Turning Moment of Rotating Inner Cylinder in the Entry Region of Concentric Annuli*

M.A.L. EL-SHAARAWI**, M.O. BUDAIR**
and M.S.A. AL-QAHTANI**

This paper is concerned with calculating the tangential shear stress and the torque required to turn the inner shaft of concentric annuli having a laminar flow with simultaneously developing tangential and axial boundary layers. The nondimensional governing equations have been numerically solved over a wide range of the annulus radius ratio ($N=0.5 - 0.95$) and the ratio of the square of Reynolds number to Taylor number ($Re^2/Ta=0.3 - 10$). The results clarify the effect of these two controlling parameters (N and Re^2/Ta) on the torque and show that the assumption of whole-channel fully developed flow leads to a considerable underestimation of the values of the torque.

Key Words: Annulus, Rotating Inner Boundary, Turning Torque

1. Introduction

Fluid flow between concentric cylinders with inner rotating walls is of practical importance in many industrial applications. In the field of electric motors and generators, knowledge of the hydrodynamic features of such fluid flow is needed to limit the rotor temperature to less than the maximum allowable value. In the fields of axial flow pumps, mixers and other rotary machines, the study of this type of flow can assist in estimating the performance of such machines and the torque required to rotate them. In addition to other applications in the fields of journal bearings, fiber coating and paper manufacturing, there are suggestions of possible future applications for compact rotary heat exchangers and combustion chambers. These possible future applications depend on the phenomenon that when the rotational speed of the inner cylinder becomes sufficiently large, centrifugal forces cause the fluid to be moved radially

outward and Taylor vortices appear[1]. These vortices can enhance significantly the rates of heat transfer and momentum transportation in the system. On the other hand, provided laminar flow conditions prevail, it is known[2]-[4] that the heat transfer characteristics are slightly affected by the inner cylinder rotation.

The flow of a viscous fluid in the entry region of an annulus with a rotating inner cylinder has three simultaneously developing hydrodynamic boundary layers. Two of these boundary layers are related to the axial velocity component. They develop on the inner and outer walls of the annulus. The third boundary layer is relevant to the tangential (azimuthal) velocity component and develop on the inner rotating cylinder. Astill[5],[6] investigated the development of these boundary layers experimentally and by means of a momentum-integral analysis; he developed an empirical stability criterion for the transition of tangentially developing flows from the laminar regime to the secondary-laminar (Taylor-vortex) flow regime. This stability criterion is the Taylor number based on the tangential boundary-layer displacement thickness rather than on the annular gap width. A pertinent semi-empirical stability criterion

has recently been presented[7],[8]. Both the aforesaid empirical and semi-empirical stability criteria require knowledge of the developing laminar velocity profiles in the entry region of the annular channel. The empirical criterion requires knowledge of the developing tangential velocity profiles, while the semi-empirical one needs the developing tangential and axial velocity profiles at the same time.

Astill et al.[9] obtained the developing tangential velocity profiles for an axially fully developed flow by uncoupling the interaction between the axial and tangential profiles through the use of a uniform axial velocity distribution in the tangential momentum equation. Martin and Payne[10] utilized a finite-difference technique to numerically solve the governing Navier-Stokes equations and presented some developing velocity profiles. However, the effect of all the hydrodynamic similarity parameters which govern this problem (Reynolds number, Taylor number or ratio between Taylor and Reynolds numbers, and the annulus radius ratio) was not investigated in their work. Coney and El-Shaarawi[11] developed a finite-difference scheme to solve the governing boundary-layer equations and presented results for the axial and tangential velocity profiles developing over a wide range of the flow-controlling parameters. Recently an improvement of this-difference technique has been proposed[12].

The main goal of the studies which deal with tangentially developing laminar flows[3]–[6],[9]–[11] was the determination of the axial growth of the tangential boundary-layer displacement thickness; knowledge of this axial growth is essential to locate the axial position of the point of origin of hydrodynamic instability. These papers and a careful search of the literature failed to uncover any prior data on the torque needed to rotate the inner cylinder of concentric annular channels with simultaneously developing axial and tangential boundary layers. On the other hand, the turning moment of the rotating inner cylinder for tangentially fully developed flows, which have the well-known Couette velocity profile, can be easily calculated[13]. However, the use of such fully developed torque values gives a conservative estimation for the real moment needed to operate machines having flows with simultaneously developing tangential and axial velocity boundary layers. One of the objectives of this paper is to present the torque values for such flows developing over a wide range of the controlling parameters, Re^2/Ta and N.

