Chapter 13:

GAS-VAPOR MIXTURES AND AIR-CONDITIONING

INTRODUCTION

At temperatures below the critical temperature, the gas phase of a substance is frequently referred to as a *vapor.*

The term *vapor* implies a gaseous state that is close to the saturation region of the substance, raising the possibility of condensation during a process.

In the previous chapter, we discussed mixtures of gases that are usually above their critical temperatures.

Therefore, we were not concerned about any of the gases condensing during a process.

Not having to deal with two phases greatly simplified the analysis.

INTRODUCTION

When we are dealing with a gas–vapor mixture, however, the vapor may condense out of the mixture during a process, forming a two-phase mixture.

This may complicate the analysis considerably.

Therefore, a gas–vapor mixture needs to be treated differently from an ordinary gas mixture.

Several gas–vapor mixtures are encountered in engineering.

In this chapter, we consider the *air–water-vapor mixture,* which is the most commonly encountered gas–vapor mixture in practice.

We also discuss *air-conditioning,* which is the primary application area of air–water-vapor mixtures.

DRY AND ATMOSPHERIC AIR

Air is a mixture of nitrogen, oxygen, and small amounts of some other gases.

Air in the atmosphere normally contains some water vapor (or *moisture*) and is referred to as **atmospheric air.**

By contrast, air that contains no water vapor is called **dry air.**

It is often convenient to treat air as a mixture of water vapor and dry air since the composition of dry air remains relatively constant, but the amount of water vapor changes as a result of condensation and evaporation from oceans, lakes, rivers, showers, and even the human body.

Although the amount of water vapor in the air is small, it plays a major role in human comfort.

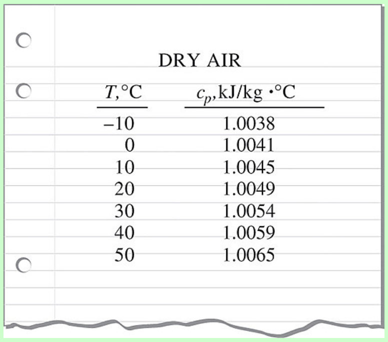
Therefore, it is an important consideration in air-conditioning applications.

DRY AND ATMOSPHERIC AIR

The temperature of air in air-conditioning applications ranges from about -10 to about 50°C.

In this range, dry air can be treated as an ideal gas with constant C_p value of 1.005 kJ/kg · K [0.240 Btu/lbm · R] with negligible error (under 0.2 percent).

The enthalpy and enthalpy change of dry air can be determined from:



DRY AIR

T,°C	c_p,kJ/kg ·°C
−10	1.0038
0	1.0041
10	1.0045
20	1.0049
30	1.0054
40	1.0059
50	1.0065

$$h_{\text{dry air}} = c_p T = (1.005\ \text{kJ/kg}\cdot{}^\circ\text{C})T \qquad (\text{kJ/kg})$$

$$\Delta h_{\text{dry air}} = c_p \Delta T = (1.005\ \text{kJ/kg}\cdot{}^\circ\text{C})\ \Delta T \qquad (\text{kJ/kg})$$

NOTICE: T in °C

DRY AND ATMOSPHERIC AIR

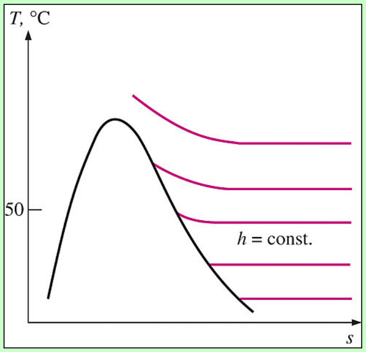
It certainly would be very convenient to also treat the water vapor in the air as an ideal gas and you would probably be willing to sacrifice some accuracy for such convenience.

At 50°C, the saturation pressure of water is 12.3 kPa.

At pressures below this value, water vapor can be treated as an ideal gas with negligible error (under 0.2 percent), even when it is a saturated vapor.

