when referring to this cumulative transport, and the term *advection* when referring to transport due to bulk fluid motion.

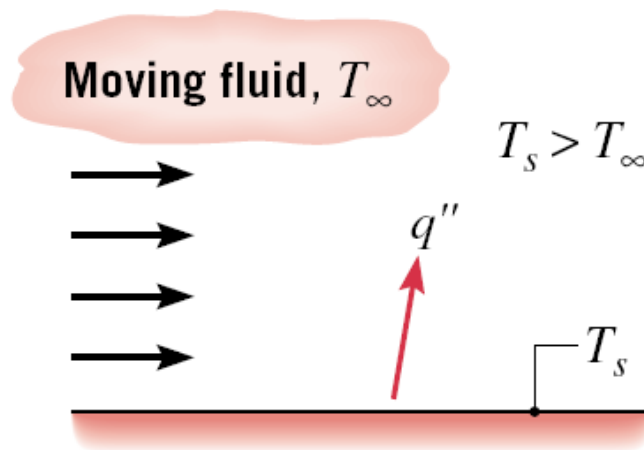


Figure 3.1: Convection from a surface to a moving fluid [1].

You learned in the fluids course that, with fluid flow over a surface, viscous effects are important in the hydrodynamic (velocity) boundary layer and, for a Newtonian fluid, the frictional shear stresses are proportional to the velocity gradient. In the treatment of

convection in the heat transfer course, you have been exposed to the concept of *thermal boundary layer*, the region that experiences a temperature distribution from that of the free stream T_∞ to the surface T_s (refer to Figure 3.2). Appreciation of boundary layer phenomena is essential to understanding of convection heat transfer. It is for this reason that the discipline of fluid mechanics plays a vital role in our analysis of convection mechanism [1-3].

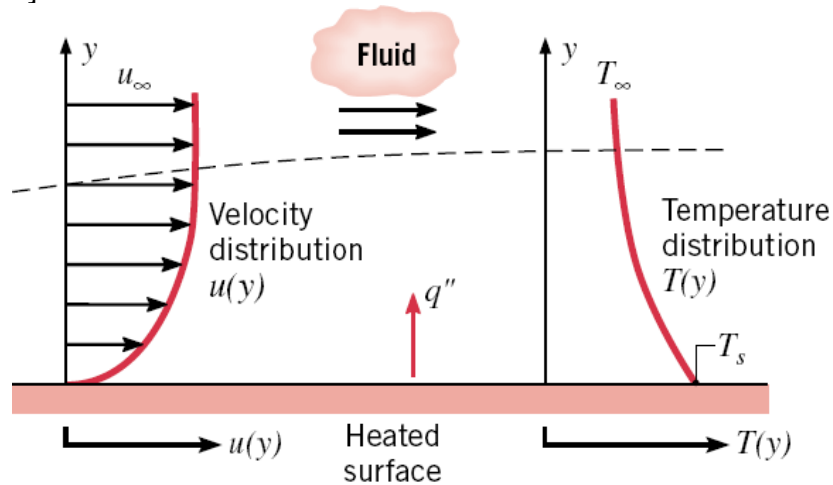


Figure 3.2 Hydrodynamic and thermal boundary development in convection heat transfer [1].

It is important to emphasize that convection heat transfer may be classified according to the nature of the flow. We speak of **forced convection** when the flow is caused by external means, such as a fan, a pump, or atmospheric winds. In contrast, for **free** (or *natural*) **convection**, the flow is induced by buoyancy forces, which arise from density differences caused by temperature variations in the fluid. We speak also of **external** and **internal** flow. As you learned in fluid mechanics course, external flow is associated with immersed bodies for situations such as flow over plates, cylinders and foils. In internal flow, the flow is constrained by the tube or duct surface. You saw that the corresponding hydrodynamic boundary layer phenomena are quite different, so it is reasonable to expect that the convection processes for the two types of flow are distinctive.

Regardless of the particular nature of the convection heat transfer process, the appropriate *rate equation*, known as **Newton's law of cooling**, is of the form

$$q'' = \frac{q}{A_s} = h(T_s - T_\infty) \quad (3.1a)$$

where q'' , is the convective **heat flux** (W/m^2), is proportional to the difference between the surface and fluid temperatures, T_s and T_∞ , respectively, and the proportionality constant h (in $\text{W/m}^2\cdot\text{K}$) is termed the **convection heat transfer coefficient**. When using Equation (3.1a), the convection heat flux is presumed to be **positive** if the heat transfer is

from the surface ($T_s > T_\infty$) and **negative** if the heat transfer is to the surface ($T_\infty > T_s$). However, if in situations ($T_\infty > T_s$), there is nothing to prevent us from expressing Newton's law of cooling as

$$q'' = h(T_\infty - T_s) \quad (3.1a)$$

in this case heat transfer is positive to the surface. The choice of Equation (3.1a) or (3.1b) is normally made in the context of a particular problem as appropriate. It is important to note that the convection coefficient depends on conditions in the boundary layer, which is influenced by surface geometry, the nature of fluid motion, and an assortment of fluid thermodynamic and transport properties. Any study of convection ultimately reduces to a study of the means by which h may be determined. One of the means to estimate the value of h under turbulent flow conditions will be carried out in the present experiment.

Convection Theory

It is important to that internal flow that we are going to study may be a hydrodynamically or thermally developing flow (in the entrance region), or a fully developed flow. For instance, when fluid enters the tube at a uniform temperature (less than tube surface temperature), convection heat transfer occurs. The thermal boundary layer begins to develop (refer to Figure 3.3). If the tube surface condition is fixed by imposing either a Uniform Wall Temperature (UWT) or Uniform Heat Flux (UHF), a thermally fully developed condition is eventually reached. Notice that fully developed temperature profiles differ according to whether the case is UWT or UHF. However, for both cases, fluid temperature increases with increasing x , but the relative shape of temperature profile no longer changes. So the ratio of heat transfer coefficient to thermal conductivity of the fluid, h/k remains constant. Therefore, the local heat transfer coefficient under fully developed conditions remains constant with x .

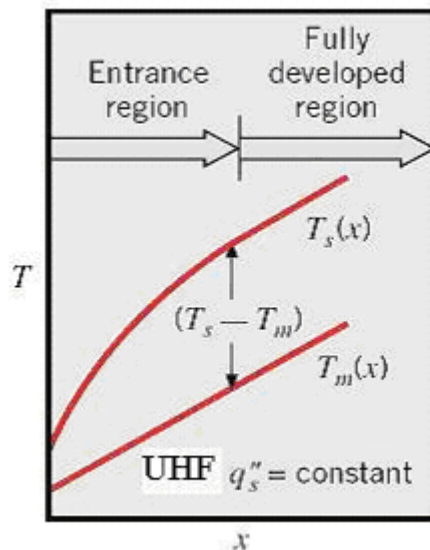


Figure 3.3: Uniform heat flux heating of a fluid

It can be seen from the above figure that, the difference between surface temperature and mean temperature is small at the entrance of the pipe because of the large value of " h " (small boundary thermal layer thickness). As the value of " h " decreases, the temperature difference increases with x until it becomes constant for fully developed flow.

Experimental Apparatus

The objectives of this experiment are achieved through the use of Convection Heat Transfer Unit (refer to Figures 3.4) available in our laboratory. The system consists of a fan, orifice plate and thermally insulated copper pipe heated over approximately 6 ft. of its length and provided with thermocouples, selector switches, Cambridge Potentiometer, three manometers, variable transformer (Variac), 0-5 amp ammeter, 0-300 V voltmeter, 0-50°C mercury in glass thermometer. The pipe is provided with two pressure tapping at a pitch of 5 ft., and also 7 thermocouples..

It should be noted that there are 3 cold junctions for the thermocouples, so that each thermocouple can be used in conjunction with anyone *of* the three cold junctions. The cold junctions are fixed to the brass sheath around the bulb of the thermometer in the air stream immediately before the entry to the copper pipe. Using this arrangement implies that all thermocouple readings are relative to the air temperature at inlet to the pipe.



Figure 3.4: Convection heat transfer test unit

Experimental Procedure

Switch on the fan with the inlet valve fully open. When this has been done the heater current can be switched on with the **Variac set at zero**. Increase the VARIAC voltage to give a maximum current of about 4 amps. Allow the flow to reach steady state before taking any reading. When the experiment is completed the heater must be switched off first and the apparatus is allowed to cool for 2 or 3 minutes before the fan is switched **off**. It is very important that the airflow through the duct reaches stable and steady state conditions before recording the following parameters:

1. Air static pressure before the orifice plate;
2. Pressure drop (ΔP) across the orifice plate;
3. Air temperature after the orifice plate (cold junction temperature);
4. Barometric pressure (refer to the wall barometer);
5. Pressure drop over the length of test section;
6. Thermocouple readings on the pipe. These are numbered from 1 to 7. Each should be checked against cold junction thermocouples.
7. Measure current from the ammeter; and
8. Voltage from the voltmeter.

Data Analysis

Notice that thermocouples, and pressure sensors located along the length of the test section as well as the voltage and current readings, enable the student to measure temperature, pressure drop, and power input under stable and steady state conditions. The electrical input is determined by measuring (with laboratory meters) voltage and current. The heat input to the air, for example up to section 3 by an electric heater through the pipe wall, is given by equation (3.1a),

$$q_{0-3} = hA_{s(0-3)}(T_s - T_\infty) \quad (3.2)$$

Where

- q_{0-3} is defined as the heat input to the air up to section 3, in watts;
 $A_{s(0-3)}$ is the convection heat transfer area up to section 3, in m^2 ;
 H is the convection heat transfer coefficient, in $w/m^2.k$;
 T_s is the pipe surface temperature, in $^{\circ}c$; and
 T_m is the bulk-mean temperature of air, in $^{\circ}c$.

From the first law of thermodynamics, the amount of heat energy input to the air can also be expressed as

$$q_{0-3} = \dot{m}C_p(T_{m,3} - T_{m,0}) = VI \frac{L_{0-3}}{L_{0-7}} \quad (3.3)$$

In the above equation, L_{0-3} and L_{0-7} are the lengths of the test section up to section 3 and 7, respectively. Combining equation (3.2) and (3.3), we get the heat transfer coefficient as

$$h = \frac{\dot{m}C_p (T_{m,3} - T_{m,0})}{A_{s(0-3)} (T_s - T_m)} \quad (3.4)$$

Once h is determined, one can easily calculate Nusselt Number (Nu), Reynolds Number (Re), Prandtl Number (Pr), and Stanton Number (St) by the following expressions,

$$Nu = \frac{hD}{k} \quad (3.5)$$

$$Re = \frac{\dot{m}D}{A_{flow}\mu} \quad (3.6)$$

$$Pr = \frac{\mu C_p}{k} \quad (3.7)$$

$$St = \frac{Nu}{Re Pr} \quad (3.8)$$

It is important to note that equation (3.5) and (3.8) give the experimental values of Nu and St that we found from our experiment. These are to be compared with the following correlations given in your textbook:

$$Nu = 0.023 Re^{0.8} Pr^{0.4}, \quad Re \geq 10^4 \quad (3.9)$$

$$St = 0.023 Re^{-0.2} Pr^{-0.6}, \quad Re \geq 10^4 \quad (3.10)$$

The friction factor, f that you were earlier introduced in your fluid course can be expressed as,

$$f = -\frac{(dp/dx)D}{\rho U_m^2 / 2} \quad (3.11)$$

Where the mean fluid velocity, $U_m = \dot{m} / \rho A_{flow}$, and in the hydro-dynamically fully developed flow condition, dp/dx is constant. Therefore, we can write

$$\frac{dp}{dx} = \frac{\Delta p}{\Delta x} \quad (3.11a)$$

The mass flow rate of air can be calculated from an orifice meter installed in the test section. Therefore the flow through the orifice meter is given by

$$\dot{m} = \rho A_o C_d \sqrt{2g\Delta h_{air}} \quad (3.12)$$

And the column of air can be related in terms of column of water by the expression

$$\rho_{air} \Delta h_{air} = \rho_{water} \Delta h_{water} \quad (3.12a)$$

where

A_o is the cross-sectional area of orifice, in m^2 ; and

C_d is the coefficient of discharge for the orifice. In our case $C_d = 0.613$

It is important to note that equation (3.11) gives an experimental value of f . It should be compared with the following value for a hydro-dynamically fully developed turbulent flow in pipe,

$$f = 0.184 Re^{-0.2}, \quad Re \geq 10^4 \quad (3.13)$$

Finally, the relation given below can verify the Reynolds analogy that relates heat transfer to momentum transfer,

$$St = \frac{f}{8} \quad (3.14)$$

From the graph, find out the value of bulk stream temperature at location 3.

- (1) From an experimental data, on a single graph plot the surface and bulk mean temperature versus position along the duct length.
- (2) Using data for voltage and current, determine the heat input to the pipe, and calculate heat transfer coefficient.
- (3) Find the Nusselt number and Stanton number from the experimental data and compare it with the empirical equations (3.9) and (3.10).
- (4) Calculate the friction factor f from both the experimental data and compare it with empirical equation (3.13).

Suggestions for Discussion

- (i) What effect does the heat loss have on the value of heat transfer coefficient?
- (ii) Is it important to consider heat losses from the insulation?
- (iii) What errors may be introduced by measuring heat transfer coefficient by this method?
- (iv) Carry out the sensitivity analysis of heat transfer coefficient in terms of important variables that you have measured during this experiment.
- (v) What would you recommend to improve the reliability of this experiment?

References:

- [1] Incropera, F. P. and D. P. DeWitt, *Fundamentals of Heat and Mass Transfer, 4th edition*, John Wiley and Sons Inc., New York, 1996.
 - [2] Çengel, Yunus A., *Heat Transfer: A Practical Approach, 2nd edition*, McGraw-Hill, New York, 2003.
 - [3] Bejan, A., *Convection Heat Transfer, 2nd edition*, John Wiley and Sons Inc., New York, 1995.
-

LAB DATA SHEET**Experiment 3: Forced Convection Heat Transfer**

Name : _____ Id# : _____ Date : _____

Note: Make sure that the process has reached steady state before recording any readings.

Table 3.1(a): Experimental data

Heater Voltage, V	
Heater Current, amp	
Cold junction temperature, $T_{m,0}$ (in °C)	
Manometer liquid sp. gravity, S	0.784
$\Delta h_{\text{orifice}}$ (in cm)	
$\Delta h_{\text{test section}}$ (in cm)	
Coefficient of discharge of orifice meter, C_d	0.613

Table 3.1(b): Tube temps, T_s

TC #	x mm	Tube surface temps, T_s (in °C)
1	370	
2	767	
3	1081	
4	1249	
5	1417	
6	1585	
7	1753	

Table 3.2: Area calculation

Pipe internal diameter	D	31.75 mm
Orifice diameter	D_o	40 mm
Length of pipe up to TC # 3	L_{0-3}	1081 mm
Test section cross section area, m^2	A	
Test section surface area, m^2	$A_{s,0-3}$	
Orifice cross-sectional area, m^2	A_o	

Table 3.3: Flow rate calculation (properties at $T_{m,0}$)

Density of air	ρ_{air} (kg/m ³)	
Density of water	ρ_{water} (kg/m ³)	
$\Delta h_{air} = \frac{(S \rho_{water}) \Delta h_{orifice}}{\rho_{air}}$	(m)	
Mass flow rate of air $\dot{m} = \rho_{air} A_{orifice} C_d \sqrt{2g\Delta h_{air}}$	(kg/s)	
Bulk stream velocity, $U_b = \frac{\dot{m}}{\rho A}$	(m/s)	

Table 3.4: Properties of air at an average of $T_{m,0}$ and $T_{m,3}$

Density ρ (kg/m ³)	Sp. Heat C_p (J/kg-C)	Viscosity μ (N-s/m ²)	Thermal conductivity k (W/m-K)

Table 3.5: Heat transfer calculations (for the section L_{0-3})

Description	Equation	Result
Heat absorbed by air (in Watts)	$q_{0-3} = \dot{m} C_p (T_{m,3} - T_{m,0})$	
Heat transfer coefficient, h (in W/m ² .K)	$h = \frac{q_{0-3}}{A_{s(0-3)} (T_s - T_b)}$	
Nusselt number, Nu	$Nu = hD / k$	
Reynolds number, Re	$Re = \rho U_m D / \mu$	

Prandtl number, Pr	$Pr = \mu C_p / k$	
Stanton number, St	$St = \frac{Nu}{Re Pr}$	
Nusselt number from correlation, Nu_{corr}	$Nu = 0.023 Re^{0.8} Pr^{0.4}$	
% difference in Nu	$\frac{Nu - Nu_{corr}}{Nu_{corr}} \times 100$	
Stanton number from correlation, St_{corr}	$St = 0.023 Re^{-0.2} Pr^{-0.6}$	
% difference in St	$\frac{St - St_{corr}}{St_{corr}} \times 100$	

Table 3.6: Determination of friction factor

Length of test section, Δx (in m)	$\Delta x = x_7 - x_1$	
Pressure drop across the test section, ΔP (in Pa)	$\Delta P = (S \gamma_{water}) \Delta h_{test\ section}$	
Friction factor by experiment, f_{exp}	$f_{exp} = \left(\frac{2D}{\rho U_b^2} \right) \left(\frac{\Delta P}{\Delta x} \right)$	
Friction factor by correlation, f_{corr}	$f_{corr} = 0.184 Re^{-0.2}$	
% difference	$\frac{f_{exp} - f_{corr}}{f_{corr}} \times 100$	

Table 3.7: Verification of Reynolds Analogy

Stanton number by experiment (refer to Table 3.5)	$St = \frac{Nu}{Re Pr}$	
Stanton number by Reynolds analogy for turbulent flow of air in a pipe	$St = \frac{f_{exp}}{8}$	
% difference		

Table 3.8: Summary of Results

Parameter	Experimental	Theoretical	% Difference
Nusselt number, Nu			
Stanton number, St			
Friction factor, f			
Stanton number, St (Reynolds analogy)			

EXPERIMENT 4

Double-Pipe Heat-Exchanger Characteristics

Objective

To evaluate the performance characteristics of a double-pipe heat exchanger under parallel and counter-flow arrangements.

Background

The technology of heating and cooling of systems is one of the most basic areas of mechanical engineering. Wherever steam is used, or wherever hot or cold fluids are required we will find a heat exchanger. They are used to heat and cool homes, offices, markets, shopping malls, cars, trucks, trailers, aeroplanes, and other transportation systems. They are used to process foods, paper, petroleum, and in many other industrial processes. They are found in superconductors, fusion power labs, spacecrafts, and advanced computer systems. The list of applications, in both low and high tech industries, is practically endless.

Heat exchangers are typically classified according to *flow arrangement* and *type of construction*. In this introductory treatment, we will consider three types that are representative of a wide variety of exchangers used in industrial practice. The simplest heat exchanger is one for which the hot and cold fluids flow in the same or opposite directions in a **concentric-tube** (or *double-pipe*) construction. In the *parallel-flow* arrangement of Fig. 4.1a, the hot and cold fluids enter at the same end, flow in the same direction, and leave at the same end. In the *counter flow* arrangement, Fig. 4.1b, the fluids enter at opposite ends, flow in opposite directions, and leave at opposite ends. A common configuration for power plant and large industrial applications is the **shell-and-tube heat exchanger**, shown in Fig. 4.1c. This exchanger has one shell with multiple tubes, but the flow makes one pass through the shell. Baffles are usually installed to increase the convection coefficient of the shell side by inducing turbulence and a cross-flow velocity component. The **cross-flow heat exchanger**, Fig. 4.1d, is constructed with a stack of thin plates bonded to a series of parallel tubes. The plates function as fins to enhance convection heat transfer and to ensure cross-flow over the tubes. Usually it is a gas that flows over the fin surfaces and the tubes, while a liquid flows in the tube. Such exchangers are used for air-conditioner and refrigeration heat rejection applications.