Nomenclature

b : annular gap width, $(r_2 - r_1)$;

D : equivalent (hydraulic) diameter of annulus,

$(2b)$;

N : annulus radius ratio, r_1/r_2 ;

M^* : dimensional torque (moment) required to rotate the inner cylinder ;

M : dimensionless torque, $M^*/2\rho\pi r_1^3 r_2 \Omega u_0$;

M_{fd} : dimensionless torque required to rotate the inner cylinder if the flow were fully developed right from the annulus entrance ;

p : pressure of fluid at any point ;

p_0 : pressure of fluid at annulus entrance ;

P : dimensionless pressure of fluid at any point ;

r : radial coordinate ;

r_1 : inner radius of annulus ;

r_2 : outer radius of annulus ;

R : dimensionless radial coordinate, r/r_2 ;

Re : axial Reynolds number, $\rho u_0 D/\mu$;

Ta : Taylor number, $2\Omega^2 r_1^2 b^3 \rho^2/\mu^2(r_1+r_2)$;

u : axial velocity component ;

u_0 : entrance or mean axial velocity,
$$\int_{r_1}^{r_2} u r \, dr \Big/ \int_{r_1}^{r_2} r \, dr ;$$

U : dimensionless axial velocity component, u/u_0 ;

v : radial velocity component ;

V : dimensionless radial velocity component, $\rho v r_2/\mu$;

w : tangential (azimuthal) velocity component ;

W : dimensionless tangential velocity component, $w/\Omega r_1$;

W_{fd} : fully developed W ;

z : axial coordinate ;

Z : dimensionless axial coordinate $2z(1-N)/r_2 Re$;

∇ : torque increment, $M - M_{fd}$;

∇_{fd} : fully developed ∇ ;

Ω : angular velocity of inner cylinder ;

μ : dynamic viscosity of fluid ;

ρ : fluid density ;

τ^* : tangential shear stress at the annulus inner wall, $\mu(\partial w/\partial r)_{r_1}$;

τ : dimensionless shear stress, $\tau^*/\mu\Omega N = (\partial W/\partial R)_{R=N}$;

τ_{fd} : fully developed τ ;

2. Governing Equations and Method of Solution

We consider the forced flow in the entry region of a concentric annulus whose inner cylinder rotates with a constant angular velocity Ω. Figure 1 illustrates the geometry, coordinate system and the developing tangential boundary layer on the inner rotating wall. It is assumed that the flow is steady, axisymmetric, and enters the annular channel with a uniform axial velocity profile of a value equal to the mean axial velocity in the annular gap $(u = u_0)$. The fluid is assumed to
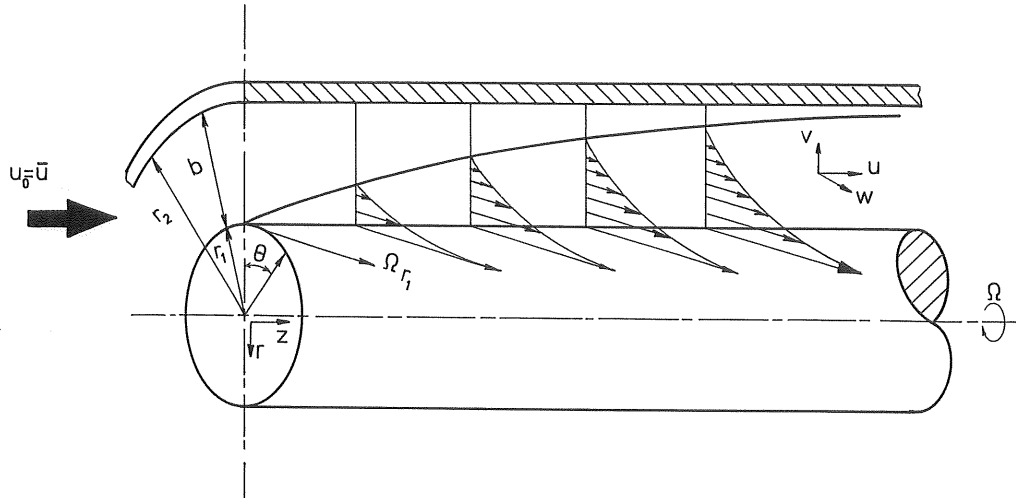
Fig. 1 Problem geometry and coordinate system

have constant physical properties and body forces are absent. The boundary-layer equations which govern this flow are given in Ref.(11). However, these equations in their dimensionless forms, using the dimensionless parameters given in the nomenclature, are repeated hereafter for the sake of completeness.