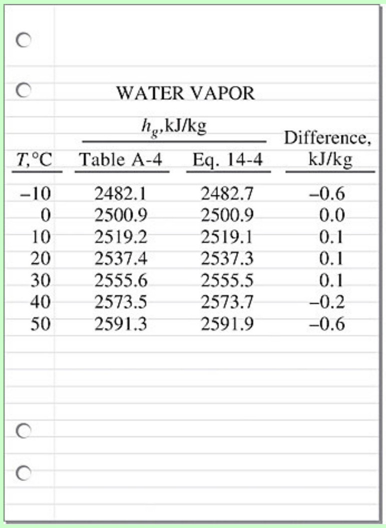
Since water vapor is an ideal gas, the enthalpy of water vapor is a function of temperature only, that is, $h = h(T)$.

This can also be observed from the T-s diagram of water given in Fig. A–9 where the constant enthalpy lines coincide with constant-temperature lines at temperatures below 50°C.



DRY AND ATMOSPHERIC AIR

Therefore, *the enthalpy of water vapor in air can be taken to be equal to the enthalpy of saturated vapor at the same temperature.*



WATER VAPOR

T,°C	h_g,kJ/kg Table A-4	h_g,kJ/kg Eq. 14-4	Difference, kJ/kg
−10	2482.1	2482.7	−0.6
0	2500.9	2500.9	0.0
10	2519.2	2519.1	0.1
20	2537.4	2537.3	0.1
30	2555.6	2555.5	0.1
40	2573.5	2573.7	−0.2
50	2591.3	2591.9	−0.6

That is,

$$h_v(T, \text{low } P) \cong h_g(T)$$

The enthalpy of water vapor at 0°C is 2501.3 kJ/kg. The average C_p value of water vapor in the temperature range -10 to 50°C can be taken to be 1.82 kJ/kg · °C. Then the enthalpy of water vapor can be determined approximately from:

$$h_v = h_g(T) \cong 2501.3 + 1.82T \left(\frac{kJ}{kg_v}\right) \quad T \text{ in } {}^{o}C$$

NOTICE: T in °C

DRY AND ATMOSPHERIC AIR

Since water vapor can be treated as an ideal gas, therefore, water vapor in air behaves as if it existed alone and obeys the ideal-gas relation $Pv = RT$.

Then the atmospheric air can be treated as an ideal-gas mixture whose pressure is the sum of the partial pressure of dry air* P_a and that of water vapor P_v:

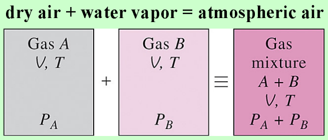
*Throughout this chapter, the subscript *a* denotes dry air and the subscript *v* denotes water vapor.

$$P = P_a + P_v$$

Dalton's law

The partial pressure of water vapor P_v is usually referred to as the **vapor pressure.**

It is the pressure water vapor would exert if it existed alone at the temperature and volume of atmospheric air.



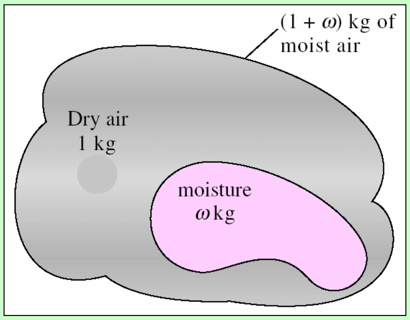
SPECIFIC AND RELATIVE HUMIDITY OF AIR

The amount of water vapor in the air can be specified in various ways.

Probably the most logical way is to specify directly the mass of water vapor present in a unit mass of dry air.

This is called **absolute** or **specific humidity** (also called *humidity ratio*) and is denoted by ω:

$$\omega = \frac{Mass\ of\ water\ vapor\ in\ air}{Mass\ of\ dry\ air} = \frac{m_v}{m_a}$$



The specific humidity can also be expressed as:

$$\omega = \frac{m_v}{m_a} = \frac{P_vV/(R_vT)}{P_aV/(R_aT)} = \frac{P_v/R_v}{P_a/R_a} = 0.622\frac{P_v}{P_a}$$

or

$$\boxed{\omega = \frac{0.622\,P_v}{P - P_v}}$$

where P is the total pressure.

SPECIFIC AND RELATIVE HUMIDITY OF AIR

Consider 1 kg of dry air.