Heat Exchanger Analysis

The concept of overall heat transfer resistance or coefficient that you were introduced earlier in your heat transfer course, if we apply this concept to a, for example, double-pipe heat exchanger, the total resistance is the sum of the individual components; i.e.,

resistance of the inside flow, the conduction resistance in the tube material, and the outside convective resistance, given by

$$R_{total} = \frac{1}{A_i h_i} + \frac{t}{k A_{ln}} + \frac{1}{A_o h_o} \quad (4.1)$$

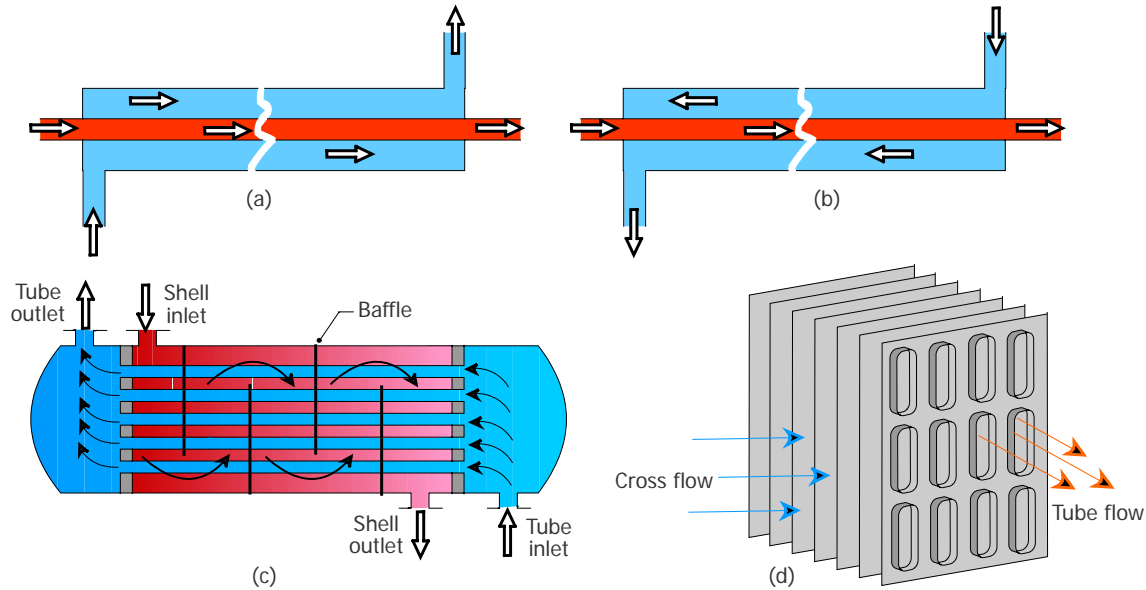


Figure 4.1 Types of Heat Exchangers – concentric tube (a) Parallel flow, and (b) counter-flow; (c) Shell-and-tube: one shell pass and one tube pass; (d) Cross-flow.

where subscripts i and o refer to inner and outer heat-transfer surface areas, respectively, t is the wall thickness, and A_{ln} is the logarithmic mean heat transfer area, defined as

$$A_{ln} = \frac{(A_o - A_i)}{\ln\left(\frac{A_o}{A_i}\right)} \quad (4.2)$$

The total heat transfer resistance can be defined in terms of overall heat transfer coefficient based on either outer or inner areas, as long as the basis is clearly spelled out. For example, based on outer area, we have

$$\frac{1}{U_o} = A_o R_{total} = \frac{A_o}{A_i h_i} + \frac{t A_o}{k A_{ln}} + \frac{1}{h_o} \quad (4.3)$$

which after simplifying yields the overall heat transfer coefficient based on inner and outer areas, respectively as [1]

$$U_i = \frac{1}{\frac{1}{h_i} + \frac{D_i \ln(D_o / D_i)}{2k} + \frac{D_i}{D_o h_o}} \quad (4.4)$$

$$U_o = \frac{1}{\frac{D_o}{D_i h_i} + \frac{D_o \ln(D_o / D_i)}{2k} + \frac{1}{h_o}} \quad (4.5)$$

where the inner and outer heat-transfer areas, as well as the wall thickness, and the logarithmic mean heat transfer area, in terms of tube inner and out diameters and length L , are given, respectively, as

$$A_i = \pi D_i L \quad (4.6a)$$

$$A_o = \pi D_o L \quad (4.6b)$$

$$t = \frac{D_o - D_i}{2} \quad (4.6c)$$

$$A_{\ln} = \frac{\pi(D_o - D_i)L}{\ln\left(\frac{D_o}{D_i}\right)} \quad (4.6d)$$

We note from the above equations that if the wall thickness is negligible ($D_o \approx D_i$), for example, in thin tube heat exchangers or the thermal conductivity of the tube material is very high, the conduction resistance through the tube may be neglected in Equations (4.4) and (4.5) to give

$$\frac{1}{U_i} \approx \frac{1}{U_o} = \frac{1}{h_i} + \frac{1}{h_o} \quad (4.7)$$

The convection coefficients for the inlet and outlet side of the heat exchanger tube can be estimated using empirical correlations appropriate for the flow geometry and conditions. During normal heat exchanger operation, surfaces are subjected to fouling by fluid impurities, rust formation, and scale depositions, which can markedly increase the resistance to heat transfer between the fluids. For such situations, one would add the *fouling resistance* (inside and/or outside-side) to Equations 4.3 to give

$$R_{total} = \frac{1}{A_i h_i} + \frac{R_{f,i}''}{A_i} + \frac{t}{k A_{\ln}} + \frac{R_{f,o}''}{A_o} + \frac{1}{A_o h_o} \quad (4.8)$$

where $R_{f,i}''$ and $R_{f,o}''$ are the inside and outside fouling resistances per unit respective heat-transfer areas, in $\text{m}^2 \cdot \text{K/W}$. In actual applications, fouling is normally on one-side of

the heat-transfer surface. Therefore, if the overall heat transfer coefficient based on the clean condition is, U_c , determined typically based on outer tube area (refer to Equation 4.5), the time-dependent U based on fouled condition can be written as

$$\frac{1}{U_f(t)} = \frac{1}{U_c} + R_f''(t) \quad (4.9)$$

Heat Transfer

The general heat exchanger equation is written in terms of the mean-temperature difference between the hot and cold fluid, ΔT_m as

$$\dot{q} = UA \Delta T_m \quad (4.10)$$

This equation, combined with the First Law equations, defines the energy flows for a heat exchanger. It can be expressed in terms of the temperature change of the hot and cold fluids, as

$$\dot{q} = -(\dot{m}C_p)_h \Delta T_h = -\dot{C}_h \Delta T_h = \dot{C}_c \Delta T_c \quad (4.11)$$

where \dot{C}_h and \dot{C}_c are the hot and cold fluid capacitance rates, respectively.

Log Mean Temperature Difference (LMTD)

Heat flows between the hot and cold streams due to the temperature difference across the tube acting as a driving force. As seen in Fig. 4.2, the difference will vary with axial location so that one must speak in terms of the effective or integrated average temperature differences.

The form of the average temperature difference, ΔT_m , may be determined by applying an energy balance to differential control volumes (elements) in the hot and cold fluids. As shown in Fig. 4.2, for the case of parallel flow arrangement, each element is of length dx and the heat transfer surface area is dA . It follows for the *hot and cold fluid* as [1-3]

$$d\dot{q} = -\dot{C}_h dT_h = \dot{C}_c dT_c \quad (4.12)$$

The heat transfer across the surface area dA may be expressed by the convection rate equation in the differential form as

$$d\dot{q} = U dA dT \quad (4.14)$$

where $dT = T_h - T_c$ is the *local* temperature difference between the hot and cold fluids.

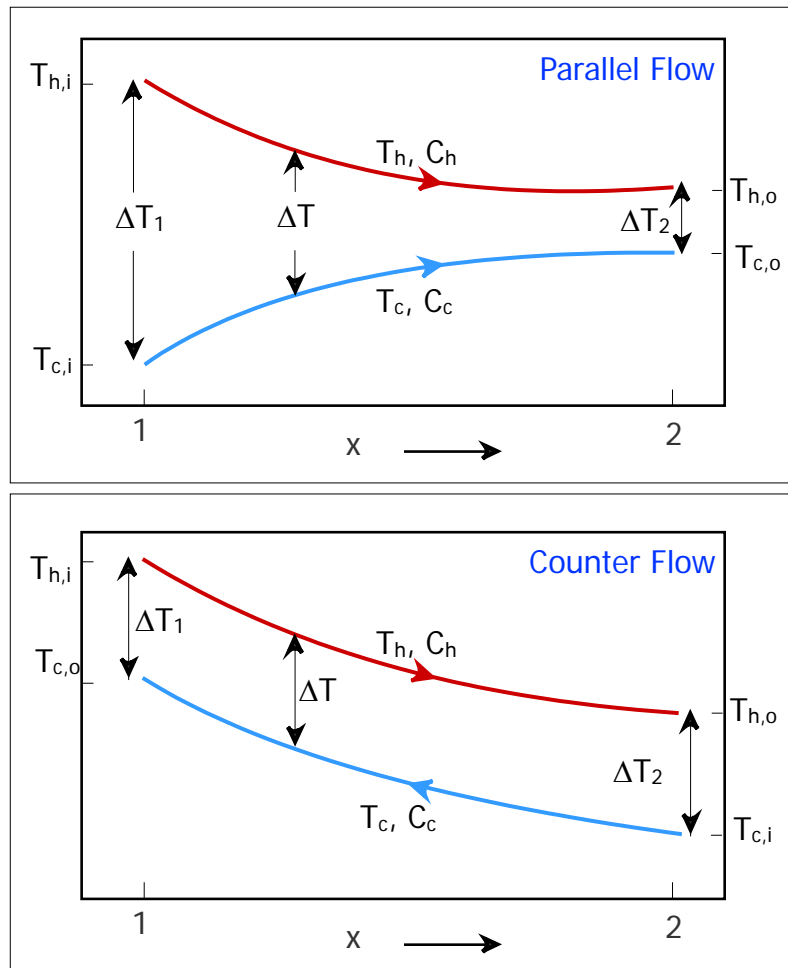


Figure 4.2: Temperature differences between hot and cold process streams

To determine the integrated form of Equation 4.14, we begin by substituting Equation 4.12 into the differential form for the temperature difference,

$$d(\Delta T) = d(T_h - T_c) \quad (4.15)$$

to obtain

$$\int_1^2 \frac{d(\Delta T)}{\Delta T} = -U \left(\frac{1}{\dot{C}_h} + \frac{1}{\dot{C}_c} \right) \int_0^A dA$$

$$\ln \left(\frac{\Delta T_2}{\Delta T_1} \right) = -UA \left(\frac{1}{\dot{C}_h} + \frac{1}{\dot{C}_c} \right) \quad (4.18)$$

Substituting \dot{C}_h and \dot{C}_c from the fluid energy balances, Equations 4.12 and integrating, we get after some manipulation,

$$Q = UA \frac{(\Delta T_2 - \Delta T_1)}{\ln\left(\frac{\Delta T_2}{\Delta T_1}\right)} \quad (4.19)$$

Comparing the above expression with Equation 4.10, we conclude that the appropriate mean temperature difference is the *log mean temperature difference*, ΔT_{lm} . Accordingly, we have

$$\Delta T_m = \Delta T_{lm} = \frac{(\Delta T_2 - \Delta T_1)}{\ln\left(\frac{\Delta T_2}{\Delta T_1}\right)} = \frac{(\Delta T_1 - \Delta T_2)}{\ln\left(\frac{\Delta T_1}{\Delta T_2}\right)} \quad (4.20)$$

where

$$\Delta T_1 = (T_{h,i} - T_{c,i}) \quad \text{and} \quad \Delta T_2 = (T_{h,o} - T_{c,o}) \quad (4.20)$$

A similar derivation can be shown for *counter-flow heat exchangers*; however, the temperature difference as shown in Figure 4.2, will be

$$\Delta T_1 = (T_{h,i} - T_{c,o}) \quad \text{and} \quad \Delta T_2 = (T_{h,o} - T_{c,i}) \quad (4.21)$$

As discussed above, the effective mean temperature difference calculated from this equation is known as the log mean temperature difference, frequently abbreviated as LMTD, based on the type of mathematical average, which it describes. While the equation applies to either parallel or counter flow, it can be shown that ΔT_m will always be greater in the counter flow arrangement.

Effectiveness – NTU Method

In this method first, we define effectiveness of a heat exchanger as an actual over maximum possible heat transfer [1-3],

$$\varepsilon = \frac{q}{q_{\max}} = \frac{\dot{C}_c (T_{c,o} - T_{c,i})}{\dot{C}_{\min} (T_{h,i} - T_{c,i})} = \frac{\dot{C}_h (T_{h,i} - T_{h,o})}{\dot{C}_{\min} (T_{h,i} - T_{c,i})} \quad (4.22)$$

where q_{\max} is obtained from an infinitely long pure counter-flow heat exchanger that will have maximum possible temperature difference. We choose an infinitely long exchanger since that will yield the *maximum* heat transfer that can take place (i.e., eventually, enough heat will be transferred so that the driving force will *disappear* - the streams will reach the same temperature).

It can be easily shown by simple analysis that effectiveness of a parallel flow heat exchangers can be expressed as

$$\varepsilon = \frac{1 - \exp[-NTU(1 + C_r)]}{(1 + C_r)} \quad (4.23)$$

and for a counter flow,

$$\varepsilon = \frac{1 - \exp[-NTU(1 - C_r)]}{1 - C_r \exp[-NTU(1 - C_r)]} \quad \text{for } C_r < 1 \quad (4.24a)$$

$$\varepsilon = \frac{NTU}{1 + NTU} \quad \text{for } C_r = 1 \quad (4.24b)$$

where NTU is defined as the number of transfer units, and C_r as a ratio of fluid capacitance rate ratio. They are dimensionless quantities defined as

$$NTU = \frac{UA}{\dot{C}_{\min}} \quad \text{and} \quad C_r = \frac{\dot{C}_{\min}}{\dot{C}_{\max}} \quad (4.25)$$

Experimental Apparatus and Procedure

The objectives of the heat exchanger experiments are achieved through the use of a bench mounted double-pipe heat-exchanger unit (refer to Fig. 4.3). The unit consists of six passes, but only two passes will be used during the experiment. The hot fluid flows in the inner tube, while the cold in outer tube. The cold fluid circuit has four valves that are used for changing flow arrangement from parallel flow to counter flow. It is important that water at constant temperature is provided to both fluid circuits by means of a water bath. Three thermocouples are available for the hot fluid temperature measurements and seven for the cold fluid temperature measurements. Using two selector switches, fixed on the instrument panel, easily makes the temperature readings from these thermocouples. The flow rates of hot and cold fluids are measured using rotameters, connected to the inner and outer tubes. Notice that the measurements would be taken for temperatures and flow rates for both (a) parallel and (b) counter-flow arrangements. It is very important that the water flow through the tubes reaches stable and steady state conditions before recording the following parameter for both parallel and counter flow conditions:

1. Set the flow rate at the lowest (stable) reading and then monitor the difference between the inlet and outlet temperatures for both hot and cold water ($C_r = 1.00$) until a steady state is established.
2. Measure and record the inlet, outlet and temperature difference for both hot and cold flows.
3. Change the cold water flow to give $C_r = 0.75$, then .5 and .25, each time repeating (1) and (2) above.

4. Repeat the above sequence by raising the hot-side water flow rate.

Data Analysis

- Organize your lab data and calculated values in a neat spreadsheet array. Use only the **SI system of units**;
- Plot the heat transfer to the cold fluid versus the log-mean-temperature difference.
- Calculate the heat loss by the hot fluid and heat gain by the cold fluid;
- From the experiments performed, determine the average overall heat transfer coefficient, U ;
- On one plot, present the effectiveness, ε , versus NTU and curve fit the data where C is a constant by making a plot similar to $\varepsilon - NTU$. Remember effectiveness from your data is determined from the four measured temperatures using equation (4.22).

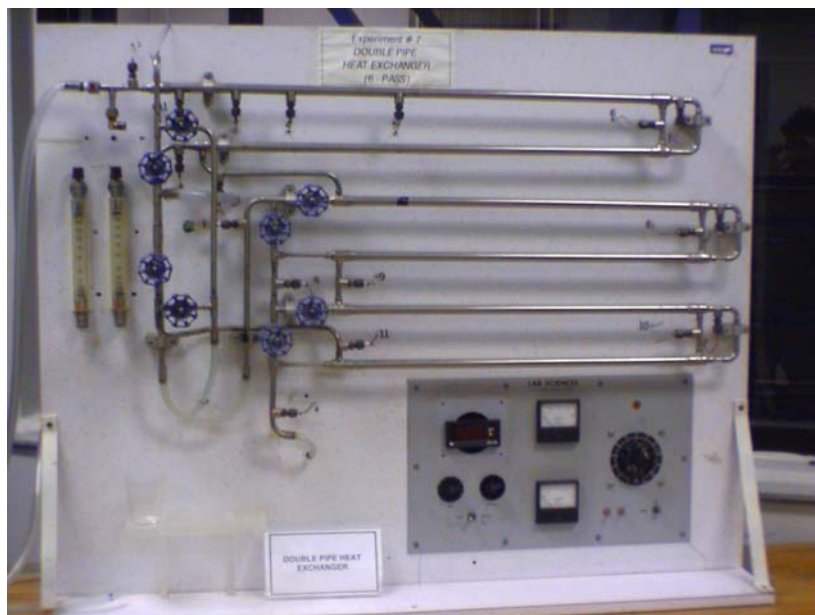


Figure 4.3: A bench mounted double-pipe heat-exchanger unit.

Suggestions for Discussion

- (i) What effect does the heat loss have on the performance of heat exchangers?
 - (ii) Is it important to consider heat losses from the counter- or parallel-flow arrangement?
 - (iii) What errors may be introduced in calculating heat exchanger effectiveness from the experimental measurements?
 - (iv) What would you recommend to improve the reliability of this experiment?
-

References:

- [1] Incropera, F. P. and D. P. DeWitt, *Fundamentals of Heat and Mass Transfer, 4th edition*, John Wiley and Sons Inc., New York, 1996.
- [2] Çengel, Yunus A., *Heat Transfer: A Practical Approach, 2nd edition*, McGraw-Hill, New York, 2003.
- [3] Thomas, L.C., *Heat Transfer – Professional Version, 2nd edition*, Capstone Publishing Corp, 1999

LAB DATA SHEET
Experiment 4: Double-pipe heat exchanger characteristics

Name : _____ Id# : _____ Date : _____

Note: Make sure that the process has reached steady state conditions before recording any readings.

Fluid Temperature and Properties			
	Temp (°C)	Density (kg/m ³)	Sp. Heat (J/kg.K)

Useful Information
1 liter = 0.001 m ³
1 liter/min = 1.667 x 10 ⁻⁵ m ³

Dimensions of Heat Exchanger:

D_i = _____ mm D_o = _____ mm L = _____ m

Surface area $A_{s,o}$ = _____ m²

Table 4.1: Parallel flow arrangements

Distance x from hot end (in m)	Hot fluid temperature (°C)	Cold fluid temperature (°C)
0	TC # 1	TC # 1
0.04	--	TC # 2
0.13	--	TC # 3
0.22	--	TC # 4
0.39	--	TC # 5
0.84	TC # 2	TC # 6
1.61	TC # 3	TC # 7
Flow rate (liter/min)		

Table 4.2: Counter flow arrangement

Distance x from hot end (cm)	Hot fluid temperature ($^{\circ}\text{C}$)	Cold fluid temperature ($^{\circ}\text{C}$)
0	TC # 1	TC # 1
0.04	--	TC # 2
0.13	--	TC # 3
0.22	--	TC # 4
0.39	--	TC # 5
0.84	TC # 2	TC # 6
1.61	TC # 3	TC # 7
Flow rate (lit/min)		

The following table will help you to report thermal calculations for the two fluid arrangements:

Table 4.3: Experimental and calculated values of temperatures, flow rates, and heat transfer rates

Parameter	Parallel-flow		Counter-flow	
	Hot fluid	Cold fluid	Hot fluid	Cold fluid
Inlet temperature (in $^{\circ}\text{C}$)				
Outlet temperature (in $^{\circ}\text{C}$)				
Average temperature (in $^{\circ}\text{C}$)				
Density at avg. temp. (in kg/m^3)				
Sp. heat at avg. temp. (in $\text{J}/\text{kg}\cdot\text{K}$)				
Volume flow rate (in m^3/s)				
Mass flow rate (in kg/s)				
Thermal capacitance rate, $\dot{m} C_p$ (in W/K)				
Heat transfer rate $q = \dot{m} C_p T_{in} - T_{out} $ (in W)				
Heat loss from the exchanger, transfer, $\dot{q}_{loss} = \dot{q}_h - \dot{q}_c$ (in W)				

Table 4.4: Calculated values of temperature differences, LMTD, overall heat transfer coefficient, heat exchanger effectiveness and NTUs

Parameter	Parallel flow	Counter flow
ΔT_1 (in C)		
ΔT_2 (in C)		
$LMTD = \frac{\Delta T_1 - \Delta T_2}{\ln\left(\frac{\Delta T_1}{\Delta T_2}\right)}$ (in C)		
Overall heat transfer coefficient $U_o = \frac{q_{avg}}{A_{s,o} (LMTD)}$ (in W/m ² .K)		
Fluid capacitance ratio, $R = \frac{(\dot{m}C_p)_{min}}{\dot{m}C_p}$		
Maximum possible heat transfer $q_{max} = (\dot{m}C_p)_{min} (T_{h,i} - T_{c,i})$ (in W)		
Heat exchanger effectiveness, $\varepsilon = \frac{q}{q_{max}}$		
Number of transfer units, $NTU = \frac{UA_s}{(\dot{m}C_p)_{min}}$		

EXPERIMENT 5

Stefan-Boltzmann Law and Radiation Properties

Objective

The objectives for this experiment are:

- (a) To derive the Stefan-Boltzmann law, which states that the emissive power of a black or gray body is proportional to the fourth power of its absolute temperature; and
- (b) To determine the emissivity, absorptivity, and reflectivity of a plate.

Background

At temperatures above absolute zero all matter emits radiant energy. Such radiant energy is the result of kinetic energy of the atoms or molecules, which is a function of the temperature of the substance. Radiant energy is transmitted in the form of electromagnetic waves having a wide range of wavelengths. The ability of a substance to absorb incident radiant energy is intimately related to its ability to radiate its own energy. Radiation phenomena like emission, absorption, reflection and transmission are widely affected by material characteristics, surface conditions and geometrical shape.

An understanding of radiation phenomena is useful to nearly all scientists and engineers. The following paragraphs summarize the major physical laws of radiation. No attempt is made in this handout to derive these laws; they are merely stated for reference purposes. Most of these laws were obtained experimentally from test data and only later were the laws theoretically derived. All of these basic laws can be demonstrated and even derived using only the Scott Radiation and Temperature Measurement System, Model 9053.

The Scott Radiation and Temperature Measurement System is a bench-top laboratory for studying heat transfer by radiation. The unit is simple to use, yet sophisticated enough for advanced work. After describing the equipment, this manual summarizes the basic theory of radiation and gives the procedures for conducting experiments to derive the classical laws of radiation. The laws are derived experimentally by taking test measurements of voltage (emf) from a thermopile receiving radiation from a radiating body in the form of a hot plate. These test points are then compared to the theoretical fourth power curve.

The emissive power E of a radiating blackbody depends only on its temperature T . The second law of thermodynamics may be used to prove that the emissive power is also proportional to the fourth power of the absolute temperature and to a universal constant σ , called the Stefan-Boltzmann constant. For a blackbody [1-3],

$$E = \sigma T^4 \quad (5.1)$$

where the constant σ is the Stefan-Boltzman constant.

The emissivity ε of a surface is the ratio of the emissive power of the surface to that of a blackbody surface,

$$\varepsilon = \frac{E_{\text{surface}}}{E_{\text{blackbody}}} \quad (5.2)$$

Therefore, a more general form of the Stefan-Boltzmann law for any type of surface is,

$$E = \varepsilon \sigma T^4 \quad (5.3)$$

To prove this law, we need some type of sensor that could measure emitted radiation from a source of known temperature. As the emitted radiation changes, the resistance of the sensor also changes in proportion. The voltage drop across the resistor will allow us to quantify the radiation emitted. As a result, Equation (5.3) can be related to the voltage drop across the resistor according to,

$$V = CE = \varepsilon \sigma T^4 \quad (5.4)$$

where C is some proportionally constant. By combining all the constants into a "new" C, we get,

$$V = CT^4 \quad (5.5)$$

We must first find C and then using this C, we can see if this law holds true. Keep in mind sources of error that might influence your results, for example, convection, other radiation sources, etc.