$$\frac{\partial V}{\partial R}+\frac{V}{R}+\frac{\partial U}{\partial Z}=0 \qquad (1)$$

$$\frac{W^2}{R}=\frac{(1-N)}{2(1+N)}\frac{Re^2}{Ta}\frac{\partial P}{\partial R} \qquad (2)$$

$$V\frac{\partial W}{\partial R}+U\frac{\partial W}{\partial Z}=\frac{\partial^2 W}{\partial R^2}+\frac{1}{R}\frac{\partial W}{\partial R}-\frac{W}{R^2} \qquad (3)$$

$$V\frac{\partial U}{\partial R}+U\frac{\partial U}{\partial Z}=-\frac{\partial P}{\partial Z}+\frac{\partial^2 U}{\partial R^2}+\frac{1}{R}\frac{\partial U}{\partial R} \qquad (4)$$

It is noteworthy that the term W/R^2 on the right-hand side of Eq.(3) was neglected in Ref.(11). Such a negligence, as a result of a classical order-of-magnitude analysis for the derivation of boundary-layer equations, may be acceptable in narrow-gap annuli. However, it has been shown[14] that this term should be taken into consideration for greater accuracy, especially with annuli of small radius ratio, and also in order that the developing flow can reach the fully developed Couette tangential velocity profile far from the channel entrance.

The four coupled equations above are subject to the following boundary conditions.

for $Z=0$ and $N<R<1$, $P=V=W=0$, $U=1$
for $Z\geq0$ and $R=N$, $U=V=0$, $W=1$
for $Z\geq0$ and $R=1$, $U=V=W=0$

$$(5)$$

It is worthy of note that due to the negligence of the axial diffusions of momentum $(\partial^2 U/\partial Z^2=\partial^2 W/\partial Z^2=0)$, the Reynolds number is inherent in the dimensionless formulation of the problem and thus it is not explicitly needed for the solution. However, two other similarity parameters are explicitly required in order to solve the problem under consideration: the annulus radius ratio (N) and the ratio between the square of Reynolds number and the Taylor number (Re^2/Ta). The former is relevant to geometrical similarity considerations while the latter is concerned with kinematic similarity and represents the ratio of inertia to centrifugal forces. Values of these two controlling similarity parameters must, therefore, be chosen for each computer run. The above equations have been numerically solved using the finite-difference scheme presented in Refs.(3) and (11). The obtained numerical results regarding only the developing tangential shear stress and the torque will be presented and discussed hereafter since the developing velocity profiles, pressure and coefficient of friction were previously reported[11].

For a given axial distance from the annulus entrance (z), the infinitesimal torque (dM^*) exerted by the fluid on a surface element of the inner cylinder having an area $dA=2\pi r_1 dz$ is due to the local tangential shear stress $\tau^*=\mu(\partial w/\partial r)_{r_1}$ and is given by

$$dM^*=2\pi\mu r_1^2\left(\frac{\partial w}{\partial r}\right)_{r_1} dz.$$

Hence upon integration from the annulus entrance $(z=0)$ to any axial distance z, and using the dimensionless parameters given in the nomenclature, one obtains the following expression for the dimensionless torque (M) required to rotate a dimensionless length Z of the inner cylinder.

$$M=\int_0^Z \tau dZ=\int_0^Z\left(\frac{dW}{dR}\right)_{R=N}dZ \qquad (6)$$

The fully developed tangential (or Couette) velocity profile (W_{fd}) can be obtained by solving Eq.(3) with the inertia terms on its left-hand side set at zero.