By definition, dry air contains no water vapor, and thus its specific humidity is zero.

Now let us add some water vapor to this dry air.

The specific humidity will increase.

As more vapor or moisture is added, the specific humidity will keep increasing until the air can hold no more moisture.

At this point, the air is said to be saturated with moisture, and it is called **saturated air.**

Any moisture introduced into saturated air will condense.

SPECIFIC AND RELATIVE HUMIDITY OF AIR

The amount of water vapor in saturated air at a specified temperature and pressure can be determined from ω by replacing P_v by P_g, the saturation pressure of water at that temperature.

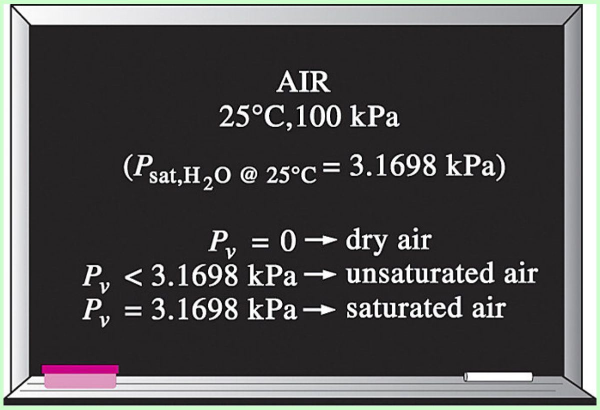
$$\omega = \frac{0.622 P_v}{P - P_v}$$

Becomes

$$\omega = \frac{0.622 P_g}{P - P_g}$$

Where

$$P_g = P_{sat\ @T}$$

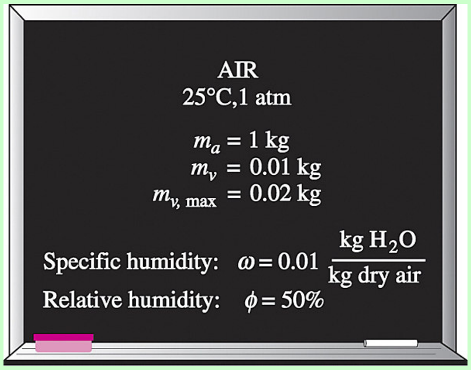


SPECIFIC AND RELATIVE HUMIDITY OF AIR

The amount of moisture in the air has a definite effect on how comfortable we feel in an environment.

However, the comfort level depends more on the amount of moisture the air holds (m_v) relative to the maximum amount of moisture the air can hold at the same temperature (m_g).

The ratio of these two quantities is called the **relative humidity** ϕ



$$\phi = \frac{Mass\ of\ vapor\ in\ air}{Mass\ of\ saturated\ air} = \frac{m_v}{m_g}$$

$$\phi = \frac{m_v}{m_g} = \frac{P_v V/(R_v T)}{P_g V/(R_v T)} = \frac{P_v}{P_g}$$

SPECIFIC AND RELATIVE HUMIDITY OF AIR

Combining ω and ϕ, we can also express the relative humidity as:

$$\omega = \frac{0.622\,P_v}{P - P_v} \qquad \phi = \frac{P_v}{P_g}$$

$$\omega = \frac{0.622\phi P_g}{P - \phi P_g} \quad and \quad \phi = \frac{\omega P}{(0.622+\omega)P_g}$$

The relative humidity ranges from 0 for dry air to 1 for saturated air.

Note that the amount of moisture that air can hold depends on its temperature.

Therefore, the relative humidity of air ϕ changes with temperature even when its specific humidity remains constant.

SPECIFIC AND RELATIVE HUMIDITY OF AIR

Atmospheric air is a mixture of dry air and water vapor, and thus the enthalpy of air is expressed in terms of the enthalpies of the dry air and the water vapor.

The total enthalpy (an extensive property) of atmospheric air is the sum of the enthalpies of dry air and the water vapor:

$$H = H_a + H_v = m_a h_a + m_v h_v$$

In most practical applications, the amount of dry air (m_a) in the air–water-vapor mixture remains constant, but the amount of water vapor changes.

Therefore, the enthalpy of atmospheric air is expressed *per unit mass of dry air* instead of per unit mass of the air–water vapor mixture.