It is important to note that net rate of radiant exchange between two surfaces; in particular a plate in a room can be expressed as

$$q_{p-r} = \varepsilon \sigma (T_p^4 - T_r^4) \propto V \quad (5.6)$$

where

- T_p is the plate temperature, K
- T_r is the room temperature, K
- V is the thermopile voltage, milli Volts
- ε is the emissivity of the plate

To compare a plate of equal area with the selected standard etched plate coated with lampblack, we get

$$\frac{q_{p(a)-r}}{q_{p(1)-r}} = \frac{V_a}{V_1} = \frac{\varepsilon_a (T_a^4 - T_r^4)}{\varepsilon_1 (T_1^4 - T_r^4)} \quad (5.7)$$

where V_1 is the thermopile voltage of the standard etched plate coated with lampblack, while V_a is the voltage of plate under investigation. The emissivity of the reference lampblack plate is taken to be 0.95. Therefore, emissivity of the the test plate can be calculated from the following equation:

$$\varepsilon_a = \frac{V_a}{V_1} \frac{\varepsilon_1 (T_1^4 - T_r^4)}{(T_a^4 - T_r^4)} \quad (5.8)$$

Notice that under thermal equilibrium condition, the amount of radiation absorption is equal to radiation emitted plus reflected and transmitted. Therefore, for a gray surface, one can write

$$\varepsilon = \alpha \quad (5.9)$$

$$\alpha + \rho + \tau = 1.00 \quad (5.10)$$

where

- α is defined as the absorptivity of a surface
- ρ is defined as the reflectivity of a surface; and
- τ is defined as the transmissivity of a surface, $\tau = 0$, for a surface if it is totally opaque; i.e., no radiation can pass through the surface.

EXPERIMENTAL APPARATUS

The Scott Radiation and Temperature Measurement System, Model 9053 (refer to Fig. 5.1) organizes a number of sources of heat and light radiation, various types of instrumentation, a number of fixtures, and certain controls for use in an integrated group of experiments designed to give insight into radiation phenomena and common means for obtaining temperature measurements. Major components are mounted on a portable laboratory table.

The control panel includes an auto transformer, a voltmeter, an ammeter and appropriate switches. Temperature measurements are performed with thermocouples connected to a precision potentiometer (optional accessory); or with a two-axis, direction-sensitive, thermal-electric pyrometer (thermopile) which may be positioned along either of two tracks provided with ruled scales, or hand held. For measuring light intensities in the visible band, a photoelectric cell with an integral galvanometer is provided as standard equipment. It, too, may be positioned along the two scaled tracks or hand held. In addition, several radiation targets with various absorption abilities and geometrical shapes are provided.

Some accessories that are used in conjunction with the Scott Radiation System are one hand held thermocouple meter and a voltmeter. Please ask the instructor for these accessories.

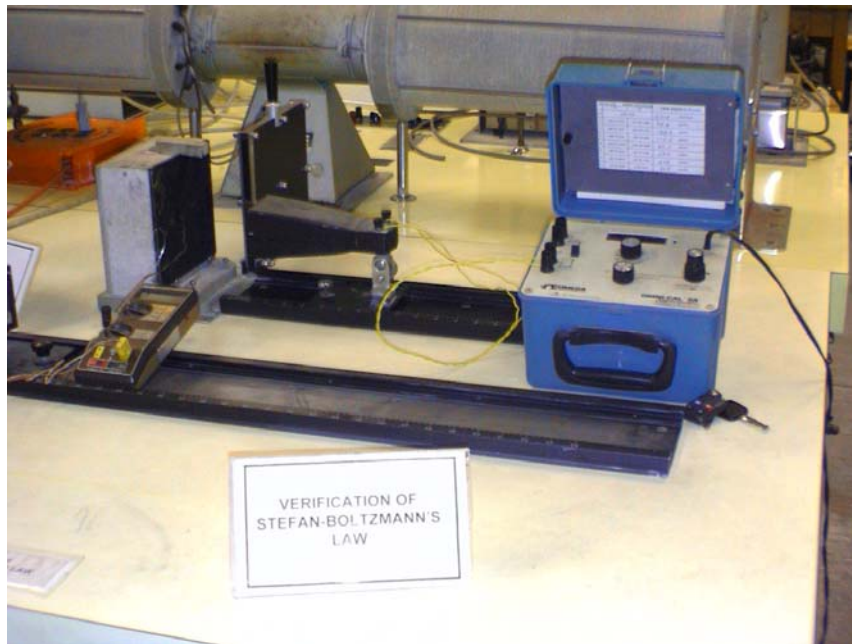


Figure 5.1: Scott radiation and temperature measurement system - model 9053

Experimental Procedure

Place the thermopile on the short track at the specified mark (note a mark on thermopile base), facing the plate heater. Mount the etched stainless steel plate as an emitter in front of the plate heater after coating both sides with a layer of lampblack. The side of the plate having the thermocouple attached to it faces the thermopile. Connect the thermocouple to the thermocouple meter provided. Attach the leads from the thermopile to the voltmeter. Ensure that the voltmeter is on the proper setting, mV. Now follow the following test procedure to collect the data:

- (a) Start with a setting of 120 on the powerstat. Please do not go above 120 on the powerstat because it may cause the fuse to blow. Allow the plate to come to steady state (when the temperature of plate no longer changes), then take readings of the thermocouple temperature, thermopile voltage (millivolts DC), voltmeter and ammeter. Higher voltages, thus higher plate temperatures, appear to yield better results. When you are done with the 120 setting, go to 100 then 80 etc. Take as much data as you have time for, preferably at least 4 settings of the powerstat.
- (b) Plot the thermopile voltage (Emf) as a function of the plate temperature (K) on linear-linear graph paper (refer to sample Figure 5.2).
- (c) Make the same plot as in step (b) above on log-log paper (refer to sample Figure 5.3).

- (d) Take all relevant data for radiation properties at 80 volts.

Data Analysis

The thermopile voltage is proportional to the plate emissive power. Figure 5.3 shows a typical curve obtained from actual measurements plus the theoretical curve according to the Stefan-Boltzmann Law. Theoretically, thermopile voltage = CT^4 where C is a constant. To make this plot, determine this constant from your highest temperature data. Then use that constant to plot the theoretical relation between thermopile voltage and plate temperature according to this equation. Note that this constant is not the Stefan-Boltzmann constant σ but is related to σ since $E = \sigma T^4$ and the thermopile voltage is proportional to E . The log-log plot for a fourth power function has a slope of 4.0 (see Figure 5.3). (Note that some computer log-log plots are scaled differently, so your slope may be different.) Determine the actual slope using your data and compare it to 4.0. Discuss each plot qualitatively, commenting on the shape of the curve against what it theoretically should be. Try to discuss reasons for the differences, etc. Keep a copy of your plot for future reference. For radiation properties, first calculate the emissivity of the etched plate, and then determine both absorptivity and reflectivity from equations (5.9) and (5.10), respectively.

References:

- [1] Incropera, F. P. and D. P. DeWitt, *Fundamentals of Heat and Mass Transfer*, 4th edition, John Wiley and Sons Inc., New York, 1996.
- [2] Çengel, Yunus A., *Heat Transfer: A Practical Approach*, 2nd edition, McGraw-Hill, New York, 2003.
- [3] Thomas, L.C., *Heat Transfer – Professional Version*, 2nd edition, Capstone Publishing Corp, 1999.

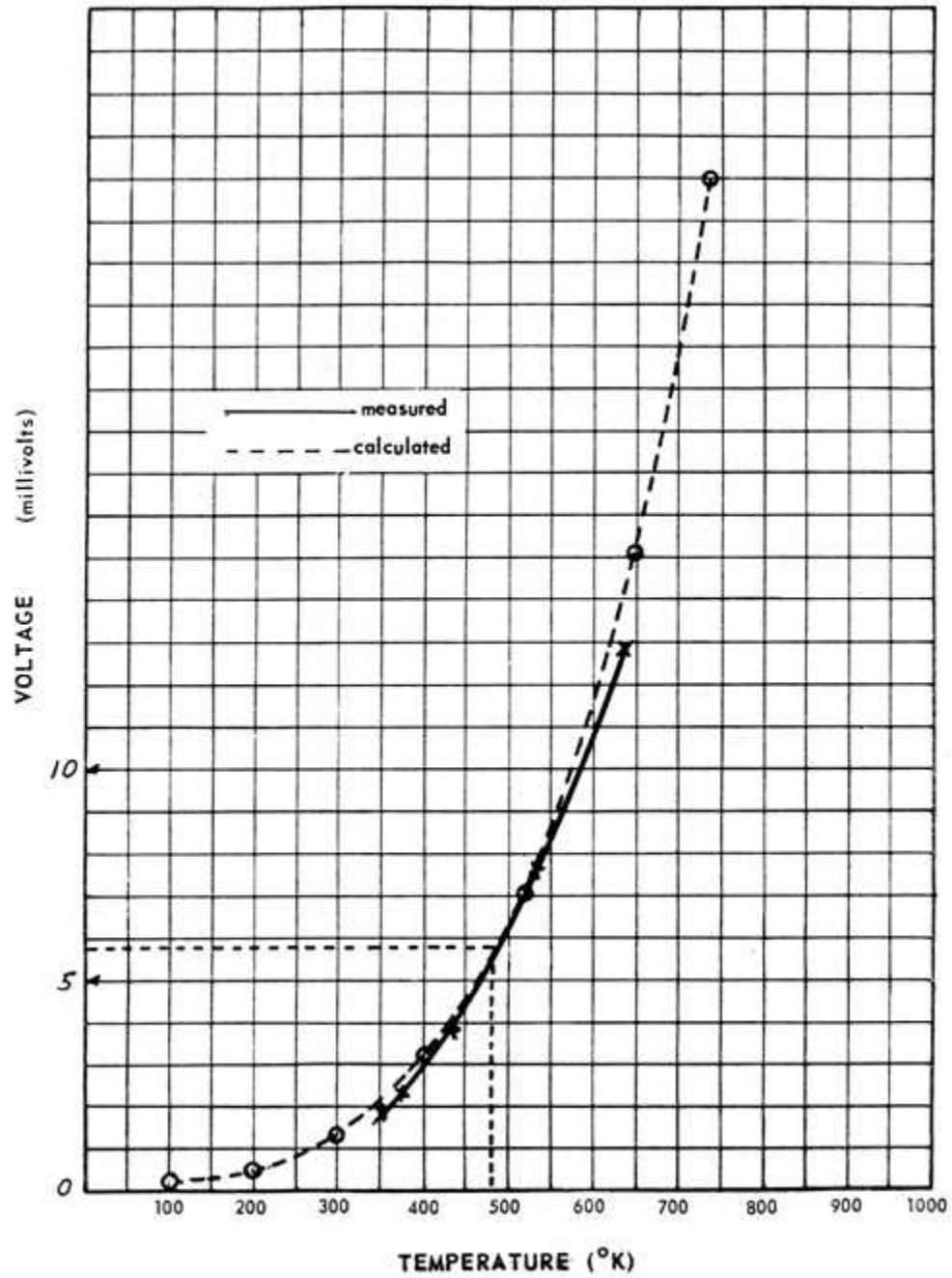


Figure 5.2: Stefan-Boltzmann law, linear plot

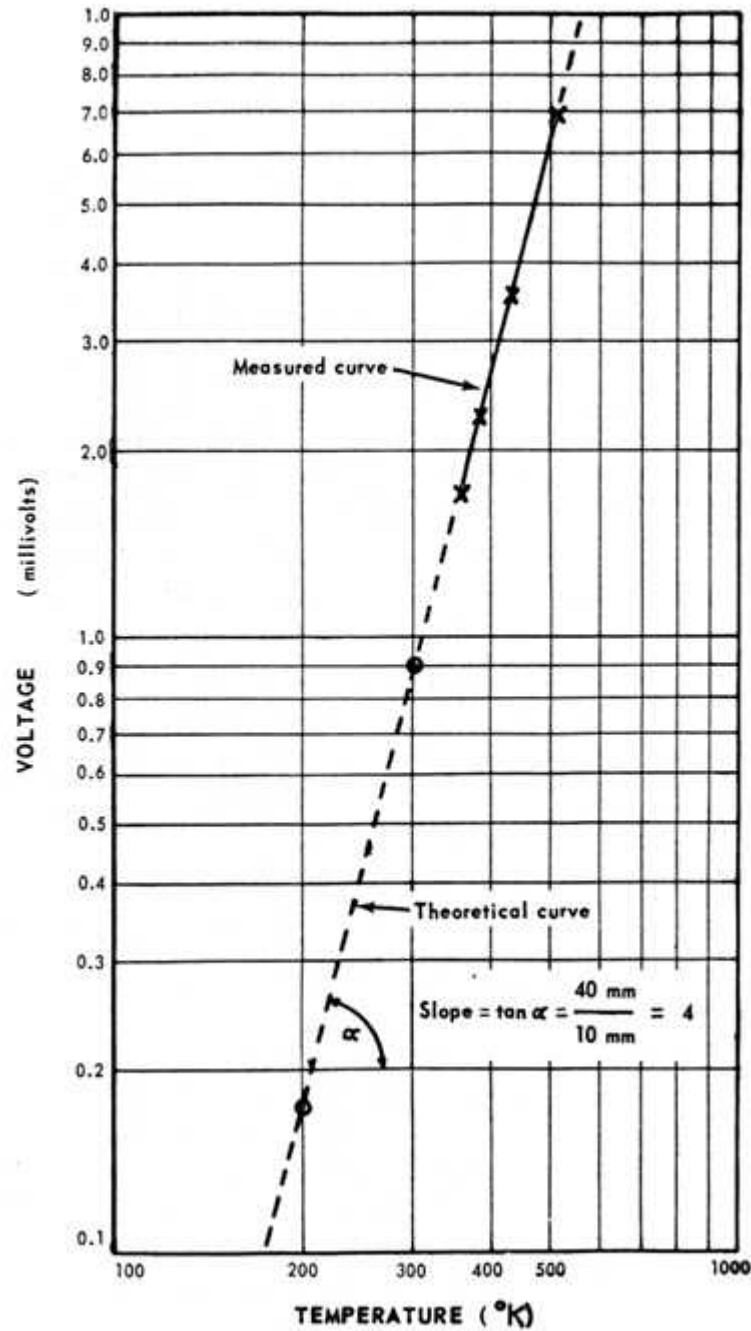


Figure 5.3: Stefan-Boltzmann law, logarithmic plot

LAB DATA SHEET**Experiment 5: Stefan-Boltzmann Law and Radiation Properties**

Name : _____ Id# : _____ Date :

Note: Make sure that the process has reached steady state before recording any readings.

Table 5.1: Black body temperature and potentiometer voltage.

Heater Voltage (Volts)	Plate Temperature		Potentiometer Reading (mV)
	(°C)	(K)	
120 V			
100 V			
80 V			

- Plot potentiometer voltage (V , in mV on y-axis) versus plate temperature (T , in Kelvin on x-axis) on a log-log graph. Obtain the slope n of the line.

Slope n = _____Calculate % error = $\frac{n - 4}{4} \times 100 =$ _____

- Complete the table shown below. (Note: V_{ref} and T_{ref} are the values of V and T at a reference heater voltage (say, at 80V))

T (K)	$V_{\text{expt}} = \left(\frac{T}{T_{\text{ref}}} \right)^n V_{\text{ref}}$ (mV)	$V_{\text{theo}} = \left(\frac{T}{T_{\text{ref}}} \right)^4 V_{\text{ref}}$ (mV)
0		
100		
200		
300		
400		
500		
600		

Table 5.2: Emissivity of a gray body

Heating plate voltage (V)		80 V	
Black body temperature	T_b	C	K
Black body emf	V_b (mV)		
Emissivity of black body	ε_b	0.95	
Gray body temperature	T_g	C	K
Gray body emf	V_g (mV)		
Room temperature	T_R	C	K

Calculate emissivity, absorptivity (α), reflectivity (ρ) and transmittivity (τ) for the gray body from equations (5.8) – (5.10).

EXPERIMENT 6

Free Convection over Solid Cylinders

Objective

The objective of this experiment is to study unsteady state heat transfer by the lumped capacitance method and to determine free convection heat transfer coefficients.

Background

In view of the fact that the density of most fluids varies with temperature, temperature gradients within a fluid medium will give rise to density gradients. If these density gradients are such that the fluid is in an unstable situation, heavy fluid on top of light fluid, the fluid will begin to move. This motion is termed natural (or free) convection. Newton's law of cooling describes the heat transfer in this physical process. It states that the heat flux from the surface is directly proportional to the temperature difference between the surface and the fluid far away from the surface or

$$q'' \propto (T_{surface} - T_{fluid}) \quad (6.1)$$

Introducing the convective heat transfer coefficient as the constant of proportionality, we get

$$q'' = \frac{q}{A_s} = h(T_{surface} - T_{fluid}) \quad (6.2)$$

It should be noted that the major complication in utilizing equation (6.2) for convective heat transfer calculations is the evaluation of the convective heat transfer coefficient, h . There are three standard methods used to evaluate h . The first involves a mathematical solution to the conservation equations in differential form. For problems where these equations are too complicated to be solved analytically, we can employ the second method, a computational solution. Finally, for problems so complicated that we cannot even write the appropriate conservation equations, we must go into the laboratory and make measurements in employing an experimental solution. Before we consider employing one of these solution methods, it is useful to use our intuition to determine what affects the convective heat transfer coefficient. Our fundamental understanding of heat transfer tells us that the three major factors associated with the calculation of h [1-3],

- (a) Fluid Mechanics;
- (b) Fluid Transport Properties; and
- (c) System Geometry

Considering the fluid mechanics, we recognize that there are a variety of ways to characterize the flow. We can anticipate what is driving the flow and classify it as forced or natural convection. Next, we consider how many boundaries the fluid flow interacts with and classify it as external or internal flow. Recall that the difference between internal flows and external flows is often one of viewpoint. A third way to characterize the flow is by the presence or absence of turbulence (or instability).

One of the most important considerations in the handling of convective heat transfer coefficients is the notion of dynamic similarity. It is found that certain systems in fluid mechanics or heat transfer have similar behaviours even though the physical situations may be quite different. For example, consider the fluid mechanics of flow in a pipe or conduit. What we are able to normally perform is to take data as shown in Fig. 6.1 for different fluids and pipe diameters and by appropriately scaling collapse these curves into one curve.

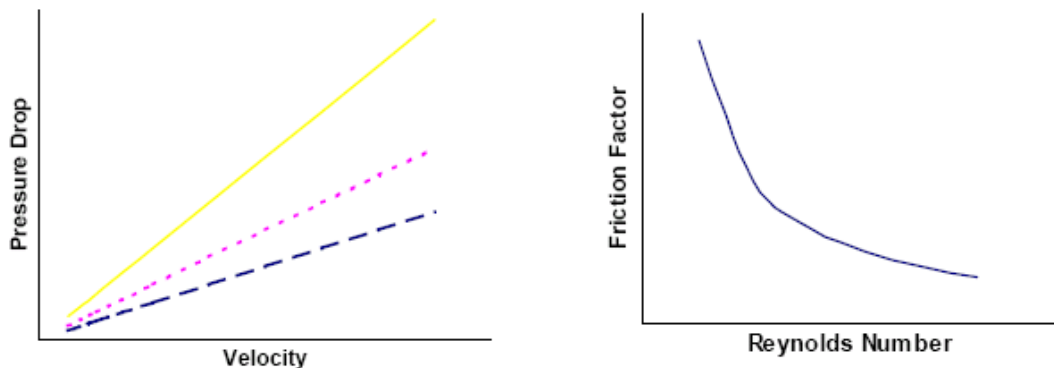


Figure 6.1: Dynamic similarity for a pipe flow problem

In convective heat transfer we may apply dynamics scaling to make a somewhat parallel transformation (refer to Fig. 6.2)

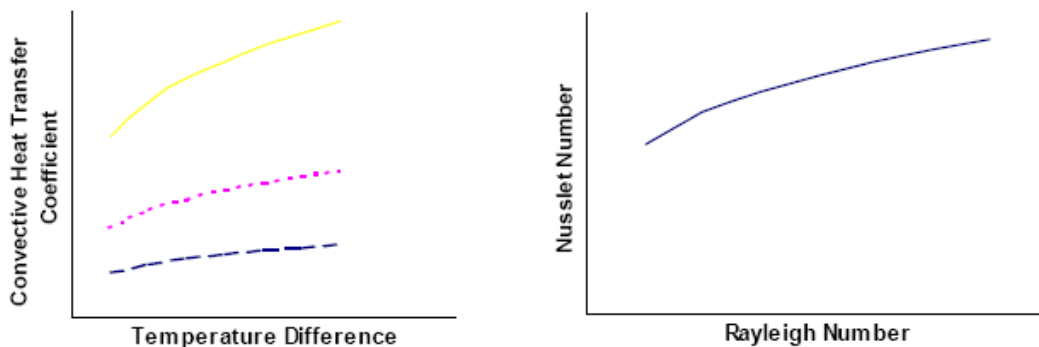


Figure 6.2: Dynamic similarity for a convective heat transfer problem

In your heat transfer text; dimensionless convective heat transfer coefficient called the Nusselt number is defined as

$$Nu = \frac{hL_c}{k} \quad (6.3)$$

where

h is convective heat transfer coefficient, $\text{W/m}^2\cdot\text{K}$

L_c is characteristic length, m

k is thermal conductivity of the fluid, $\text{W/m}\cdot\text{K}$

The characteristic length is chosen as the system length that most affects the fluid flow. For flow around a cylinder our characteristic length is the diameter, D , we can write as

$$Nu = \frac{hD}{k} \quad (6.4)$$

The Rayleigh number which is defined as a ratio of buoyancy force to viscous force, is an indicative of the buoyancy force that is driving the flow is given by

$$Ra = \frac{g\beta|T_{\text{surface}} - T_{\text{fluid}}|L_c}{\nu\alpha} \quad (6.5)$$

where

g is acceleration due to gravity, m/s^2

k is thermal conductivity of the fluid, $\text{W/m}\cdot\text{K}$

T_{surface} is surface temperature, $^{\circ}\text{C}$

T_{fluid} is fluid temperature, $^{\circ}\text{C}$

β is fluid thermal expansion coefficient, K^{-1}

ν is fluid kinematic velocity, m^2/s

α is fluid thermal diffusivity, m^2/s

The dimensionless parameter that characterizes the affect of fluid properties is defined as Prandtl number,

$$Pr = \frac{\nu}{\alpha} \quad (6.6)$$

The influence of geometry may be seen in a couple of ways. First, for those configurations that have two length dimensions, such as a cylinder, we introduce a dimensionless geometric parameter in terms of diameter and length of cylinder as,

$$X = \frac{D}{L} \quad (6.7)$$

The second way in which we may see geometrical influences is through the functional form of the Nusselt number correlation. In general, we express as [3]

$$Nu = f(Ra, Pr, X) \quad (6.8)$$

For example, for simple situations having only one length dimension, we often express Nusslet number as

$$Nu = a Ra^n Pr^n \quad (6.9)$$

where the constants a, m, and n will change for different geometries.