$$W_{fd}=\frac{N}{1-N^2}\left(\frac{1}{R}-R\right) \qquad (7)$$

If the flow were fully developed right from the annulus entrance, the dimensionless tangential shear stress (τ) would have, for a given radius ratio (N), a constant value (τ_{fd}) regardless of the axial distance Z. This fully developed value of the dimensionless tangential shear stress (τ_{fd}) is given by the following equation.

$$\tau_{fd}=\left(\frac{\partial W_{fd}}{\partial R}\right)_{R=N}=\frac{N^2+1}{N(1-N^2)} \qquad (8)$$

Moreover, the variation of the dimensionless torque with the axial distance under the assumption of whole-channel fully developed flow can be obtained by substituting Eq.(8) in Eq.(6). This yields the following linear relationship.

$$M_{fd}=\tau_{fd}Z=\frac{N^2+1}{N(1-N^2)}Z \qquad (9)$$

If the annulus is sufficiently long, the developing local tangential shear stress (τ) should asymptotically approach (far from the duct entrance) the corresponding fully developed τ_{fd}. Moreover, the developing torque (M) should finally (at large values of Z) follow a linear relationship with Z, having the same slope $[(N^2+1)/\{N(1-N^2)\}]$ as that given by Eq.(9) for the fully-developed torque (M_{fd}). The difference between M and M_{fd} will be referred to as the torque increment (∇). This torque increment is due to the inertia forces in the developing region. For an annulus of a given radius ratio (N), the torque increment must asymptotically approach a constant value which will be referred to as the fully developed torque increment (∇_{fd}).

Besides the other means of verification which have been reported in Ref.(11), the Couette velocity profile, given by Eq.(7), provides an analytical check of the present computer code and the obtained numerical results. The obtained numerical results for the developing tangential velocity profiles asymptotically approached (at large values of Z) the Couette velocity profile. Moreover, validation of the computer code and its numerical results were also checked on the basis of the trend of the numerical results at large values of Z; the torque (M) and the torque increment (∇), for a given N, were found to follow the aforesaid linear relationship and to have a constant value, respectively.

3. Results and Discussion

Computations were carried out for seven annulus radius ratios, namely, $N=0.5, 0.6, 0.7, 0.8, 0.85, 0.9$, and 0.95, at three values of the rotational parameter $Re^2/Ta=0.3, 1$, and 10. Figures 2(a) and 2(b) give the variation of the local dimensionless tangential shear stress τ with the axial distance Z for some of

the annuli under consideration at $Re^2/Ta=1$. These two figures focus on the developing region in which the shear stress varies nonlinearly with Z. However, the axial distance required for the flow to reach its state of full development is shorter for narrow-gap annuli (i.e., annuli of large radius ratio) than for annuli of small radius ratio. Therefore, Fig.2(b) shows the τ-Z variation over a larger axial distance (Z) than does Fig.2(a); thus more useful information is presented especially for the annulus of radius ratio 0.5. In both figures, the tangential shear stress value at the entrance cross section ($Z=0$) should be infinite. This is due to the step change in the value of the tangential velocity component (W) from zero everywhere to unity on the inner wall at this particular cross section. Also, this infinite value can be attributed to the fact that the tangential boundary-layer thickness is zero at $Z=0$.

As can be seen from Figs.2(a) and 2(b), for a given annulus (N), the local tangential shear stress (τ) decreases gradually as Z increases, until it
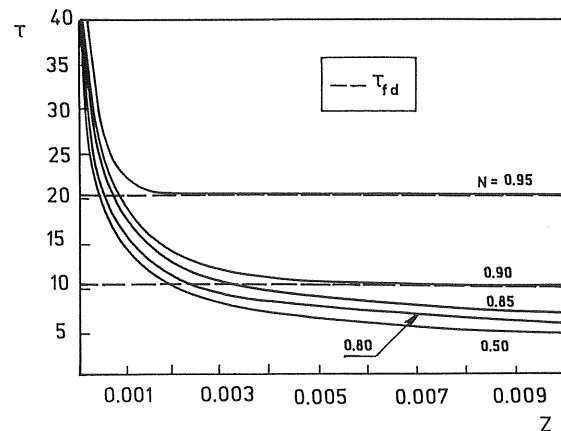
(a) Shear stress versus axial distance ($0 \le Z \le 0.01$) for various values of annulus radius ratio, $Re^2/Ta=1$
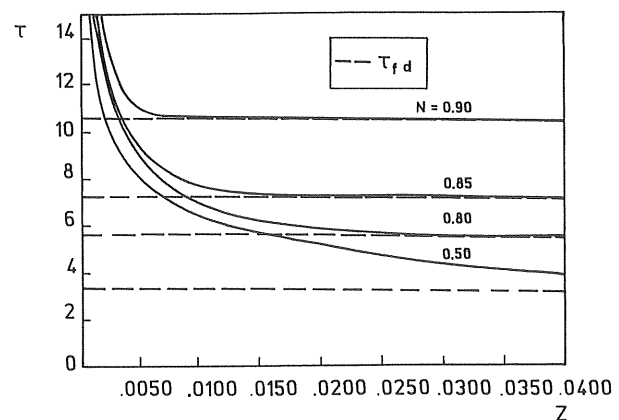
(b) Shear stress versus axial distance ($0 \le Z \le 0.04$) for various values of annulus radius ratio, $Re^2/Ta=1$