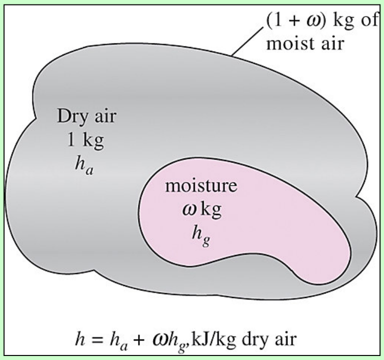
Dividing by m_a gives: $$h = \frac{H}{m_a} = h_a + \frac{m_v}{m_a} h_v = h_a + \omega h_v$$

SPECIFIC AND RELATIVE HUMIDITY OF AIR

$$h = \frac{H}{m_a} = h_a + \frac{m_v}{m_a}h_v = h_a + \omega h_v$$

since $h_v = h_g$ (see this Figure).

$$\boxed{h = h_a + \omega h_g}$$



$h = h_a + \omega h_g$, kJ/kg dry air

Also note that the ordinary temperature of atmospheric air is frequently referred to as the **dry-bulb temperature** to differentiate it from other forms of temperatures that shall be discussed.

Example 13-1
The Amount of Water Vapor in Room Air

A 5-m X 5-m X 3-m room shown contains air at 25°C and 100 kPa at a relative humidity of 75 percent. Determine:
(*a*) the partial pressure of dry air,
(*b*) the specific humidity,
(*c*) the enthalpy per unit mass of the dry air, and
(*d*) the masses of the dry air and water vapor in the room.

$P_a = P - P_v$

$P_v = \phi P_g = \phi P_{\text{sat @ 25°C}}$

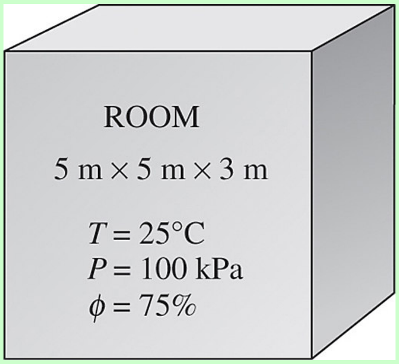
$$\omega = \frac{0.622P_v}{P - P_v}$$

$h = h_a + \omega h_v \cong c_pT + \omega h_g$

$V_a = V_v = V_{\text{room}}$

$$m_a = \frac{P_aV_a}{R_aT}$$

$$m_v = \frac{P_vV_v}{R_vT}$$

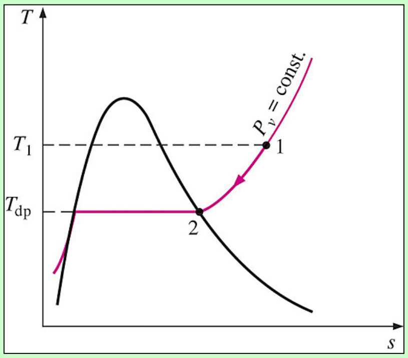


DEW-POINT TEMPERATURE

The **dew-point temperature** T_{dp} is defined as *the temperature at which condensation begins when the air is cooled at constant pressure*.

In other words, T_{dp} is the saturation temperature of water corresponding to the vapor pressure:

$$\boxed{T_{dp} = T_{sat\,@\,P_v}}$$

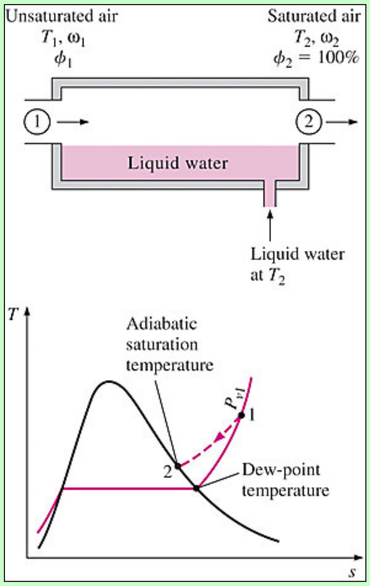


If the temperature drops any further from (2), some vapor condenses out.
As a result, the amount of vapor in the air decreases, which results in a decrease in P_v.