EXPERIMENTAL APPARATUS

The apparatus consists of an electric heater to heat solid cylinders ($D = 25$ mm, $L = 25$ mm) to certain specified temperature (300 °C). A string in still air then suspends the heated cylinders. The two cylindrical specimens are made of aluminium and brass respectfully and both have a thermocouple imbedded within them. The drop in temperature of the cylinder is recorded as a function of time using a temperature recorder or a data acquisition system available in the laboratory. A set up of the system is shown in Figure 6.1.

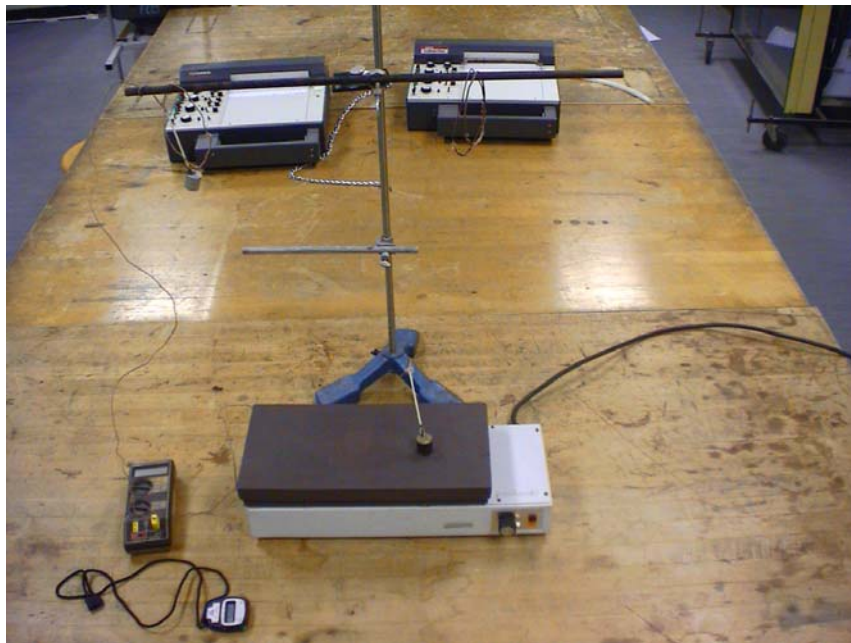


Figure 6.1: Test equipment to study natural convection from a cylindrical object

Experimental Procedure

Place the specimens on the electrically heated hot plate and monitor the temperature of the specimen until it reaches about 300 oC. The heated specimens are then suspended in the still air, and the temperature is recorded as a function of time. Now follow the following test procedure to collect the data:

- (1) Place the specimen on an electric hot plate;
- (2) Set up the data acquisition system by the supplemental instructions (consult your lab instructor);
- (3) Suspend the first cylinder in the air and obtain temperature data as a function of time as the cylinder cools down;
- (4) It will **not** be necessary to make several runs for this experiment; just one for each is sufficient. Make sure you always start with the same initial specimen temperature during these additional runs, if needed;
- (5) Repeat the above procedures for the second cylinder.

It is very important in this simple experiment to discuss whether assumptions made are valid (keep in mind all approximations used, especially in using the equations and finding slopes, etc). Comment on the sources of error, and carry out sensitivity analysis using EES software. Keep in mind the correlations used are models of a slightly different physical situation. Also discuss errors, if any, within the equipment and methodology of the experiment.

Theory and Data Analysis

There are many heat transfer application where unsteady heat flow causes temperature and other variables to change with time. We will see, however, that in some unsteady situations, for which a certain criterion is met, the use of the lumped capacitance theory greatly simplifies the analysis. The criterion as we will see is based on the assumption that temperature gradients within a solid are negligible compared to the temperature gradients between the solid and fluid. To verify whether this assumption is true or not depends on calculating the Biot number, Bi , which is defined as the ratio of temperature differences across the solid itself, and between the solid and fluid. It can also be defined as the ratio of thermal resistance of the solid by conduction and the fluid by convection, given as

$$Bi = \frac{T_{s,1} - T_{s,2}}{T_{s,2} - T_{\infty}} = \frac{R_{cond}}{R_{conv}} = \frac{(L / kA)}{(1 / hA)} = \frac{hL_c}{k} \quad (6.10)$$

The value, L_c , is the characteristic length which is defined as the solid's volume over the surface area, $L_c = V/A_s$. In order to use the **Lumped Capacitance Method**, the Biot number must be calculated and confirmed to be less than **0.1**. Unless this requirement is satisfied, use of this method would create too much error.

To understand the lumped heat capacity theory we must start by considering a hot metal cylinder, which is suspended, in a still air environment. The essence of this theory is that the temperature within the solid cylinder is assumed to be *spatially* uniform at any instant throughout the unsteady cooling process. This implies that the temperature gradient within the solid is negligible compared to that of the gradient across the solid-fluid interface. The nice thing about this assumption is that we no longer have to use the conduction heat transfer analysis, which becomes very difficult. Instead we simply use the first law of thermodynamics on the solid. Writing the first law for unsteady heat flow with no work we have heat rejected from the cylinder due to free convection is equal to rate of change of temperature of the solid with respect to time. This gives

$$-q = \rho VC \frac{dT}{dt} \quad (6.11)$$

where q is the heat transferred from the system to the surroundings (in Watts), C is the specific heat of the metal (in J/kg.K), A_s is the total surface area (in m²), V is the volume (in m³), and ρ is the density of the cylinder (in m³/kg). Substituting q from equation (6.2), we get,

$$hA_s(T_s - T_\infty) = -\rho VC \frac{dT}{dt} \quad (6.12)$$

This equation states that the rate of heat lost by convection from the block surfaces equals the negative of the time rate of change of energy stored in the block.

Now, if we define the time constant, ($\tau = hA_s / \rho VC$) in the above equation as a constant value with respect to time, equation (6.12) could be easily integrated to give

$$\theta(t) = \frac{T_s(t) - T_\infty}{T_i - T_\infty} = \exp(-t / \tau) \quad (6.13)$$

Simplifying the above equation, we get

$$\ln(\theta) = -t / \tau = mt \quad (6.14)$$

where T_i is defined as the initial temperature of the cylinder at $t = 0$, and m is the slope of the $\ln(\theta)$ versus t plot. It can be observed from equation (6.14) that the time constant is the negative reciprocal of the slope m . Once we find the time constant from the experimental data, gives us the average heat transfer coefficient,

$$h = \frac{\rho VC \tau}{A_s} \quad (6.15)$$

For this experiment we are to compare the h values with expected values from the literature. Note that the value of Nu for free convection on a long horizontal cylinder vertical surface can be found using equation (9.34) in ref. [1]. It is given by

$$Nu = 0.60 + \left\{ \frac{0.387 Ra^{1/6}}{[1 + (0.559 / Pr)^{9/16}]^{8/27}} \right\}^2 \quad Ra \leq 10^{12} \quad (6.16)$$

where the Prandtl number, Pr can be found from the tables in the back of your textbook, and the Rayleigh number, Ra is defined by equation (6.6) . The values of ν and α that are needed in Ra can also be found from the tables in the back of your textbook.

Room air can be treated as an ideal gas, therefore fluid thermal expansion coefficient, $\beta = 1/T_{film}$. Note the film temperature is the average temperature between the surface and ambient. All properties of air are should be evaluated at the film temperature.

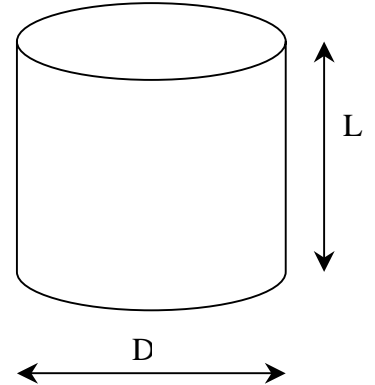
References:

- [1] Incropera, F. P. and D. P. DeWitt, *Fundamentals of Heat and Mass Transfer, 4th edition*, John Wiley and Sons Inc., New York, 1996.
- [2] Çengel, Yunus A., *Heat Transfer: A Practical Approach, 2nd edition*, McGraw-Hill, New York, 2003.
- [3] Bejan, A., *Convection Heat Transfer, 2nd edition*, John Wiley and Sons Inc., New York, 1995.

LAB DATA SHEET
Experiment 6: Free Convection over Solid Cylinders

Name : _____ Id# : _____ Date : _____

Parameter		Aluminium	Brass
Ambient temperature, $T_{ambient}$			
Diameter	D (m)	0.02515	0.02520
Length	L (m)	0.02520	0.02510
Density,	ρ (kg/m ³)	2705	8530
Specific heat	Cp (J/kg.K)	977	377.5
Thermal conductivity	k (W/m.K)	237	110
Total surface area	A_s (m ²)		
Volume	V (m ³)		

**1. Aluminum**

Time t (minutes)	Temperature T	$\frac{\theta}{\theta_i} = \frac{T - T_F}{T_i - T_F}$
0		
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		
11		
12		
13		

2. Brass

Time t (minutes)	Temperature T	$\frac{\theta}{\theta_i} = \frac{T - T_F}{T_i - T_F}$
0		
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		
11		
12		
13		

14		
15		
16		
17		
18		
19		
20		

14		
15		
16		
17		
18		
19		
20		

Some Sample Calculations

Sample	Time constant τ (minutes)	Time constant τ (seconds)	Heat transfer coefficient $h = \frac{\rho V C_p}{A_s} \cdot \frac{1}{\tau}$	Biot Number $Bi = \frac{h(V / A_s)}{k}$
Aluminium				
Brass				

Note: You must compare and discuss your experimentally calculated heat transfer coefficient with the theoretical value [refer to equation (6.16)].

EXPERIMENT 7

Numerical Analysis: Counter and Parallel Flow Heat Exchangers

Objective

The objective of this laboratory period is to study design and performance analysis of counter- and parallel-flow heat exchangers numerically using EES software.

Background

It should be noted that, standard analytical approaches to the analysis of heat exchangers that we discussed in our heat transfer class as well as in heat exchanger experiment hand out, are restricted to approximate uniform property conditions and generally involve restrictions on the variation in overall coefficient of heat transfer. Furthermore, except for cases involving basic parallel- and counter-flow arrangements, the details required in developing the underlying analytical relations for the effectiveness – NTU and temperature correction factor F_T associated with the *LMTD* methods, are normally quite involved. On the other hand, similar to the situation encountered in the analysis of heat transfer to fluid flowing in a tubular passage with specified wall temperature, basic numerical methods provide the means for developing solutions that account for variation in bulk-stream fluid properties and non-uniform distribution in overall coefficient of heat transfer. A more realistic finite-difference method patterned after the basic numerical approach developed in Thomas [1], is particularly useful for developing solutions for various types of heat exchangers.

Numerical Method

To introduce this approach, we consider a simple one-dimensional parallel-flow arrangement. As we know from our previous experience, the key relations associated with the numerical formulation for energy transfer in heat exchangers include the heat-exchanger-governing equation [1-3],

$$dQ = U_x dA_s (T_h - T_c) \quad (7.1)$$

where the heat transfer from the hot to the cold fluid is given by,

$$dQ = -(\dot{m}C_p)_h dT_h = (\dot{m}C_p)_c dT_c \quad (7.2)$$

Following the approach discussed in Thomas [1], a shifted finite-difference grid is constructed by subdividing the heat exchanger into $M_1 = M - 1$ sub-volumes along its

length, as shown in Figure 7.1 in the context of a parallel-flow arrangement. Each sub-volume is identified by a sub-volume index m , such that $m = 1, 2, \dots, M_1$. It also follows that $\Delta x = L / M_1$, $\Delta A_s = A_s / M_1 = p \Delta x$, and $x = (m-1) \Delta x$. To relate Equation (7.1) and (7.2) to sub-volume m , we approximate dQ by ΔQ_m , dT_h by $\Delta T_{h,m} = T_{h,m+1} - T_{h,m}$ and dT_c by $\Delta T_{c,m} = T_{c,m+1} - T_{c,m}$. Substitution of these approximations in to Equations (7.1) and (7.2) gives

$$\Delta Q_m = U_m \Delta A_s (T_h - T_c)_m = U_m P \Delta x (T_h - T_c)_m \quad (7.3)$$

and

$$\Delta Q_m = -C_{h,m} (T_{h,m+1} - T_{h,m}) = C_{c,m} (T_{c,m+1} - T_{c,m}) \quad (7.4)$$

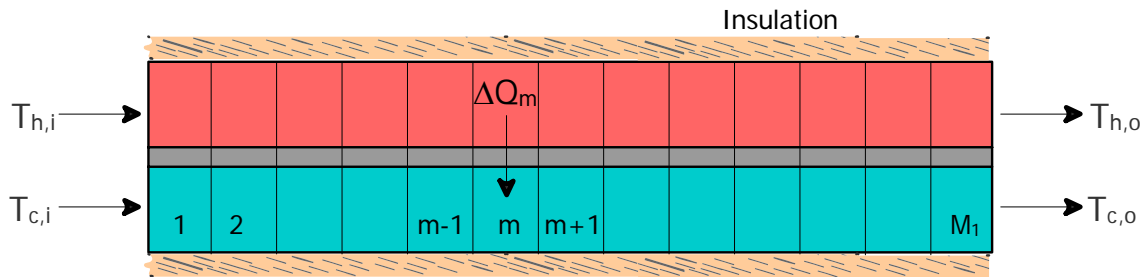


Figure 7.1: A parallel-flow heat exchanger is subdivided into equal sub-volumes along its length.

For $m = 1$, $(T_h - T_c)_m$ is set equal to $(T_{h,i} - T_{c,i})$. Equations (7.3) and (7.4) can be sequentially solved for ΔQ_m , $T_{h,m+1}$ and $T_{c,m+1}$ for $m = 1, 2, \dots, M_1$. The following steps can be used for the solution of the above equations to obtain standard rating (or performance evaluation) calculations:

Numerical Method – Parallel Flow

In this section, standard rating procedure, wherein $(Q, T_{h,o}, T_{c,o})$ is to be calculated for a parallel flow arrangement, is summarized below:

Inputs:

- Specify geometric parameters;
- Specify process parameters $\dot{m}_h, \dot{m}_c, T_{h,i}, T_{c,i}$;

- Specify fluid specific heats $C_{p,h}, C_{p,c}$;
- Specify heat transfer coefficients h_h, h_c (or fluid properties μ_h, μ_c, k_h, k_c and convection correlations); fouling resistances or factors $R_{f,h}, R_{f,c}$; wall thermal conductivity k_w ;

Calculations

- Set M_1 to sub-divide the heat exchanger in to sub-volumes;
- Set $m = 1, T_{h,1} = T_{h,i}, T_{c,1} = T_{c,i}$;
- Calculate ΔQ_m from Equation (7.3);
- Calculate $T_{h,m+1}$ and $T_{c,m+1}$ from Equation (7.4);
- Continue the above two steps for $m = 2, 3, \dots, M_1$; and
- Set $T_{h,o} = T_{h,M+1}$ and $T_{c,o} = T_{c,M+1}$ and calculate Q by integrating from equation (7.2), or $Q = \sum_{m=1}^{M_1} \Delta Q_m$.

These steps are similar to those outlined in Thomas [1] for convection in a passageway with specified wall temperature and is featured in the EES program presented in sample example 7.1. This simple approach is readily adapted to counter flow heat exchangers by employing a simple iteration scheme and can be extended to any heat exchanger configuration such as multi-pass shell-and-tube and cross-flow arrangements. In addition, this basic numerical approach can be formulated for the purpose of performing rating, testing, or design calculations.

Numerical Method – Counter Flow

We will follow exactly the same procedure as discussed above for a parallel flow, with the following modifications:

Since the cold fluid is moving opposite to the hot fluid, Equation (7.4) becomes

$$\Delta Q_m = -C_{h,m}(T_{h,m+1} - T_{h,m}) = -C_{c,m}(T_{c,m+1} - T_{c,m}) \quad (7.4m)$$

Set $m = 1, T_{h,1} = T_{h,i}, T_{c,1} = T_{c,o}$;

Assume the cold fluid outlet temperature, $T_{c,o}$ and do the following iterative steps:

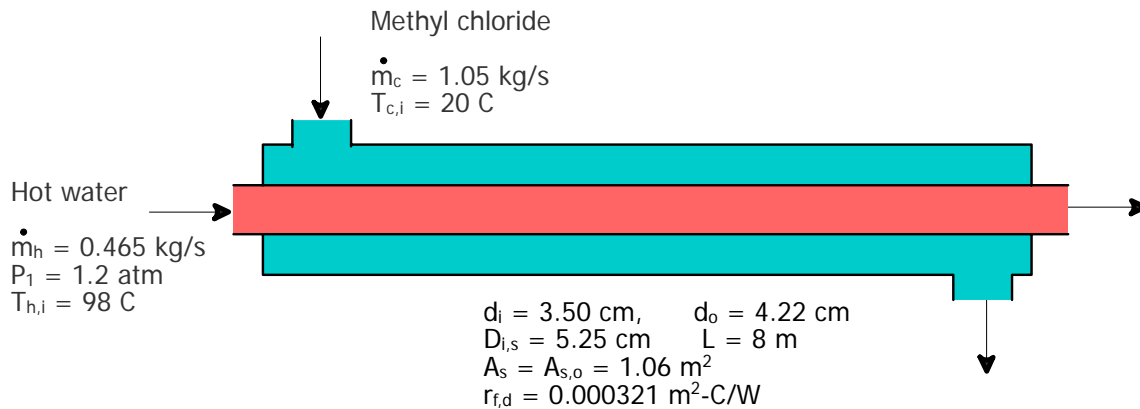
- Compare $T_{c,M}$ with $T_{c,i}$ by calculating $DIFF = T_{c,M} - T_{c,i}$;
- If $DIFF$ is greater than the specified value, adjust $T_{c,o}$ by setting, for instance, $T_{c,o} = T_{c,o} - 0.50(DIFF)$ and then re-start the calculations again;
- If $DIFF$ is less than the specified value, conclude that the convergence has been achieved and calculate the total heat transfer, and the outlet fluid temperatures.

Example 7.1: Methyl chloride at 20°C and 1.4 atmospheres flowing at a rate of 1.05 kg/sec to be heated in a double-pipe heat exchanger of 8 m length with tube diameters given by $d_i = 3.50$ cm, $d_o = 4.22$ cm, and $D_{i,s} = 5.25$ cm. The heating fluid is water at 98°C and 1.2 atmospheres flowing in the tube with mass-flow rate of 0.465 kg/sec. The tubing is made of a chrome steel alloy with thermal conductivity equal to 65 W/m.K. The process engineer has specified the fouling resistances on both the hot and cold sides, respectively as $R_{f,h} = 0.0001$ m².K/Watt, and $R_{f,c} = 0.0002$ m².K/Watt, while the convective heat transfer coefficients are: $h_h = 3,770$ W/m².K, $h_c = 3,420$ W/m².K Using the numerical approach discussed above, solve for parallel and counter-flow arrangements, the temperature distributions in the heat exchanger.

Solution Procedure

Purpose: Use the numerical method to calculate Q , $T_{h,o}$, and $T_{c,o}$ for the double-pipe heat exchanger for the given conditions

Schematic: Double-pipe heat exchanger: parallel and counter flow. The convective heat transfer coefficients are: $h_h = 3,770$ W/m².K, $h_c = 3,420$ W/m².K.



Assumptions

- Uniform overall heat transfer coefficient
- Uniform properties
- Standard conditions

Properties: The specific heat of the hot and cold fluids, respectively are:
 $C_{p,h} = 4.21$ kJ/(kg.K), $C_{p,c} = 1.59$ kJ/(kg.K).

Analysis:

Following the numerical approach discussed in the handout, the heat-exchanger-solution relations are represented in finite-difference form by equations (7.3), (7.4), and (7.4m)

$$\Delta Q_m = U_m \Delta A_s (T_h - T_c)_m = U_m P \Delta x (T_h - T_c)_m$$

$$\Delta Q_m = -C_{h,m} (T_{h,m+1} - T_{h,m}) = C_{c,m} (T_{c,m+1} - T_{c,m})$$

for parallel flow, and

$$\Delta Q_m = -C_{h,m} (T_{h,m+1} - T_{h,m}) = -C_{c,m} (T_{c,m+1} - T_{c,m})$$

for counter-flow arrangement. In the above equations, $m = 1, 2, 3, \dots, M_1$, $T_{h,1} = T_{h,i}$, and $T_{c,1} = T_{c,i}$ for the parallel-flow, while $T_{c,1} = T_{c,o}$ for the counter-flow arrangement.