Fig. 2

asymptotically reaches its fully developed value (τ_{fd}). This expected behavior of τ is related to the shape of the tangential velocity profile (W); recall that τ is equal to the radial gradient of W at the inner wall. The steep tangential velocity gradient near the entrance results in higher values of τ. As the fluid moves away from the entrance, tangential momentum diffuses into the annular gap, and hence the tangential velocity gradient decreases as Z increases.

The effect of the annulus radius ratio (N) on the developing tangential shear stress can also be seen from Figs. 2(a) and 2(b). For a given Z, the larger the annulus radius ratio, the higher the value of the tangential shear stress. This is because the annular gap width decreases as the radius ratio increases. Hence the radial gradient of W at the inner wall increases as the radius ratio increases; recall that W decreases from unity at the inner rotating wall to zero at the outer stationary wall in a smaller annular gap width as N increases.

The effect of the spin or rotational parameter Re^2/Ta on the tangential shear stress τ originates from the effect of this parameter on the developing tangential velocity profiles (W). It is known[5],[6],[11] that this parameter has a slight effect on tangential velocity development. Consequently, it is anticipated that this parameter would similarly have a slight effect on the tangential shear stress τ. To clarify such an effect, values of τ are given in Table 1 for an annulus of radius ratio 0.9 at various axial positions for three values of Ra^2/Ta (0.3, 1, and 10).

One of the objectives of the present paper is to present results for the torque required to rotate the inner cylinder in a laminar flow with simultaneously developing axial and tangential boundary layers. Such developing-torque results are very much needed by engineers working in the field of rotary machinery and, to the best of the authors' knowledge, these results are not available in the literature. Therefore, some detailed results will be given in this regard. Figures 3 through 6 show the variation of this developing torque (M) with the axial distance Z for four of the chosen annulus radius ratios $(N=0.5, 0.7, 0.8,$ and $0.9)$. For the sake of comparison, the corresponding fully developed torque, given by the linear relationship (9), is also shown in each of these figures. Thus, Figs. 3 through 6 indicate that the assumption of whole-channel fully developed flow would lead to a considerable error in the estimation of the torque needed to rotate the inner cylinder of the annulus; this assumption gives considerably underestimated values for the torque. The largest value of Z in each of these figures is chosen so that the figure mainly
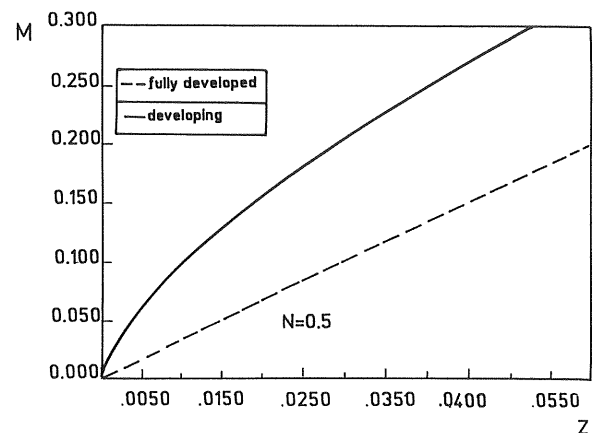
Table 1 Effect of the spin parameter Re^2/Ta on the tangential shere stress τ for an annulus of $N=0.9$