The air remains saturated during the condensation process and thus follows a path of 100 percent relative humidity (the saturated vapor line).

The ordinary temperature and the dew-point temperature of saturated air are identical.

ADIABATIC SATURATION AND WET-BULB TEMPERATURES



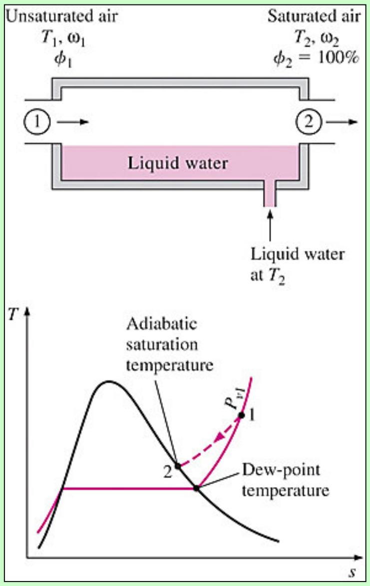
Relative humidity and specific humidity are frequently used in engineering and atmospheric sciences, and it is desirable to relate them to easily measurable quantities such as temperature and pressure.

One way of determining the relative humidity is to determine the dew-point temperature of air.

This approach is simple, but not quite practical.

Another way of determining the absolute or relative humidity is related to an *adiabatic saturation process,* shown schematically and on a *T*-*s* diagram.

ADIABATIC SATURATION AND WET-BULB TEMPERATURES



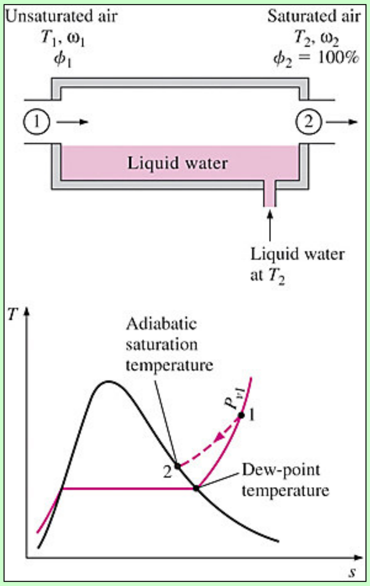
The system consists of a long insulated channel that contains a pool of water.

A steady stream of unsaturated air that has a specific humidity of ω_1 (unknown) and a temperature of T_1 is passed through this channel.

As the air flows over the water, some water evaporates and mixes with the airstream. The moisture content of air increases during this process, and its temperature decreases.

If the channel is long enough, the airstream exits as saturated air (ϕ = 100 percent) at temperature T_2, which is called the **adiabatic saturation temperature.**

ADIABATIC SATURATION AND WET-BULB TEMPERATURES



Mass balance:

$$\dot{m}_{a_1} = \dot{m}_{a_2} = \dot{m}_a$$

$$\dot{m}_{w_1} + \dot{m}_f = \dot{m}_{w_2}$$

or

$$\dot{m}_a\omega_1 + \dot{m}_f = \dot{m}_a\omega_2$$

Thus, $\boxed{\dot{m}_f = \dot{m}_a(\omega_2 - \omega_1)}$

Energy balance:

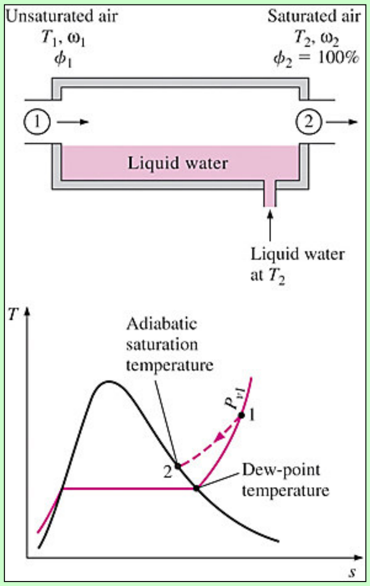
$$\dot{E}_{\text{in}} = \dot{E}_{\text{out}}$$

$$\dot{m}_a h_1 + \dot{m}_f h_{f_2} = \dot{m}_a h_2$$

$$\dot{m}_a h_1 + \dot{m}_a(\omega_2 - \omega_1)h_{f_2} = \dot{m}_a h_2$$

$$\boxed{(c_p T_1 + \omega_1 h_{g_1}) + (\omega_2 - \omega_1)h_{f_2} = (c_p T_2 + \omega_2 h_{g_2})}$$