The EES computer program given below solves these equations sequentially for $m = 1, 2, 3, \dots, M_1$ following the steps outlined above. In addition to calculating the distributions in fluid-stream temperatures $T_{h,m}$ and $T_{c,m}$ and the heat-transfer rate Q the program also calculates the distribution in wall temperature at the hot fluid interface using the finite-difference form of the general Newton Law of Cooling,

$$\Delta Q_m = h_h \Delta A_s (T_h - T_{s,h})_m \quad T_{s,h,m} = T_{h,m} - \frac{\Delta Q_m}{h_h P \Delta x}$$

Counter-Flow EES Program

Note: In this program, we have considered $m_h = m_c = 0.465$ kg/sec to get an interesting temperature distribution. All other values are same as that specified in problem statement

```
FUNCTION U(h_h,h_c,R_f_h,R_f_c,D_o,D_i,k_w)
```

```
  R_w:=D_o/(2*k_w)*LN(D_o/D_i)    "[C*m^2/W]"
  R_f:=R_f_h+(D_o/D_i)*R_f_c      "[C*m^2/W]"
  U:=1/(1/h_h+R_w+(D_o/D_i)/h_c+R_f) "[(W/C)/m^2]"
  END
```

```
PROCEDURE
```

```
OUT(M_1,J_final,L,P,h_h,h_c,R_f_h,R_f_c,D_o,D_i,k_w,C_h,C_c,T_h_i,T_c_i:Q,T_h[1..J_final],T_c[1..J_final],T_w[1..M_1],Length[1..J_final],T_c_o,DELTAT[1..M_1])
```

```
D_x:=L/M_1    "[m]";T_c_o=90    "[C]"
T_h[1]:=T_h_i "[C]";
1: T_c[1]:=T_c_o    "[C]";Q:=0    "[W]";J:=0;Length[1]:=0    "[m]"
```

```
REPEAT
```

```
  J:=J+1
```

```
  U:=U(h_h,h_c,R_f_h,R_f_c,D_o,D_i,k_w)    "[(W/C)/m^2]"
```

ME 316: Thermo-Fluids Laboratory 7.6

```

    DELTAQ:=U*P*D_x*(T_h[J]-T_c[J]) "[W]"
    T_h[J+1]:=T_h[J]-DELTAQ/C_h "[C]"
    T_c[J+1]:=T_c[J]-DELTAQ/C_c "[C]"
    DELTAT[J]:=T_h[J]-T_c[J]
    T_w[J]:=(T_h[J]+T_h[J+1])/2-DELTAQ/(h_h*P*D_x) "[C]"
    Length[J+1]:=Length[J]+D_x "[m]"
    { T_w[J]:=T_h[J]-DELTAQ/(h_h*P*D_x) "[C]"
      Q:=Q+DELTAQ "[W]"
    }
UNTIL (J=>M_1)

DIFF:=T_c[J+1]-T_c_i "[C]"
IF (DIFF>0.01) THEN GOTO 15
IF (DIFF<-0.01) THEN GOTO 15
GOTO 20
15: T_c_o:=T_c_o-0.5*DIFF "[C]"
GOTO 1
20:
END

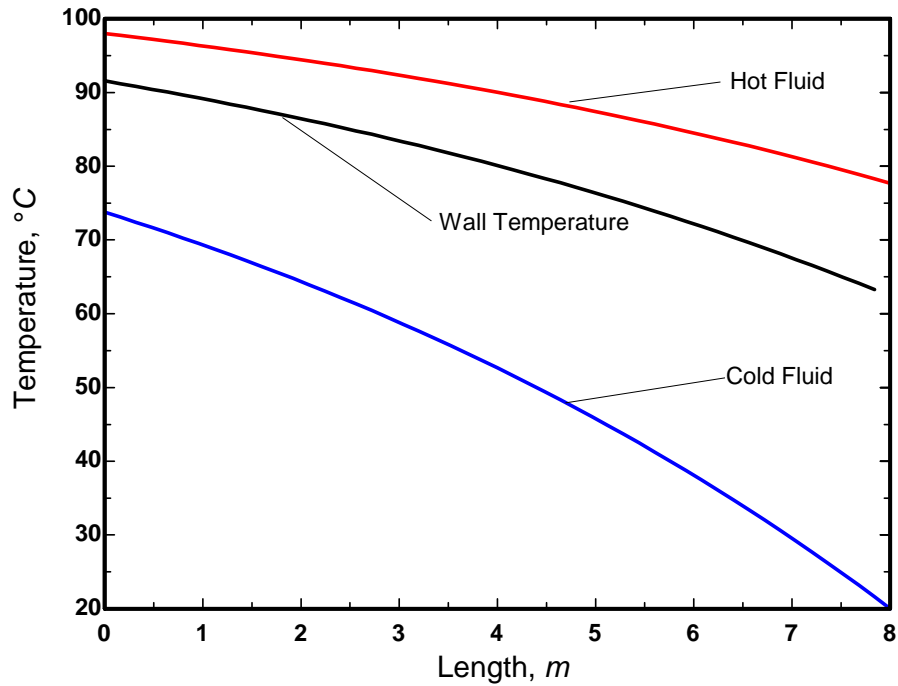
"INPUTS"
T_h_i=98 "[C]";T_c_i=20 "[C]"
m_dot_h=0.465 "[kg/s]";m_dot_c=0.465 "[kg/s]"
c_p_h=4210 "[J/(kg*C)]";c_p_c=1590 "[J/(kg*C)]"
C_h=c_p_h*m_dot_h "[W/C]";C_c=c_p_c*m_dot_c "[W/C]";
k_w=65 "[W/(m*C)]";h_h=3770 "[W/C)/m^2]";h_c=3420 "[W/C)/m^2]"
R_f_h=0.0001 "[C*m^2/W]";R_f_c=0.0002 "[C*m^2/W]"
L=8 "[m]";D_o=0.0422 "[m]";D_i=0.035 "[m]"
P=Pi*D_o "[m]"

"SET NUMBER OF INCREMENTS TO 1"
M_1=50;J_final=M_1+1

CALL
OUT(M_1,J_final,L,P,h_h,h_c,R_f_h,R_f_c,D_o,D_i,k_w,C_h,C_c,T_h_i,T_c_i:Q,T_h[1..J_final],T_c[1..J_final],T_w[1..M_1],Length[1..J_final],T_c_o,DELTAT[1..M_1])

```


Counter-flow Temperature Distribution



Parallel-Flow EES Program

```
FUNCTION U(h_h,h_c,R_f_h,R_f_c,D_o,D_i,k_w)
```

```

R_w:=D_o/(2*k_w)*LN(D_o/D_i)    "[C*m^2/W]"
R_f:=R_f_h+(D_o/D_i)*R_f_c      "[C*m^2/W]"
U:=1/(1/h_h+R_w+(D_o/D_i)/h_c+R_f) "[W/C)/m^2]"
END

```

```
PROCEDURE
```

```
OUT(M_1,J_final,L,P,h_h,h_c,R_f_h,R_f_c,D_o,D_i,k_w,C_h,C_c,T_h_i,T_c_i:Q,T_h[1..J_final],T_c[1..J_final],T_w[1..M_1],Length[1..J_final])
```

```
D_x:=L/M_1    "[m]"
```

```
T_h[1]:=T_h_i "[C]"; T_c[1]:=T_c_i    "[C]"; Q:=0    "[W]"; J:=0; Length[1]:=0    "[m]"
```

```
REPEAT
```

```
    J:=J+1
```

```
    U:=U(h_h,h_c,R_f_h,R_f_c,D_o,D_i,k_w)    "[W/C)/m^2]"
```

```
    DELTAQ:=U*P*D_x*(T_h[J]-T_c[J])    "[W]"
```

```
    T_h[J+1]:=T_h[J]-DELTAQ/C_h    "[C]"
```

```
    T_c[J+1]:=T_c[J]+DELTAQ/C_c    "[C]"
```

```
    T_w[J]:=(T_h[J]+T_h[J+1])/2-DELTAQ/(h_h*P*D_x)    "[C]"
```

```
    Length[J+1]:=Length[J]+D_x    "[m]"
```

```
{    T_w[J]:=T_h[J]-DELTAQ/(h_h*P*D_x)    "[C]"}
```

```

Q:=Q+DELTAQ      "[W]"
UNTIL (J=>M_1)
END

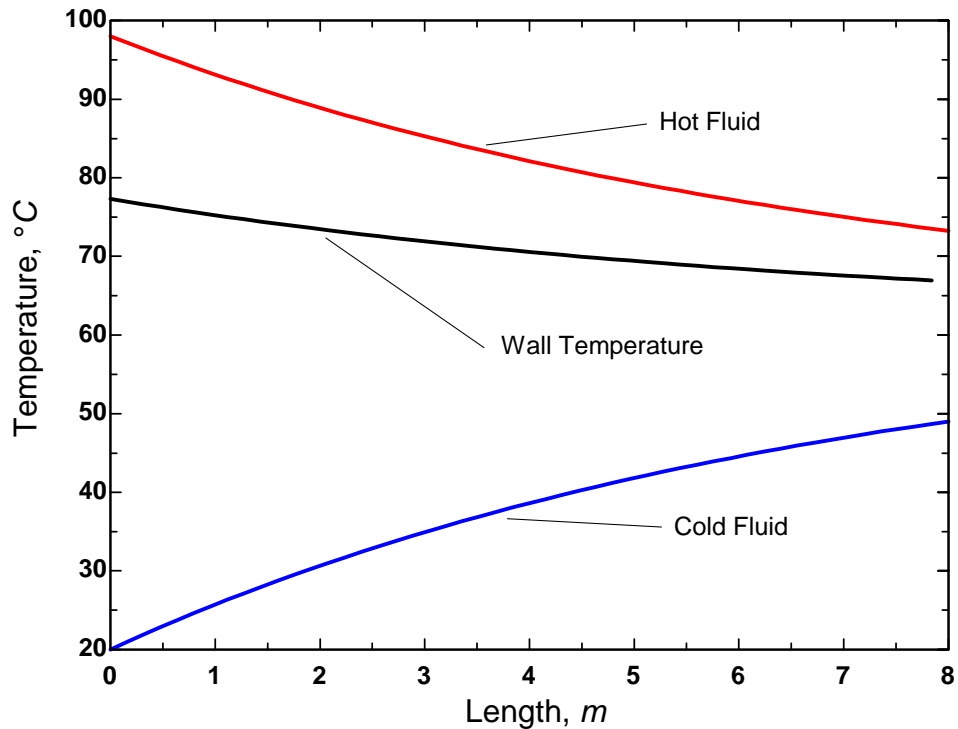
"INPUTS"
T_h_i=98      "[C]";T_c_i=20 "[C]"
m_dot_h=0.465 "[kg/s]";m_dot_c=1.05 "[kg/s]"
c_p_h=4210 "[J/(kg*C)]";c_p_c=1590 "[J/(kg*C)]"
C_h=c_p_h*m_dot_h "[W/C]";C_c=c_p_c*m_dot_c "[W/C]";
k_w=65 "[W/(m*C)]";h_h=3770 "[W/(C)/m^2]";h_c=3420 "[W/(C)/m^2]"
R_f_h=0.0001 "[C*m^2/W]";R_f_c=0.0002 "[C*m^2/W]"
L=8 "[m]";D_o=0.0422 "[m]";D_i=0.035 "[m]"
P=Pi*D_o "[m]"

"SET NUMBER OF INCREMENTS TO 1"
M_1=50;J_final=M_1+1

CALL
OUT(M_1,J_final,L,P,h_h,h_c,R_f_h,R_f_c,D_o,D_i,k_w,C_h,C_c,T_h_i,T_c_i:Q,T_h[1..J_final],T_c[1..J_final],T_w[1..M_1],Length[1..J_final])

```

Parallel-flow Temperature Distribution



Numerical Experimental Procedure

The students are expected to carry out the following for both counter- and parallel-flow heat exchangers:

- (1) Confirm that key heat exchanger equations are satisfied for the problem discussed in this handout.
- (2) Calculate the overall heat transfer coefficient both under clean and fouled conditions.
- (3) Show the effect of fouling on heat exchanger performance.
- (4) Compare your result with effectiveness-NTU method of analysis.
- (5) Identify input parameters that can vary and then carry out sensitivity analysis
- (6) Convert this problem in to a heat exchanger design problem wherein heat transfer area need to be calculated provided heat duty, flow rates, and the exit temperatures are known.

References:

- [1] Thomas, L.C., *Heat Transfer – Professional Version*, 2nd edition, Capstone Publishing Corp, 1999.
- [2] Incropera, F. P. and D. P. DeWitt, *Fundamentals of Heat and Mass Transfer*, 4th Edition, John Wiley and Sons Inc., New York, 1996.
- [3] Çengel, Yunus A., *Heat Transfer: A Practical Approach*, 2nd edition, McGraw-Hill, New York, 2003.

LAB DATA SHEET**Experiment 7: Numerical Analysis: Counter and Parallel Flow Heat Exchangers**

Name: _____ Id# : _____ Date: _____

Given below is the data for a double-pipe heat exchanger.

		Hot fluid (tube side)	Cold fluid (shell side)
Flow rate	kg/s	0.465	1.05
Specific heat	J/kg-°C	4210	1590
Inlet temperature	°C	98	20
Heat transfer coeff..	W/m ² -°C	3770	3420

Inner tube inside diameter	3.5 cm
Inner tube outside diameter	4.22 cm
Outer tube diameter	5.25 cm
Heat exchanger length	8 m
Tube wall thermal conductivity, k	65 W/m.K
Assume: A clean heat exchanger ($r_f = 0$ W/m ² -°C)	

Calculations

Inner surface area, A_i m ²	$\pi d_i L$	
Outer surface area, A_o m ²	$\pi d_o L$	

Total thermal resistance, R_{tot}	$R_{tot} = \frac{1}{(UA_s)}$ $= \frac{1}{h_i A_i} + \frac{\ln(d_o / d_i)}{2\pi k L} + \frac{1}{h_o A_o} + r_f$	
Overall heat transfer coefficient based on A_o , U_o	$U_o = \frac{1}{R_{tot} A_o}$	

	Parallel flow		Counterflow	
	Hot fluid	Cold fluid	Hot fluid	Cold fluid
Heat capacity rate (in W/K) $\dot{C} = (\dot{m}C_p)$				
Capacity ratio, $R = \frac{(\dot{m}C_p)_{\min}}{(\dot{m}C_p)_{\max}}$				
NTU				
Effectiveness, ε				
\dot{Q}_{\max}				
$\dot{Q}_{analytical}$				
Outlet temperatures				

Table 7.1: Use the EES program to complete the following table for both parallel- and counter-flow configurations using effectiveness – NTU method

Table 7.2: Complete the following tables using the numerical method of solution - EES software.

Parallel flow:

Nodes	$T_{h,o}$ (°C)	$T_{c,o}$ (°C)	$\dot{Q}_{numerical}$ (W)	$\dot{Q}_{analytical}$ (from table 7.1)	% error in \dot{Q} $\frac{\dot{Q}_{numerical} - \dot{Q}_{analytical}}{\dot{Q}_{analytical}} \times 100$
5					
20					
80					

Counter flow:

Nodes	$T_{h,o}$ (°C)	$T_{c,o}$ (°C)	$\dot{Q}_{numerical}$ (W)	$\dot{Q}_{analytical}$ (from table 7.1)	% error in \dot{Q} $\frac{\dot{Q}_{numerical} - \dot{Q}_{analytical}}{\dot{Q}_{analytical}} \times 100$
5					
20					
80					

Design Programs:

Parallel Flow

ΔA_s	\dot{Q} (Watts)	No of Sub-Volumes (M)	$A_s = M \times \Delta A_s$	% error in \dot{Q}	% error in A_s
0.20					
0.10					
0.01					

Counter Flow

ΔA_s	\dot{Q} (Watts)	No of Sub-Volumes (M)	$A_s = M \times \Delta A_s$	% error in \dot{Q}	% error in A_s
0.20					
0.10					
0.01					

APPENDIXES

A.1 Introduction to EES Software

References

The EES manual and on-line help system is the best source.

Objectives

To provide basic information on Engineering Equation Solver (EES) software.

Computer Based Problem Solving

Mathematical software packages have had an enormous impact on engineering education practice. Today most engineering problems are solved in whole or in part using computer softwares. Certainly, industrial employers expect ME graduates to be proficient with the use of computers to solve complex engineering problems. There are a number of mathematical software packages available today, including MathCad, Mathematica, MATLAB, EXCEL, EES, and others. More complicated problems can be solved using FORTRAN or C⁺⁺ programs.

There is obviously no consensus about which software is "the best" (if there were agreement then there wouldn't be so many choices). Many people feel that there is no one software packages that are ideal for every task. I use EES for most everyday problems but sometimes use EXCEL to handle large quantities of data, MatLab to implement numerical modeling techniques, and FORTRAN for large numerical models. The point is that you will be required to use mathematical software and probably will learn several packages before you settle on the one (or more than one) that meets your needs as a practicing engineer.

Engineering Equation Solver

In this laboratory course we are going to use the Engineering Equation Solver (EES) package because it is a single package that can meet all of the needs of ME 316. You may be familiar with EES from your thermodynamics course and will use it again in ME 311 and ME 315 (even if you are not required to use it you will find it helpful for many other classes). An academic version of EES is available free of charge to KFUPM students and is installed on the ME Department computers.

In this lab I hope you become familiar with EES and some of its special features that are particularly useful for the experimentalist in the lab. If you have previous experience with EES you will find much of this document to be review (and therefore probably pretty boring) in this case move over and share your knowledge and experience with other students.

Getting Started

1. The computers should always be on; the monitors are sometimes turned off.
 - a. Log in to ME computers with your regular user name and password.
 - b. Double click on the EES icon on the desktop to start the program.
2. EES begins by displaying a dialog window that shows registration information, the version number and other information. Click the OK button to dismiss the dialog window.
3. Detailed help is available at any point in EES. Pressing the F1 key will bring up a Help window relating to the foremost window. Clicking the Contents button will present the Help index shown in Figure 1. Clicking on an underlined word (shown in green on color monitors) will provide help relating to that subject.

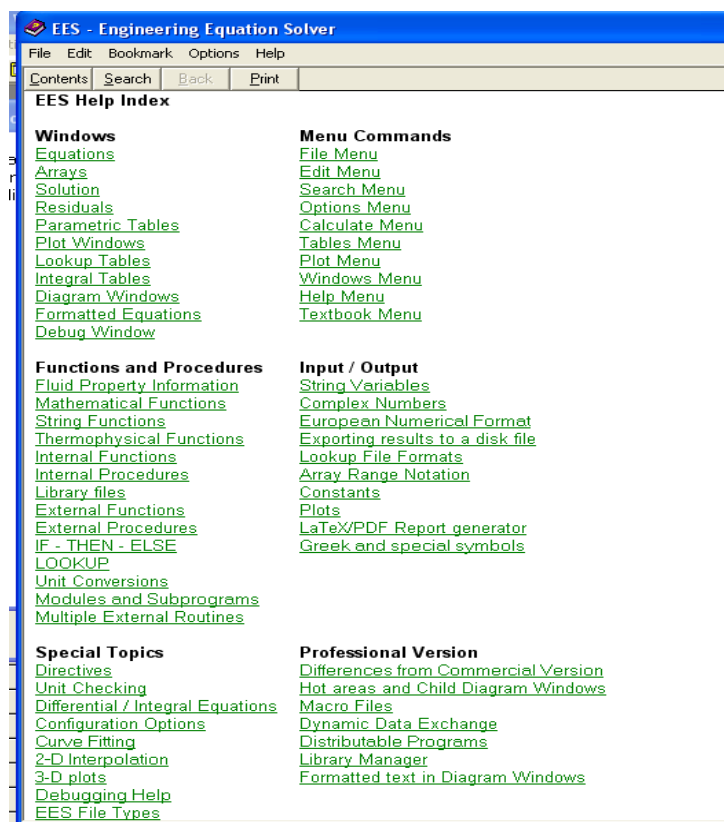


FIGURE 1: EES HELP MENU

4. EES commands are distributed among nine pull-down menus, shown in Figure 2.

**FIGURE 2: PULL-DOWN MENUS**

Note that a toolbar is provided below the menu bar. The toolbar contains small buttons that provide rapid access to many of the most frequently used EES menu commands. If you move the cursor over a button and wait for a few second, a few words will appear to explain the function of that button.

- The **File** menu provides commands for loading, merging and saving work files and libraries, and printing.
- The **Edit** menu provides the editing commands to cut, copy, and paste information.
- The **Search** menu provides Find and Replace commands for use in the Equations window.
- The **Options** menu provides commands for setting the guess values and bounds of variables, the unit system, default information, and program preferences. A command is also provided for displaying information on built-in and user-supplied functions.
- The **Calculate** menu contains the commands to check, format and solve the equation set.
- The **Tables** menu contains commands to set up and alter the contents of the Parametric and Lookup Tables and to do linear regression on the data in these tables. The Parametric Table, similar to a spreadsheet, allows the equation set to be solved repeatedly while varying the values of one or more variables. The Lookup table holds user-supplied data, which can be interpolated and used in the solution of the equation set.
- The **Plot** menu provides commands to modify an existing plot or prepare a new plot of data in the Parametric, Lookup, or Array tables. Curve-fitting capability is also provided.
- The **Windows** menu provides a convenient method of bringing any of the EES windows to the front or to organize the windows.

- The **Help** menu provides commands for accessing the online help documentation.
5. The basic capability provided by EES is the solution of a set of non-linear algebraic equations.
- a. Start EES and enter the simple example problem shown in Figure 3 in the Equations window. Note that EES makes no distinction between upper and lower case letters and the ^ sign (or **) is used to signify raising to a power.

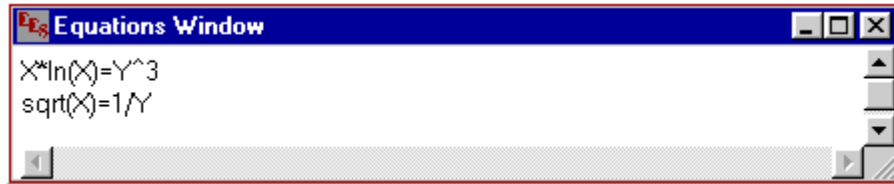


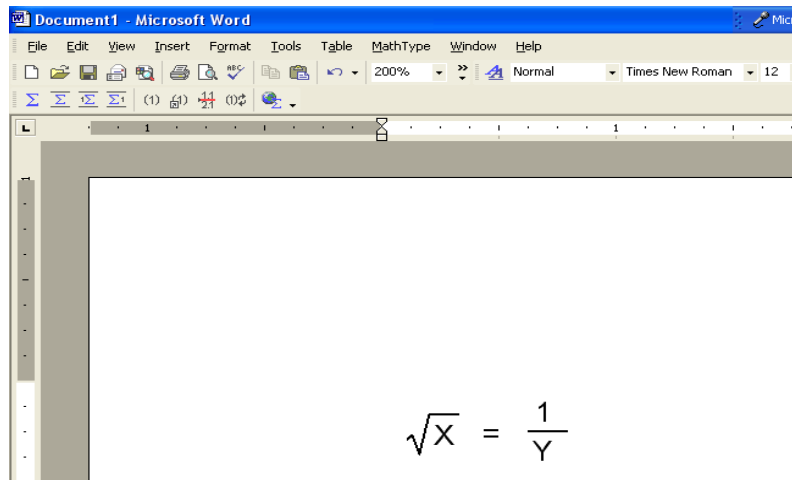
FIGURE 3: EXAMPLE EES PROBLEM

- b. Select the Formatted Equations window under the Windows pull down menu to view the equations in mathematical notation, as shown in Figure 4.



FIGURE 4: FORMATTED EQUATIONS

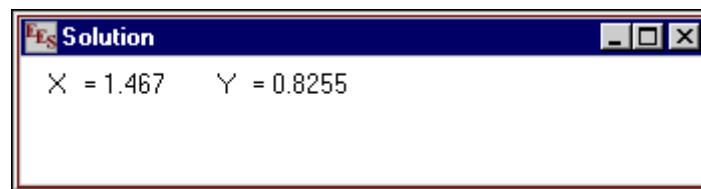
- c. Select one of the equations in the formatted equation window by clicking on it. Copy the equation either using Ctrl-C or by selecting Copy using the Edit pull-down menu.
- d. Start Microsoft Word by selecting the Word icon on the desk top.
- d. Paste the equation into Word. You should have a presentation quality mathematical equation, as shown in Figure 5. This is an entirely acceptable method of producing equations for your lab reports.