$Z \times 10^4$	τ		
	$Re^2/Ta = 0.3$	$Re^2/Ta = 1$	$Re^2/Ta = 10$
0.5	70.4407	67.6522	66.5212
1.0	48.5206	45.9299	44.6672
1.5	40.3886	38.4630	37.5953
2.0	35.7782	34.2324	33.5722
3.0	30.3996	29,2940	28.8526
4.0	27.2008	26.6673	27.2916
6.0	23.3360	22.7696	22.5665
8.0	20.9199	20.5082	20.3667
15.0	16.2070	16.0723	16.0312
25.0	13.0123	13.0822	13.1110
40.0	11.2040	11.3265	11.3690
60.0	10.6798	10.7435	10.7598
∞ (τ_{fd})	10.5946	10.5946	10.5946

Fig. 3 Development and fully developed torques versus axial distance, $N=0.5$, $Re^2/Ta=1$

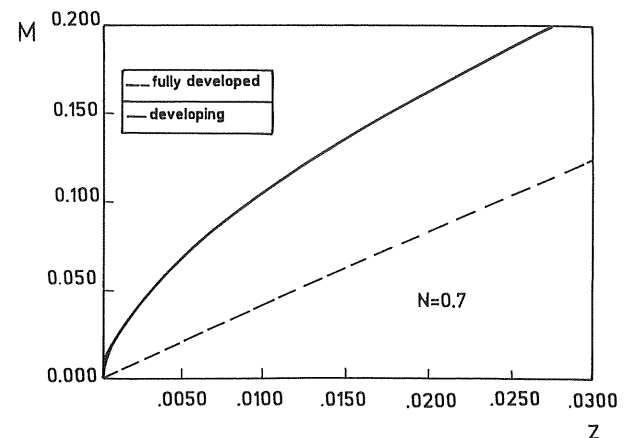

Fig. 4 Development and fully developed torques versus axial distance, $N=0.7$, $Re^2/Ta=1$

represents the entry region in which the developing torque varies nonlinearly with Z, i.e., before the tangential velocity gradient at the inner wall becomes hydrodynamically fully developed with linear variation of torque with Z.

To show the effect of the radius ratio N on the torque values more clearly, each of Figs. 7(a) and 7(b) shows comparison of such torque values for a pair of annulus radius ratios over larger value of Z than that presented in the corresponding Figs. 3 through 6. Thus, each of these two figures can show the behavior of the developing torque at large values of Z. As can be seen from these two figures, for a given value of Z, the larger the radius ratio (N), the higher the value of the torque. This is attributed to the higher values of tangential shear stress in narrow-gap annuli, as explained previously. For a given anulus (N), each of Figs. 7(a) and 7(b) clearly shows the developing-torque variation at large values of Z becomes linear, and consequently, both the developing and the fully developed solutions become

asymptotically parallel. Thus the developing torque values ultimately become higher than the corresponding fully developed torque values by a constant increment (the fully developed torque increment, ∇_{fd}). Since, according to Eq.(6), the torque is the area under the tangential shear stress curve, the fully developed torque increment (∇_{fd}) is indeed equal to the hatched area shown in Fig. 8, which is a schematic diagram of τ against Z.

To expand the idea behind the torque increment (∇), the torque results presented in Figs. 5, 6, and 7(a) are replotted in Fig. 9 in terms of torque increment. This figure shows the variation of the torque increment with Z. For a given N, the torque increment, which is due to the inertia forces in the entry region, varies nonlinearly with Z and asymptotically reaches its fully developed value (∇_{fd}) at large values of Z where the inertia forces vanish.

More torque results, especially for some annular radius ratios which have not been presented in the
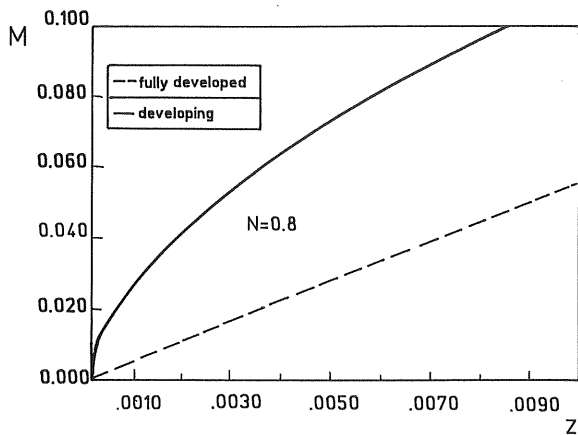