ADIABATIC SATURATION AND WET-BULB TEMPERATURES



Applying the conservation of mass and conservation of energy principal for this adiabatic saturator reduces to the following:

$$\omega_1 = \frac{c_p(T_2 - T_1) + \omega_2 h_{fg_2}}{h_{g_1} - h_{f_2}}$$

where $\omega_2 = \frac{0.622 P_{g_2}}{P_2 - P_{g_2}}$ since $\phi_2 = 100\%$

Thus we conclude that the specific humidity (and relative humidity) of air can be determined by measuring the pressure and temperature of air at the inlet and the exit of an adiabatic saturator.

ADIABATIC SATURATION AND WET-BULB TEMPERATURES

The adiabatic saturation process requires a long channel or a spray mechanism to achieve saturation conditions at the exit.

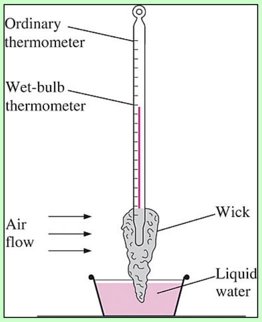
A more practical approach is to use a thermometer whose bulb is covered with a cotton wick saturated with water and to blow air over the wick, as shown.

The temperature measured in this manner is called the **wet-bulb temperature** T_{wb}, and it is commonly used in air-conditioning applications.

The basic principle involved is similar to that in adiabatic saturation process.

Therefore, the wet-bulb temperature T_{wb} can be used in place of T_2 to determine the specific humidity of air.

$$\omega_1 = \frac{c_p(T_2 - T_1) + \omega_2 h_{fg_2}}{h_{g_1} - h_{f_2}}$$



ADIABATIC SATURATION AND WET-BULB TEMPERATURES

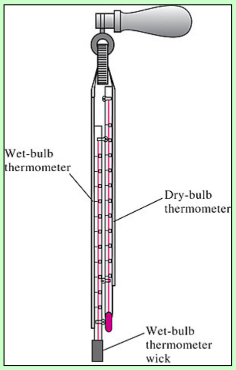
The wet-bulb temperature can also be measured by placing the wet-wicked thermometer in a holder attached to a handle and rotating the holder rapidly, that is, by moving the thermometer instead of the air.

A device that works on this principle is called a *sling psychrometer* and is shown.

Usually a dry-bulb thermometer is also mounted on the frame of this device so that both the wet- and dry-bulb temperatures can be read simultaneously.

Advances in electronics made it possible to measure humidity directly in a fast and reliable way.

It appears that sling psychrometers and wet-wicked thermometers are about to become things of the past.



Example 13-3
The Specific and Relative Humidity of Air

The dry- and the wet-bulb temperatures of atmospheric air at 1 atm (101.325 kPa) pressure are measured with a sling psychrometer and determined to be 25 and 15°C, respectively. Determine:

(*a*) the specific humidity,
(*b*) the relative humidity, and
(*c*) the enthalpy of the air.

$$\omega_1 = \frac{c_p(T_2 - T_1) + \omega_2 h_{fg_2}}{h_{g_1} - h_{f_2}}$$

$$\omega_1 = \frac{(1.005\ \text{kJ/kg}\cdot{}^\circ\text{C})[(15 - 25)^\circ\text{C}] + (0.01065)(2465.4\ \text{kJ/kg})}{(2546.5 - 62.982)\ \text{kJ/kg}}$$

$$\phi_1 = \frac{\omega_1 P_2}{(0.622 + \omega_1)P_{g_1}} = \frac{(0.00653)(101.325\ \text{kPa})}{(0.622 + 0.00653)(3.1698\ \text{kPa})}$$

$$h_1 = h_{a_1} + \omega_1 h_{v_1} \cong c_p T_1 + \omega_1 h_{g_1}$$

THE PSYCHROMETRIC CHART

The state of the atmospheric air at a specified pressure is completely specified by two independent intensive properties.