A screenshot of the Microsoft Word interface. The title bar reads 'Document1 - Microsoft Word'. The menu bar includes File, Edit, View, Insert, Format, Tools, Table, MathType, Window, and Help. The toolbar shows various icons for document editing. The main text area contains the equation $\sqrt{X} = \frac{1}{Y}$ centered on the page.

Figure 5: Formatted Equation in Word

- e. Select the **Solve** command from the **Calculate** menu. A dialog window will appear indicating the progress of the solution. When the calculations are completed, the button changes from Abort to Continue. Click the Continue button. The solution to this equation set will then be displayed, as shown in Figure 6.



A screenshot of the 'EES Solution' dialog box. The title bar is blue with the EES logo and the text 'Solution'. The main area is white and displays the solved values: $X = 1.467$ and $Y = 0.8255$.

FIGURE 6: SOLUTION WINDOW

Example: Thermodynamics Problem from EES Manual

A simple thermodynamics problem will be set up and solved in this section to illustrate the property functions, units, and equation solving capability of EES. The problem, typical of that which may be encountered in an undergraduate thermodynamics course, is as follows.

Refrigerant-134a enters a valve, shown in Figure 7, at 700 kPa, 50°C with a velocity of 15 m/s. At the exit of the valve, the pressure is 300 kPa. The inlet and outlet fluid areas are both 0.0110 m². Determine the temperature, mass flow rate and velocity at the valve exit.

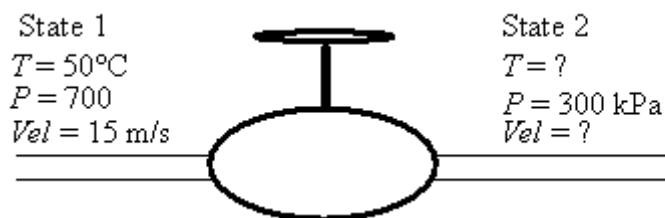


FIGURE 7: REFRIGERANT VALVE

6. Start EES and select the New command from the File menu. A blank Equations window will appear.
7. Set the unit system for the built-in thermophysical properties functions. To view or change the unit system, select Unit System from the Options menu. You should see the window shown in Figure 8.

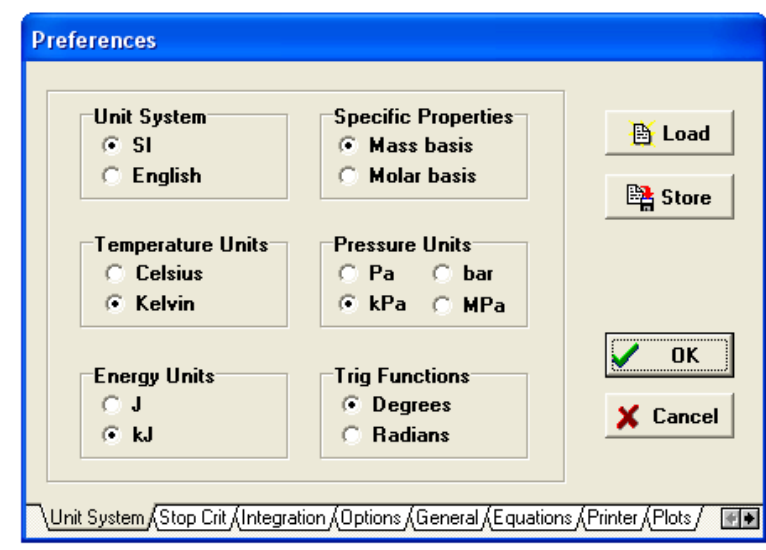


Figure 8: Units Window

8. Select SI units with T in $^{\circ}\text{C}$, P in kPa, and specific property values in their customary units on a mass basis. These defaults may have been changed during a previous use. Click on the controls to set the units as shown above. Click the OK button (or press the Return key) to accept the unit system settings.
9. The equations can now be entered into the Equations window. Text is entered in the same manner as for any word processor. Formatting rules are as follows:
 - Upper and lower case letters are not distinguished. EES will (optionally) change the case of all variables to match the manner in which they first appear.
 - Blank lines and spaces may be entered as desired since they are ignored.

- Comments must be enclosed within braces { } or within quote marks " ". Comments may span as many lines as needed. Comments within braces may be nested in which case only the outermost set of { } are recognized. Comments within quotes will also be displayed in the Formatted Equations window.
 - Variable names must start with a letter and consist of any keyboard characters except () ' | * / + - ^ { } : " or ;. Array variables are identified with square braces around the array index or indices, e.g., X[5,3]. String variables are identified with a \$ as the last character in the variable name. The maximum length of a variable name is 30 characters.
 - Multiple equations may be entered on one line if they are separated by a semi-colon (;). The maximum line length is 255 characters.
 - The caret symbol ^ or ** is used to indicate raising to a power.
 - The order in which the equations are entered does not matter.
 - The position of knowns and unknowns in the equation does not matter.
10. The inlet conditions to the valve are known and can be entered:

$$\begin{aligned}T1 &= 50 \text{ [C]} \\P1 &= 700 \text{ [kPa]} \\V1 &= 15 \text{ [m/s]} \\A1 &= 0.011 \text{ [m}^2\text{]}\end{aligned}$$

These statements assign quantities to the variables T1, P1, V1, and A1. The last part of the statements also assigns units to these variables. You will find that units become an enormous headache in the lab – you are constantly reading instruments that give quantities in esoteric and essentially useless units (like torr or °F) and then trying to convert these to more useful SI units (like Pa or K). EES can do this unit conversion work for you, reducing the number of unit conversions that you need to carry around in your head and also preventing many potential errors. If you select Solve under the Calculate menu you will see that the different variables have units attached to them in the solution window. Note that it is often a good idea to Solve your problem as you go rather than wait until you've entered the whole thing and find it harder to debug.

11. The inlet density and enthalpy are also required. We can obtain these using EES's built-in thermodynamic property functions for R134a (and many other substances). The thermodynamic property functions, such as **enthalpy** and **volume** require a special format. The first argument of the function is the substance name, R134a in this case. The following arguments are the independent variables preceded by a single identifying letter and an equal sign. Allowable letters are T, P, H, U, S, V, and X, corresponding to temperature, pressure, specific enthalpy, specific internal energy, specific entropy, specific volume, and quality.

```
h1=enthalpy(R134a, T=T1, P=P1)
rho1=density(R134a, T=T1, P=P1)
```

An easy way to enter functions, without needing to recall the format, is to use the Function Information command in the Options menu. This command will bring up the dialog window shown below. Click on the 'Thermophysical props' radio button. The list of built-in thermophysical property function will appear on the left with the list of substances on the right, as shown in Figure 9. Select the property function by clicking on its name, using the scroll bar, if necessary, to bring it into view. Select a substance in the same manner. An example of the function showing the format will appear in the Example rectangle at the bottom. The information in the rectangle may be changed, if needed. Clicking the Paste button will copy the Example into the Equations window at the cursor position. Additional information is available by clicking the Info button.

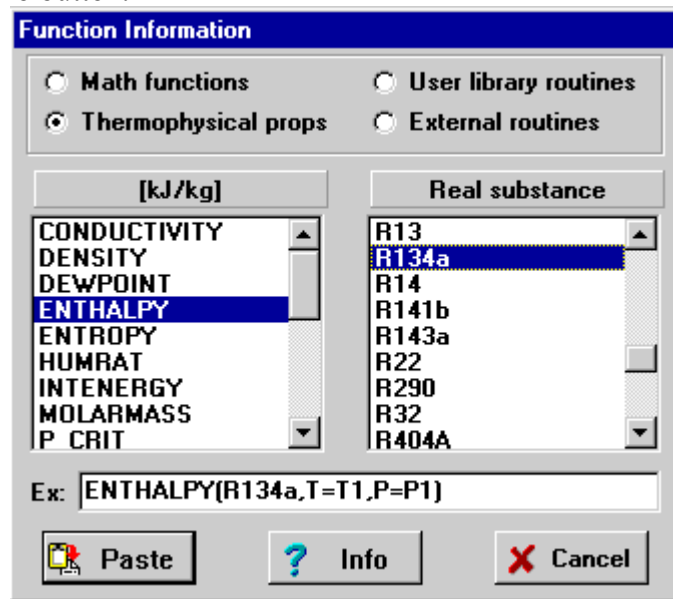


FIGURE 9: THERMOPHYSICAL PROPERTY WINDOW

If you solve the problem now you will see that $h1$ and $\rho1$ are both assigned values based on the properties of $r134a$. This is great – you’ve avoided the chore of looking up these values in a table somewhere. You should also notice that units have been assigned to these quantities that are consistent with the unit system that you selected.

12. The inlet mass flow rate can be computed using valve inlet area, density, and velocity.

$$m1 = \rho1 * v1 * A1$$

Units have already been assigned to the variables rho1, v1, and A1. If you highlight the variable m1 in the Equations window and right click on it you will get a pop-up menu. Select Variable Info from this menu and enter the units lbm/s. This tells EES that you think m1 should have units of lbm/s. EES will check to see that all units are self consistent. Select Check Units under Calculate and you will get the message shown in Figure 10.

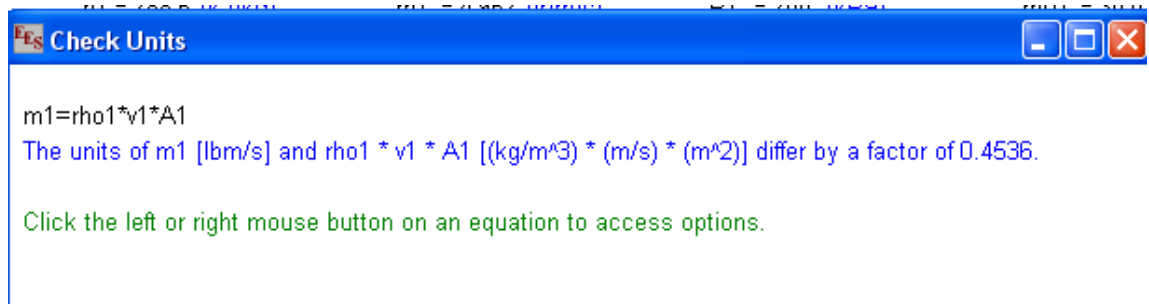


Figure 10: Error Message Indicating Unit Problem

In this example, EES has determined that m1 should not have units of lbm/s, given the units you entered for v1 and A1 together with the units associated with density. Select m1 again and enter the correct units, kg/s. You should check that this does not give a units error.

13. The two equations that govern the valve's behavior are mass conservation and energy conservation:

$$m2=m1$$

$$h2+v2^2*\text{convert}(m^2/s^2,kJ/kg)/2=h1+v1^2*\text{convert}(m^2/s^2,kJ/kg)/2$$

In the above equations, we have made use of one of EES's most useful features – built in unit conversions. We know that h1 and $0.5*v1^2$ have different units (if we didn't know this then EES would tell us) so we simply convert the units of kinetic energy (m^2/s^2) to those of enthalpy (kJ/kg) using the convert function. You will find this function extremely useful in the lab when, for example, you want to convert a pressure reading taken from a manometer (in inch Hg) to units of Pascal. Note that if you tried to Solve your problem now you would not be able to because you have an under constrained problem and therefore you would get the message shown in Figure 11.

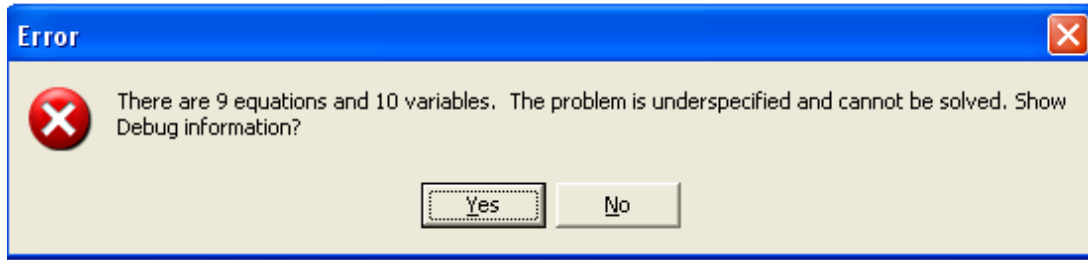


Figure 11: Error Message Associated with Under Constrained Problem

14. The pressure and area at the exit of the valve are given in the problem statement.

$$P_2 = 300 \text{ [kPa]}$$

$$A_2 = 0.011 \text{ [m}^2\text{]}$$

15. The density and enthalpy at the exit of the valve are related to the (unknown) exit temperature and (known) exit pressure.

$$h_2 = \text{enthalpy}(\text{R134a}, T=T_2, P=P_2)$$

$$\rho_2 = \text{density}(\text{R134a}, T=T_2, P=P_2)$$

16. Finally, the exit mass flow rate is related to the density, area, and velocity.

$$\dot{m}_2 = \rho_2 \cdot A_2 \cdot v_2$$

17. If you select Solve under the Calculate menu you will see that EES can obtain a solution to the set of equations you supplied. You should also verify that you have specified appropriate units for each variable by selecting Check Units under the Calculate menu.
18. One of the most useful features of EES is its ability to provide parametric studies. For example, in this problem, it may be of interest to see how the throttle outlet temperature and outlet velocity vary with outlet pressure. A series of calculations can be automated and plotted using the commands in the **Tables** menu. Select the **New Table** command. A dialog will be displayed listing the variables appearing in the Equations window. In this case, we will construct a table containing the variables P_2 , T_2 , v_2 , and h_2 . Click on P_2 from the variable list on the left. This will cause P_2 to be highlighted and the Add button will become active. Now click the Add button to move P_2 to the list of variables on the right. Repeat for T_2 , h_2 , and v_2 , using the scroll bar to bring the variable into view if necessary. (As a short cut, you can double-click on the variable name in the list on the left to move it to the list on the right.).

The table setup dialog should now appear as shown in Figure 12. Click the OK button to create the table.

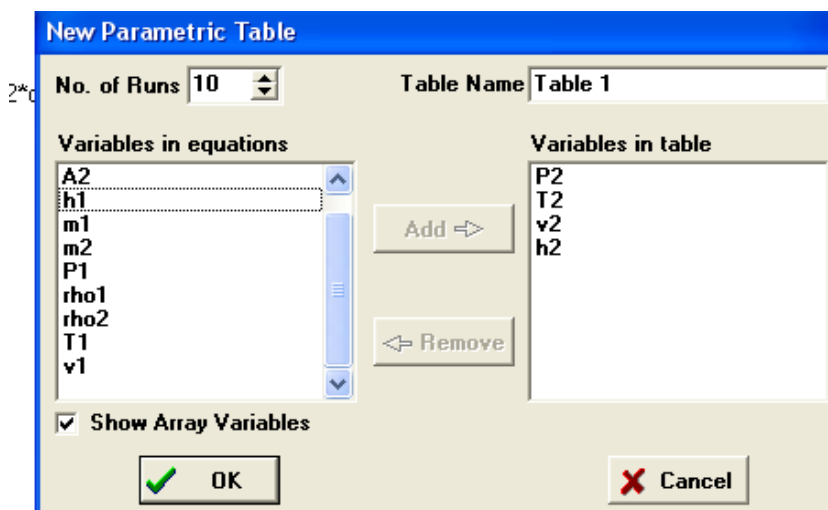
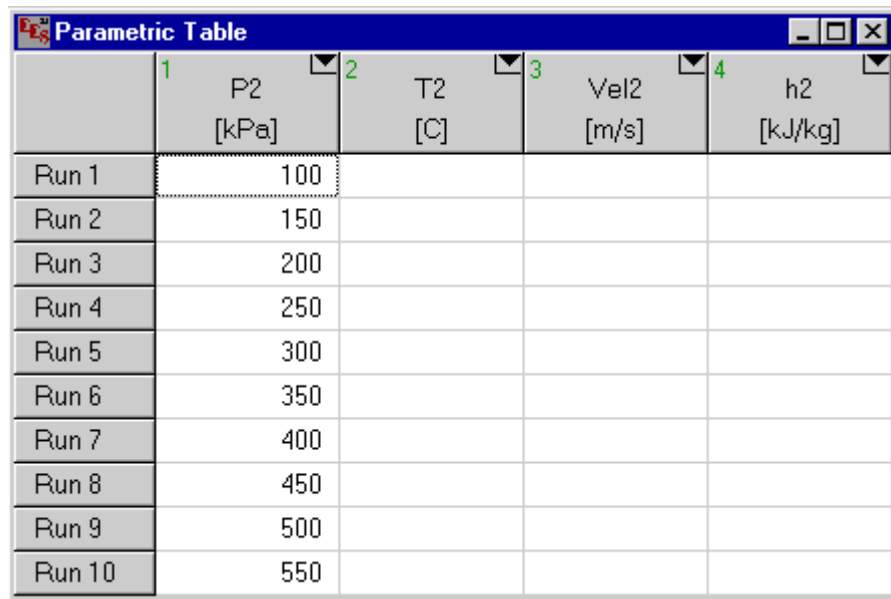


Figure 12: Table Setup Dialog

19. The Parametric Table works much like a spreadsheet. You can type numbers directly into the cells. Numbers which you enter are shown in black and produce the same effect as if you set the variable to that value with an equation in the Equations window. Delete the $P2 = 300$ equation that is currently in the Equations window (or, better yet, enclose it in comment brackets { }). A comment bracket tells EES to ignore this text yet keeps it around so that you don't have to retype it if you need it in the future. Comments are also useful to organize your work and make it clear what each statement means (for example if you need to turn it in to your lab instructor). The equation setting $P2$ will not be needed because the value of $P2$ will be set in the table. Now enter the values of $P2$ for which $T2$ is to be determined. Values of 100 kPa to 550 kPa have been chosen for this example. (The values could also be automatically entered using **Alter Values** in the **Tables** menu or by using the **Alter Values** control at the upper right of each table column header) The Parametric Table should now appear as shown in Figure 13.



	1 P2 [kPa]	2 T2 [C]	3 Vel2 [m/s]	4 h2 [kJ/kg]
Run 1	100			
Run 2	150			
Run 3	200			
Run 4	250			
Run 5	300			
Run 6	350			
Run 7	400			
Run 8	450			
Run 9	500			
Run 10	550			

Figure 13: Parametric Table.

20. Now, select **Solve Table** from the **Calculate** menu. The Solve Table dialog window, shown in Figure 14, will appear allowing you to choose the runs for which the calculations will be done.



Solve Table

First Run

Last Run

☒ Update guess values

☒ Stop if warning occurs

☐ Use input from Diagram

FIGURE 14: SOLVE TABLE DIALOG BOX

When the Update Guess Values control is selected, as shown, the solution for the last run will provide guess values for the following run. Click the OK button. A status window will be displayed, indicating the progress of the solution. When the calculations are completed, the values of T2, v2, and h2 will be entered into the table, shown in Figure 15. The values calculated by EES will be displayed in blue, bold or italic type depending on the setting made in the Screen Display tab of the Preferences dialog window in the Options menu.

	1 P2 [kPa]	2 T2 [C]	3 Vel2 [m/s]	4 h2 [kJ/kg]
Run 1	100	31.46	109.9	278.9
Run 2	150	36.32	73.79	282.2
Run 3	200	38.7	55.29	283.4
Run 4	250	40.34	44.08	284
Run 5	300	41.67	36.57	284.3
Run 6	350	42.86	31.19	284.4
Run 7	400	43.97	27.15	284.6
Run 8	450	45.03	24	284.6
Run 9	500	46.06	21.48	284.7
Run 10	550	47.07	19.42	284.7

Figure 15: Parametric Table

21. The relationship between variables such as P2 and T2 is now apparent, but it can more clearly be seen with a plot. Select **New Plot Window** from the **Plot** menu. The New Plot Window dialog window shown in Figure 16 will appear. Choose P2 to be the x-axis by clicking on P2 in the x-axis list. Click on T2 in the y-axis list. Select the scale limits for P2 and T2, and set the number of divisions for the scale as shown. Grid lines make the plot easier to read. Click on the Grid Lines control for both the x and y axes. When you click the OK button, the plot will be constructed and the plot window will appear as shown in Figure 17.

X-Axis		Y-Axis		Table	
P2		P2		<input checked="" type="radio"/> Parametric	
T2		T2		<input type="radio"/> Lookup	
Vel2		Vel2		<input type="radio"/> Arrays	
h2		h2			
Format	F 0	Format	F 0	First Run	1
Minimum	100.0	Minimum	30.00	Last Run	10
Maximum	550.0	Maximum	48.00	<input type="checkbox"/> Spline fit	
Interval	50.0	Interval	2.00	<input type="checkbox"/> Automatic update	
<input checked="" type="radio"/> Linear <input type="radio"/> Log		<input checked="" type="radio"/> Linear <input type="radio"/> Log		<input type="checkbox"/> Add legend item	
<input type="checkbox"/> Grid lines		<input type="checkbox"/> Grid lines		<input type="checkbox"/> Show error bars	
				Line	
				Symbol	◇
				Color	
				<input checked="" type="checkbox"/> OK	<input type="checkbox"/> Cancel

Figure 16: Plot Dialog Window

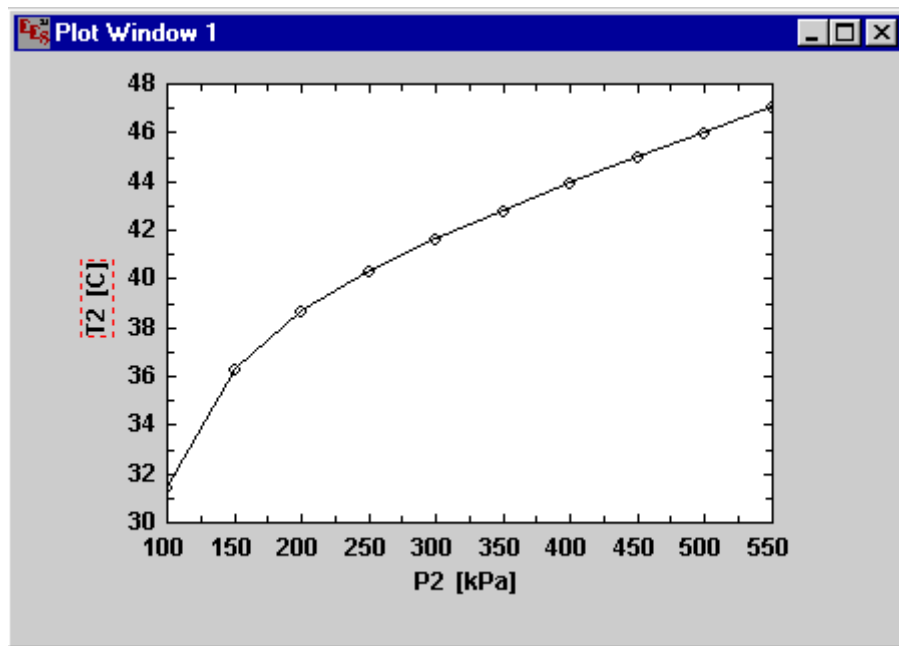


FIGURE 17. PLOT WINDOW

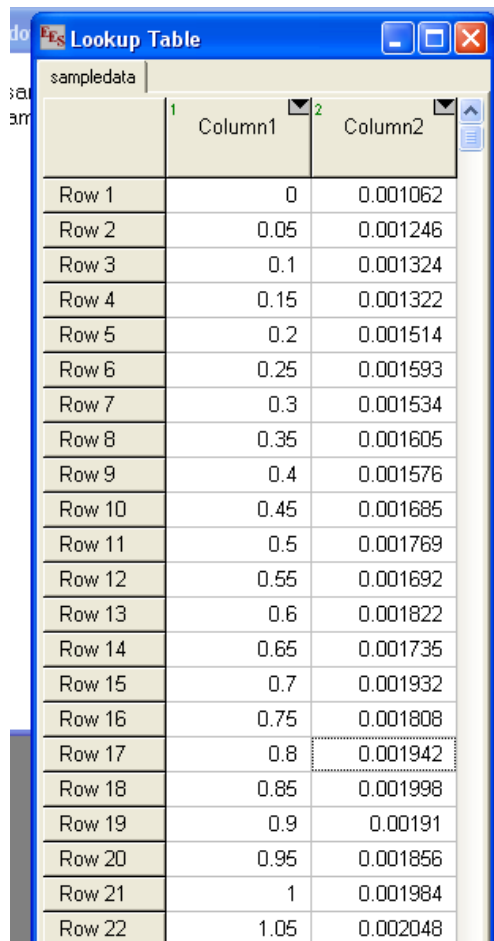
Once created, there are a variety of ways in which the appearance of the plot can be changed and formatted by double-clicking on the plot itself, the axes, and the labels.