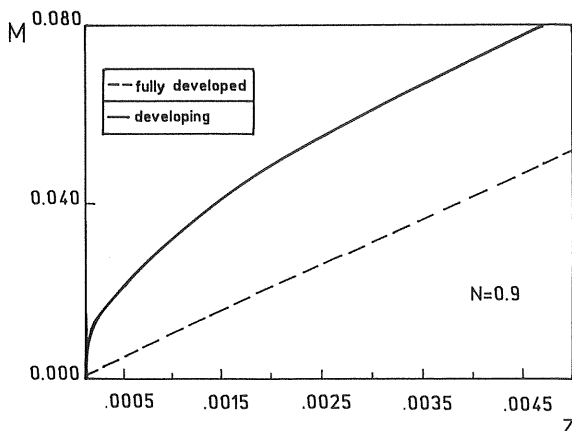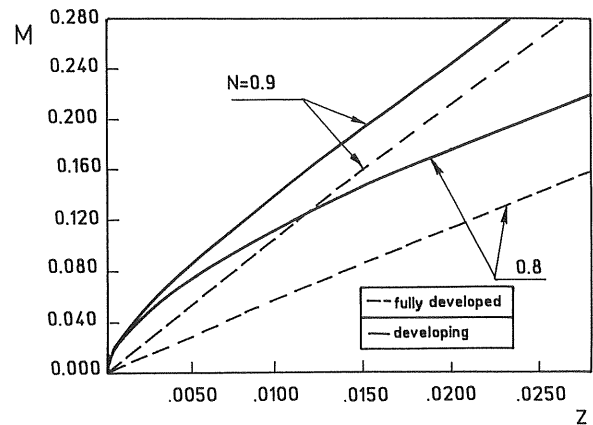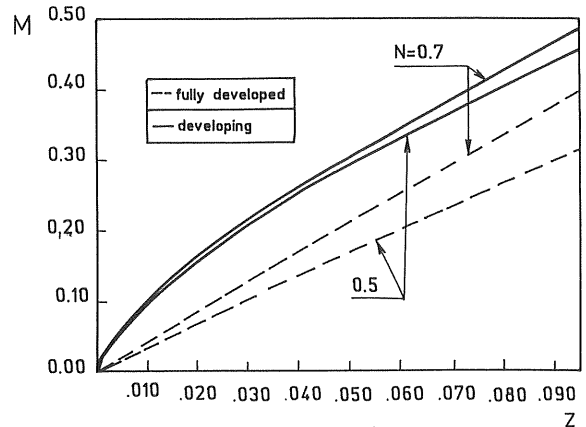Fig. 5 Developing and fully developed torques versus axial distance, $N=0.8$, $Re^2/Ta=1$

Fig. 6 Developing and fully developed torques versus axial distance, $N=0.9$, $Re^2/Ta=1$

(a) Comparison of torque for $N=0.8$ and 0.9, $Ra^2/Ta=1$

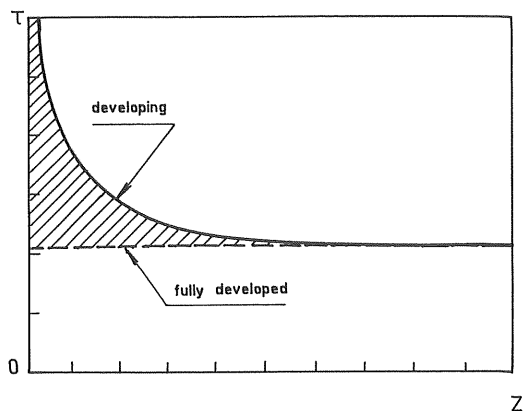(b) Comparison of torque for $N=0.5$ and 0.7, $Re^2/Ta=1$