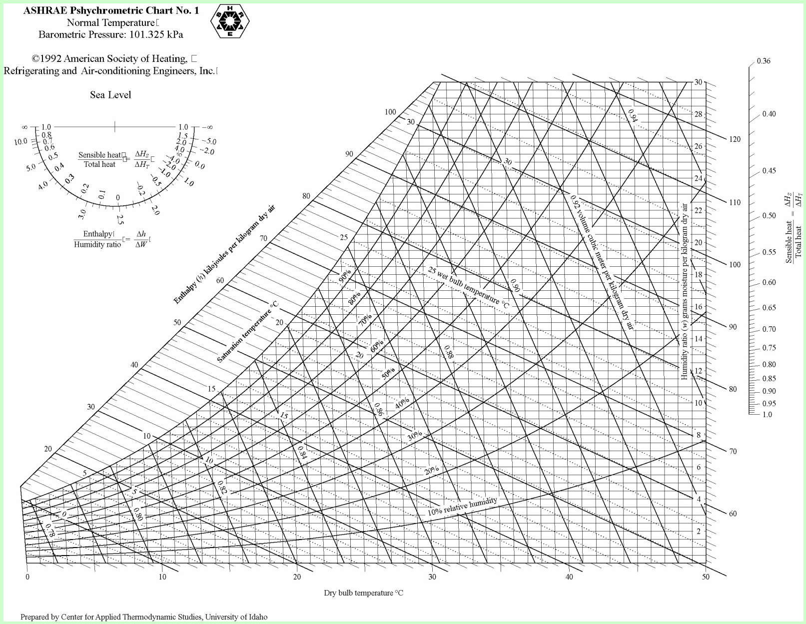
The rest of the properties can be calculated easily from the previous relations.

The sizing of a typical air-conditioning system involves numerous such calculations, which may eventually get on the nerves of even the most patient engineers.

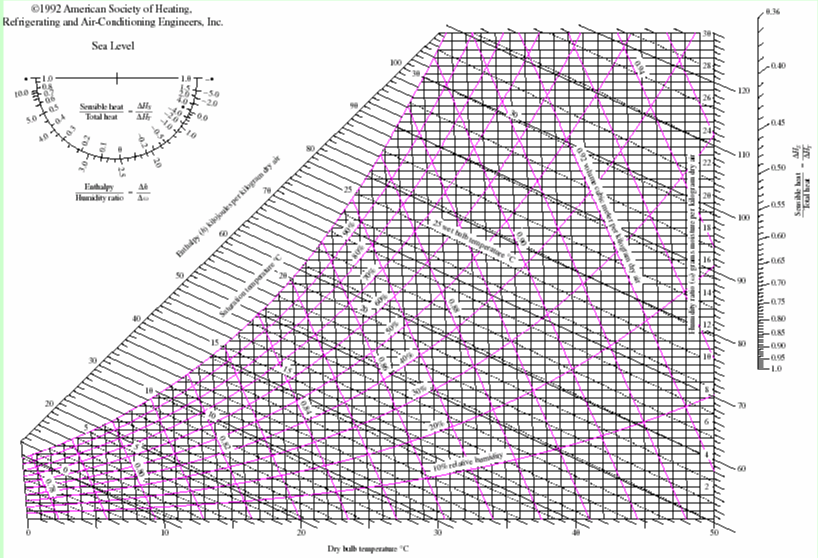
Therefore, there is clear motivation to computerize calculations or to do these calculations once and to present the data in the form of easily readable charts.

Such charts are called **psychrometric charts**, and they are used extensively in air-conditioning applications.

THE PSYCHROMETRIC CHART



THE PSYCHROMETRIC CHART



THE PSYCHROMETRIC CHART

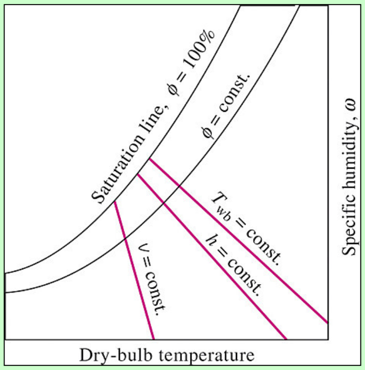
A psychrometric chart for a pressure of 1 atm (101.325 kPa or 14.696 psia) is given in Fig. A–33 in SI units and in Fig. A–33E in English units.