22. While you are in the plot window, select Copy Plot from the Edit menu. Return to your Word document and select paste special and then bitmap from the Edit menu. This is a good way to insert plots into your lab reports.
23. Return to EES and select Print from the File menu. Select Equations, Solution, Parametric Tables, and Plots. Turn your printouts in to your lab instructor to be graded.

This example problem has illustrated some of the capabilities of EES. With this example behind you, you should be able to solve many types of problems.

Manipulating Data in EES

24. Open a new EES file and select Open Lookup Table under the Tables menu. Select files of type CSV data files and open the data file sampledata.csv on the C: drive. The lookup table sampledata.csv is read into EES, as shown in Figure 18. This is data from a thermocouple attached to a copper block that was placed in an oven. The first column is the elapsed time and the second is the voltage from the thermocouple.



	Column1	Column2
Row 1	0	0.001062
Row 2	0.05	0.001246
Row 3	0.1	0.001324
Row 4	0.15	0.001322
Row 5	0.2	0.001514
Row 6	0.25	0.001593
Row 7	0.3	0.001534
Row 8	0.35	0.001605
Row 9	0.4	0.001576
Row 10	0.45	0.001685
Row 11	0.5	0.001769
Row 12	0.55	0.001692
Row 13	0.6	0.001822
Row 14	0.65	0.001735
Row 15	0.7	0.001932
Row 16	0.75	0.001808
Row 17	0.8	0.001942
Row 18	0.85	0.001998
Row 19	0.9	0.00191
Row 20	0.95	0.001856
Row 21	1	0.001984
Row 22	1.05	0.002048

Figure 18: Lookup Table of Data in EES

25. We would like to manipulate this data in the EES environment. To do this we will setup our equations window to operate on each data point individually, starting with the 1st. The data point will be indicated using the variable *i*. We can pull a piece of data out of a table of data using the LOOKUP command:

```

i=1
time=LOOKUP('sampledata',i,1)
Vtc=LOOKUP('sampledata',i,2)

```

The LOOKUP function has the format $x = \text{LOOKUP}(\text{Tablename}, \text{row}, \text{column})$. You should set the units of these variables by right clicking on them.

26. We would like to turn the thermocouple's voltage into a temperature. The thermocouple was a type T thermocouple, one of several types for which the calibration curves suggested by the National Institute of Standards and Technology (NIST) are built in to EES's library of functions. To access these calibrations (and many other useful functions), select Function Information in the Options menu and then select EES library routines. The dialog window shown in Figure 19 should appear.

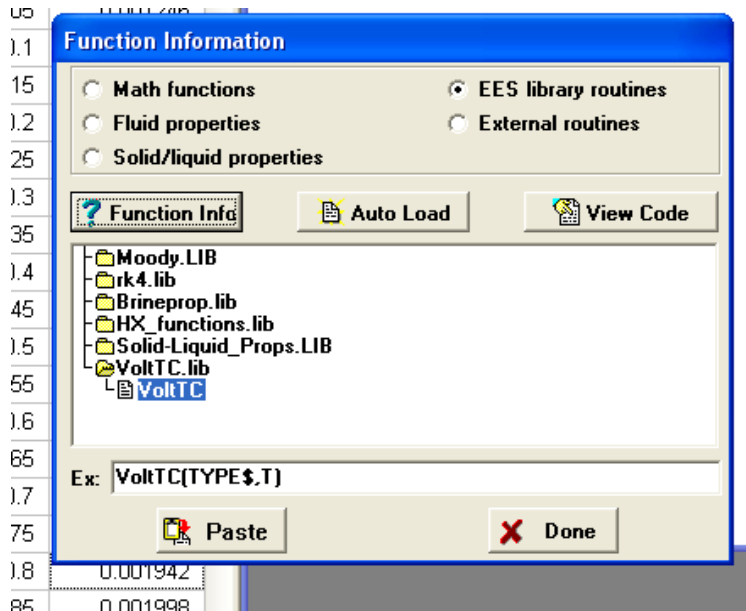


Figure 19. Function Information Dialog Window

Click on the Function Info button to learn about the VoltTC function. You will find that this function requires the thermocouple type and temperature (in °C) and will return the output voltage (in milliVolt). The following statement will convert the thermocouple voltage to a temperature.

$$V_{tc} * \text{convert}(V, \text{milliVolt}) = \text{VoltTC}('T', T_{tc})$$

Set the units of T_{tc} to C by right clicking on the variable. If you solve the problem you will find that the temperature at time zero was 26.72°C.

27. We are interested in temperature and time for all of the data points, not just the first one. Therefore we will make a Parametric Table. Select New Parametric Table from the Tables menu. Choose i , time, and T_{tc} as your variables. Specify 41 runs for this table. We need to make i go from 1 to 41 in your parametric table (the number of data points we are going to process). Right-click on the column i and selecting Alter Values. You can specify that i go from 1 to 41 as shown in Figure 20.

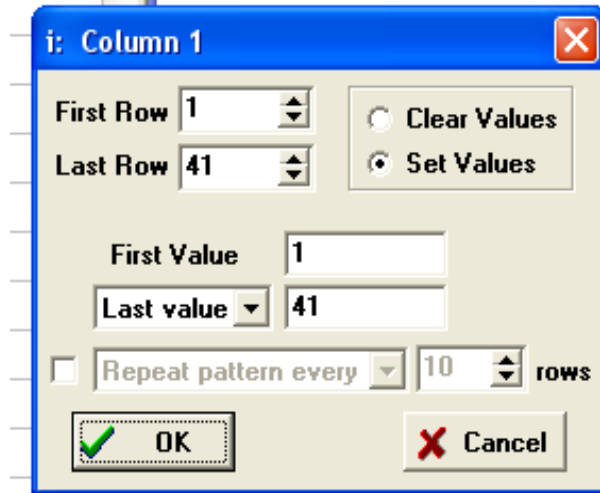


Figure 20: Alter Values Dialog Window

28. Comment out the $i=1$ equation in the equation windows and select Solve Table from the Calculate Window. Your Parametric Table should be filled with the data after having reduced it to temperature vs time.
29. Prepare a plot of temperature vs time, it should end up looking like Figure 21 after you remove the lines between the data points, add gridlines, fix the labels, etc.

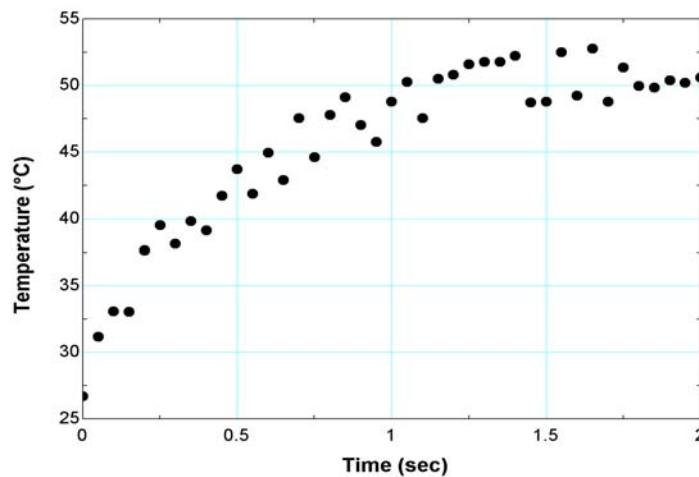


Figure 21. Temperature vs Time Data

30. The temperature vs time data is just that, data. The curve resembles an exponential function, but there is some obvious scatter or uncertainty. The uncertainty of a Type T thermocouple is nominally $\pm 2^\circ\text{C}$ and we should indicate this on the graph using error bars. Double-click on the plot to obtain the Modify Plot window and then select Y-axis under Error Bars. You should see the Error Bar Information window, shown in Figure 22.

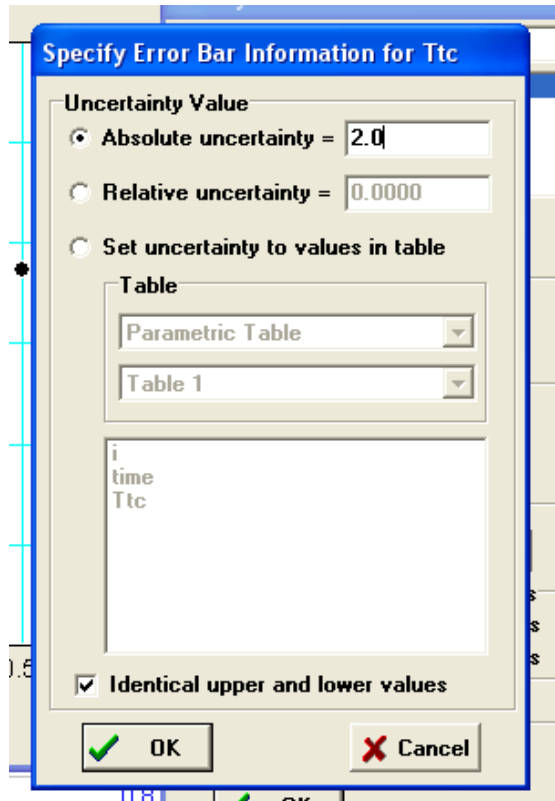


Figure 22: Error Bar Information Window

Specify the absolute uncertainty to be 2.0 and press OK. The appropriate error bars have been placed on your graph.

31. We can try to fit the data to a function using the curve fitting functions built in to EES. Select Curve Fit under the Plots menu and then select 4th order polynomial, as shown in Figure 23.

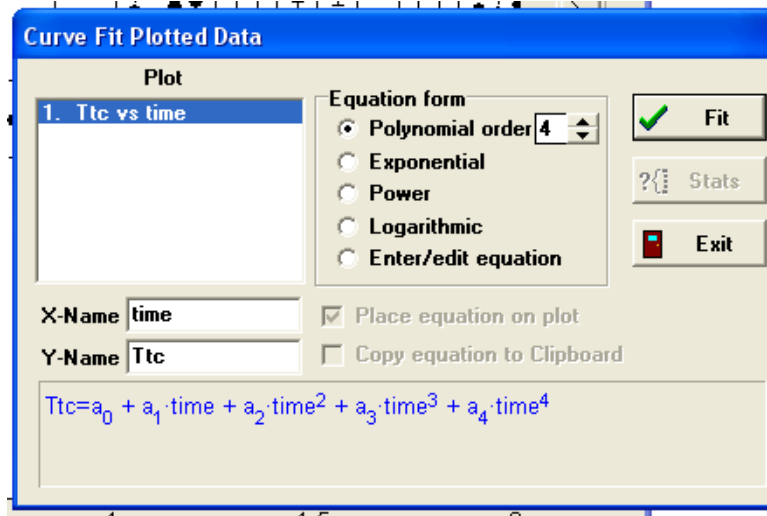


Figure 23: Curve Fit Dialog Window

Press the Fit button and the undetermined coefficients will be selected based on a least-squares fit to the data. If you select the Place equation on plot option and press the Plot button then EES will place the best fit curve on the plot and give the equation.

32. Print out your graph with your curve fit and hand it in with your lab to your lab instructor to be graded.
-

A.2 Analysis of Experimental Uncertainty

References

1. *Introduction to Fluid Mechanics*, Fox, R.W. and McDonald, A.T., John Wiley & Sons, 4th edition, 1994, Appendix F.
2. Taylor B.N. and Kuyatt, C.E., *Guidelines for Evaluating and Expressing the Uncertainty of NIST Measurement Results*, National Institute of Standards and Technology Technical Note 1297, 1994

Types of Error and the Estimation of Uncertainty

Results of experimental tests are often used for engineering analysis and design purpose. Not all data are equally reliable. The validity of data should be documented before test results are used for design and analysis. Actually, experimental uncertainty should be considered in the course of setting up an experiment.

Errors always are present when experimental measurements are made. Aside from gross blunders by the experimenter, experimental error may be of two types. **Fixed** (or systematic) error is the same for each reading and can be removed by proper calibration or correction. **Random** error (non-repeatability) is different for every reading and hence cannot be removed. The factors that introduce random error are uncertain by their nature. The objective of uncertainty analysis is to estimate the probable random error in experimental results. To do this we must first estimate the error for each measured quantity. We then analyze the propagation of uncertainty into results calculated from the experimental data.

Uncertainty and the Measurement Error

Designate the measured variable in an experiment as x . One possible way to find the measurement error for the variable would be to repeat each measurement many times. The result would be a distribution of data. Random errors in measurement usually produce a normal (Gaussian) frequency distribution of measured values. The data scatter for a normal distribution is characterized by the standard deviation, σ . The uncertainty

interval for the measured variable, x , may be stated as $\pm \sigma$, $\pm 2\sigma$, and $\pm 3\sigma$. For example, in the case of normally distributed data, over 99% of measured values of x lie within $\pm 3\sigma$ of the mean value, 95% lie within $\pm 2\sigma$, and about 70% lie within $\pm \sigma$ of the mean value of the data set. Thus it would be possible to quantify expected errors within any desired confidence limit if a statistically significant data set were available.

It is important to note that the method of repeated measurements usually is not practical. In most applications it is impossible to obtain enough data for a statistically significant sample owing to the excessive time and cost involved. However, the normal distribution suggests several important concepts to note:

- Small errors are more likely than large ones.
- Plus and minus errors are almost equally likely.
- No finite maximum error can be specified.

A more typical situation in engineering design and analysis is a "single-sample" experiment, where only one measurement is made for each point. A reasonable estimate of the measurement error due to random error in a single-sample experiment usually is plus or minus half the smallest scale division (the least count) of the instrument.

Propagation of Uncertainty in Calculations

Consider that R is the value of the function $R = R(x_1, x_2)$ and ΔR represents the uncertainty in the value of the function. Measurements of the two independent variables, x_1 and x_2 , are made. The actual uncertainty of each separately measured quantity is estimated as Δx_1 and Δx_2 . A plot as shown in Fig. 1 will illustrate how errors in the measurements propagate into the calculation of R , where R represents the value of the function $R = R(x_1, x_2)$.

The effect on R of an error in a measured quantity may be estimated by analogy to the derivative of a function. A variation, Δx_1 , in x_1 would cause R to vary by an amount ΔR_1 according to the relation

$$\Delta R_1 = \frac{\partial R}{\partial x_1} \Delta x_1 \quad (\text{A.1})$$

and the uncertainty in R caused by the uncertainty in x_2 is given by

$$\Delta R_2 = \frac{\partial R}{\partial x_2} \Delta x_2 \quad (\text{A.2})$$

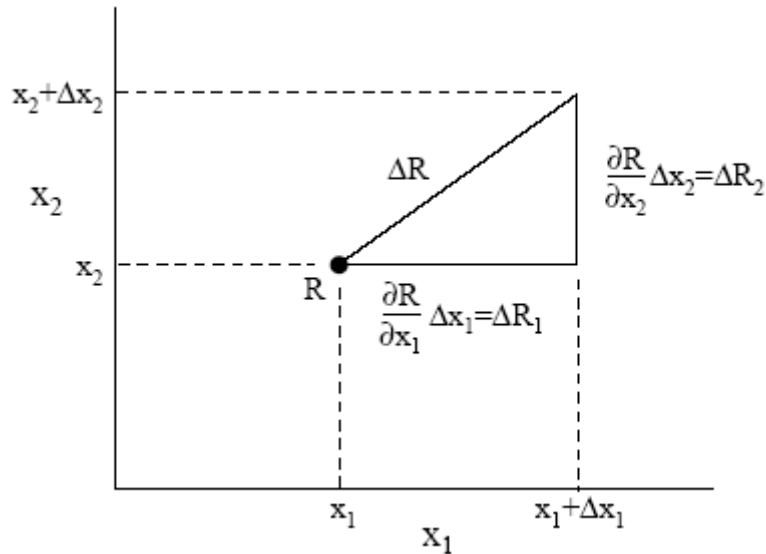


Figure A.1 Uncertainty of a function

The effects of the measurement errors in x_1 and x_2 affect the calculated value of R as shown in Figure 1. Using the Pythagorean relation we get

$$\Delta R = \pm \sqrt{(\Delta R_1)^2 + (\Delta R_2)^2} \quad (\text{A.3})$$

or

$$\Delta R = \pm \sqrt{\left(\frac{\partial R}{\partial x_1} \Delta x_1 \right)^2 + \left(\frac{\partial R}{\partial x_2} \Delta x_2 \right)^2} \quad (\text{A.4})$$

Now consider that measurements of independent variables, x_1, x_2, \dots, x_n , are made and the actual uncertainty of each independently measured quantity is estimated as Δx_i . In general, R may be expressed mathematically as $R = R(x_1, x_2, \dots, x_n)$. A variation, Δx_i , in x_i would cause R to vary by an amount ΔR_i according to the relation,

$$\Delta R_i = \frac{\partial R}{\partial x_i} \Delta x_i \quad (\text{A.5})$$

Expanding the development used to produce equation (3), the uncertainty in R due to the combined effects of uncertainty in all the variables is

$$\Delta R = \pm \sqrt{\sum_{i=1}^n (\Delta R_i)^2} \quad (\text{A.6})$$

which can be expanded to

$$\Delta R = \pm \sqrt{\left(\frac{\partial R}{\partial x_1} \Delta x_1\right)^2 + \left(\frac{\partial R}{\partial x_2} \Delta x_2\right)^2 + \dots + \left(\frac{\partial R}{\partial x_n} \Delta x_n\right)^2} \quad (\text{A.7})$$

Example A.1 - Uncertainty in liquid mass flow rate

The mass flow rate of water through a tube is to be determined by collecting water in a beaker. It is calculated from the net mass of water collected divided by the time interval.

$$\dot{m} = \frac{m_f - m_e}{t} \quad (\text{A.8})$$

The measured quantities and the error estimates, or actual uncertainties, are:

- Mass of full beaker, $m_f = 400 \pm 2$ g
- Mass of empty beaker, $m_e = 200 \pm 2$ g
- Collection time interval, $t = 10.0 \pm 0.2$ sec

Therefore the experimentally-calculated mass flow rate is given by

$$\dot{m} = \frac{400g - 200g}{10s} = 20 \text{ grams.s}^{-1} \quad (\text{A.9})$$

We note that there are three independent variables in this problem; therefore the actual uncertainty in the calculated value of mass flow rate is calculated by using Equation (A.7). This gives

$$\frac{\partial \dot{m}}{\partial m_f} = \frac{1}{t} \quad (\text{A.10a})$$

$$\frac{\partial \dot{m}}{\partial m_e} = -\frac{1}{t} \quad (\text{A.10b})$$

$$\frac{\partial \dot{m}}{\partial t} = -\frac{(m_f - m_e)}{t^2} \quad (\text{A.10c})$$

The actual uncertainty in the calculated value of mass flow rate is calculated using Equation (A.7), this gives

$$\begin{aligned} \Delta \dot{m} &= \pm \sqrt{\left(\frac{\partial \dot{m}}{\partial m_f} \Delta m_f \right)^2 + \left(\frac{\partial \dot{m}}{\partial m_e} \Delta m_e \right)^2 + \left(\frac{\partial \dot{m}}{\partial t} \Delta t \right)^2} \\ &= \pm \sqrt{\left(\frac{1}{t} \Delta m_f \right)^2 + \left(-\frac{1}{t} \Delta m_e \right)^2 + \left(-\frac{m_f - m_e}{t^2} \Delta t \right)^2} \\ &= \pm \sqrt{\left(\frac{1}{10s} (\pm 2g) \right)^2 + \left(-\frac{1}{10s} (\pm 2g) \right)^2 + \left(-\frac{(400 - 200)g}{(10s)^2} (\pm 0.2s) \right)^2} \\ &= \pm 0.49 \text{ gms.s}^{-1} \quad (\text{A.11}) \end{aligned}$$

To determine if this magnitude of error is significant we can look at the relative uncertainty, which can be defined as the ratio of the actual uncertainty to the actual value, usually in percent. Thus, the relative uncertainty in mass flow rate can be described as

$$\frac{\Delta \dot{m}}{\dot{m}} = \pm \frac{0.49 \text{ gms} / \text{s}}{20 \text{ gms} / \text{s}} = \pm 0.0245 = \pm 2.54 \text{ percent} \quad (\text{A.12})$$

This example illustrates an important point for experimental design. The mass of water collected is calculated from two measured masses, m_f and m_e , and the time t . For any stated uncertainty in the measurements of m_f and m_e , the relative uncertainty in m_f can be decreased by making $(m_f - m_e)$ larger. This might be accomplished by using larger containers or a longer measuring interval, t , which would also reduce the relative uncertainty in the measured value of t .