Fig. 7

Fig. 8   Schematic diagram for τ versus Z

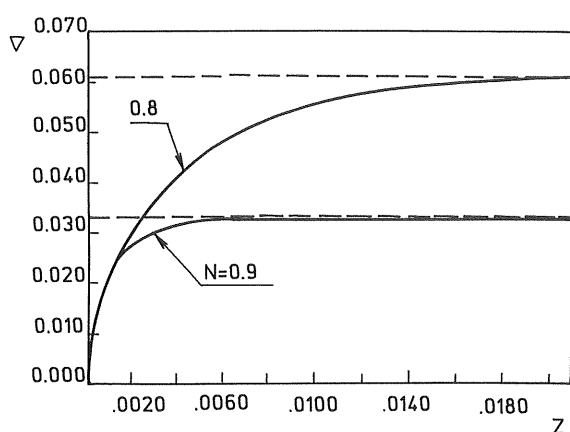
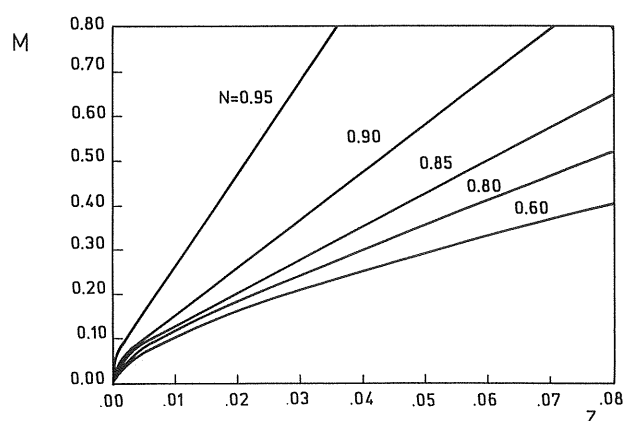Fig. 9   Torque increment versus axial distance for $N=0.8$ and 0.9, $Re^2/Ta=1$

Fig. 10   Developing torque versus axial distance for various values of annulus radius ratio

Table 2   Effect of the spin parameter Ra^2/Ta on the developing torque M for an annulus of $N=0.9$

$Z \times 10^4$	M		
	$Re^2/Ta = 0.3$	$Re^2/Ta = 1$	$Re^2/Ta = 10$
0.5	0.68630	0.68260	0.68112
1.0	0.96181	0.94338	0.93464
1.5	1.17775	1.14856	1.13480
2.0	1.36502	1.32742	1.30999
3.0	1.69049	1.64008	1.61739
4.0	1.97576	1.91581	1.88939
6.0	2.47553	2.40188	2.37042
8.0	2.91537	2.83215	2.79732
15.0	4.19128	4.09046	4.04971
25.0	5.62966	5.52677	5.48654
40.0	7.47456	7.38903	7.35577
60.0	9.64907	9.58094	9.55320

axial positions for three values of the spin parameter Re^2/Ta (0.3, 1, and 10). Thus, the torque results corresponding to $Re^2/Ta=1$, which are presented in Figs. 3 through 6 and 10, can be used with satisfactory accuracy for other values of the spin parameter (Re^2/Ta).

4. Conclusions

This investigation provides data not available in the literature, which, however, are needed in many engineering applications, for the tangential shear stress and the torque required to turn the inner shaft of an annulus having a laminar flow with simultaneously developing axial and tangential boundary layers. The problem under investigation is governed by three controlling parameters, namely, the Reynolds number (Re), the annulus radius ratio (N), and the ratio between Reynolds and Taylor numbers. The governing equations were formulated such that the first of these parameters (Re) becomes inherent in the dimensionless axial coordinate and hence only the latter two parameters $(N$ and $Re^2/Ta)$ are explicitly needed for the numerical solution of the problem. The effects of these two similarity parameters $(N$ and $Re^2/Ta)$ on the developing tangential shear stress and torque have been investigated. The results show that the first of these two parameters (N) has marked effects while the latter parameter (Re^2/Ta) has only slight effects on the developing tangential shear stress and torque. The negligence of the developing region (i.e., the assumption of whole-channel fully developed flow) leads to considerable error in estimating the torque, especially in short annular configurations which might exist in many practical situations.

previous figures, are given in Fig. 10. This figure also clarifies once more the effect of the annulus radius ratio on the torque values. Finally, the effect of the spin parameter on the torque values originates from its effect on the tangential shear stress τ. Therefore, this effect is slight, as clarified in Table 2 which gives values of M in an annulus of radius 0.9 at various

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