Psychrometric charts at other pressures (for use at considerably higher elevations than sea level) are also available.

The basic features of the psychrometric chart are illustrated in this Figure.

The dry-bulb temperatures are shown on the horizontal axis, and the specific humidity is shown on the vertical axis.

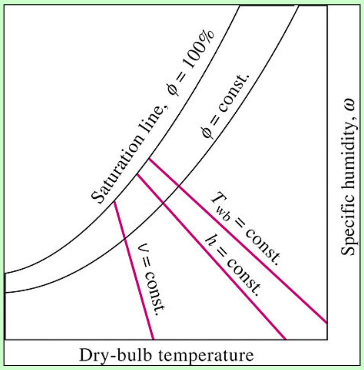
On the left end of the chart, there is a curve (called the *saturation line*) instead of a straight line.



THE PSYCHROMETRIC CHART

All the saturated air states are located on this curve. Therefore, it is also the curve of 100 percent relative humidity.
Other constant relative-humidity curves have the same general shape.

Lines of constant wet-bulb temperature have a downhill appearance to the right.



Lines of constant specific volume (in m^3/kg dry air) look similar, except they are steeper.

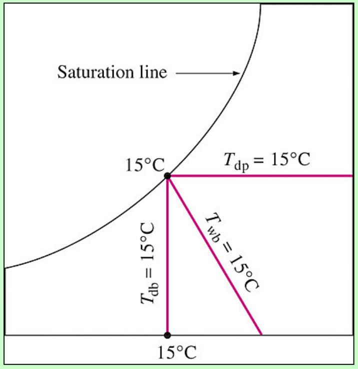
Lines of constant enthalpy (in kJ/kg dry air) lie very nearly parallel to the lines of constant wet-bulb temperature.

Therefore, the constant wet-bulb-temperature lines are used as constant-enthalpy lines in some charts.

THE PSYCHROMETRIC CHART

The dew-point temperature of atmospheric air at any point on the chart can be determined by drawing a horizontal line (a line of ω = constant or P_v = constant) from the point to the saturated curve.
The temperature value at the intersection point is the dew-point temperature.

For saturated air, the dry-bulb, wet-bulb, and dew-point temperatures are identical.



The psychrometric chart also serves as a valuable aid in visualizing the air-conditioning processes.

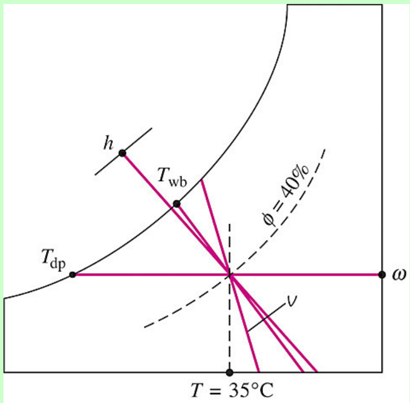
An ordinary heating or cooling process, for example, appears as a horizontal line on this chart if no humidification or dehumidification is involved (that is, ω = constant).

Any deviation from a horizontal line indicates that moisture is added or removed from the air during the process.

Example 13-4
The Use of the Psychrometric Chart

Consider a room that contains air at 1 atm, 35°C, and 40 percent relative humidity. Using the psychrometric chart, determine:

(*a*) the specific humidity,
(*b*) the enthalpy,
(*c*) the wet-bulb temperature,
(*d*) the dew-point temperature, and
(*e*) the specific volume of the air.



$\omega = \mathbf{0.0142\ kg\ H_2O/kg\ dry\ air}$

$h = \mathbf{71.5\ kJ/kg\ dry\ air}$

$T_{wb} = \mathbf{24°C}$

$T_{dp} = \mathbf{19.4°C}$

$v = \mathbf{0.893\ m^3/kg\ dry\ air}$