Example A.2 - Uncertainty in Reynolds number for water flow rate

The Reynolds number is to be calculated for flow of water in a tube of the previous example. It can be calculated by the equation,

$$\text{Re} = \frac{4\dot{m}}{\pi\mu D} = \text{Re}(\dot{m}, D, \mu) \quad (\text{A.13})$$

The uncertainty in calculating the mass flow rate was determined in the earlier example. What about uncertainties in μ and D ? The tube diameter is $D = 6.35 \text{ mm}$. Do we assume that it is an exact value? The diameter might be measured to the nearest of $\pm 0.1 \text{ mm}$. With no other available information, we assume that the uncertainty in diameter is $\Delta D = \pm 0.05 \text{ mm}$.

It is important to note that the viscosity of water depends on temperature. The temperature is measured as $24 \pm 0.5^\circ\text{C}$. An obvious question arises that how will the uncertainty in temperature affect the uncertainty in μ ? One way to estimate this is to write

$$\Delta\mu_T = \frac{\partial\mu}{\partial T} \Delta T \quad (\text{A.14})$$

The derivative can be estimated from tabulated viscosity data near the nominal temperature, 24°C. We can get this value from EES that is introduced to you earlier. This gives,

$$\begin{aligned}\frac{\partial \mu}{\partial T} &\approx \frac{\Delta \mu}{\Delta T} = \frac{\mu(25^\circ\text{C}) - \mu(23^\circ\text{C})}{(25 - 23)\text{C}} = \frac{(0.00089 - 0.000932)\text{N.s.m}^{-2}}{2\text{C}} \\ &= -0.000021\text{N.s.m}^{-2}\text{C}^{-1}\end{aligned}\quad (\text{A.15})$$

It follows from Equation (A.14) that the actual uncertainty in viscosity due to temperature is

$$\Delta \mu_T = \frac{\partial \mu}{\partial T} \Delta T = -0.000021\text{N.s.m}^{-2}.\text{C}^{-1}(\pm 0.5\text{C}) = \pm 0.0000105\text{N.s.m}^{-2} \quad (\text{A.16})$$

It is important to emphasize that tabulated viscosity data themselves have some uncertainty, although we usually neglect this contribution. Viscosity, in particular, is difficult to accurately measure. If we assume that the uncertainty in the viscosity value at a given temperature is $\Delta \mu_{tab} = \pm 0.000005\text{N.s.m}^{-2}$, the overall uncertainty in viscosity may be computed by Equation (A.3), to give

$$\begin{aligned}\Delta \mu &= \pm \sqrt{(\Delta \mu_T)^2 + (\Delta \mu_{tab})^2} \\ \Delta \mu &= \pm \sqrt{(\pm 0.0000105)^2 + (\pm 0.000005)^2} = \pm 0.000012\text{N.s.m}^{-2}\end{aligned}\quad (\text{A.17})$$

The actual uncertainties in mass flow rate, tube diameter and viscosity needed to compute the uncertainty for the calculated Reynolds number are now known. The required partial derivatives, determined from Equation (A.13), are

$$\frac{\partial \text{Re}}{\partial \dot{m}} = \frac{4}{\pi \mu D}, \quad \frac{\partial \text{Re}}{\partial \mu} = -\frac{4 \dot{m}}{\pi \mu^2 D}, \quad \text{and} \quad \frac{\partial \text{Re}}{\partial D} = -\frac{4 \dot{m}}{\pi \mu D^2} \quad (\text{A.18})$$

Each term in the Reynolds number uncertainty equation can now be easily determined by Equation (A.5), to give

$$\begin{aligned}\Delta \text{Re}_{\dot{m}} &= \frac{\partial \text{Re}}{\partial \dot{m}} \Delta \dot{m} = \frac{4}{\pi \mu D} \Delta \dot{m} \\ &= \frac{4}{\pi(0.000911)6.35} \frac{m^2}{N.s.mm} \left(\pm 0.49 \frac{gms}{s} \right) = \pm 107.85\end{aligned}\quad (\text{A.19})$$

$$\begin{aligned}\Delta \text{Re}_D &= \frac{\partial \text{Re}}{\partial D} \Delta D = -\frac{4\dot{m}}{\pi \mu D^2} \Delta D \\ &= -\frac{4(20)}{\pi(0.000911)(6.35)^2} \frac{m^2.gm}{N.s^2.mm^2} (\pm 0.05mm) = \mp 34.66\end{aligned}\quad (\text{A.20})$$

and

$$\begin{aligned}\Delta \text{Re}_{\mu} &= \frac{\partial \text{Re}}{\partial \mu} \Delta \mu = -\frac{4\dot{m}}{\pi \mu^2 D} \Delta \mu \\ &= -\frac{4(20)}{\pi(0.000911)^2 (6.35)} \frac{m^4.gm}{N.s^3.mm} (\pm 0.000012 mm) = \mp 57.98\end{aligned}\quad (\text{A.21})$$

The uncertainty in the calculation of Reynolds number resulting from the uncertainties in all of the quantities involved in calculating Reynolds number can now be calculated:

$$\begin{aligned}\Delta \text{Re} &= \pm \sqrt{\left(\frac{\partial \text{Re}}{\partial \dot{m}} \Delta \dot{m} \right)^2 + \left(\frac{\partial \text{Re}}{\partial D} \Delta D \right)^2 + \left(\frac{\partial \text{Re}}{\partial \mu} \Delta \mu \right)^2} \\ &= \pm \sqrt{(\pm 107.85)^2 + (\mp 34.66)^2 + (-57.98)^2} = \pm 127\end{aligned}\quad (\text{A.22})$$

The calculated Reynolds number is

$$\text{Re} = \frac{4\dot{m}}{\pi \mu D} = \frac{4(20)}{\pi(0.000911)(6.35)} \frac{m^2 gm}{s^2.N.mm} = 4,402\quad (\text{A.23})$$

Finally, the relative uncertainty for the Reynolds number is

$$\frac{\Delta \text{Re}}{\text{Re}} = \pm \frac{127}{4,402} = \pm 0.029 = \pm 2.9 \text{ percent}\quad (\text{A.24})$$

It is important to note that the uncertainty in tabulated property data may be significant. Also, the data uncertainty is increased by the uncertainty in fluid temperature measurements.

Example A.3 - Uncertainty in Air Speed Problem

Air speed is calculated from pitot tube measurements in an air-conditioning duct, using Bernoulli's equation,

$$V = \left(\frac{2gh\rho_{\text{water}}}{\rho_{\text{air}}} \right)^{1/2} \quad (\text{A.25})$$

where h is the observed height of the manometer column. The only new element in this example is the square root. The variation in V due to the uncertainty in h can be expressed as

$$\begin{aligned} \Delta V_h &= \frac{\partial V}{\partial h} \Delta h = \frac{1}{2h} \left(\frac{2gh\rho_{\text{water}}}{\rho_{\text{air}}} \right)^{-1/2} \frac{2gh\rho_{\text{water}}}{\rho_{\text{air}}} \Delta h \\ &= \pm \frac{1}{2h} \left(\frac{2gh\rho_{\text{water}}}{\rho_{\text{air}}} \right)^{1/2} \Delta h \end{aligned} \quad (\text{A.26})$$

If uncertainties due to other independent variables are negligible, the relative uncertainty in velocity is given by

$$\frac{\Delta V}{V} = \pm \frac{1}{2} \frac{\Delta h}{h} \quad (\text{A.27})$$

We thus find that the relative uncertainty in the calculated velocity is half the relative uncertainty of the height.

Some Comments

A statement of the possible uncertainty of data is an important part of reporting experimental results completely and clearly in engineering experiments. Estimating uncertainty in experimental results requires care, experience and judgment, in common with many endeavours in engineering.

Uncertainty Analysis Using EES

As discussed earlier, uncertainty analysis is an important part of any experimental investigation. However, uncertainty analysis can require significant computational effort, particularly if the partial derivatives in Equation (A.7) cannot be determined analytically. This computational burden can be eliminated using the EES program.

An uncertainty analysis can be conducted in EES using either the **Uncertainty Propagation** or **Uncertainty Propagation Table** commands in the **Calculate** menu. The purpose of these commands is to calculate the effect of uncertainties in all of the measured variables , x_1 , x_2 , ..., x_n , on the uncertainty of the calculated quantity, R , as described in the preceding section of this appendix.

This uncertainty is calculated using Equation (A.7) where Δx_i is the uncertainty of measured variable x_i and ΔR is the propagated uncertainty in calculated variable R .

$$\Delta R = \pm \sqrt{\left(\frac{\partial R}{\partial x_1} \Delta x_1\right)^2 + \left(\frac{\partial R}{\partial x_2} \Delta x_2\right)^2 + \dots + \left(\frac{\partial R}{\partial x_n} \Delta x_n\right)^2}$$

After selecting **Uncertainty Propagation** or the **Uncertainty Propagation Table** command, EES will present two lists of variables. Select the variable for which the uncertainty propagation is to be determined from the list on the left. Select one or more measured variables from the list on the right. In general, the variables appearing in the measured variables list must be constants so that their values are set to a numerical constant with an equation in the EES Equations window (for the Uncertainty Propagation command) or in a Parametric Table (for the Uncertainty Propagation Table command). EES will, however, allow some variables that are not specified as constants to be moved from the **Calculated** variables to the **Measured** variables list. The variables that can be moved are shown in red. Variables that are read from a Lookup table, for example, may be moved from the left to the right list.

To specify the uncertainty associated with the measured variables, click the **Set Uncertainties button** below the right list. A second dialog window will appear in which the absolute or relative fraction of the measured value) uncertainties for each selected measured variable can be specified. An uncertainty value for each measured variable must be provided. Click the OK button to set the uncertainties and close the Specify Uncertainties dialog window. Click the OK button in the Uncertainty Propagation dialog window to start the calculations.

After the calculations initiated with the Uncertainty Propagation command are completed, EES will display an abbreviated Solution Window containing the calculated and measured variables and their respective uncertainties. The partial derivative of the calculated variable with respect to each measured variable will also be displayed. If the Uncertainty Propagation Table command is used, the value and uncertainty for the calculated variable and each measured variable will be displayed in the Parametric Table after the calculations are completed. The calculated variable can then be plotted with error bars representing the propagated uncertainty using the New Plot Window command in the EES Plot menu.

Example A.1 - Uncertainty in liquid mass flow rate using EES

Example 1 in the previous section is resolved here to illustrate how the uncertainty calculations are done in EES. Start EES and enter the following equations to calculate the mass flow rate using measured values of the final and initial masses and the elapsed time.

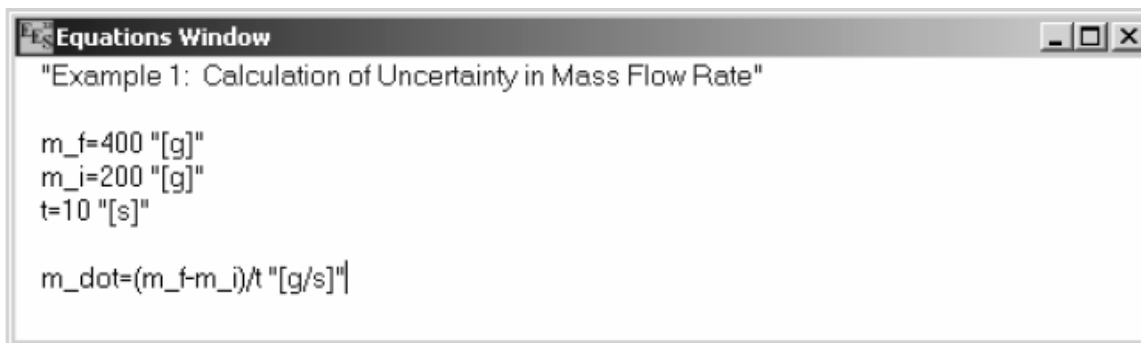


Figure A.2 Equations needed to solve for the mass flow rate

Now select the **Uncertainty Propagation** in the **Calculate** menu. The following dialog will appear. Click on the three variables displayed in the measured variables list to select them.

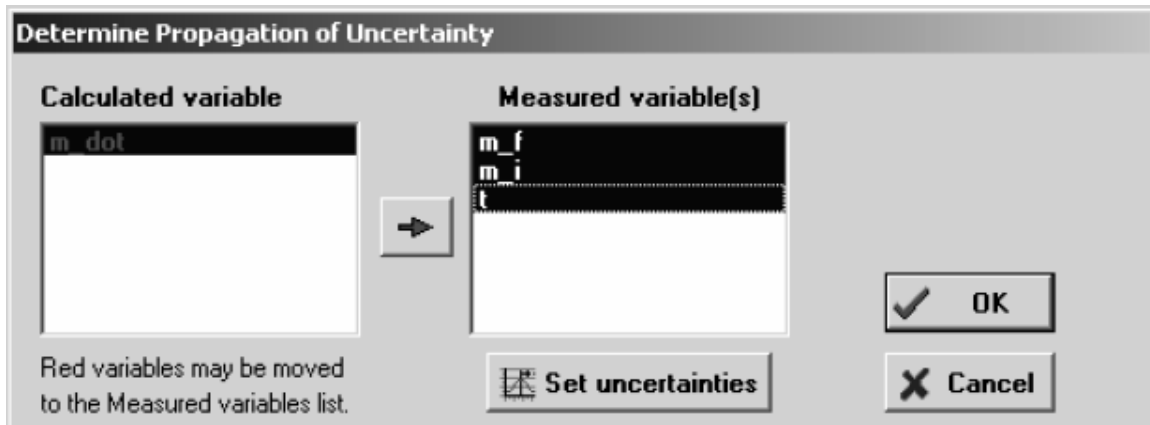


Figure A.3 Uncertainty propagation Window

Next, click the Set Uncertainties button. The following dialog window will appear in which you can enter the absolute uncertainties that were given above for each selected variable.

Variable	Value	Units	Absolute Uncertainty	Relative Uncertainty
m_f	400	g	2	
m_i	200	g	2	
t	10	s	0.2	

Figure A.4 Dialog window for entering uncertainties of measured variables

Click the OK button after entering the uncertainty values. Then click the OK button in the Uncertainty Propagation dialog to initiate the calculations. The solution window will appear.

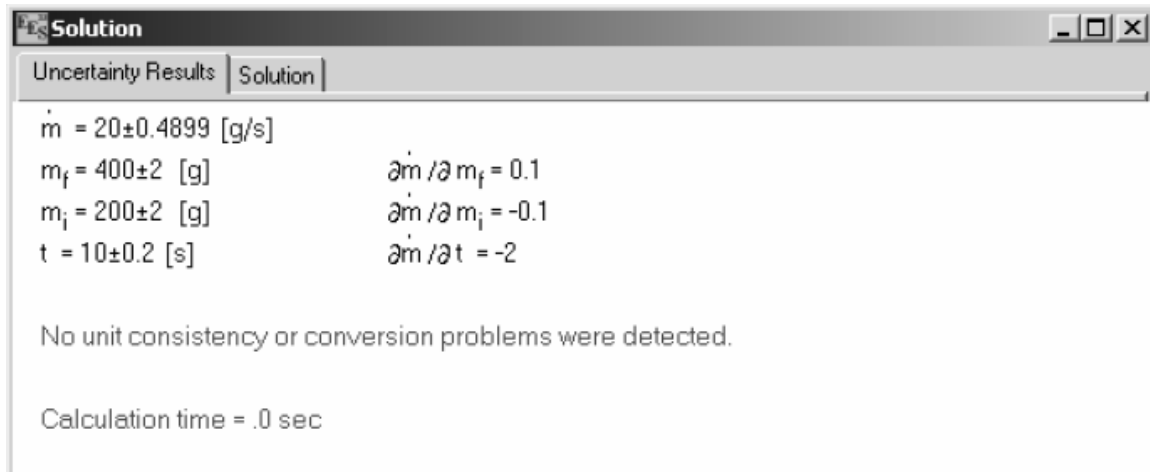


Figure A. 5 Solution window showing uncertainty calculation results

The calculated value of the mass flow rate is displayed with an uncertainty of ± 0.4899 , the same value found in equation (A.11). Note that EES also displays the partial derivatives of the calculated variable with respect to each of the measured variables.

Example A.2 - Uncertainty in Reynolds number for water flow using EES

Example A.2 of the previous section can be solved using EES as described below. First, enter the equations shown in Figure A.6 into the Equations window. Then select Uncertainty Propagation from the Calculate menu.

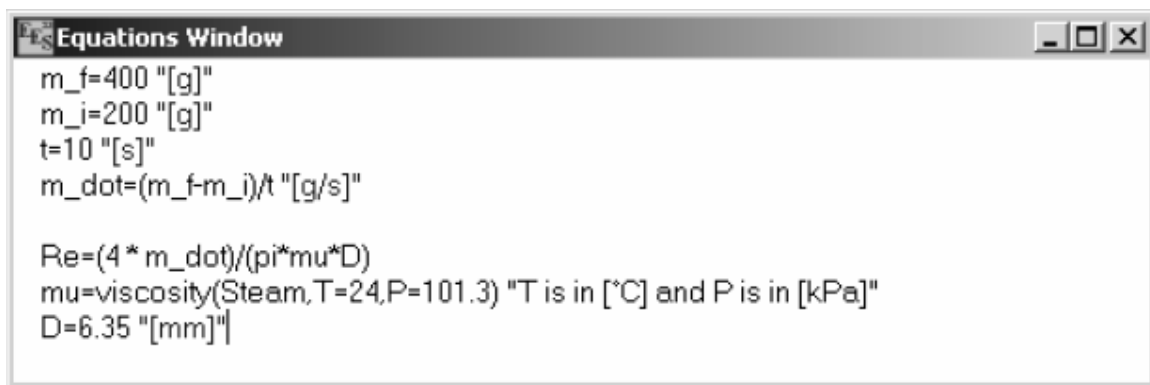


Figure A.6 Equations needed to solve for the Reynolds number

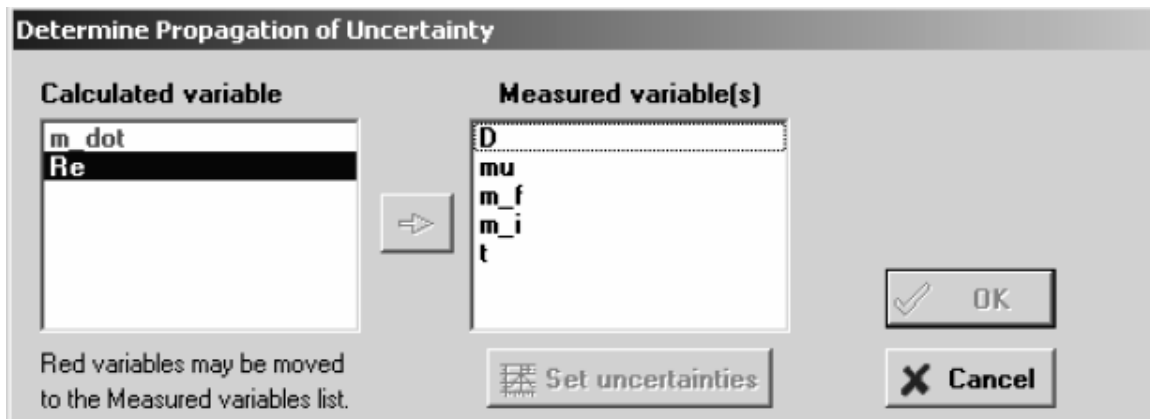


Figure A.7 Uncertainty propagation dialog window initially

In example A.2, the uncertainty in Reynolds number is determined in terms of uncertainties in D , viscosity (μ) and mass flow rate (\dot{m}). To repeat these calculations, just as they were done in the previous section, click on m_dot in the left list and then on the arrow between the lists to move it to the right list. Then, select D , μ , and \dot{m} in the right list. The dialog window now appears as:

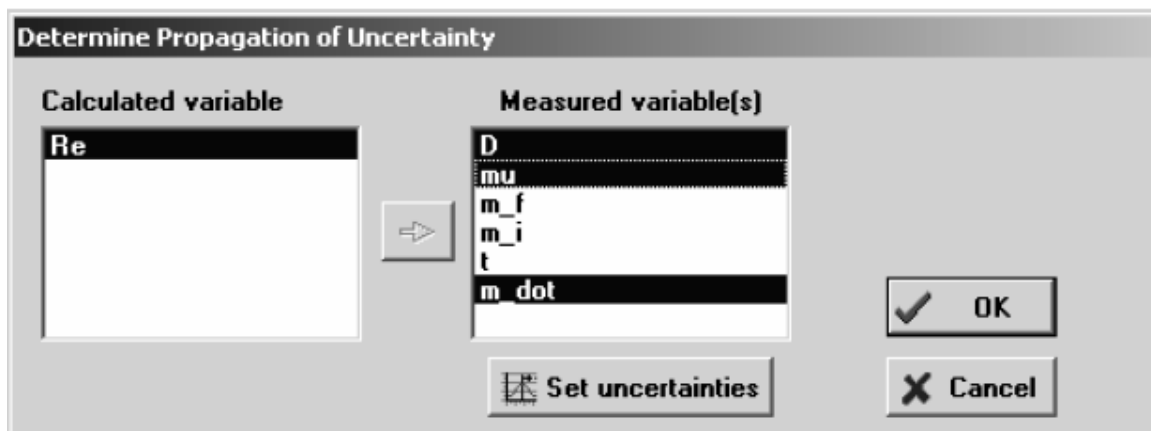


Figure A.8 Uncertainty propagation dialog window for the Reynolds number problem

Now, set the uncertainties for D , μ , and \dot{m}

Variable	Value	Units	Absolute Uncertainty	Relative Uncertainty
D	6.35	mm	0.05	
μ	0.0009112	kg/m-s	0.000012	
\dot{m}	20	g/s	0.4899	

Figure A.9 Dialog window for entering uncertainties of measured variables for the Reynolds number problem

Click OK to dismiss this dialog and OK again to do the calculations. The solution will appear as shown below. Note that the uncertainty in Re is ± 127.2 , which agrees with the result in equation (A.22).

Results of uncertainty analysis for Re	
Uncertainty Results	Solution
Unit Settings: [C]/[kPa]/[kg]/[degrees]	
D = 6.35 \pm 0.05 [mm]	$\partial \text{Re} / \partial D = -693.1$
$\mu = 0.0009112 \pm 0.000012$ [kg/m-s]	$\partial \text{Re} / \partial \mu = -4.830\text{E}+06$
$\dot{m} = 20 \pm 0.4899$ [g/s]	$\partial \text{Re} / \partial \dot{m} = 220.1$
Re = 4401 \pm 127.2	
No unit consistency or conversion problems were detected.	
Calculation time = .0 sec	

Figure A. 10 Solution for the Reynolds number problem